Interpretation of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ Mass Spectra in the Reaction $\bar{p}p \to \bar{p}p\pi^+\pi^-$ at 2.7 and 2.9 GeV/c†

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The reaction $\bar{p}p \to \bar{p}p\pi^+\pi^-$ has been studied for $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ enhancements using 719 events at 2.7 GeV/c and 1015 events at 2.9 GeV/c. We find that the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra can be adequately explained by assuming that the reaction proceeds 100% through the $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ channel, and that it is not necessary to assume the production of $p\pi^+\pi^-$ or $\bar{p}\pi^+\pi^-$ resonances in this reaction at these energies.

I. INTRODUCTION

There have been many reports of a low-mass enhancement in the $p\pi^+\pi^-$ system produced in hadron-nucleon interactions.[1] The general assumption is that this is the $P_{11}(1470)$ resonance reported at approximately the same mass in phase-shift analyses of πN scattering data.[2] Recently Bacon *et al.*[3] reported the observation of an effect in the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass distributions in the reaction $\bar{p}p(n) \to \bar{p}p\pi^+\pi^-(n)$ using a 2.8-GeV/c incident $\bar{p}$ beam and the 20-in. Brookhaven National Laboratory bubble chamber filled with deuterium. They obtain a mass and width of 1400 ± 10 and 80 ± 20 MeV, respectively, for their resonance effects. Ferbel *et al.*[4] have studied the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass distributions in the reaction $\bar{p}p \to \bar{p}p\pi^+\pi^-$ at 3–4 GeV/c. Their interpretation is that there is no evidence for $p\pi^+\pi^-$ or $\bar{p}\pi^+\pi^-$ resonance production in their data, and that their $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra can be adequately explained by assuming that the reaction proceeds through the intermediate state $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$. We report here an analysis of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra from the reaction

$$\bar{p}p \to \bar{p}p\pi^+\pi^- \qquad (1)$$

at incident momenta 2.7 and 2.9 GeV/c. The Iowa State group has previously presented a detailed study of reaction (1) at 2.7 GeV/c.[5] However, the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass histograms, while examined,[6] were not presented in that work. In addition, we are now completing a study of this reaction at incident beam momenta of 2.4 and 2.9 GeV/c. These recent investigations enable a direct comparison of the results of Bacon *et al.* at 2.8 GeV/c, our results at 2.7 and 2.9 GeV/c, and the results of Ferbel *et al.* at 3–4 GeV/c for the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra.

In Sec. II is presented a brief discussion of the experimental details, while Sec. III contains the report of an analysis of the amounts of $\Delta(1236)$ and $\bar{\Delta}(1236)$ production in these data. The $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra are presented and discussed in detail in Sec. IV, and the general conclusions are given in Sec. V.

II. EXPERIMENTAL PROCEDURE

The 2.7-GeV/c experiment was performed using the 20-in. Brookhaven National Laboratory hydrogen bubble chamber in the Yale-BNL separated beam. A total of 43 500 pictures was used for the data collection at this energy. Full details of the data-selection procedure have been published,[5] and they will not be repeated here. A final sample of 719 events was accepted as being due to reaction (1) and losses from all sources (nonmeasurable events, scan losses, etc.) were estimated to account for another 80 events. This corresponds to a cross section for reaction (1) of 1.93 ± 0.16 mb at 2.7 GeV/c.

The 2.9-GeV/c experiment was performed using the 31-in. Brookhaven National Laboratory hydrogen bubble chamber in an electrostatically separated beam. Approximately 40 000 pictures were scanned for four-prong events, yielding a measurable sample of 9400 events in this topology. From these 9400 events, a final sample of 1015 events was accepted as being due to reaction (1). These events all have good kinematic fits to reaction (1) and are all ionization consistent with this reaction. We estimate that losses from all sources (nonmeasurable events, scan losses, etc.) correspond to approximately 235 events. With this correction, the

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cross section for reaction (1) at 2.9 GeV/c is found to be 2.53±0.10 mb.

