

Branching ratios and CP violations of $B \rightarrow \rho(\omega)\gamma$ decays in the modified perturbative QCD approach and the relevant dark photonic decays of $B \rightarrow \rho(\omega)\gamma'$

Shang Qi^{*} and Mao-Zhi Yang[†]

School of Physics, Nankai University, Tianjin 300071, People's Republic of China



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In this work we study the radiative decays of $B \rightarrow \rho(\omega)\gamma$ processes in the modified perturbative QCD approach, where the transverse momenta of the quark and gluons involved in the interaction process are kept, and the Sudakov factor is incorporated in the theoretical calculation, which are helpful to suppress the infrared end point contribution. A critical infrared momentum cutoff scale is introduced, which is used to separate the soft and hard contributions in QCD. Then the form factors involved in the radiative decays should be separated as hard and soft form factors. The hard form factors can be calculated with the perturbative QCD approach, while the soft form factors should be viewed as soft input parameters. The soft form factors can be obtained by confronting the theoretical calculation of the branching ratios of the radiative decay of B meson with the experimental data. Summing the soft and hard form factors, we find that the total form factors are in fine agreement with that obtained by nonperturbative theoretical method, such as the light-cone sum rules of QCD. We also study $B \rightarrow \rho(\omega)\gamma'$ decays by using the form factors obtained in the $B \rightarrow \rho(\omega)\gamma$ decay processes, where γ' is the dark photon introduced by the extra $U(1)$ symmetry model. The upper limit of the branching ratios of these decays are estimated.

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I. INTRODUCTION

Radiative decays $B \rightarrow \rho(\omega)\gamma$ are processes induced by the flavor-changing-neutral-current (FCNC) transition of $b \rightarrow d\gamma$ in the quark level, which is an interesting process for investigations of flavor physics. On one hand, it is an excellent platform to test the electroweak interaction and QCD effect in the standard model (SM); on the other hand, it is one of the most appropriate process to detect new physics because it is a pure loop effect which is sensitive to physics beyond the SM [1]. The FCNC process has attracted large amount of interest in theory. It has been considered up to next-to-leading order in QCD within the frame work of QCD factorization (QCDF) [2,3], and in effective theory of QCD, such as the large-energy-effective theory [4]. QCD factorization is a theoretical approach based on the factorization theorem with respect to the collinear part of the parton momentum [5–8]. Some dominant long-distance contributions beyond QCDF are considered in Ref. [9]. There are also studies of

$B \rightarrow (K^*, \rho, \omega)\gamma$ decays with perturbative QCD (PQCD) method in Refs. [10–12], which is based on K_T factorization, where the transverse momenta of the partons are included in the calculation of the decay amplitude [13–15].

The branching ratios and CP violations for these radiative decays of B meson have been measured in experiments. The world averaged data are [16]

$$\begin{aligned} \text{Br}(B^+ \rightarrow \rho^+\gamma) &= (1.29 \pm 0.20) \times 10^{-6} \\ \text{Br}(B^0 \rightarrow \rho^0\gamma) &= (0.82 \pm 0.13) \times 10^{-6} \\ \text{Br}(B^0 \rightarrow \omega\gamma) &= (0.44 \pm 0.17) \times 10^{-6} \end{aligned} \quad (1)$$

for the branching ratios, and

$$A_{CP}(B^+ \rightarrow \rho^+\gamma) = (-0.08 \pm 0.15) \quad (2)$$

for the CP violation.

The branching ratios of $B \rightarrow (\rho, \omega)\gamma$ predicted by QCDF can approximately agree with the experimental data [2,3,9]. However, the predictions to the branching ratios of $B \rightarrow (\rho, \omega)\gamma$ by PQCD approach in Ref. [11] suffer large uncertainty, and the central values are too large compared with the experimental data. Therefore, it is necessary to improve the calculations of the branching ratios and CP violations for these radiative decays in PQCD approach.

$B \rightarrow \pi$ transition form factor has been analyzed in PQCD approach with several modifications in Ref. [17], where the

^{*}Contact author: qis@mail.nankai.edu.cn

[†]Contact author: yangmz@nankai.edu.cn

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wave function for B meson is changed to be the one that is obtained by solving the wave equation in QCD-inspired relativistic potential model [18–21], and the transition form factor is separated as hard and soft parts. The hard part of the form factor can be calculated with perturbative method, where the strong coupling constant is small $\alpha_s/\pi < 0.2$. On the other hand, the soft part is controlled by nonperturbative dynamics. This is equivalent to introduce a critical infrared momentum cutoff scale μ_c . The hard form factor is controlled by interactions with the scale $\mu > \mu_c$, and the soft form factor is relevant to interactions of lower scale $\mu < \mu_c$. The cutoff scale can be approximately taken as 1 GeV in practice.

