Vector-Dominance Model and Photoproduction of Pions*

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(Received 2 September 1969)

The relation between the processes $\gamma+N \to \pi+N$ and $\pi+N \to \rho+N$, according to the vector-dominance model (VDM), is reexamined. The VDM is defined in terms of the behavior of the invariants in the matrix elements of the vector current. The consequences of the VDM, and assumptions about asymptotic properties, for measurable quantities are discussed. The inability of the VDM to account for the results of polarized-photoproduction experiments is confirmed. An appendix provides a detailed discussion of the relation between invariants and density matrix elements in various frames.

I. INTRODUCTION

THE vector-dominance model (VDM) has been used[1] to obtain relations between electromagnetic processes and hadronic reactions involving vector mesons (ρ, ω, and ϕ). Early successes have been followed by recent apparent failures.[2] One of these involves the comparison of the two processes[3]

$$\gamma(k)+N(p_1) \to \pi(q)+N(p_2)\,, \tag{1.1}$$

$$\pi(q)+N(p_2) \to \rho(k)+N(p_1)\,. \tag{1.2}$$

These processes, of course, are related to each other by time reversal and the substitution $\rho \leftrightarrow \gamma$. Recent photoproduction experiments with polarized photons have shown striking disagreements with predictions based on VDM.[4] There have been a number of subsequent efforts to account for the discrepancy without abandoning VDM.[5–8]

The question of whether a particular modified theory is still VDM is, of course, partially subjective, since it depends on how one chooses to define the phrase "VDM." Our viewpoint is as follows: Consider the matrix element $\langle a|V^\mu|b\rangle$, where, for example, V^μ is the isovector current, and the states a and b are $a=\pi(q)N(p_2)$ and $b=N(p_1)$. This may be expanded in some set of invariants A_i. Then the A_i will have poles at $k^2=m_\rho^2$ $(k=p_a-p_b)$; the residues will be related to the matrix element for $\rho+b\to a$ [eg., (1.1)]. The matrix element for $a+\gamma\to b$ [e.g., (1.2)] will be related to the A_i $(k^2=0)$. This so far is independent of the VDM and will be true of any theory. The simplest version of the VDM is the assumption that

$$A_i \approx C_i/(k^2-m_\rho^2)\,, \tag{1.3}$$

where C_i is independent of k^2. In a more sophisticated version, we may be willing to tolerate some k^2 dependence. Clearly, as long as we are only concerned with processes (1.1) and (1.2), we can at most measure $C_i(k^2=0)$ and $C_i(k^2=m_\rho^2)$; thus, if we were to allow just linear terms in *each* of the C_i, we would already have destroyed the predictive power of the VDM. We therefore only allow k^2 dependence in C_i, when we have a specific reason for it. We will certainly allow no *singularities* in the A_i other than the ρ pole and kinematic singularities.

This definition of the VDM still leaves some vagueness, especially in the choice of an appropriate set of invariants A_i. We attempt to use the simplest definition possible; some complications due to gauge invariance intrude, however.

In Sec. II, we introduce a number of useful definitions, and present relations among the defined quantities. Section III is devoted to a discussion of asymptotic (large-s) properties. In Sec. IV, we discuss what k^2 dependence can be expected according to our criteria, and whether the experimental discrepancy can be accounted for in this way. Our general conclusion is that it cannot, i.e., if the experimental results[4] hold up, then the VDM should be regarded as having failed.

This is a quantitative failure rather than a gross qualitative one. Thus VDM does provide the right order of magnitude, i.e., the missing terms in (1.3) are not orders of magnitude larger than the one present. This is characteristic of all the recent failures[2] of VDM. The failure appears more dramatic for *polarized* photoproduction because polarization results depend sensitively on the relative sizes of the omitted terms in *different* A_i.

In an appendix we provide a careful discussion of ρ

* Work supported in part through funds provided by the U. S. Atomic Energy Commission, under Contract No. AT(30-1)-2098.

[1] For a review, see, e.g., J. J. Sakurai, in *Lectures in Theoretical Physics* (Gordon and Breach, Science Publishers, Inc., New York, 1969), Vol. XI.

[2] These include: (1) nucleon form factors (dipole fit) [see, e.g., G. Weber and R. E. Taylor, in *Proceedings of the 1967 International Symposium on Electron and Phonon Interactions at High Energies* (Stanford Linear Accelerator Center, Stanford, Calif., 1967)]; (2) photoproduction of ρ^0 mesons from complex nuclei [G. McClellan *et al.*, Phys. Rev. Letters **22**, 377 (1969); F. Bulos *et al.*, *ibid.* **22** 490 (1969)]; (3) the ratio $\Gamma(\omega\to\pi\gamma)/\Gamma(\omega\to 3\pi)$ [J. E. Augustin *et al.*, Phys. Letters **28B**, 503 (1969)]; and (4) π photoproduction, discussed in the present paper.

[3] We shall, in the case of photoproduction, be concerned with the sum $d\sigma/dt(\gamma p\to\pi^+n)+d\sigma/dt(\gamma n\to\pi^-p)$, since this sum has no interference terms between isovector photons (ρ's) and isoscalar photons (ω's).

[4] C. Geweniger *et al.*, Phys. Letters **28B**, 155 (1968); R. Diebold and J. A. Poirier, Phys. Rev. Letters **22**, 255 (1969); L. J. Gutay *et al.*, *ibid.* **22**, 424 (1969).

[5] A. Bialas and K. Zalewski, Phys. Letters **28B**, 436 (1969).

[6] F. T. Meiere, Phys. Rev. **182**, 1561 (1969).

