Analysis of the S_{31} πN Partial Wave*

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With the notable exception of the S waves, reasonably good fits to the πN phase-shift analyses had been obtained with a phenomenological one-pole, multichannel ND^{-1} model. We show that a two-channel $(\pi N,\pi\Delta)$ model with a second pole in only the diagonal πN potential yields a good fit to the detailed behavior of the S_{31} partial wave.

THERE has been difficulty in understanding the complicated behavior of the S_{31} and S_{11} πN amplitudes. In particular, with the notable exception of the S waves, we had obtained[1,2] good fits to the recent phase-shift analyses[3] up to moderate energies with a phenomenological multichannel ND^{-1} model using a one-pole approximation for the generalized input potential B. In the work reported in Ref. 1, we approximated all the effects of the inelastic reactions by a purely phenomenological second channel[4] and found reasonably good fits except for the S waves. Improvements on this model were made[2] by approximating the inelastic scattering channels by realistic quasi-two-body channels such as σ-N,[5] $\pi\Delta(1236)$, and ρN. Improved fits were obtained, again with the exception of the S waves. Here, we report that the addition of a second pole in only the diagonal πN potential greatly improves the nature of the S-wave solutions. This is very reasonable since we expect that the S waves, because of the absence of the angular momentum barrier, will be more sensitive to the far away left-hand cut singularities. We present a two-channel [πN denoted by a and $\pi\Delta(1236)$ denoted by b] solution with a second pole in B_{aa} which yields a good fit to the detailed behavior of the S_{31} partial wave.[6]

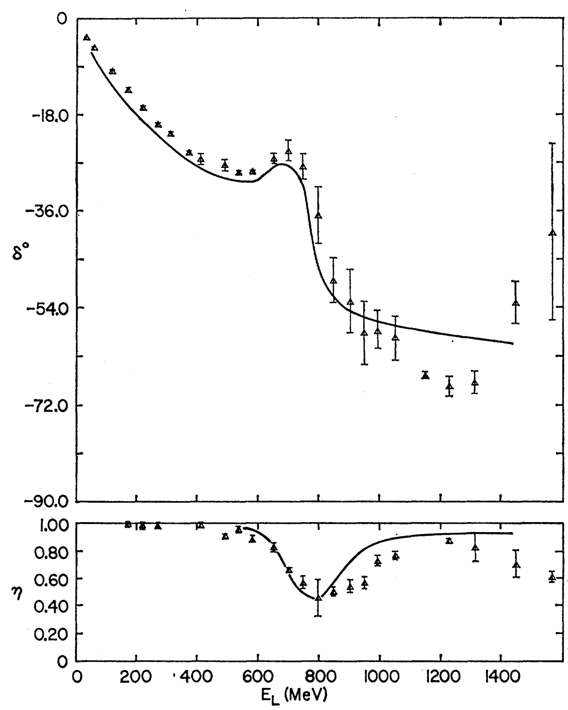

FIG. 1. Solid curves are calculated values of δ and η versus incident lab pion kinetic energy E_L for the S_{31} partial wave from Eqs. (1)–(3). The data points are an average of the Berkeley, CERN, and Saclay phase-shift analyses (Ref. 3). The bars represent an estimate of the errors.

The elastic πN partial-wave amplitude is given by[1]

$$(\eta e^{2i\delta}-1)/2i=\rho_a(ND^{-1})_{aa},$$

$$N_{ij}(W)=B_{ij}(W)+\sum_{k=a,b}\frac{1}{\pi}\int_{\alpha_i}^{\infty}dW'\times\left(B_{ik}(W')-\frac{W-W_0}{W'-W_0}B_{ik}(W)\right)\times\rho_k(W')\frac{1}{W'-W}N_{kj}(W'),\quad(1)$$

$$D_{ij}(W)=\delta_{ij}-\frac{W-W_0}{\pi}\int_{\alpha_i}^{\infty}dW'\times\frac{\rho_i(W')N_{ij}(W')}{(W'-W_0)(W'-W-i\epsilon)},$$

where ρ is a kinematical factor and α_i the threshold for the ith channel. (Note that ND^{-1} is symmetric for symmetric input B and is independent of the subtraction point W_0 in D.) In Fig. 1, we present a fit to the

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[1] J. Ball, R. Garg, and G. Shaw, Phys. Rev. **177**, 2258 (1969).
[2] R. Garg, Ph.D. thesis, University of California, Irvine, Calif., 1969 (unpublished).
[3] C. Johnson, University of California Lawrence Radiation Laboratory Report No. UCRL-17683, 1967 (unpublished); C. Lovelace, CERN Report No. Th-837, 1967 (unpublished); P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968).
[4] It is known that the inelastic production for the S_{11} partial wave is mainly in the ηN channel.
[5] We refer to an $I=0$, S-wave $\pi\pi$ enhancement as a σ.
[6] The addition of a second pole in B_{aa} in a two-channel $(\pi N,\eta N)$ model for the S_{11} partial wave greatly improves the fit. We are able to obtain both the cusp effect in δ and the rise through 90°. However, there are more low-mass inelastic channels in the $I=\frac{1}{2}$ amplitudes (as compared with $I=\frac{3}{2}$) which must be considered in order to obtain a good quantitative fit.