In this work, we revisit the radiative decays of $B \rightarrow \rho(\omega)\gamma$ in PQCD approach with the above modifications. The B meson wave function is the one that is obtained from the QCD-inspired relativistic potential model [18–21], and an infrared momentum cutoff scale μ_c is introduced to separate the hard and soft interactions. The contributions to the form factors with the scale larger than μ_c are calculated by the perturbative method, while the contributions with the scale lower than μ_c is replaced by the soft form factor, which is viewed as nonperturbative input.

The remainder of the paper is organized as follows. Section II is for the calculation in perturbative QCD in leading order (LO). Section III is devoted to the calculations of the next-to-leading order (NLO) in QCD. The annihilation contributions are considered in Sec. IV. Section V is for the contribution of soft form factor. Section VI is for the numerical treatment and discussion. The radiative decays of B meson to dark photon are presented in Sec. VII by using form factors obtained in Sec. VI. Finally Sec. VIII is a brief summary.

II. THE HARD AMPLITUDE OF LEADING-ORDER IN PERTURBATIVE QCD

A. The effective Hamiltonian

The effective Hamiltonian describing $b \rightarrow d$ transition process is [22]

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[V_{ub} V_{ud}^* (C_1^{(u)} O_1^{(u)} + C_2^{(u)} O_2^{(u)}) + V_{cb} V_{cd}^* (C_1^{(c)} O_1^{(c)} + C_2^{(c)} O_2^{(c)}) - V_{tb} V_{td}^* \cdot \left(\sum_{i=3}^6 C_i O_i + C_{7\gamma} O_{7\gamma} + C_{8g} O_{8g} \right) \right] \quad (3)$$

where G_F is the Fermi constant, $V_{qb} V_{qd}^*$ ($q = u, c, t$) the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, C_i 's the Wilson coefficients, and O_i 's the effective operators, which are

$$\begin{aligned} O_1^{(q)} &= (\bar{d}_i q_j)_{V-A} (\bar{q}_j b_i)_{V-A}, \\ O_2^{(q)} &= (\bar{d}_i q_i)_{V-A} (\bar{q}_j b_j)_{V-A}, \\ O_3^{(q)} &= (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V-A}, \\ O_4^{(q)} &= (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V-A}, \\ O_5^{(q)} &= (\bar{d}_i b_i)_{V-A} \sum_q (\bar{q}_j q_j)_{V+A}, \\ O_6^{(q)} &= (\bar{d}_i b_j)_{V-A} \sum_q (\bar{q}_j q_i)_{V+A}, \\ O_{7\gamma} &= \frac{e}{8\pi^2} m_b \bar{d}_i \sigma^{\mu\nu} (1 + \gamma_5) b_i F_{\mu\nu}, \\ O_{8g} &= \frac{g}{8\pi^2} m_b \bar{d}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j G_{\mu\nu}^a, \end{aligned} \quad (4)$$

with i, j being the color indices, e the electromagnetic coupling, g the strong coupling, and $F_{\mu\nu}$ and $G_{\mu\nu}^a$ the strength tensors of the electromagnetic and color fields, respectively. And the current $(\bar{q}_i q_j)_{V\pm A}$ denotes $\bar{q}_i \gamma^\mu (1 \pm \gamma_5) q_j$.

With the effective Hamiltonian given in Eq. (3), the decay amplitude of the $B \rightarrow \rho(\omega)\gamma$ decay can be written as

$$A = \langle F | H_{\text{eff}} | B \rangle = \frac{G_F}{\sqrt{2}} \sum_{i,q} V_{qb}^* V_{qd} C_i(\mu) \cdot \langle F | O_i(\mu) | B \rangle, \quad (5)$$

where F stands for the final state $\rho\gamma$ and $\omega\gamma$ in the decays.