[7] F. T. Meiere, Phys. Letters **30B**, 44 (1969).

[8] Z. G. T. Guiragossian and A. Levy, Phys. Letters **30B**, 48 (1969).

density matrix elements in different reference frames and their relation to the invariants.

II. DEFINITIONS AND CONSEQUENCES

We are concerned with the two processes (1.1) and (1.2), for which we have adopted a common notation with regard to the momenta. We define s, t, and u as usual:

$$s=(p_1+k)^2=(p_2+q)^2, \tag{2.1a}$$

$$t=(q-k)^2=(p_1-p_2)^2, \tag{2.1b}$$

$$u=(p_1-q)^2=(p_2-k)^2. \tag{2.1c}$$

Both processes are related to the matrix element

$$\langle\pi(q)N(p_2)|eV^\mu|N(p_1)\rangle, \tag{2.2}$$

where V^μ is the isovector current. We expand (2.2) in two ways:

$$\langle\pi(q)N(p_2)|eV^\mu|N(p_1)\rangle \equiv\sum_{i=1}^{6}\bar{u}(p_2)\Gamma_i{}^\mu u(p_1)A_i(k^2,s,t) \tag{2.3a}$$

$$\equiv\sum_{i=1}^{8}\bar{u}(p_2)\bar{\Gamma}_i{}^\mu u(p_1)\bar{A}_i(k^2,s,t), \tag{2.3b}$$

$$\begin{aligned}
\Gamma_1{}^\mu&=\gamma^5(\boldsymbol{k}\gamma^\mu-k^\mu),\\
\Gamma_2{}^\mu&=2\gamma^5(q\cdot kP^\mu-P\cdot kq^\mu),\\
\Gamma_3{}^\mu&=\gamma^5(\boldsymbol{k}q^\mu-q\cdot k\gamma^\mu),\\
\Gamma_4{}^\mu&=2\gamma^5(\boldsymbol{k}P^\mu-P\cdot k\gamma^\mu)-2M\Gamma_1{}^\mu,\\
\Gamma_5{}^\mu&=\gamma^5(q\cdot kk^\mu-k^2q^\mu),\\
\Gamma_6{}^\mu&=\gamma^5(\boldsymbol{k}k_\mu-k^2\gamma^\mu),
\end{aligned} \tag{2.4}$$

$$\begin{aligned}
\bar{\Gamma}_1{}^\mu&=\gamma^5P^\mu, & \bar{\Gamma}_5{}^\mu&=\gamma^5\boldsymbol{k}P^\mu,\\
\bar{\Gamma}_2{}^\mu&=\gamma^5q^\mu, & \bar{\Gamma}_6{}^\mu&=\gamma^5\boldsymbol{k}q^\mu,\\
\bar{\Gamma}_3{}^\mu&=\gamma^5k^\mu, & \bar{\Gamma}_7{}^\mu&=\gamma^5\boldsymbol{k}k^\mu,\\
\bar{\Gamma}_4{}^\mu&=\gamma^5\gamma^\mu, & \bar{\Gamma}_8{}^\mu&=\gamma^5\boldsymbol{k}\gamma^\mu,
\end{aligned} \tag{2.5}$$

where $P=\frac{1}{2}(p_1+p_2)$, M is the mass of the nucleon, and μ is the mass of the pion. The $\bar{\Gamma}_i$ are a kinematically simple set of covariants. However, because of current conservation, they are not all independent. When current conservation is used to eliminate two of them, one obtains a gauge-invariant set, such as the conventionally chosen Γ_i. The A_i, however, will, as a result, contain kinematic singularities. The $\bar{A}_i$ can, of course, be written as a function of the A_i:

$$\begin{aligned}
&\bar{A}_1=2q\cdot kA_2,\\
&\bar{A}_2=-2P\cdot kA_2-k^2A_5,\\
&\bar{A}_3=q\cdot kA_5-A_1+2MA_4,\\
&\bar{A}_4=-q\cdot kA_3-k^2A_6-P\cdot kA_4,\\
&\bar{A}_5=2A_4,\quad \bar{A}_6=A_3,\\
&\bar{A}_7=A_6,\quad \bar{A}_8=A_1-2MA_4.
\end{aligned} \tag{2.6}$$

The amplitude for photoproduction [process (1.1)] will be (2.2) (evaluated at $k^2=0$) times ϵ_μ (the polarization vector of the photon). (Note that A_5 and A_6 do not contribute.) The amplitude for process (1.2) is similar, but, instead of the A_i, involves the quantities[9]

$$a_i(s,t)\equiv\lim_{k^2\to m_\rho^2}\frac{k^2-m_\rho{}^2}{ef_\rho}A_i(k^2,s,t). \tag{2.7}$$

The rate for the process is of course proportional to the square of the amplitude. Further, the experiments of interest do not measure nucleon polarizations, but do use polarized photons (or measure the ρ density matrix). We are therefore interested in the quantity

$$A^{\mu\nu}\equiv\sum_{N\text{ spins}}\langle\pi N|V^\mu|N\rangle^*\langle\pi N|V^\nu|N\rangle \tag{2.8a}$$

$$\begin{aligned}
&\equiv g^{\mu\nu}B_1+P^\mu P^\nu B_2+q^\mu q^\nu B_3+(P^\mu q^\nu+q^\mu P^\nu)B_4+(P^\mu q^\nu-q^\mu P^\nu)B_5+k^\mu k^\nu B_6+(k^\mu P^\nu+P^\mu k^\nu)B_7\\
&\qquad+(k^\mu q^\nu+q^\mu k^\nu)B_8+(k^\mu P^\nu-P^\mu k^\nu)B_9+(k^\mu q^\nu-q^\mu k^\nu)B_{10}.
\end{aligned} \tag{2.8b}$$

Since $A^{\mu\nu}=A^{\nu\mu*}$, B_5, B_9, and B_{10} are pure imaginary, while the other B_i are real. Further, since the rates are proportional to $\epsilon_\mu{}^*A^{\mu\nu}\epsilon_\nu$, and $\epsilon\cdot k=0$, only the first five B_i's are measurable.

