Evidence for Current Divergences Belonging to the $(3,3^*)+(3^*,3)$ Representation of $SU(3)\times SU(3)$

RIAZUDDIN AND FAYYAZUDDIN
Institute of Physics, University of Islamabad, Rawalpindi, Pakistan
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We present evidence that current divergences belong to the $(3,3^*)+(3^*,3)$ representation of $SU(3)\times SU(3)$. We also give various estimates of the parameter c that describes the strength of the $SU(3)$-breaking term relative to the chiral-symmetry-breaking term, using the Veneziano model for π-π and π-K scattering. The ratio f_K/f_π is also estimated.

IN this paper we wish to present some evidence in support of certain commutation relations[1] of the scalar and pseudoscalar densities S_i and P_i with vector and axial-vector charges, namely,

$$[F_i(t),S_j(\mathbf{x},t)]=if_{ijk}S_k(x)\,,\tag{1a}$$

$$[F_i(t),P_j(\mathbf{x},t)]=if_{ijk}P_k(x)\,,\tag{1b}$$

$$[F_i^5(t),S_j(\mathbf{x},t)]=id_{ijk}P_k(x)\,,\tag{1c}$$

$$[F_i^5(t),P_j(\mathbf{x},t)]=-id_{ijk}S_k(x)\,.\tag{1d}$$

These relations specify the transformation properties of the scalar and pseudoscalar nonets S_i and P_i as belonging to the representation $(3,3^*)+(3^*,3)$ of $SU(3)\times SU(3)$. The assumption is also made that S_0 and S_8 components of the nonet S_i specify the symmetry-breaking terms in the hadron energy density except for scale factors; i.e., one writes the hadron energy density as[1]

$$H=H_0+\delta m_0 S_0+\delta m S_8\,,\tag{2}$$

where H_0 is invariant under $SU(3)\times SU(3)$, and δm_0 and δm express the scales of $SU(3)\times SU(3)$- and $SU(3)$-symmetry breakings, respectively. The current divergences can be simply obtained in terms of S_i and P_i from Eq. (2), and the commutation relations (1a) and (1b) and are given by

$$\partial_\mu V_{i\mu}=-i[F_i,H]=-\delta m f_{8ik}S_k\,,\tag{3}$$

$$\partial_\mu A_{i\mu}=-i[F_i^5,H]=\delta m_0[(\sqrt{\tfrac{2}{3}})P_i+cd_{8ik}P_k]\,.\tag{4}$$

Recently, attempts have been made to estimate the scale parameter[2,3] c ($c=\delta m/\delta m_0$ measures the strength of the $SU(3)$-symmetry-breaking term relative to the $SU(3)\times SU(3)$-symmetry-breaking term) and the change[4] in the average mass of the baryon octet due to $SU(3)\times SU(3)$-symmetry breaking. In fact, it is easy to see from Eq. (3) that[5]

$$(m_K^2-m_\pi^2)=-\tfrac{1}{2}\sqrt{3}\,\delta m s_d'\,,\tag{5}$$

where s_d' is defined by ($\mathcal{P}_i$ denotes the 0^- octet)

$$\langle\mathcal{P}_k(p')\,|\,S_i\,|\,\mathcal{P}_j(p)\rangle=(2\pi)^{-3}(4p_0'p_0)^{-1/2}[\tfrac{1}{2}\sqrt{2}s_d''\delta_{i0}\delta_{jk}+d_{ijk}s_d']\,,\tag{6}$$

and j, $k=1,\ldots,8$ and $i=0,\ldots,8$. By using the soft-meson technique, partial conservation of axial-vector current (PCAC), and the commutation relation (1c), s_d'' and the parameter c have been estimated by Gell-Mann, Oakes, and Renner[2] to be

$$s_d''\approx 0\,,\quad c=-2\sqrt{2}\frac{m_K^2-m_\pi^2}{2m_K^2+m_\pi^2}\approx -1.256\,.\tag{7}$$

It is important to note that in this estimate, the η-η' mixing has been neglected.