The decay amplitude A can be decomposed into the following from according to its Lorentz structure

$$A = (\varepsilon_V^* \cdot \varepsilon_\gamma^*) M^S + \frac{i}{P_V \cdot P_\gamma} \epsilon_{\mu\nu\rho\sigma} \varepsilon_\gamma^{*\mu} \varepsilon_V^{*\nu} P_\gamma^\rho P_V^\sigma M^P, \quad (6)$$

where P_V is the momentum of the vector meson ρ and/or ω , P_γ the momentum of the photon, ε_V and ε_γ the polarization vectors of the vector meson and photon, respectively.

The matrix element $\langle F | H_{\text{eff}} | B \rangle$ can be calculated in PQCD approach for the contributions from the interactions transferred by hard gluon. It is convenient to use the light-cone coordinate in the calculations. Then any momentum p can be expressed in the form

$$p = (p^+, p^-, \vec{p}_T) = \left(\frac{p^0 + p^3}{\sqrt{2}}, \frac{p^0 - p^3}{\sqrt{2}}, (p^1, p^2) \right), \quad (7)$$

and the product of any two momenta A and B can be expressed in terms of the light cone components equivalently as $A \cdot B = A_\mu B^\mu = A^+ B^- + A^- B^+ - \vec{A}_\perp \cdot \vec{B}_\perp$.

In the B meson rest frame, the momenta of B , ρ (ω) and the photon γ can be written as

$$\begin{aligned}
P_B &= (P_B^+, P_B^-, \vec{P}_{B\perp}) = \frac{M_B}{\sqrt{2}}(1, 1, \vec{0}_\perp), \\
P_V &= (P_V^+, P_V^-, \vec{P}_{V\perp}) = \frac{M_B}{\sqrt{2}}(0, 1, \vec{0}_\perp), \\
P_\gamma &= (P_\gamma^+, P_\gamma^-, \vec{P}_{\gamma\perp}) = \frac{M_B}{\sqrt{2}}(1, 0, \vec{0}_\perp),
\end{aligned} \quad (8)$$

where the vector mesons ρ or ω are chosen as moving in the “−” direction of the z -axis and the photon in the “+” direction. The momenta of the spectator quarks in B and the vector mesons are

$$\begin{aligned}
k_1 &= (k_1^+, k_1^-, \vec{k}_{1T}) = \left(\frac{M_B}{\sqrt{2}}x_1, 0, \vec{k}_{1T} \right), \\
k_2 &= (k_2^+, k_2^-, \vec{k}_{2T}) = \left(0, \frac{M_B}{\sqrt{2}}x_2, \vec{k}_{2T} \right),
\end{aligned} \quad (9)$$

where x_1 and x_2 are the momentum fractions defined by $x_1 = k_1^+/P_B^+$ and $x_2 = k_2^-/P_V^-$, respectively.

B. The factorization formula for the decay amplitude and the meson wave functions

The mass of b quark is significantly larger than that of light quarks u , d and s , so the energy release in the decays of B meson to light mesons is usually large. Then the momenta transferred by the gluons between the spectator quark and the other quarks (or antiquarks) in B or the final mesons are also generally large to kick the light quark in the rest B meson to a speed which matches the fast moving light final particles. Therefore, hard interaction generally dominates in B decays, which can be treated by perturbative QCD. The PQCD factorization scheme has been developed for B decays several decades before [13–15]. Soft gluon exchanges between the quark lines can generate the large double logarithms like $\ln^2(Pb)$ as the collinear and soft divergences overlap, with P being the dominant light-cone component of a meson momentum, and b the

conjugate variable of the transverse momentum k_T of the quark or antiquark in a meson. The resummation of these double logarithms results in the Sudakov factor $\exp[-s(P,b)]$. The Sudakov factor can suppress the long-distance contributions as b being large, and vanishes when $b > 1/\Lambda_{\text{QCD}}$. Loop corrections can also result in large double logarithms as $\alpha_s \ln^2 x$ in the end point region $x \rightarrow 0$, where x is the momentum fraction of the quark or antiquark in a meson, and the summation of these logarithms leads to the threshold factor $S_t(x)$ [23]. Both the Sudakov and threshold factors can help to suppress the long-distance contributions and the explicit form of them can be found in the Appendix A.