We similarly define

$$a^{\mu\nu}\equiv\lim_{k^2\to m_\rho^2}\frac{(k^2-m_\rho{}^2)^2}{(ef_\rho)^2}A^{\mu\nu} \tag{2.9}$$

and expand this in terms of b_i's ($i=1,\ldots,10$).

The B's (b's) can, of course, be written in terms of the A's (a's). We give here the relationship for the first four B_i's:

$$-M^2B_1=|P\cdot kA_1+Mq\cdot kA_3+Mk^2A_6|^2-t|P\cdot kA_4+\tfrac{1}{2}q\cdot kA_3+\tfrac{1}{2}k^2A_6|^2-[(q\cdot k-k^2)^2-k^2t]|\tfrac{1}{2}A_1-MA_4|^2, \tag{2.10}$$

$$M^2B_2=|q\cdot kA_3+k^2A_6|^2-t\left|q\cdot kA_2+\frac{q\cdot k-k^2}{t}A_1\right|^2+(1/t)[(q\cdot k-k^2)^2-k^2t][|A_1|^2-t|A_4|^2], \tag{2.11}$$

[9] We define f_ρ by the equation $\langle\rho^0|V_\mu|0\rangle=f_\rho\epsilon_\mu$.

$$M^2B_3=[(P\cdot k)^2-k^2(M^2-\tfrac{1}{4}t)]|A_3|^2-t|P\cdot kA_2+\tfrac{1}{2}k^2A_5|^2-|P\cdot kA_4+\tfrac{1}{2}k^2(A_3+A_6)|^2+\tfrac{1}{4}k^2|A_1-2MA_4|^2$$
$$-P\cdot k2\,\mathrm{Re}A_1(P\cdot kA_2+\tfrac{1}{2}k^2A_5)^*-Mk^22\,\mathrm{Re}(P\cdot kA_2+\tfrac{1}{2}k^2A_5)(A_3+A_6)^*, \quad (2.12)$$

$$M^2B_4=-q\cdot kP\cdot k[|A_3|^2+\mathrm{Re}(A_3A_6^*-A_1A_2^*)]+q\cdot kk^2M\,\mathrm{Re}A_2(A_3+A_6)^*+Mk^4\,\mathrm{Re}A_1A_3^*$$
$$+q\cdot kt\,\mathrm{Re}A_2(P\cdot kA_2+\tfrac{1}{2}k^2A_5)^*+(q\cdot k-k^2)\,\mathrm{Re}A_1(P\cdot kA_2+\tfrac{1}{2}k^2A_5)^*$$
$$+(q\cdot k-k^2)\,\mathrm{Re}A_4[P\cdot kA_4+\tfrac{1}{2}k^2(A_3+A_6)]^*. \quad (2.13)$$

The results of ρ experiments are given in terms of density matrix elements. For some choice of ρ polarization vectors $\epsilon_\mu^{(i)}$ (satisfying $\epsilon_\mu^{(i)}k^\mu=0$), we define an "un-normalized density matrix":

$$r_{ij}\equiv\epsilon_\mu^{(i)*}a^{\mu\nu}\epsilon_\nu^{(j)}. \quad (2.14)$$

The usual density matrix is simply the r_{ij} normalized:

$$\rho_{ij}=r_{ij}/\sum_{k=1}^{3}r_{kk}. \quad (2.15)$$

As discussed in the Appendix, the ρ_{ij} will depend on the reference frame (i.e., on the choice of the $\epsilon_\mu^{(i)}$). We always choose $\epsilon^{(1)}$ and $\epsilon^{(2)}$ as transverse ($\boldsymbol{\varepsilon}^{(i)}\cdot\mathbf{k}=0$, $i=1, 2$) and $\epsilon^{(3)}$ as longitudinal; we also choose $\epsilon^{(2)}$ as normal to the production plane ($\boldsymbol{\varepsilon}^{(2)}\cdot\mathbf{p}=\boldsymbol{\varepsilon}^{(2)}\cdot\mathbf{q}=0$). The calculation of the r_{ij} in terms of b_i for various frames is discussed in the Appendix. Two frames of interest are the helicity or center-of-mass frame (p_1+k at rest) and the Gottfried-Jackson frame (q at rest). (These frames may be defined equivalently in terms of the choice of a quantization axis in the ρ rest frame. See Appendix for discussion.) Here we state the results for these two frames at large s. For the helicity frame,

$$r_{11}^h=-b_1-t(\tfrac{1}{4}b_2+b_3-b_4), \quad (2.16a)$$

$$r_{22}^h=-b_1, \quad (2.16b)$$

$$r_{33}^h=-b_1+(1/k^2)[(P\cdot k)^2b_2+(q\cdot k-k^2)^2b_3+2(q\cdot k-k^2)P\cdot kb_4], \quad (2.16c)$$

$$\mathrm{Re}r_{13}^h=(-t/k^2)^{1/2}\{-\tfrac{1}{2}P\cdot kb_2+(q\cdot k-k^2)b_3+[P\cdot k-\tfrac{1}{2}(q\cdot k-k^2)]b_4\}, \quad (2.16d)$$

