Comments and Addenda

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Remarks on the Unitarity Condition in the Impact-Parameter Representation

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The kernel needed in the application of the unitarity condition in the Blankenbecler-Goldberger impact-parameter representation is reduced to a simple sum suitable for numerical work. We also comment briefly on a modified version of the representation.

RECENTLY there has been considerable interest in the use of the impact-parameter representation for scattering amplitudes at high energy.[1] In an important paper[2] (referred to as BG below), Blankenbecler and Goldberger obtained an integral equation for the scattering amplitude in potential theory with only a right-hand cut. The unitarity condition, however, involves an unwieldy double integral [$G(b;b_1,b_2;s)$ in the notation of BG] over angles. In this paper, we show how to transform this double integral into a single sum whose terms are rather simple and appear to be amenable to numerical work. We also note briefly the complications that arise when similar calculations are made in an alternative representation[3] which uses $J_0(kb\sin\theta)$ instead of the usual $J_0(2kb\sin\frac{1}{2}\theta)$.

We follow the notation of BG and write $H(b^2,s)$ for the Fourier-Bessel transform of the scattering amplitude. As was shown in BG, $H(b^2,s)$ satisfies the integral equation

$$H(b^2,s)=\text{Born term}+\int_0^\infty \frac{ds'}{\pi}\frac{\operatorname{Im}H(b^2,s')}{s'-s},$$

where

$$\operatorname{Im}H(b^2,s)=\int_0^\infty b_1db_1\int_0^\infty b_2db_2H^*(b_1^2,s)\times H(b_2^2,s)G(b;b_1,b_2;s)$$

and

$$G(b;b_1,b_2;s)=\frac{2s}{\pi}\int_0^\pi d\phi\int_0^\pi d\theta\,\sin\tfrac{1}{2}\theta\,\cos\tfrac{1}{2}\theta J_0(2kb\sin\tfrac{1}{2}\theta)\times J_1(2kbB(\theta,\phi))/B(\theta,\phi)\,, \quad (1)$$

with

$$B^2(\theta,\phi)=b_1^2+b_2^2-2b_1b_2\cos\tfrac{1}{2}\theta\cos\phi\,.$$

Let us write $\rho=kb$, $\rho_1=kb_1$, $\rho_2=kb_2$, and use Eq. 5.22(1) of Watson[4] with $z=4(\rho_1^2+\rho_2^2)$ and $h=-8\rho_1\rho_2\times\cos\frac{1}{2}\theta\cos\phi$, so that

$$\frac{J_1(2kB)}{B}=\sum_{m=0}^\infty\frac{(4\rho_1\rho_2\cos\frac{1}{2}\theta\cos\phi)^m}{m!}\times\frac{J_{m+1}(2(\rho_1^2+\rho_2^2)^{1/2})}{[4(\rho_1^2+\rho_2^2)]^{(m+1)/2}}\,. \quad (2)$$

The integration over ϕ is now trivial, with

$$\int_0^\pi \cos^m\phi d\phi=\pi(2n)!2^{-2n}/(n!)^2$$

when $m=2n$ and zero when m is odd. The final integration over θ gives for the $2n$th term

$$\begin{aligned}\int_0^\pi &\sin\theta d\theta\, J_0(2\rho\sin\tfrac{1}{2}\theta)(\cos\tfrac{1}{2}\theta)^{2n}d\theta\\ &=4\int_0^{\pi/2}\cos^{2n+1}t\,\sin t J_0(2\rho\sin t)dt\ (\theta=2t) \quad (3)\\ &=2n!J_{n+1}(2\rho)/\rho^{n+1},\end{aligned}$$

where use has been made of Eq. 13.3.2(11) of Luke.[5] The unitarity kernel G thus reduces to a sum

$$G(b;b_1,b_2;s)=s\sum_{n=0}^\infty\frac{(\rho_1\rho_2)^{2n}}{n!}\frac{J_{2n+1}(2(\rho_1^2+\rho_2^2)^{1/2})}{(\rho_1^2+\rho_2^2)^{n+1/2}}\frac{J_{n+1}(2\rho)}{\rho^{n+1}}\,, \quad (4)$$

which can be readily evaluated on a computer.

The unitarity condition takes a considerably more complicated form if the kernel $J_0(\rho\sin\theta)$ is used instead of $J_0(2\rho\sin\frac{1}{2}\theta)$. Because $J_0(\rho\sin\theta)$ contains only even

[1] See, e.g., the papers cited in J. D. Jackson, University of California Lawrence Radiation Laboratory Report No. UCRL-19205, 1969 (unpublished).