The factorization formula for $B \rightarrow \rho(\omega)\gamma$ decay can be written in b -space as

$$\begin{aligned}
\mathcal{M} &= \int_0^1 dx_1 dx_2 \int_0^{1/\Lambda} d^2 b_1 d^2 b_2 C(\mu) \\
&\times \exp^{-S(x_1, x_2, b_1, b_2, \mu)} \Phi^V(x_2, b_2) \\
&\times H(x_1, x_2, b_1, b_2, \mu) \Phi^B(x_1, b_1),
\end{aligned} \quad (10)$$

where $C(\mu)$ stands for a collect of the relevant Wilson coefficients, and $\exp^{-S(x_1, x_2, b_1, b_2, \mu)}$ the Sudakov factor. $\Phi^V(x_2, b_2)$ and $\Phi^B(x_1, b_1)$ are the wave functions of the vector and B mesons in the conjugate b -space, respectively. $H(x_1, x_2, b_1, b_2, \mu)$ is the hard kernel from the hard scattering contribution. b_1 and b_2 are the conjugate variables to the transverse momenta of the spectator quarks in B and vector mesons, and $\Lambda = \Lambda_{\text{QCD}}$.

The wave function of B meson in the momentum space can be define by the matrix element

$$\langle 0 | \bar{q}(z)_\beta [z, 0] b(0)_\alpha | \bar{B} \rangle = \int d^3 k \Phi_{\alpha\beta}^B(\vec{k}, \mu) e^{-ik \cdot z}, \quad (11)$$

where α and β are the spinor indices, and $\Phi_{\alpha\beta}^B(\vec{k})$ being the spinor wave function of B meson, which is given by

$$\begin{aligned}
\Phi_{\alpha\beta}^B(\vec{k}) &= \frac{-if_B m_B}{4} K(\vec{k}) \cdot \left\{ (E_Q + m_Q) \frac{1 + \not{v}}{2} \left[\left(\frac{k^+}{\sqrt{2}} + \frac{m_q}{2} \right) \not{v}_+ + \left(\frac{k^-}{\sqrt{2}} + \frac{m_q}{2} \right) \not{v}_- - k_\perp^\mu \gamma_\mu \right] \gamma^5 \right. \\
&\quad \left. - (E_q + m_q) \frac{1 - \not{v}}{2} \left[\left(\frac{k^+}{\sqrt{2}} - \frac{m_q}{2} \right) \not{v}_+ + \left(\frac{k^-}{\sqrt{2}} - \frac{m_q}{2} \right) \not{v}_- - k_\perp^\mu \gamma_\mu \right] \gamma^5 \right\}_{\alpha\beta},
\end{aligned} \quad (12)$$

where f_B is the decay constant of B meson, m_B the B meson mass, E_Q and m_Q the energy and mass of the heavy quark b , respectively. k is the momentum of the light quark in the B meson. E_q and m_q are the energy and mass of the light quark in B meson, respectively. The light quark mass m_q is neglected in the numerical calculation. v is the four-speed of the B meson, such that $p_B = m_B v$. Two lightlike

vectors are defined as $n_\pm^\mu = (1, 0, 0, \mp 1)$, and $k^\pm = \frac{E_q \pm k^3}{\sqrt{2}}$, $k_\perp^\mu = (0, k^1, k^2, 0)$.

The function $K(\vec{k})$ is

$$K(\vec{k}) = \frac{2N_B \Psi_0(\vec{k})}{\sqrt{E_q E_Q (E_q + m_q) (E_Q + m_Q)}}, \quad (13)$$

where $N_B = \frac{i}{f_B} \sqrt{\frac{3}{(2\pi)^3 m_B}}$, and $\Psi_0(\vec{k})$ is the wave function for B meson, which has been obtained by numerically solving the wave equation in the relativistic potential model [18–20]. An analytical form for this wave function has been obtained in Refs. [21,24] by fitting to the numerical solution in the potential model, which is

$$\Psi_0(\vec{k}) = a_1 e^{a_2 |\vec{k}|^2 + a_3 |\vec{k}| + a_4} \quad (14)$$

where the parameters are [21]

$$\begin{aligned} a_1 &= 4.55_{-0.30}^{+0.40} \text{ GeV}^{-3/2}, & a_2 &= -0.39_{-0.20}^{+0.15} \text{ GeV}^{-2}, \\ a_3 &= -1.55 \pm 0.20 \text{ GeV}^{-1}, & a_4 &= -1.10_{-0.05}^{+0.10}. \end{aligned} \quad (15)$$

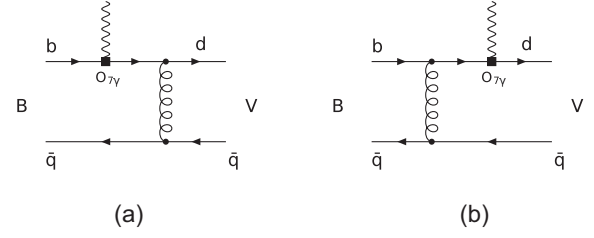


FIG. 1. Contribution of operator $O_{7\gamma}$ to $B \rightarrow \rho(\omega)\gamma$ decay.