$$r_{23}^h=r_{21}^h=0. \quad (2.16e)$$

For the Gottfried-Jackson (GJ) frame,

$$r_{11}^{\mathrm{GJ}}=-b_1-\frac{t(P\cdot k)^2}{(q\cdot k)^2-\mu^2k^2}b_2, \quad (2.17a)$$

$$r_{22}^{\mathrm{GJ}}=-b_1, \quad (2.17b)$$

$$r_{33}^{\mathrm{GJ}}=-b_1+\{k^2[(q\cdot k)^2-\mu^2k^2]\}^{-1}\{(P\cdot k)^2\times(q\cdot k-k^2)^2b_2+[(q\cdot k)^2-\mu^2k^2]^2b_3+2P\cdot k(q\cdot k-k^2)[(q\cdot k)^2-\mu^2k^2]b_4\}, \quad (2.17c)$$

$$\mathrm{Re}r_{13}^{\mathrm{GJ}}=P\cdot k\left(\frac{-t}{k^2}\right)^{1/2}\left[\frac{P\cdot k(q\cdot k-k^2)}{(q\cdot k)^2-\mu^2k^2}b_2+b_4\right], \quad (2.17d)$$

$$r_{21}^{\mathrm{GJ}}=r_{23}^{\mathrm{GJ}}=0. \quad (2.17e)$$

For a real photon there are, of course, only two polarization states, both transverse (i.e., states 1 and 2 in the notation introduced above for the ρ). The rates for photoproduction for photons in polarization state i will be proportional to R_{ii}, where R is the same function of the B's that r is of the b's. [It can be shown that $R_{ij}(k^2=0)$ is independent of the frame chosen and that $R_{i3}(k^2=0)=0$.] To use the VDM to compare polarized-photoproduction and ρ-production experiments, we will need relations between $R_{ii}(k^2=0)$ ($i=1, 2$), on the one hand, and the $r_{ij}(k^2=m_\rho^2)$, in some frame, on the other, since these are the measured quantities. This will be discussed in Sec. IV; as a preliminary, we discuss asymptotic (large-s) properties of the invariants, since these will enable us to simplify the expressions for the R's (r's).

III. ASYMPTOTIC PROPERTIES

We now wish to consider the behavior of the B's and A's (b's and a's) in the large-s limit. We will avoid the explicit use of the language of Regge theory.

The differential cross section $d\sigma/d\Omega$ for $\pi N\to\rho N$ with ρ in the ith polarization state is proportional to $s^{-1}r_{ii}$. Since we do not expect the cross section for this process to remain constant at large s, we assume

$$r_{ii}=O(1) \quad \text{as} \quad s\to\infty. \quad (3.1a)$$

This is, of course, an unproved assumption for vector-meson production. We have the empirical fact[10] that $d\sigma/d\Omega$ for photoproduction appears to be approximately $\sim s^{-1}$, which suggests that (for $k^2=0$)

$$R_{ii}=O(1) \quad \text{as} \quad s\to\infty. \quad (3.1b)$$

We assume that (3.1) holds for all k^2.

From (2.16b) [or (2.17b)], the assumption (3.1) implies

$$B_1=O(1). \quad (3.2)$$

From (2.17a), since

$$2P\cdot k=\tfrac{1}{2}(s-u)\to s, \quad (3.3)$$

we now find

$$B_2=O(1/s^2). \quad (3.4)$$

Now (2.16a) implies

$$B_3-B_4=O(1), \quad (3.5)$$

[10] See, e.g., B. Richter, in *Proceedings of the 1967 International Symposium on Electron and Photon Interactions at High Energies* (Stanford Linear Accelerator Center, Stanford, Calif., 1967).

and (2.16c) leads to

$$(q\cdot k-k^2)B_3+P\cdot kB_4=O(1). \tag{3.6}$$

Combining (3.5) and (3.6) leads to

$$B_4=O(1/s), \tag{3.7}$$

$$B_3=O(1). \tag{3.8}$$

It is readily seen that with (3.2), (3.4), (3.7), and (3.8), all of the R_{ij} calculated in Sec. II are $O(1)$.

We now consider what these imply for the A_i. Consider first what we learn from the requirement on the behavior of B_1. Equation (2.10) gives $-B_1$ as the sum of three terms, of which the first two are, in the physical region ($t<0$), necessarily positive, and the last is necessarily negative. Thus if the last term increases as $s\to\infty$, at least one of the first two must increase, and at the same rate, in order for the total B_1 to be constant. Similarly, if *either* of the first two terms increases, so must the last one.

Suppose

$$\tfrac{1}{2}A_1-MA_4\sim\beta s^\alpha, \quad \alpha>0. \tag{3.9}$$

Then at least one of A_1, A_4 must increase at least as fast as s^α. We write

$$A_{1,4}\sim\beta_{1,4}s^\alpha, \tag{3.10}$$

where

$$\tfrac{1}{2}\beta_1-M\beta_4=\beta \tag{3.11}$$

and, of course, one (but not both) of the β_i may be zero. If, say, β_1 is not zero, then the first term in (2.10) will increase as $s\to\infty$, and faster than the last one, *unless*

$$A_7\equiv k^2A_6+q\cdot kA_3\sim\beta_7 s^{\alpha+1} \tag{3.12a}$$

$$\beta_7=-(1/2M)\beta_1. \tag{3.12b}$$

Similarly, the second term will increase too rapidly unless

$$\beta_7=-\beta_4. \tag{3.13}$$

Combining these, we find

$$\beta=0, \tag{3.14}$$

and the last term in (2.10) does not increase. Each of the remaining terms (both positive) must now separately be constant at large s, so that