phase-shift analyses for the S_{31} partial wave using Eq. (1) with the potential[7]

$$B=\begin{bmatrix} \dfrac{0.72}{W-6.7}-\dfrac{10.0}{W-2} & \dfrac{5.48}{W-6.7} \\ \dfrac{5.48}{W-6.7} & \dfrac{140}{W-6.7} \end{bmatrix}. \qquad (2)$$

$\alpha_a=m_N+1$ and ρ_a is taken[8] to be simply the momentum k_a, whereas for the quasi-two-body D-wave $\pi\Delta$ channel, we have $\alpha_b=m_N+2$ and

$$\rho_b=\int_{m_N+1}^{W-1} dm\left(\frac{[W^2-(m+1)^2][W^2-(m-1)]}{4W^2}\right)^{5/2} \times\frac{1}{W^4}\frac{\Gamma_\Delta{}^2/4}{(m-m_\Delta)^2+\Gamma_\Delta{}^2/4}, \qquad (3)$$

where Γ_Δ and m_Δ are the width and mass of the $\Delta(1236)$. With the choice of B as a sum of poles, the integral equations for N_{ij} in (1) are easily reduced to quadrature.

[7] We use units $m_\pi=1$ and denote the nucleon mass by m_N.

[8] The nature of the fit is not sensitive to the detailed choice of the asymptotic behavior of ρ.

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Theorem on K_{l3} Form Factors

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A simple proof of a theorem derived recently by Dashen and Weinstein on K_{l3} form factors is presented. With the further assumption that $D(t)\equiv\langle\pi|\partial_\lambda V_\lambda{}^1|K\rangle$ is dominated by the 0^+ κ meson, a relation between κ-meson mass and $f_K/f_\pi F_+(0)$ is obtained.

RECENTLY Dashen and Weinstein[1] have proved a theorem on the form factors in K_{l3} decay which leads to the result

$$\xi(0)\equiv\frac{F_-(0)}{F_+(0)}=\frac{1}{2}\left(\frac{f_K}{f_\pi}-\frac{f_\pi}{f_K}\right)-\frac{m_K{}^2-m_\pi{}^2}{m_\pi{}^2}\lambda_++O(\epsilon^2), \qquad (1)$$

where $F_+(t)$ and $F_-(t)$ are the two form factors which determine the hadronic part of the matrix elements of K_{l3},

$$\lambda_+=m_\pi{}^2[d\ln F_+(t)/dt]_{t=0}, \qquad (2)$$

and ϵ denotes a parameter which characterizes the simultaneous breaking of $SU(3)$ and $SU(3)\times SU(3)$. The purpose of this paper is to present a simple proof of relation (1). With the further assumption that

$$D(t)\equiv\langle\pi|\partial_\lambda V_\lambda{}^1|K\rangle=(m_K{}^2-m_\pi{}^2)F_+(t)+tF_-(t) \qquad (3)$$

is dominated by a $\kappa(0^+)$ meson, a relation between the experimentally known number $f_K/f_\pi F_+(0)$ and the κ-meson mass is obtained.

By using the current-algebra techniques, we have

$$-k_\nu\Gamma_{\nu\lambda}=f_\pi\Gamma_\lambda+i\int d^4x \times e^{-ik\cdot x}\langle 0|\delta(x_0)[A_0{}^{1-i2},V_\lambda{}^{4+i5}(0)]|\bar{K}^0(p)\rangle, \qquad (4)$$

where $\Gamma_{\nu\lambda}$ and Γ_λ are defined by

$$i\int d^4x\, e^{-ik\cdot x}\langle 0|T(A_\nu{}^{1-i2}(x)V_\lambda{}^{4+i5}(0))\bar{K}^0(p)\rangle =i\Gamma_{\nu\lambda}+i\frac{f_\pi k_\nu}{k^2+m_\pi{}^2}\Gamma_\lambda, \qquad (5a)$$

$$i\int d^4x\, e^{-ik\cdot x}\langle 0|T(\partial_\nu A_\nu{}^{1-i2}(x)V_\lambda{}^{4+i5}(0))|\bar{K}^0(p)\rangle =\frac{f_\pi m_\pi{}^2}{k^2+m_\pi{}^2}\Gamma_\lambda, \qquad (5b)$$

$$\Gamma_\lambda=\langle\pi^+(k)|V_\lambda{}^{4+i5}|\bar{K}^0(p)\rangle =[F_+(t)(p+k)_\lambda+F_-(t)(p-k)_\lambda]. \qquad (5c)$$

Using the current commutation relation

$$\delta(x_0)[A_0{}^{1-i2}(x),V_\lambda{}^{4+i5}(0)]=V_\lambda{}^{6+i7}(0)\delta^4(x) \qquad (6a)$$

and the definition

$$\langle 0|V_\lambda{}^{6+i7}|\bar{K}^0(p)\rangle=if_Kp_\lambda, \qquad (6b)$$

we obtain from (4)

$$-k_\nu\Gamma_{\nu\lambda}=f_\pi[F_+(t)(p+k)_\lambda+F_-(t)(p-k)_\lambda]-f_Kp_\lambda. \qquad (7)$$

[1] R. Dashen and M. Weinstein, Phys. Rev. Letters 22, 1337 (1969).