In the processes of $B \rightarrow \rho(\omega)\gamma$ decays, the vector mesons ρ and ω are transversely polarized. The spinor wave function of vector meson in transverse polarization is defined as [25,26]

$$\langle \rho/\omega(P, \epsilon_V^T) | \bar{d}_\alpha(z) u_\beta(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP \cdot z} \left[M_V \epsilon_V^{*T} \phi_V^v(x) + \epsilon_V^{*T} P \phi_V^T(x) - \frac{M_V}{P \cdot n_+} i \epsilon_{\mu\nu\rho\sigma} \gamma^5 \gamma^\mu \cdot \epsilon_V^{T\nu} P^\rho n_+^\sigma \phi_V^a(x) \right], \quad (16)$$

where ϵ_V^T is the polarization vector of the vector meson in transverse polarization. $\phi_V^v(x)$ is the distribution amplitude of twist-2, and $\phi_V^T(x)$ and $\phi_V^a(x)$ are the distribution amplitude of twist-3. Their expressions in terms of Gegenbauer polynomials can be found in the Appendix B.

C. The leading order contribution

In this section we present the decay amplitudes of the leading order contributions in QCD, which are contributed by the operators of $O_{7\gamma}$ and O_{8g} .

The diagrams of the contribution of $O_{7\gamma}$ are depicted in Fig. 1, where hard gluon is exchanged between the spectator and other quarks. The contributions of the operator $O_{7\gamma}$ to the scalar and pseudoscalar parts of the amplitude M^S and M^P defined in Eq. (6) are

$$\begin{aligned} \mathcal{M}_{7\gamma}^{S(a)} &= -\frac{2em_B^4 f_B G_F m_0}{\sqrt{27}} \eta_t \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_7^a) C_{7\gamma}(t_7^a) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ &\quad \times ((E_q - k_z) E_Q + m_Q) [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] K(k) J_0(k_T b_1) H_7^a(x_1, x_2, b_1, b_2) \exp[-S_B(t_7^a) - S_V(t_7^a)] S_t(x_1), \end{aligned} \quad (17)$$

$$\begin{aligned} \mathcal{M}_{7\gamma}^{S(b)} &= -\frac{2em_B^4 f_B G_F}{\sqrt{27}} \eta_t \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_7^b) C_{7\gamma}(t_7^b) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_Q + m_Q) \\ &\quad \times \{ m_B [E_q(1 + x_2) + k_z(1 - x_2)] \phi_t(x_2, b_2) - m_0 [E_q(2x_2 - 1) + k_z] [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] \} K(k) J_0(k_T b_1) \\ &\quad \times H_7^b(x_1, x_2, b_1, b_2) \exp[-S_B(t_7^b) - S_V(t_7^b)] S_t(x_2), \end{aligned} \quad (18)$$

$$\mathcal{M}_{7\gamma}^{P(a)} = -\mathcal{M}_{7\gamma}^{S(a)}, \quad (19)$$

$$\mathcal{M}_{7\gamma}^{P(b)} = -\mathcal{M}_{7\gamma}^{S(b)}, \quad (20)$$

where $\eta_t = V_{tb} V_{td}^*$. The superscripts (a) and (b) mean the contributions of the diagrams (a) and (b) in Fig. 1, respectively. The integration of x_1 is from x_1^d to x_1^u with $x_1^{u,d} = 1/2 \pm \sqrt{1/4 - |\vec{k}_\perp|^2/m_B^2}$, which can keep the virtual mass of the b quark in B meson to be all positive. The details of this can be found in Ref. [17]. The functions H_7^a and H_7^b are as follows:

$$H_7^a(x_1, x_2, b_1, b_2) = K_0(\sqrt{x_1 x_2} m_B b_2) [\theta(b_2 - b_1) I_0(\sqrt{x_1} m_B b_1) K_0(\sqrt{x_1} m_B b_2) + \theta(b_1 - b_2) I_0(\sqrt{x_1} m_B b_2) K_0(\sqrt{x_1} m_B b_1)],$$