As has been emphasized in Ref. 2, it is desirable to confirm the universality of c. Moreover, one should point out that in the above estimate of c, the commutation relation (1d) has not been used. It is, therefore, important that one consider such processes where the commutation relation (1d) is explicitly used and obtain an independent estimate of the parameter c. It is now well known[6] that, by using the commutation relation (1d), PCAC, and the soft-meson technique, one can express a scattering amplitude involving at least two pseudoscalar mesons (where these two pseudoscalar mesons are off the mass shell) in terms of S_0 and S_8 and the parameter c. An attempt to estimate c in this way has made it possible[3] to establish only a lower bound on c, $|c|\geqslant 0.7$. In this paper we wish to consider π-π and π-K scattering, where the Veneziano models[7] for these amplitudes enable us to extrapolate the amplitude concerned to the unphysical values of the scattering variables s, u, and t, at which values the amplitude is expressible in terms of c and thus provide a measure of c. We get a value of c in an excellent agreement with that determined in Ref. 2, and thus our calculation provides a confirmation for the universality of c and for the commutation relations (1). We also obtain an estimate of the ratio f_K/f_π, where f_K and f_π are decay constants for $K\to\mu\nu$ and $\pi\to\mu\nu$, respectively. We give details of our calculation below.

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We consider the scattering amplitude

$$\mathcal{P}_l(q)+\mathcal{P}_j(p)\rightarrow\mathcal{P}_m(q')+\mathcal{P}_k(p') \quad (8)$$

for which we can write the T matrix as

$$T_{jk}{}^{lm}(s,u,t)=(2\pi)^{-6}(16p_0p_0'q_0q_0')^{-1/2}A_{jk}{}^{lm}(s,u,t)\,, \quad (9)$$

where

$$s=-(p+q)^2=-(p'+q')^2\,,$$
$$u=-(p-q')^2=-(p'-q)^2\,,$$
$$t=-(p-p')^2=-(q'-q)^2\,,$$

and

$$s+u+t=m_l^2+m_j^2+m_m^2+m_k^2\,. \quad (10)$$

The soft-meson technique and the use of PCAC give

$$\partial_\lambda A_\lambda{}^0=\partial_\lambda A_{1+i2,\lambda}=f_\pi m_\pi{}^2\pi^-\,,$$
$$\partial_\lambda A_\lambda{}^1=\partial_\lambda A_{4+i5,\lambda}=f_K m_K{}^2K^-\,, \quad (11)$$

$$A_{jk}{}^{\pi^-\pi^-}(s=m_j^2,\ u=m_j^2,\ t=0)$$
$$=f_\pi{}^{-2}(2\pi)^3 2p_0 i\langle\mathcal{P}_k(p)|[\partial_\lambda A_\lambda{}^0,F_{1-i2}{}^5]|\mathcal{P}_j(p)\rangle\,, \quad (12)$$

$$A_{jk}{}^{K^-K^-}(s=m_j^2,\ u=m_j^2,\ t=0)$$
$$=f_K{}^{-2}(2\pi)^3 2p_0 i\langle\mathcal{P}_k(p)|[\partial_\lambda A_\lambda{}^1,F_{4-i5}{}^5]|\mathcal{P}_j(p)\rangle\,. \quad (13)$$

Taking $j,\ k=(1\pm i2)/\sqrt{2}$ and $(4\pm i5)/\sqrt{2}$ in turn in (12), and $j,\ k=(1\pm i2)/\sqrt{2}$ in (13), we obtain, on using (4) and the commutation relation (1d),

$$A_{\pi^+\pi^+}{}^{\pi^-\pi^-}(s=u=m_\pi{}^2,\ t=0)$$
$$=-f_\pi{}^{-2}\tfrac{2}{3}(1+\sqrt{2}/c)\delta m(2\pi)^3 2p_0$$
$$\times\langle\pi^+(p)|S_8+\sqrt{2}S_0|\pi^+(p)\rangle$$
$$=f_\pi{}^{-2}\tfrac{4}{3}(1+\sqrt{2}/c)(m_K{}^2-m_\pi{}^2)[1+\tfrac{1}{3}\sqrt{3}(s_d''/s_d')]\,, \quad (14a)$$

$$A_{K^+K^+}{}^{\pi^-\pi^-}(s=u=m_K{}^2,\ t=0)$$
$$=f_\pi{}^{-2}\tfrac{2}{3}(1+\sqrt{2}/c)(m_K{}^2-m_\pi{}^2)[1+\tfrac{2}{3}\sqrt{3}(s_d''/s_d')]\,, \quad (14b)$$

$$A_{\pi^+\pi^+}{}^{K^-K^-}(s=u=m_\pi{}^2,\ t=0)$$
$$=f_K{}^{-2}(2\sqrt{2}/c-1)\tfrac{1}{3}(m_K{}^2-m_\pi{}^2)$$
$$\times[1+\tfrac{2}{3}\sqrt{3}(s_d''/s_d')]\,, \quad (14c)$$

where we have made use of Eqs. (5) and (6).