$$sA_1+2MA_7=O(1), \tag{3.15}$$

$$sA_4+A_7=O(1), \tag{3.16}$$

which implies

$$A_1-2MA_4=O(1/s). \tag{3.17}$$

Next we examine B_2, using (2.11). Again we have three terms, the first two positive and the last negative. Now the requirement is that $B_2\to O(1/s^2)$. An analysis similar to the above leads to

$$A_{1,2,4,7}=O(1/s). \tag{3.18}$$

Now we turn to B_3. We suppose

$$A_5\sim\beta_5 s^{\alpha_5}, \tag{3.19}$$

$$A_8\equiv A_3+A_6\sim\beta_8 s^{\alpha_8}. \tag{3.20}$$

Examining B_3 [as given by (2.12)] and using (3.18), we see that if $\alpha_8\leq-1$, $\alpha_5\leq 0$, then $B_3=O(1)$ as desired. Suppose then that one or both of α_5 and $\alpha_8+1>0$. If $\alpha_8+1\geqslant\alpha_5$, then the first term in (2.12) will increase, and more rapidly than any other term with the exception (when $\alpha_8+1=\alpha_5$) of the second. However, both of the first two terms are positive and thus cannot cancel each other. Thus α_8+1 cannot be $\geqslant\alpha_5$ and >0. Similarly, if $\alpha_8+1<\alpha_5$ and $\alpha_5>0$, the second term will increase too rapidly. Thus $\alpha_5\leq 0$, $\alpha_8\leq -1$.

Thus we have found from our study of B_1, B_2, and B_3 that[11]

$$A_i=O(1/s), \quad i\neq 5 \tag{3.21a}$$

$$A_5=O(1). \tag{3.21b}$$

It is easy to see that Eqs. (3.21) imply $B_4\to O(1/s)$ as desired, so that no further information is obtained from the asymptotic requirement on B_4. [Of course, some of the A_i could vanish *more* rapidly at large s than (3.21) implies without violating our assumption (3.1).]

IV. TESTING VDM

As noted at the end of Sec. II, to test the VDM, we need predictions relating the R_{ij} at the two measured values of $k^2=0$ and m_ρ^2. Since in our view the clearest phrasing of the VDM is as a statement about matrix elements of V_μ, we first wish the R_{ij} written in terms of the A_i, the invariants in this matrix element.

Referring to (2.16), (2.10)–(2.13), and the asymptotic relations (3.21), we find that, at large s, the R_{ii} ($i=1, 2$) in the helicity frame are

$$R_{11}{}^h=\frac{(P\cdot k)^2}{M^2}\left[\left|A_1+t\left(A_2+\frac{k^2}{2P\cdot k}A_5\right)\right|^2 -t|A_3|^2\right], \tag{4.1}$$

$$R_{22}{}^h=\frac{(P\cdot k)^2}{M^2}[|A_1|^2-t|A_4|^2]. \tag{4.2}$$

We note that all terms in (4.1) and (4.2) are $O(1)$. This includes those involving A_5, since, unlike the other A_i, A_5 is $O(1)$.

Meiere has argued that the explicit k^2 multiplying A_5 in (4.1) gives rise to an extra k^2 dependence[12] of $R_{11}{}^h$, which accounts for the experimental discrepancy between polarized photoproduction and ρ production.[4,13] However, we note that k^2A_5 appears in (4.2) in the

[11] Meiere (Ref. 7) has argued from Regge theory that A_5 should behave comparably to sA_i ($i\neq 5$), in agreement with (3.21).

[12] By "extra" we mean beyond a factor $(k^2-m_\rho^2)^{-1}$ in A_i, or beyond $(k^2-m_\rho^2)^{-2}$ in the R_{ij}.

[13] This argument does not account for the discrepancy in comparing $R_{22}{}^h$ with r_{22}, which also exists, according to R. Diebold and J. A. Poirier, Phys. Rev. Letters 22, 906 (1969). For a contrary view of the data, see Ref. 8.

combination

$$A_2+\frac{k^2}{2P\cdot k}A_5=-\frac{1}{2P\cdot k}\bar{A}_2, \tag{4.3}$$

where $\bar{A}_2$ is defined in Eqs. (2.3) and (2.5). Meiere implicitly assumes that the gauge-invariant A_2 and A_5 lack extra[12] k^2 dependence. This clearly implies that the kinematically simple $\bar{A}_2$ *does* have such extra k^2 dependence. We feel that the reverse is more likely to be true within the framework of the VDM.

Why should A_2 and A_5 have extra k^2 dependence? The answer lies in kinematic singularities induced by the use of a gauge-invariant set of covariants. Thus, from (2.6) we see that if $\bar{A}_1\neq 0$ at $q\cdot k=\frac{1}{2}(\mu^2+k^2-t)=0$ (and there is no reason to expect $\bar{A}_1$ to vanish at this point), then A_2 must have a pole there. But if A_2 has such a pole, then the kinematically simple $\bar{A}_2$ will have one also—contrary to expectation—unless A_5 has a pole at $q\cdot k=0$ of the right size.

The behavior is, for example, observed in a simple Feynman-diagram model involving s- and u-channel nucleon poles and a t-channel pion pole. In this model,

$$A_2\sim\frac{s-u}{(s-M^2)(u-M^2)q\cdot k(k^2-m_\rho^2)}, \tag{4.4a}$$

$$A_5\sim\frac{2}{(t-\mu^2)q\cdot k(k^2-m_\rho^2)}, \tag{4.4b}$$

$$\bar{A}_2\sim\left[-\frac{4}{t-\mu^2}-\frac{2q\cdot k}{(s-M^2)(u-M^2)}\right]\frac{1}{k^2-m_\rho^2}. \tag{4.4c}$$

Thus at large s [where the second term in (4.4c) vanishes], there is no extra k^2 dependence of $\bar{A}_2$, although there is such a dependence in A_2 and A_5 (via the $q\cdot k$ factor).