$$H_7^b(x_1, x_2, b_1, b_2) = H_7^a(x_2, x_1, b_2, b_1). \quad (21)$$

Here K_0, I_0 are the modified Bessel functions. t_7^a and t_7^b in Eqs. (17) and (18) are the largest energy scale of the virtual quark and gluon in the diagrams of 1(a) and 1(b), which are

$$t_7^a = \max \left(\sqrt{x_1} m_B, \sqrt{x_1 x_2} m_B, \frac{1}{b_1}, \frac{1}{b_2} \right) \quad (22)$$

$$t_7^b = \max \left(\sqrt{x_2} m_B, \sqrt{x_1 x_2} m_B, \frac{1}{b_1}, \frac{1}{b_2} \right) \quad (23)$$

The functions S_B and S_V are functions related to the Sudakov factor, and S_t the threshold factor, which can be found in the Appendix A.

The diagrams for the operator O_{8g} are shown in Fig. 2, where gluons from the operator O_{8g} are connected to all the quark lines. The contributions of O_{8g} to M^S and M^P are

$$\mathcal{M}_{8g}^{S(a)} = \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_t Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^a) C_{8g}(t_8^a) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ \times (E_Q + m_Q) \{ -x_2 (E_q - k_z) m_0 [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] \} K(k) J_0(k_T b_1) H_8^a(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^a) - S_V(t_8^a)] \\ \times S_t(x_1), \quad (24)$$

$$\mathcal{M}_{8g}^{S(b)} = \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_t Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^b) C_{8g}(t_8^b) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ \times (E_Q + m_Q) x_2 \{ m_0 (3E_q + k_z) [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] + 2m_B (k_z - E_q) \phi_V^T(x_2, b_2) \} K(k) J_0(k_T b_1) H_8^b(x_1, x_2, b_1, b_2) \\ \times \exp[-S_B(t_8^b) - S_V(t_8^b)] S_t(x_2), \quad (25)$$

$$\mathcal{M}_{8g}^{S(c)} = \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_t Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^c) C_{8g}(t_8^c) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ \times (E_Q + m_Q) \{ x_2 (E_q + k_z) m_0 [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] \} K(k) J_0(k_T b_1) \\ \times H_8^c(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^c) - S_V(t_8^c)] \times S_t(x_1), \quad (26)$$

$$\mathcal{M}_{8g}^{S(d)} = \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_t Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^d) C_{8g}(t_8^d) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ \times (E_Q + m_Q) \{ m_0 x_2 [6E_q \phi_V^v(x_2, b_2) - 2k_z \phi_V^a(x_2, b_2)] + m_B [E_q (x_2 + 2) - k_z (x_2 - 2)] \\ \times \phi_V^T(x_2, b_2) \} K(k) J_0(k_T b_1) H_8^d(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^d) - S_V(t_8^d)] S_t(x_2), \quad (27)$$

$$\mathcal{M}_{8g}^{P(a)} = -\mathcal{M}_{8g}^{S(a)}, \quad (28)$$

$$\mathcal{M}_{8g}^{P(b)} = -\mathcal{M}_{8g}^{S(b)}, \quad (29)$$

$$\mathcal{M}_{8g}^{P(c)} = -\mathcal{M}_{8g}^{S(c)}, \quad (30)$$

$$\mathcal{M}_{8g}^{P(d)} = \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_t Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^d) C_{8g}(t_8^d) \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_Q + m_Q) \\ \times \{ m_0 x_2 \times [6E_q \phi_V^a(x_2, b_2) - 2k_z \phi_V^v(x_2, b_2)] + m_B [E_q (x_2 + 2) - k_z (x_2 - 2)] \phi_V^T(x_2, b_2) \} \\ \times K(k) J_0(k_T b_1) H_8^d(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^d) - S_V(t_8^d)] S_t(x_2), \quad (31)$$

$$H_8^a(x_1, x_2, b_1, b_2) = K_0(\sqrt{x_1 x_2} m_B b_2) [\theta(b_2 - b_1) I_0(\sqrt{x_1 + 1} m_B b_1) K_0(\sqrt{x_1 + 1} m_B b_2) + \theta(b_1 - b_2) I_0(\sqrt{x_1 + 1} m_B b_2) K_0(\sqrt{x_1 + 1} m_B b_1)] \quad (32)$$