We now use the Veneziano formulas[7] to estimate $A_{\pi^+\pi^+}{}^{\pi^-\pi^-}(s=u=m_\pi{}^2)$, $A_{K^+K^+}{}^{\pi^-\pi^-}(s=u=m_K{}^2,\ t=0)$, and $A_{\pi^+\pi^+}{}^{K^-K^-}(s=u=m_\pi{}^2,\ t=0)$, and obtain, respectively,

$$A_{\pi^+\pi^+}{}^{\pi^-\pi^-}(s=u=m_\pi{}^2,\ t=0)$$
$$=-2\gamma_{\rho\pi\pi}{}^2\frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}+m_\pi{}^2/2(m_\rho{}^2-m_\pi{}^2))}{\Gamma(m_\pi{}^2/2(m_\rho{}^2-m_\pi{}^2))}$$
$$=-2\gamma_{\rho\pi\pi}{}^2\times 0.0505\,, \quad (15a)$$

$$A_{K^+K^+}{}^{\pi^-\pi^-}(s=u=m_K{}^2,\ t=0)$$
$$=-4\gamma_{K^*K\pi}{}^2\times 0.0505\,, \quad (15b)$$

$$A_{\pi^+\pi^+}{}^{K^-K^-}(s=u=m_\pi{}^2,\ t=0)$$
$$=-4\gamma_{K^*K\pi}{}^2\times 0.5266\,. \quad (15c)$$

From Eqs. (14a) and (14b) and (15a) and (15b), we obtain

$$\frac{\gamma_{\rho\pi\pi}{}^2}{4\gamma_{K^*K\pi}{}^2}=\frac{1+\frac{1}{3}\sqrt{3}(s_d''/s_d')}{1+\frac{2}{3}\sqrt{3}(s_d''/s_d')}\,. \quad (16)$$

This gives, in the $SU(3)$ limit (for vertices only), in which case $4\gamma_{K^*K\pi}{}^2=\gamma_{\rho\pi\pi}{}^2$,

$$s_d''=0 \quad (17)$$

in agreement with the estimate given in Ref. 2. Experimentally, however, $4\gamma_{K^*K\pi}{}^2/\gamma_{\rho\pi\pi}{}^2\approx 1.5$, which gives

$$s_d''/s_d'=\sqrt{3}\,. \quad (18)$$

We note that $\gamma_{\rho\pi\pi}{}^2$ can be written as

$$\gamma_{\rho\pi\pi}{}^2=am_\rho{}^2/f_\pi{}^2\,, \quad (19)$$

where the Veneziano model[7] gives

$$a=2/\pi\,, \quad (20a)$$

while the current-algebra value is[8]

$$a=1\,. \quad (20b)$$

From Eqs. (14a) and (15a), we obtain

$$(1+\sqrt{2}/c)=-2\gamma_{\rho\pi\pi}{}^2(0.05)\tfrac{3}{4}[f_\pi{}^2/(m_K{}^2-m_\pi{}^2)]$$
$$\times[1+\tfrac{1}{3}\sqrt{3}(s_d''/s_d')]^{-1}\,. \quad (21)$$

In order to evaluate c from Eq. (21), we make use of Eqs. (18)–(20). The various values of c obtained in this way are given in Table I. These values are compatible with that given in Eq. (7) obtained in an entirely different way. Note that the value of c is not sensitive to the values of s_d''/s_d' and a. This supports not only the universality character of c but also the commutation relations (1).

Note that from Eqs. (14b) and (14c), (15b) and (15c)

$$\frac{0.0505}{0.5266}=\frac{2(c+\sqrt{2})}{(2\sqrt{2}-c)}\frac{f_K{}^2}{f_\pi{}^2}\,,$$

which gives

$$f_K/f_\pi\approx 1.12\,,\quad \text{for}\quad c=-1.256$$
$$\approx 1.25\,,\quad \text{for}\quad c=-1.29\,.$$

If one uses (14a) and (14c), (15a) and (15c), one again obtains the same results, showing that these equations are mutually consistent. For $c=-1.33$, f_K/f_π comes out to be quite large. Note that f_K/f_π is sensitive to the values of c and s_d''/s_d', in particular when c approaches $-\sqrt{2}$.

TABLE I. Estimated values of the parameter c.

s_d''/s_d' \ a	$2/\pi$	1
0	−1.256	−1.20
$\sqrt{3}$	−1.33	−1.29

[8] K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters **16**, 255 (1966); Fayyazuddin and Riazuddin, Phys. Rev. **147**, 1071 (1966).