Our conclusion is that $\bar{A}_2$ is more significant than A_2 or A_5, and less likely than either to have extra k^2 dependence.[14] A similar study of $A_{1,3,4}$ provides no reason to anticipate extra k^2 dependence in these quantities. Since (4.1) and (4.2) can be written as a function of $A_{1,3,4}$ and $\bar{A}_2$, we expect no extra k^2 dependence in $R_{11}{}^h$ or $R_{22}{}^h$, so we reproduce the simple VDM prediction[15]

$$R_{ii}{}^h(k^2=0,s,t)=(ef_\rho/m_\rho{}^2)^2 r_{ii}{}^h(s,t)\quad(i=1,2). \tag{4.5}$$

The superscript indicating the frame on the left-hand side of (4.5) is redundant since $R_{ij}(k^2=0)$ is independent of the frame (this is a consequence of gauge invariance). It has been argued that this amounts to an ambiguity in the VDM, since (4.5) could be written in different frames, i.e., with superscripts GJ on both sides, or DH (for Donohue-Högaasen), etc. These equations would have different—and mutually contradictory—content since $R_{ii}{}^h(k^2=0)=R_{ii}{}^{\mathrm{GJ}}(k^2=0)=R_{ii}{}^{\mathrm{DH}}(k^2=0)=\cdots$ but $r_{11}{}^h\neq r_{11}{}^{\mathrm{GJ}}$, etc. (Note, however, that r_{22} is independent of frame.) Thus it has been argued[5] that the VDM is satisfied if (4.5) is experimentally verified with *any* choice of frame.

However, we take the viewpoint that the VDM is a theory with more specific content than the above view implies and we have obtained (4.5) *with the helicity frame specified* as a consequence of the simple form which (4.1) and (4.2) take in this frame. In contrast, we find, for example, in the GJ frame

$$\begin{aligned}R_{11}{}^{\mathrm{GJ}}-R_{22}{}^{\mathrm{GJ}}=\frac{-t(P\cdot k)^2}{(q\cdot k)^2-\mu^2k^2}\{&|q\cdot kA_3+k^2A_6|^2\\&-t|q\cdot kA_2|^2-k^2|A_1|^2-q\cdot k(q\cdot k-k^2)\\&\times 2\,\mathrm{Re}A_1A_2{}^*-[(q\cdot k)^2-\mu^2k^2]|A_4|^2\}.\end{aligned} \tag{4.6}$$

The extra k^2 dependence cancels out of the term involving $|A_4|^2$, but not out of any other. There is no reason to anticipate that the extra k^2 dependence of the remaining terms will precisely cancel each other. Thus we do *not* expect (4.5) to hold with superscripts GJ.

One frame in which it has been suggested[5] that (4.5) may hold is the DH frame.[16] This is the dynamically defined frame in which the density matrix is diagonal. The eigenvalues of the R_{ij} matrix (aside from R_{22}) are (for large s)

$$\begin{aligned}&\tfrac{1}{2}\{R_{11}+R_{33}\mp[(R_{11}-R_{33})^2+4R_{13}{}^2]^{1/2}\}\\&\quad=[(P\cdot k)^2/2M^2]\{|a|^2-t|b|^2+k^2(|d|^2-t|c|^2)\\&\qquad\mp[(|a|^2-t|b|^2-k^2|d|^2+k^2t|c|^2)^2\\&\qquad\quad-4tk^2(\mathrm{Re}(ac^*+bd^*))^2]^{1/2}\},\end{aligned} \tag{4.7}$$

where

$$\begin{aligned}a&=A_1-(t/2P\cdot k)\bar{A}_2,\\b&=A_3,\\c&=-(1/2P\cdot k)(\bar{A}_2+q\cdot kA_5),\\d&=A_3+A_6.\end{aligned} \tag{4.8}$$

Note that the pole in A_5 is cancelled by the explicit $q\cdot k$ in (4.8). We know of no reason for substantial k^2 dependence in the quantities a, b, c, and d. There is certainly no reason to anticipate that the rather involved function of them which appears in (4.7) will be independent of k^2.

One reason which has been advanced[8] for preferring the frame is that in it $\mathrm{Re}r_{13}=0$, so there is no "mixing"

[14] A similar conclusion has been reached independently by M. Krammer (private communication).

[15] M. Krammer and D. Schildknecht, Nucl. Phys. **B7**, 583 (1968). H. Fraas and D. Schildknecht [*ibid.* **B6**, 395 (1968)] obtain (4.5) from a discussion which, like ours, is based on consideration of the A_i; instead of our asymptotic assumption (3.1), these authors make assumptions about the relative importance of different A_i.

[16] J. T. Donohue and H. Högaasen, Phys. Letters **25B**, 554 (1967).

of transverse or longitudinal ρ's, just as there is no mixing of transverse and longitudinal photons. However, it is true in *any* frame that $R_{13}(k^2=0)=R_{33}(k^2=0)=0$. [In the DH frame we do not, in general, have $r_{33}=0$, so that it cannot be argued that if (4.5) held in the DH frame, we would have the pleasing result that an equation similar to (4.5) held for all the components of the R and r matrices.]