III. DOMINANCE OF $\bar{\Delta}\Delta$ CHANNEL

Investigation of the two-dimensional mass scatter plots for the invariant masses of the outgoing π^+p and $\pi^-\bar{p}$ combinations in reaction (1) (not shown) indicates that the $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ channel accounts for nearly 100% of the cross section of reaction (1) at both 2.7 and 2.9 GeV/c. The π^+p and $\pi^-\bar{p}$ mass distributions are identical within statistics at both energies, and the summed distributions are shown in Figs. 1(a) (2.7 GeV/c) and 1(b) (2.9 GeV/c). The procedure used to determine the amount of $\bar{\Delta}\Delta$ dominance is described in detail in CLK[5]; the same procedure is used on the 2.9-GeV/c data. A brief description of the method follows.

The π^+p and $\pi^-\bar{p}$ two-dimensional mass scatter plots at each energy are fitted with the expression

$$L(\omega,\bar{\omega})=\alpha(\pi^+p\pi^-\bar{p})F(\pi^+p\pi^-\bar{p})+\alpha(\Delta\pi^-\bar{p})F(\Delta\pi^-\bar{p})+\alpha(\pi^+p\bar{\Delta})F(\pi^+p\bar{\Delta})+\alpha(\Delta\bar{\Delta})F(\Delta\bar{\Delta}). \quad (2)$$

In this expression, ω ($\bar{\omega}$) is the invariant mass of the outgoing π^+p ($\pi^-\bar{p}$) combination, the α's are the fractions of the channels indicated by the arguments [e.g., $\alpha(\Delta\bar{\Delta})$ is the fraction of reaction (1) going through the $\Delta\bar{\Delta}$ intermediate state], and the F's are phenomenological expressions representing the production and decay of the state indicated by the argument [e.g., the meaning of $F(\Delta\bar{\Delta})$ is: phase-space production of a $\Delta\bar{\Delta}$ state followed by Breit-Wigner decay of that state into $\pi^+p\pi^-\bar{p}$].[5,7] The results of a maximum-likelihood fit of Eq. (2) to the scatter plots at 2.7 and 2.9 GeV/c are presented in Table I. The errors quoted are purely statistical and do not reflect uncertainties in the forms used for the F's. Small variations in the forms of the F's indicate that the statistical errors are too small, and that the fraction of reaction (1) proceeding through the $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ channel at both energies is (95±5)%, with the remainder of the reaction being nonresonant double-pion production (see again Table I). There is no evidence for $\bar{\Delta}^0(1236)$ or $\Delta^0(1236)$ production.

At this point there exist differences in the conclusions of this paper and those of Bacon *et al.* The cross sections for reaction (1) given here are 1.93±0.16 (2.7 GeV/c) and 2.53±0.10 mb (2.9 GeV/c), while Bacon *et al.* give 1.7±0.2 mb (2.8 GeV/c). Our estimate of the

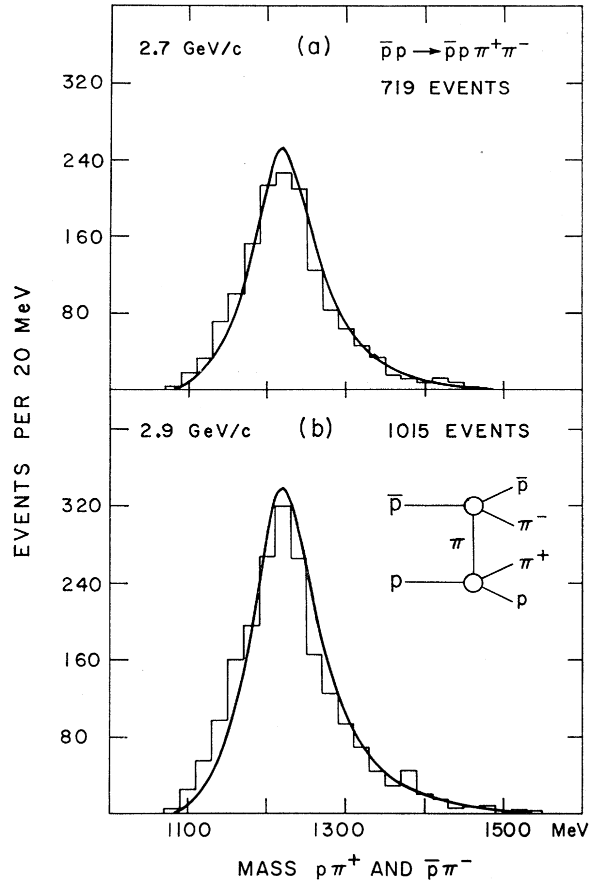

FIG. 1. Distribution of the $p\pi^+$ and $\bar{p}\pi^-$ effective masses combined. The smooth curves are the predictions of the double-isobar OPE model normalized to the data: (a) 2.7 and (b) 2.9 GeV/c.