[2] R. Blankenbecler and M. L. Goldberger, Phys. Rev. **126**, 766 (1962).

[3] N. P. Chang, Phys. Rev. **172**, 1796 (1968).

[4] G. N. Watson, *Theory of Bessel Functions* (Cambridge University Press, New York, 1966).

[5] Y. Luke, *Integrals of Bessel Functions* (McGraw-Hill Book Co., New York, 1962).

powers of $\sin\theta$ (and therefore only Legendre polynomials of even degree in $\cos\theta$), it is necessary to split the scattering amplitude into an even and an odd part and apply the Fourier-Bessel transform to them separately. The unitarity condition for the full amplitude does not leave the even and odd amplitudes uncoupled. Moreover, if one takes seriously the argument of viewing the scattering in an infinite-momentum frame,[3] the angle θ is then restricted to the interval $(0,\pi/2)$, over which the Legendre polynomials are no longer orthogonal. A calculation similar to the one leading to Eq. (4) but with $J_0(\rho\sin\theta)$ in place of $J_0(2\rho\sin\frac{1}{2}\theta)$ leads to rather complicated expressions which will not be written down here.

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Significance of Interference between Two Independent Attenuated Laser Beams

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Pfleegor and Mandel produced interference effects from two separate independent laser beams when the quanta arriving at the detector were separated in time by an average of about 1.4×10^{-7} sec. If the quanta are photons (particles), then, to produce interference, each photon must store energy in the detector for an average of at least 1.4×10^{-7} sec. But Jánossy and Náray have recently shown that over a period of 9×10^{-9} sec there is no such storage, at least when the phases are random. Thus the result of Pfleegor and Mandel is in conflict with the corpuscular concept of visible light.

In 1967 Pfleegor and Mandel[1] demonstrated that two independent He-Ne laser beams can interfere. A year later they repeated this experiment,[2] in a somewhat modified form, with the light flux so attenuated that photons from either source reached the detector spaced apart in time by about 1.4×10^{-7} sec (7×10^6 photons/sec reached the detector), whereas the transit time was about 3×10^{-9} sec.

Thus "one photon was usually received before the next one was emitted,"[2] or, in other words, "the photons may be described, rather loosely, as reaching the receiver one at a time."[2] Since the two lasers were statistically independent, their relative phases were random, so the resulting fringe pattern was transient but nevertheless, with short observation times ($20\,\mu$ sec), was observable by the correlation method they used.

The essence of our argument stems from their statement that since one photon is absorbed before the next one is emitted, "interference effects should be associated with the detection of each photon."[2] The expression "each photon" in this context can only mean a photon which was individually emitted by one laser or the other, and not a photon in two superimposed states in the sense of Dirac (see the last paragraph). But since, after one such individual photon was absorbed, another did not come along until some 1.4×10^{-7} sec later, the *detector* would have to *store* the "information" it absorbed from the first photon in order to produce any interference effects arising from the absorption of the second photon 1.4×10^{-7} sec later. In other words, if photons from two independent sources arrive, as particles, at a detector, one at a time, and still produce interference effects, the detector must "remember" the arrival of each photon, *with its phase*, at least during the time interval required for the next photon to arrive. Such "memory" implies energy storage in the cathode of the photoelectric detector (multiplier).

But Jánossy and Náray (JN) have recently shown[3] that under the conditions of their experiment there is no storage of energy in the photocathode of their multiplier for a time interval of 9×10^{-9} sec. Also using a He-Ne laser, they split its attenuated beam into two parts, one part delayed 9×10^{-9} sec behind the other. By means of a rotating mirror they produced 4.6×10^3 narrow double pulses/sec with peaks spaced 9×10^{-9} sec apart, each double pulse containing about 10 quanta. The photocathode of the multiplier was irradiated by both beams, but JN measured the photoemission from only the later beam; they found that the later beam produced the *same* photoemission whether the earlier beam was blocked off or not. If, when the earlier beam was turned on, it had left any substantial trace of energy storage in the photocathode, it would be expected that the later beam would reveal the presence of this stored energy by yielding enhanced emission. No such enhanced emission was observed, so JN concluded that there was no

[1] R. L. Pfleegor and L. Mandel, Phys. Rev. **159**, 1084 (1967).
[2] R. L. Pfleegor and L. Mandel, J. Opt. Soc. Am. **58**, 946 (1968).
[3] M. Jánossy and Zs. Náray, Phys. Letters **29A**, 479 (1969).