$$H_8^b(x_1, x_2, b_1, b_2) = i \frac{\pi}{2} K_0(\sqrt{x_1 x_2} m_B b_1) [\theta(b_2 - b_1) J_0(\sqrt{1 - x_2} m_B b_1) H_0^{(1)}(\sqrt{1 - x_2} m_B b_2) + \theta(b_1 - b_2) J_0(\sqrt{1 - x_2} m_B b_2) H_0^{(1)}(\sqrt{1 - x_2} m_B b_1)], \quad (33)$$

$$H_8^c(x_1, x_2, b_1, b_2) = [\theta(x_1 - x_2) K_0(\sqrt{x_1 - x_2} m_B b_2) + \theta(x_2 - x_1) \frac{i\pi}{2} H_0^{(1)}(\sqrt{x_2 - x_1} m_B b_2)] \times [\theta(b_2 - b_1) I_0(\sqrt{x_1} m_B b_1) K_0(\sqrt{x_1} m_B b_2) + \theta(b_1 - b_2) I_0(\sqrt{x_1} m_B b_2) K_0(\sqrt{x_1} m_B b_1)], \quad (34)$$

$$H_8^d(x_1, x_2, b_1, b_2) = \left[\theta(x_1 - x_2) \frac{i\pi}{2} K_0(\sqrt{x_1 - x_2} m_B b_1) - \theta(x_2 - x_1) \frac{\pi^2}{4} H_0^{(1)}(\sqrt{x_2 - x_1} m_B b_1) \right] \times [\theta(b_2 - b_1) J_0(\sqrt{x_2} m_B b_1) H_0^{(1)}(\sqrt{x_2} m_B b_2) + \theta(b_1 - b_2) J_0(\sqrt{x_2} m_B b_2) H_0^{(1)}(\sqrt{x_2} m_B b_1)], \quad (35)$$

where $H_0^{(1)}$ is the Hankel function of the first kind, and

$$t_{8g}^a = \max \left(\sqrt{1 + x_1} m_B, \sqrt{x_1 x_2} m_B, \frac{1}{b_1}, \frac{1}{b_2} \right), \quad (36)$$

$$t_{8g}^b = \max \left(\sqrt{1 - x_2} m_B, \sqrt{x_1 x_2} m_B, \frac{1}{b_1}, \frac{1}{b_2} \right), \quad (37)$$

$$t_{8g}^c = \max \left(\sqrt{x_1} m_B, \sqrt{|x_1 - x_2|} m_B, \frac{1}{b_1}, \frac{1}{b_2} \right), \quad (38)$$

$$t_{8g}^d = \max \left(\sqrt{x_2} m_B, \sqrt{|x_1 - x_2|} m_B, \frac{1}{b_1}, \frac{1}{b_2} \right), \quad (39)$$

Using the results of the amplitudes of $M_{7\gamma}^{j(a)}$ and $M_{7\gamma}^{j(b)}$ ($j = S, P$) given by Eqs. (17)–(20), which are contributed by the diagrams (a) and (b) in Fig. 1, the amplitudes for each decay modes caused by the operator $O_{7\gamma}$ are

$$M(B^+ \rightarrow \rho^+ \gamma)_{7\gamma}^j = M_{7\gamma}^{j(a)} + M_{7\gamma}^{j(b)}, \quad (40)$$

$$M(B^0 \rightarrow \rho^0 \gamma)_{7\gamma}^j = -\frac{1}{\sqrt{2}} [M_{7\gamma}^{j(a)} + M_{7\gamma}^{j(b)}], \quad (41)$$

$$M(B^0 \rightarrow \omega \gamma)_{7\gamma}^j = \frac{1}{\sqrt{2}} [M_{7\gamma}^{j(a)} + M_{7\gamma}^{j(b)}]. \quad (42)$$