We also note that $r_{22}^{\mathrm{DH}}=r_{22}^{h}$, and Eq. (4.5) appears to hold experimentally *neither* for $i=1$ *nor* for $i=2$.[13]

V. CONCLUDING REMARKS

We have reached a rather negative conclusion: We see no way of accounting for the results of polarized-photoproduction and ρ-production experiments within the framework of the VDM. This conclusion coincides with other apparent failures of the VDM.[2] As noted in the Introduction, these failures are not order-of-magnitude failures. Thus, apparently, the ρ pole is a major part of matrix elements of V_μ, but other contributions are (in total) comparable. We have apparently learned that the idea that electromagnetic interactions of hadrons could be entirely understood in terms of ρ-meson interactions is—perhaps disappointingly but not surprisingly—too optimistic.

APPENDIX

The form of the relations between the b's and the density matrix elements depends on the frame in which the latter are evaluated, i.e., on the way the ρ polarization states are characterized. Thus, for example, one may choose a frame in which some four-vector Q characteristic of the problem is at rest. One may choose, e.g., $Q=p_1+k$; this corresponds to the center-of-mass or helicity frame. Alternatively, one might choose $Q=q$; this is the so-called GJ frame.

In any given frame, we choose $\epsilon_\mu^{(1,2)}$ as purely spacelike (and therefore transverse); $\epsilon_\mu^{(3)}$ is longitudinal, i.e., its space component is parallel to $\mathbf{k}$, and it therefore has a time component, since all three polarization vectors are characterized by $\epsilon\cdot k=0$.

We choose $\epsilon_\mu^{(2)}$ as perpendicular to the production plane. In covariant language, this means that $\epsilon^{(2)}$ is orthogonal to all three independent four-vectors of the problem: P, q, and k. (Note that this is independent of the choice of Q.)

To give similarly a covariant characterization of $\epsilon_\mu^{(3)}$, we note that in our chosen frame, Q is purely timelike, and a linear combination of k and Q will in general have a time component, plus a space component parallel to $\mathbf{k}$—both properties of $\epsilon_\mu^{(3)}$. If we choose that linear combination of k and Q orthogonal to k, and of length -1, we will clearly have $\epsilon_\mu^{(3)}$. Thus we find

$$\epsilon_\mu^{(3)}=\gamma_Q Q_\mu+\gamma_k k_\mu\,, \tag{A1a}$$

$$\gamma_Q=\left[\frac{k^2}{(Q\cdot k)^2-Q^2k^2}\right]^{1/2}, \tag{A1b}$$

$$\gamma_k=-(Q\cdot k/k^2)\gamma_Q\,. \tag{A1c}$$

The remaining polarization vector $\epsilon_\mu^{(1)}$ is clearly the spacelike unit vector orthogonal to $\epsilon^{(2)}$, $\epsilon^{(3)}$, and k. Since it is orthogonal to $\epsilon^{(2)}$, it can be written as a linear combination of P, q, and k. Thus

$$\epsilon_\mu^{(1)}=\alpha_Q Q_\mu+\alpha_R R_\mu+\alpha_k k_\mu\,, \tag{A2}$$

where R is some combination of P and q, linearly independent of Q. $\epsilon^{(1)}$ is orthogonal to $\epsilon^{(3)}$, which is a linear combination of Q and k; since $\epsilon^{(1)}$ is also orthogonal to k, it will therefore be orthogonal to Q. We thus have

$$-1=\epsilon^{(1)}\cdot\epsilon^{(1)}=\alpha_R\epsilon^{(1)}\cdot R\,. \tag{A3}$$

Using this equation, plus $\epsilon^{(1)}\cdot k=\epsilon^{(1)}\cdot Q=0$, we find

$$\begin{aligned}Q^2k^2-(Q\cdot k)^2=\{&R^2(Q\cdot k)^2+Q^2(R\cdot k)^2\\&-2(Q\cdot R)(Q\cdot k)(R\cdot k)\\&+k^2[(Q\cdot R)^2-Q^2R^2]\}\alpha_R^2\,.\end{aligned} \tag{A4}$$

Equations (A3) and (A4) can be used to calculate $\epsilon^{(1)}\cdot R$. Since P and q can be written as a linear combination of R and Q (and $\epsilon^{(1)}\cdot Q=0$), this yields $\epsilon^{(1)}\cdot P$ and $\epsilon^{(1)}\cdot q$. This in turn gives us $A^{\mu\nu}\epsilon_\nu^{(1)}$ and enables us to calculate the r_{ij}, i.e., we do not need an explicit form for $\epsilon_\mu^{(1)}$.

We also note a way of describing the polarization states we have defined, other than as the transverse and longitudinal states in the Q rest frame. In the k rest frame, all three ϵ_μ's will be purely spatial, and

$$\boldsymbol{\epsilon}^{(1,2)}\cdot\mathbf{Q}=-\epsilon_\mu^{(1,2)}Q^\mu=0\,,$$

$$\boldsymbol{\epsilon}^{(3)}\cdot\mathbf{Q}=-\epsilon_\mu^{(3)}Q^\mu=\left[\frac{(Q\cdot k)^2-Q^2k^2}{k^2}\right]^{1/2}=|\mathbf{Q}|\,. \tag{A5}$$

Clearly $\epsilon^{(3)}$ corresponds to a state in which the component of spin along $\mathbf{Q}$ is zero, and $\epsilon^{(1)}\pm i\epsilon^{(2)}$ correspond to spin components along $\mathbf{Q}$ of ±1. Thus the use of different reference frames (choices of Q) to define the ϵ's can alternatively be viewed as the use of different quantization axes ($\mathbf{Q}$'s) in the ρ rest frame.