percentage of $\bar{\Delta}^{--}\Delta^{++}$ production is (95±5)%, while Bacon *et al.* report 72% resonance production ($\bar{\Delta}^{--}\Delta^{++}$ plus some single-resonance production). As seen in the next section, the amount of $\bar{\Delta}^{--}\Delta^{++}$ production has an important bearing on the interpretation of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra.

IV. INTERPRETATION OF $p\pi^+\pi^-$ AND $\bar{p}\pi^+\pi^-$ MASS SPECTRA

In Figs. 2(a)–2(d) are shown the outgoing $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra for reaction (1) at 2.7 and 2.9 GeV/c with and without cuts on the corresponding outgoing π^+p and $\pi^-\bar{p}$ masses. The $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$

TABLE I. Fractions of zero-, single-, and double-resonance production.

Beam (GeV/c)	$\alpha(\pi^+p\pi^-\bar{p})$ Fitted value	Best estimate	$\alpha(\Delta\pi^-\bar{p})=\alpha(\pi^+p\bar{\Delta})$ Fitted value	Best estimate	$\alpha(\Delta\bar{\Delta})$ Fitted value	Best estimate
2.7	0.07±0.01	0.05±0.05	0.00±0.02	0.00±0.02	0.93±0.01	0.95±0.05
2.9	0.06±0.01	0.05±0.05	0.00±0.02	0.00±0.02	0.94±0.02	0.95±0.05

[7] J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

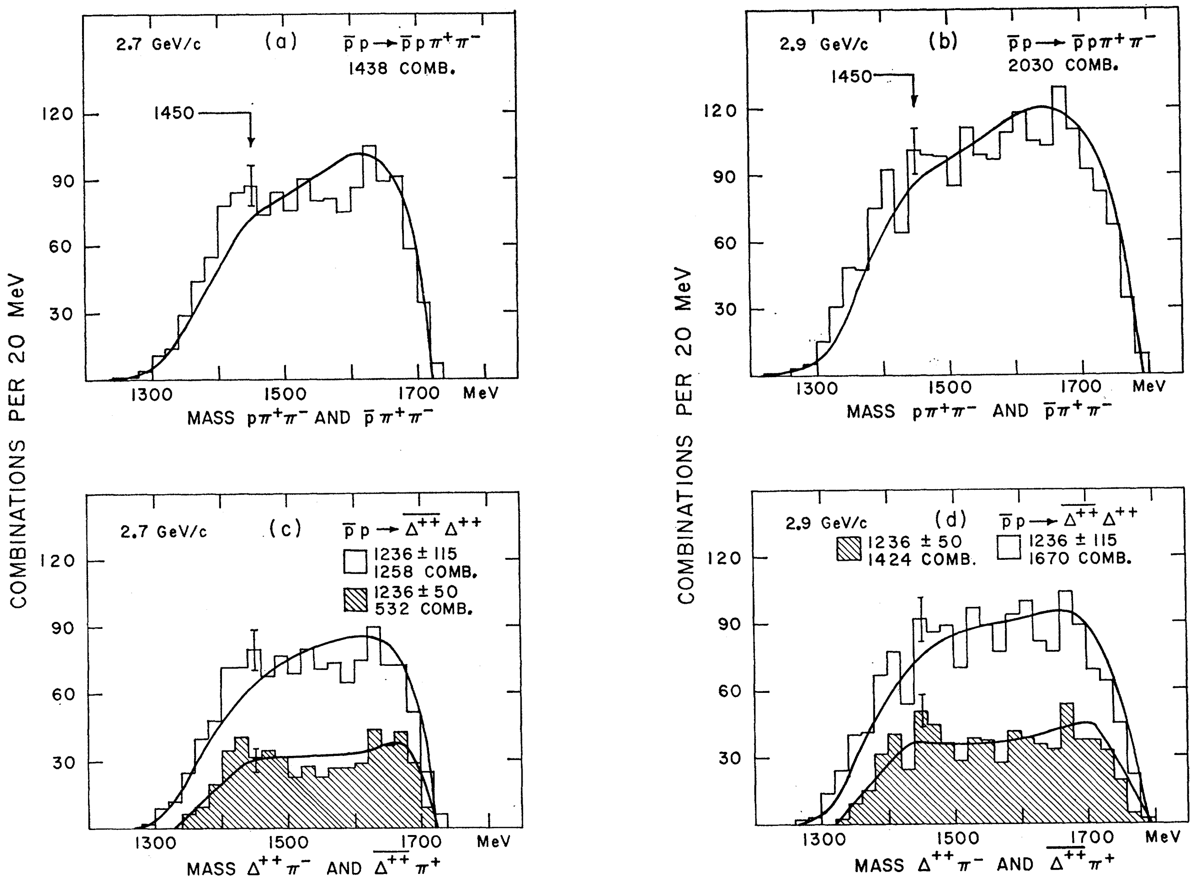

FIG. 2. Distribution of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ effective masses combined. The smooth curves are the predictions of the OPE model normalized to the data. (a) All events at 2.7 GeV/c; (b) all events at 2.9 GeV/c; (c) all events at 2.7 GeV/c for which the $p\pi^+$ and $\bar{p}\pi^-$ effective masses are in the Δ region (defined as 1236±115 MeV for the open histogram and 1236±50 MeV for the shaded histogram); (d) same as (c) for 2.9 GeV/c.

spectra were found identical within statistical uncertainties (this is expected if the strong interactions are invariant under the *CP* operation) and are summed in all cases. The numbers of events with no $\Delta(1236)$ and $\bar{\Delta}(1236)$ cuts are 719 (2.7 GeV/c) and 1015 (2.9 GeV/c), which are to be compared with Bacon *et al.*, 602 events (2.8 GeV/c), and Ferbel *et al.*, 1331 events (3–4 GeV/c). When Δ and $\bar{\Delta}$ cuts are performed on the π^+p and $\pi^-\bar{p}$ masses, the numbers of remaining events are 629 (2.7 GeV/c) and 835 (2.9 GeV/c), while Ferbel *et al.* have 892 events in this $\Delta\bar{\Delta}$ region. Bacon *et al.* do not perform any $\Delta\bar{\Delta}$ cuts.

The $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ spectra were also examined for (a) *either* the π^+p mass or the $\pi^-\bar{p}$ mass outside 1236±115 MeV and (b) *both* π^+p and $\pi^-\bar{p}$ masses outside 1236±115 MeV. While the number of events is limited, both of these restrictions yielded $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ spectra (not shown) which have the same qualitative shape as those in Figs. 2(a) and 2(b).

Preliminary examination of Figs. 2(a)–2(d) shows no outstanding enhancements in either the 2.7- or 2.9-GeV/c data, although there are possible small effects at 2.7 GeV/c in the regions 1400–1450 and 1600–1700 MeV. The shapes of these $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra are in qualitative agreement with the shapes of the data of Bacon *et al.* and Ferbel *et al.*, with the exception that the spectra of Bacon *et al.* have relatively more events in the 1300–1500-MeV region. As remarked in the Introduction, Bacon *et al.* present evidence for $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ resonance production based on their $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ spectra, while Ferbel *et al.* claim their same spectra can be adequately explained without such resonance production. Both groups base their arguments, in part, upon calculations of the expected $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra assuming a "double-isobar" one-pion-exchange (OPE) graph as the production mechanism for a part, or all, of reaction (1). In order to investigate quantitatively our spectra as presented in Figs. 2(a)–2(d) we have performed similar calculations. Our philosophy is as follows.

We take the investigations presented in Sec. III and Table I as implying that reaction (1) is nearly completely dominated by $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ production, and assume henceforth that this dominance is 100%.