We now consider specific examples. First, the helicity frame. This is characterized by $Q=p_1+k$; it is easy to see that we obtain the same $\epsilon^{(i)}$ if we use $Q=p_1$ (in general, Q is equivalent to $Q+\beta k$, since $\epsilon^{(i)}\cdot k=0$). Since, in the end, we will be interested in large s, we keep only the leading terms. We find

$$\epsilon^{(3)}\cdot q=-(q\cdot k-k^2)/\sqrt{(k^2)}+O(1/s)\,, \tag{A6a}$$

$$\epsilon^{(3)}\cdot P=-P\cdot k/\sqrt{(k^2)}+O(1)\,. \tag{A6b}$$

We choose $R=q$ and obtain

$$\begin{aligned}\alpha_R^{-2}&=-t+O(1/s)\,,\\ \epsilon^{(1)}\cdot P&=\tfrac{1}{2}\sqrt{(-t)}+O(1/s)\,,\\ \epsilon^{(1)}\cdot q&=-\sqrt{(-t)}+O(1/s)\,.\end{aligned}\tag{A7}$$

Using $\epsilon^{(2)}\cdot P=\epsilon^{(2)}\cdot q=0$ and (A6) and (A7), we derive (2.16).

In the GJ frame, we have $Q=q$, and we readily calculate

$$\epsilon^{(3)}\cdot q=-\left[\frac{(q\cdot k)^2-\mu^2k^2}{k^2}\right]^{1/2},\tag{A8a}$$

$$\epsilon^{(3)}\cdot P=-\frac{(q\cdot k-k^2)P\cdot k}{\{k^2[(q\cdot k)^2-\mu^2k^2]\}^{1/2}}.\tag{A8b}$$

Taking $R=p_1$, we find

$$\begin{aligned}\alpha_R^{-2}&=-t(P\cdot k)^2/[(q\cdot k)^2-\mu^2k^2]+O(s)\,,\\ \epsilon^{(1)}\cdot q&=0\,,\quad \epsilon^{(1)}\cdot P=-\alpha_R^{-1}.\end{aligned}\tag{A9}$$

Equation (2.17) follows.

For completeness, we note the relation between our density matrix elements, based on linearly polarized states, and those based on circularly polarized states. Since

$$\epsilon^{(\pm)}=\mp(1/\sqrt{2})(\epsilon^{(1)}\pm i\epsilon^{(2)})\,,\quad \epsilon^{(0)}=\epsilon^{(3)}\,,\tag{A10}$$

it follows that[15]

$$\begin{aligned}r_{11}&=r_{++}-r_{+-}\,,\quad & r_{22}&=r_{++}+r_{+-}\,,\\ r_{33}&=r_{00}\,, & r_{13}&=\sqrt{2}r_{-0}\,.\end{aligned}\tag{A11}$$

PHYSICAL REVIEW D VOLUME 1, NUMBER 1 1 JANUARY 1970

Veneziano Model for Compton Scattering of Pions

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(Received 30 April 1969; revised manuscript received 30 September 1969)

Compton scattering of pions is investigated in the Veneziano model. It is shown that the removal of parity doubling causes undersirable features, such as inconsistency with the low-energy theorem or the appearance of an unwanted pole in the energy variable ($s=0$) in one of the amplitudes. If we ignore the problem of parity doubling or overlook the pole at $s=0$, we get a sum rule which closely resembles that obtained from superconvergence relations or the quark model, or current algebra.

I. INTRODUCTION

The Veneziano[1] model has been used to describe meson-meson scattering with good results. As emphasized by Lovelace,[2] this model might be connected with chiral algebra. For π-π, π-K, and $A_1\rho\pi$ systems,[2–4] some of the results of current algebra and chiral symmetry have been obtained by this model.

In this paper we consider the possibility of constructing a Veneziano-type formula for Compton scattering of pions. We write two possible representations for two of the invariant amplitudes, taking care that the leading trajectory is not parity-doubled. We observe that one of the representations is inconsistent with the low-energy theorem, whereas the second introduces an unwanted pole at $s=0$ in one of the amplitudes. However, if we ignore the problem of parity doubling or overlook the pole in the s variable, we get the following relation between the decay widths of A_2, A_{2L}, and ω for the decay mode $\pi\gamma$:

$$\Gamma(A_2\to\pi\gamma):\Gamma(A_{2L}\to\pi\gamma):\Gamma(\omega\to\pi\gamma)=0.416\text{ MeV}:0.625\text{ MeV}:1.17\text{ MeV}.$$

Using ρ dominance, we then predict $\Gamma(A_2\to\rho\pi)=12.1$ MeV and $\Gamma(A_{2L}\to\rho\pi)=38.7$ MeV, where we have used $\omega\rho\pi$ coupling constant as obtained from $\Gamma(\omega\to\pi\gamma)=1.17$ MeV. These results are compatible with the experimental values.[5] We also obtain a sum rule involving $\omega\pi\gamma$ and $A_1\pi\gamma$ coupling constants. This sum rule is very similar to the sum rule obtained[6] previously by three different methods, one of which is based on the $SU(3)\times SU(3)$ algebra of currents. Thus one obtains a further support for the point of view that the Veneziano model is connected with chiral algebra. One can express $A_1\pi\gamma$ coupling constants in terms of δ (the anomalous magnetic moment of A_1). $\delta=0$ gives $\Gamma(\omega\to\pi\gamma)=1.26$ MeV, whereas for $|\delta|=1$ we get $\Gamma(\omega\to\pi\gamma)=0.95$ MeV. Both these results are consistent with the experi-

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