We next assume that the $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ production proceeds via the simple double-isobar OPE diagram shown in the corner of Fig. 1(b). The π^+p, $\pi^-\bar{p}$, $p\pi^+\pi^-$, and $\bar{p}\pi^+\pi^-$ mass spectra as predicted by this diagram are then compared with the corresponding experimental histograms to see if the data agree with the $\bar{\Delta}^{--}\Delta^{++}$-dominance and the OPE-production hypotheses.

A. Interpretation of $\bar{\Delta}^{--}\Delta^{++}$ Mass Spectra

The calculation of the prediction of the double-isobar graph for the π^+p and $\pi^-\bar{p}$ mass spectra is described in detail in CLK. Briefly, the exchanged pion is represented by a propagator, the upper and lower vertices are represented by a good approximation to real $\pi^-\bar{p}$ and π^+p scattering, respectively, and the virtuality of the exchanged pion and other off-shell effects are included by using a form-factor expression. The only difference between the present calculation and the one described in CLK is that here the form-factor expression is that of Dürr and Pilkuhn as fitted by Wolf.[8]

The theoretical predictions are shown as smooth curves in Figs. 1(a) and 1(b). It is seen that there is reasonable agreement between the theory and the data at both energies, indicating that the hypotheses of 100% $\bar{\Delta}^{--}\Delta^{++}$ and double-isobar graph production are consistent with the π^+p and $\pi^-\bar{p}$ mass histograms. It is true, however, that at both energies the data show a slight excess of events on the low-mass side of the peak, and a slight deficiency of events on the high-mass side.

The double-isobar graph also predicts the differential cross section with respect to the square of the four-momentum transfer from incident $\bar{p}$ to final $\pi^-\bar{p}$ system (called Δ^2). The experimental Δ^2 histograms (not shown) and the theoretical curves are in good agreement.

The polar angular decay distribution for $\Delta^{++}(1236) \to p\pi^+$ has the general form $W(\theta) = \text{const} \times [(\frac{3}{4} - \rho_{11}) + 3(\rho_{11} - \frac{1}{4}) \cos^2\theta]$, where θ is the polar angle between the incident proton and the outgoing proton in the Δ^{++} rest frame, and where ρ_{11} is the $(\frac{1}{2},\frac{1}{2})$ spin-density matrix element of the Δ^{++}. The simple double-isobar OPE model described above predicts $\rho_{11}=0.5$, while our data at both energies yield $\rho_{11} \sim 0.35$. This indicates that the above OPE model is too simple a description of the reaction $\bar{p}p \to \bar{\Delta}^{--}\Delta^{++}$ at our energies. It turns out, however, that the calculations of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra are relatively insensitive to values of ρ_{11} in the range 0.25–0.5. This is discussed in Sec. IV D.

It should be remarked that the theoretical mass curves of Figs. 1(a) and 1(b) are sensitive to the form-factors and to the energy dependence of the π^+p ($\pi^-\bar{p}$) scattering used at the vertices. In particular, it is necessary to reproduce faithfully the energy dependence of π^+p ($\pi^-\bar{p}$) scattering. In CLK and in the present work this is accomplished by using empirically corrected energy-dependent widths (see Secs. II B and IV D and the Appendix).

B. OPE Calculation of $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ Mass Spectra

The calculation of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra predicted by the double-isobar OPE model follows the work of Ferrari and Selleri.[9] The formula for the differential cross section with respect to the outgoing $p\pi^+\pi^-$ mass is[10]

$$\frac{d\sigma}{du} = \frac{1}{4\pi^3 F^2} \int \Omega(\omega,\bar{\omega},\Delta^2) P(u,m^2,\omega^2) \times \bar{\omega}^2 \frac{d\sigma_1}{d\Omega}(\bar{\omega}, \cos\bar{\theta}) \frac{1}{(\Delta^2+m^2)^2} \omega^2 \sigma_1(\omega) P(\omega,M^2,m^2) \times d\omega dt^2 d\cos\theta_2{}^P d\varphi_2{}^P. \quad (3)$$

In Eq. (3), u is the $p\pi^+\pi^-$ invariant mass, M (m) is the proton (charged-pion) mass, ω ($\bar{\omega}$) is the invariant mass of the final π^+p ($\pi^-\bar{p}$) system, Δ^2 is the square of the four-momentum transfer from the incoming $\bar{p}$ to the outgoing $\pi^-\bar{p}$ system, and F^2 is a kinematic factor defined by $F^2 = (p_1 \cdot p_2)^2 - M^4$, where p_2 (p_1) is the incoming p ($\bar{p}$) four-momentum. $P(\omega,M^2,m^2)$ is the magnitude of the three-momentum of the final π^+ in the final π^+p rest system, while $P(u,m^2,\omega^2)$ is the corresponding three-momentum for a system of two particles of mass m and ω with total rest system energy u (see Appendix for definition of P).

$d\sigma_1/d\Omega$ is the differential cross section for $\pi^-\bar{p} \to \pi^-\bar{p}$ (upper vertex) at c.m. energy $\bar{\omega}$ and c.m. scattering angle $\bar{\theta}$, while $\sigma_1(\omega)$ is the total cross section for $\pi^+p \to \pi^+p$ (lower vertex) at c.m. energy ω (see Appendix for the expressions used for these cross sections). $\Omega(\omega,\bar{\omega},\Delta^2)$ is a combination of form factors which has the form $G^2(\Delta^2)\Lambda(\omega,\Delta^2)\Lambda(\bar{\omega},\Delta^2)$. The form factor Λ was devised by Dürr and Pilkuhn,[8] while G^2 is an over-all form factor used by Wolf. Λ and G^2 each contain one empirical parameter and we use the values of these parameters obtained in a fit by Wolf.[8]

The variables of integration are defined in FS. Briefly, t^2 is the square of the four-momentum transfer from the incoming $\bar{p}$ to the outgoing $\bar{p}$ at the upper vertex, while $\theta_2{}^P$ and $\varphi_2{}^P$ are the polar and azimuthal scattering angles of the final $p\pi^+$ system with respect to the incoming p direction in the final $p\pi^+\pi^-$ rest frame. The limits of integration and $\bar{\omega}$ and Δ^2 are given in FS in terms of the variables of integration.

The relative importance of the limits of integration and of the various parts of the integrand of Eq. (3) in determining the exact shape of the $p\pi^+\pi^-$ mass curve is discussed in Sec. IV D.

[8] H. P. Dürr and H. Pilkuhn, Nuovo Cimento **40A**, 899 (1965); G. Wolf, Phys. Rev. Letters **19**, 925 (1967).

[9] E. Ferrari and F. Selleri, Nuovo Cimento Suppl. **24**, 453 (1962); this paper is hereafter referred to as FS.

[10] There are some differences in the notation of FS and that used in Eq. (3). In particular, the masses ω and $\bar{\omega}$ and the angle $\bar{\theta}$ are called m_2, ω, and $\theta_1{}^Q$, respectively, in FS.

C. Comparison of OPE Predictions with $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ Mass Spectra

The theoretical $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ invariant-mass curves calculated using Eq. (3) are compared to the experimental histograms in Figs. 2(a)–2(d). In Figs. 2(a) and 2(b) are shown all events from reaction (1) at 2.7 and 2.9 GeV/c, respectively, with the corresponding prediction (smooth curve) of the double-isobar OPE model. The agreement between theory and experiment is adequate at both energies; however, the histograms are somewhat flatter across the top than are the calculated curves.

As indicated in Sec. III, the data are consistent with the fraction of $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ final state being slightly less than 100%. Thus one might expect that the predictions of the double-isobar graph would agree best with data sliced on the final π^+p and $\pi^-\bar{p}$ invariant masses to ensure as high a fraction of $\bar{\Delta}^{--}\Delta^{++}$ channel as possible. In Figs. 2(c) and 2(d) are shown the data of reaction (1) with cuts on the π^+p and $\pi^-\bar{p}$ masses of both 1236 ± 115 and 1236 ± 50 MeV. The smooth theoretical curves are from Eq. (3) with appropriate modifications for the Δ and $\bar{\Delta}$ cuts. As expected, there is slight improvement in the agreement between theory and experiment over the case with no $\bar{\Delta},\Delta$ cuts; again the predictions of the model agree with the mass histograms.

The experiment of Bacon *et al.* was done with a deuterium target, so the effective beam momentum is spread somewhat about 2.8 GeV/c by the Fermi motion of the proton in the deuteron. Thus for a rough comparison with the 2.8-GeV/c data it is appropriate to combine our data at 2.7 and 2.9 GeV/c in amounts proportional to the respective cross sections for reaction (1) at these energies. The ratio of these cross sections is 0.76 (1.93/2.53 mb), while the ratio of the corresponding numbers of events is 0.71 (719/1015); hence it is sufficient to add all events at each momentum. The distribution of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ effective masses combined for all events at 2.7 and 2.9 GeV/c is shown in Fig. 3. Also shown is the prediction of the OPE model for the summed data (smooth curve).

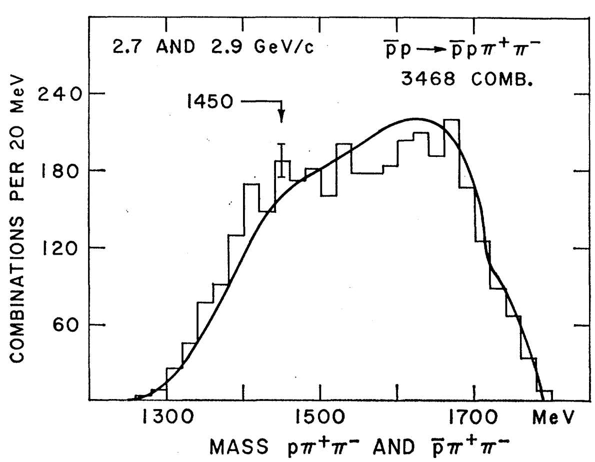

FIG. 3. Distribution of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ effective masses combined for all events at 2.7 and 2.9 GeV/c. The smooth curve is the corresponding prediction of the OPE model normalized to the data.

D. Discussion of OPE Calculation

While doing the above OPE calculation, the dependence of the exact shape of the predicted $p\pi^+\pi^-$ mass distribution on various parts of Eq. (3) was studied. In particular, the influence of the form factors, of the expressions used for $\sigma_1(\omega)$ and $d\sigma_1/d\Omega$, and of the integrand as a whole was examined. The form factor $\Omega(\omega,\bar{\omega},\Delta^2)$ as described in Sec. IV B was found to have little effect upon the $p\pi^+\pi^-$ mass distribution; while the $p\pi^+\pi^-$ mass curves with and without Ω were found to be similar, the use of the form factor did provide slightly better agreement between theory and experiment.

The calculated OPE $p\pi^+\pi^-$ mass distribution was found to be rather sensitive to the energy dependence of $\sigma_1(\omega)$ and $d\sigma_1/d\Omega=\sigma_1(\bar{\omega})(1+3\cos^2\bar{\theta})/8\pi$ [see Eq. (3) and Appendix], and for this reason special care was taken to see that $\sigma_1(\omega)$ [$\sigma_1(\bar{\omega})$] was an accurate representation of real pion-nucleon scattering. For example, whether a constant width or the energy-dependent width of Eq. (A3) is used makes a substantial difference in the predicted $p\pi^+\pi^-$ mass distribution; the energy-dependent width makes the agreement between theory and experiment adequate, while a constant width does not. It was further found that the OPE prediction for the $p\pi^+\pi^-$ mass distribution is relatively insensitive to the value of ρ_{11} in the range 0.25–0.5. Using $1+3\cos^2\theta$ ($\rho_{11}=0.5$) in Eq. (A4) caused only a slight improvement in the $p\pi^+\pi^-$ mass curve relative to using 1 ($\rho_{11}=0.25$).

It is interesting to ask how much of the agreement between theory and experiment observed in Figs. 2(a)–2(d) is due to details of the model and how much is due to kinematics. It was found that the pure phase-space prediction of the $p\pi^+\pi^-$ mass curve is a poor representation of the data. Setting the integrand of Eq. (3) to unity is another "kinematic" approximation. This was found to produce a $p\pi^+\pi^-$ mass curve in reasonable agreement with the data if no $\Delta,\bar{\Delta}$ cuts are performed, but poor agreement with $\Delta,\bar{\Delta}$ cuts. The conclusion is that the dependences of the integrand play an important role in obtaining agreement between the model predictions and the data.

As remarked in the above paragraph, the pure phase-space $p\pi^+\pi^-$ mass distribution is a poor representation of the data; in particular, the curve rises more slowly than the data for the low $p\pi^+\pi^-$ mass values. This means that the amount of $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ channel present in the final state has a critical bearing upon the interpretation of the $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra, since the more $\bar{\Delta}^{--}\Delta^{++}$ channel present, the better the agreement between the OPE calculation and the mass histograms. We disagree on this point with Bacon *et al.*[3]

V. CONCLUSIONS

We conclude that the outgoing π^+p and $\pi^-\bar{p}$ mass spectra and the outgoing $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass spectra in the reaction $\bar{p}p \to \bar{p}p\pi^+\pi^-$ at 2.7 and 2.9 GeV/c can be adequately interpreted by assuming (1) that the reaction proceeds essentially 100% through the intermediate state $\bar{\Delta}^{--}(1236)\Delta^{++}(1236)$ and (2) that the reaction mechanism is well represented by a double-isobar OPE diagram. This conclusion agrees with that of Ferbel *et al.*[4] for their $\bar{\Delta}^{--}\Delta^{++}$ events at 3–4 GeV/c. It disagrees with that of Bacon *et al.*[3] at 2.8 GeV/c; they claim that the fraction of $\bar{\Delta}^{--}\Delta^{++}$ final state in their data is 72%, and that an OPE calculation cannot adequately explain their $p\pi^+\pi^-$ and $\bar{p}\pi^+\pi^-$ mass distributions.

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APPENDIX

The definition of the three-momentum symbol $P(\omega,M^2,m^2)$ is

$$P(\omega,M^2,m^2)=(2\omega)^{-1}[\omega^4-2\omega^2(M^2+m^2)+(M^2-m^2)^2]^{1/2}, \quad \text{(A1)}$$

while the $\pi^+p \to \pi^+p$ cross section $\sigma_1(\omega)$ is represented by the Breit-Wigner expression

$$\sigma_1(\omega)=\frac{2\pi}{[P(\omega,M^2,m^2)]^2}\frac{[\Gamma(\omega)]^2}{(\omega-\omega_0)^2+[\frac{1}{2}\Gamma(\omega)]^2}, \quad \text{(A2)}$$

with the empirical resonance-width expression

$$\Gamma(\omega)=2\gamma^2[P(\omega,M^2,m^2)a]^3/\{1+[P(\omega,M^2,m^2)a]^2\}, \quad \text{(A3)}$$

where $2\gamma^2=116$ MeV, $a=0.88m^{-1}$, and $\omega_0=1236$ MeV. The expression for $\Gamma(\omega)$ and the values of the parameters $2\gamma^2$, a, and ω_0 are from Gell-Mann and Watson.[11] In spite of the fact that their fit was performed years ago, comparison with recent data[12] indicates that (A2) and (A3) provide an accurate representation of the energy dependence of π^+p scattering in the region of the $\Delta^{++}(1236)$ resonance. It is noted here that the use of the empirically corrected energy-dependent width given by (A3) produces a $\sigma_1(\omega)$ curve considerably different than what one obtains using (A2) with a constant width. This point is important in calculating both the $\pi^+p(\pi^-\bar{p})$ and the $p\pi^+\pi^-(\bar{p}\pi^+\pi^-)$ theoretical invariant-mass curves.

The expression used for the $\pi^-\bar{p} \to \pi^-\bar{p}$ differential cross section $d\sigma_1/d\Omega$ is

$$\frac{d\sigma_1}{d\Omega}(\bar{\omega}, \cos\bar{\theta})=\sigma_1(\bar{\omega})\frac{1+3\cos^2\bar{\theta}}{8\pi}, \quad \text{(A4)}$$

where $\sigma_1(\bar{\omega})$ is given by (A2) with ω replaced by $\bar{\omega}$ throughout.

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[11] M. Gell-Mann and K. M. Watson, Ann. Rev. Nucl. Sci. 4, 219 (1954).
[12] M. N. Focacci and G. Giacomelli, CERN Report No. 66-18, 1966 (unpublished).