The amplitudes caused by the operator O_{8g} which are corresponding to Fig. 2 are in the following

$$M(B^+ \rightarrow \rho^+ \gamma)_{8g}^j = M_{8g}^{j(a)}(Q_b) + M_{8g}^{j(b)}(Q_d) + M_{8g}^{j(c)}(Q_u) + M_{8g}^{j(d)}(Q_u), \quad (43)$$

$$M(B^0 \rightarrow \rho^0 \gamma)_{8g}^j = -\frac{1}{\sqrt{2}} [M_{8g}^{j(a)}(Q_b) + M_{8g}^{j(b)}(Q_d) + M_{8g}^{j(c)}(Q_d) + M_{8g}^{j(d)}(Q_d)], \quad (44)$$

$$M(B^0 \rightarrow \omega \gamma)_{8g}^j = \frac{1}{\sqrt{2}} [M_{8g}^{j(a)}(Q_b) + M_{8g}^{j(b)}(Q_d) + M_{8g}^{j(c)}(Q_d) + M_{8g}^{j(d)}(Q_d)], \quad (45)$$

where $M_{8g}^{j(a,b,c,d)}(Q_q)$, ($j = S, P$), can be found in Eqs. (24)–(31).

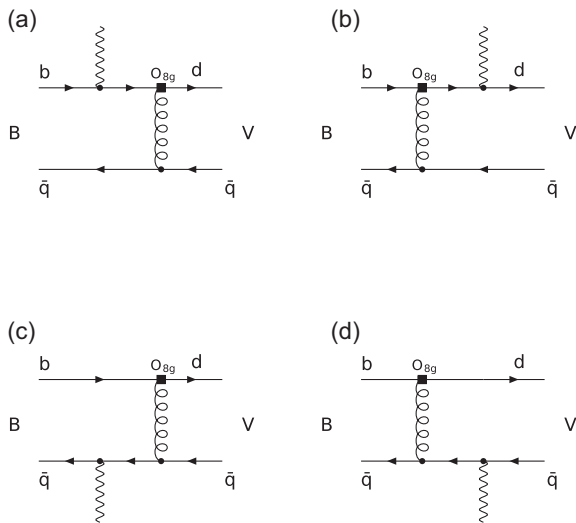


FIG. 2. Contribution of operator O_{8g} to $B \rightarrow \rho(\omega) \gamma$ decay.

III. THE CONTRIBUTION OF NEXT-TO-LEADING-ORDER CORRECTIONS

In this section, we consider the contributions of the diagrams with the effective operators O_i 's being inserted in the quark loops. These diagrams can be divided into two types. One type is for the diagrams where the photons are emitted from the external quark lines, which are shown in Fig. 3. The other type is for the diagrams with the photons being emitted from the internal quark lines in the loop, which are shown in Fig. 4. In this work we only consider the insertion of the tree-level operators O_1 and O_2 , and neglect the penguin operators $O_{3\sim 6}$, because their contributions are further suppressed by their small Wilson coefficients in addition to the loop suppression. For the tree-level operators O_1 and O_2 , only O_2 contributes because the color of O_1 does not match the quark loops with one gluon attached on it.

A. Contributions of external-quark-line emission

The function for the inner quark loop is

$$I^\nu = \frac{g}{8\pi^2} \left[\frac{2}{3} - G(m_i^2, k^2, \mu) \right] \bar{b} T_{ij}^a (k^2 \gamma^\nu - k^\nu \gamma) (1 - \gamma_5) d \quad (46)$$

with $G(m_i^2, k^2, \mu)$ is

$$G(m_i^2, k^2, \mu) = - \int_0^1 dx 4x(1-x) \times \log \left[\frac{m_i^2 - x(1-x)k^2 - i\epsilon}{\mu^2} \right], \quad (47)$$

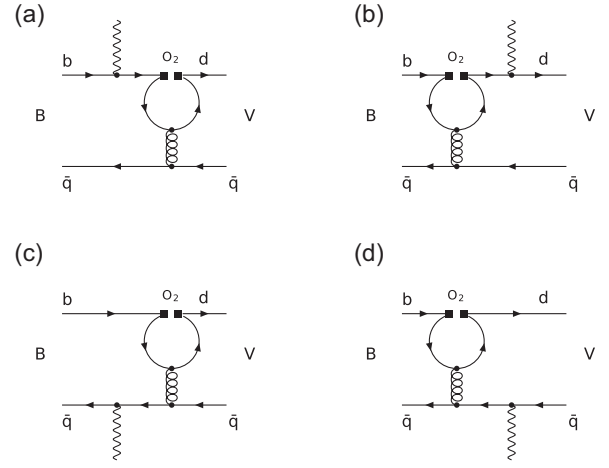


FIG. 3. Diagrams in which the operator O_2 is inserted in the loop, and a photon is emitted from the external quark line.

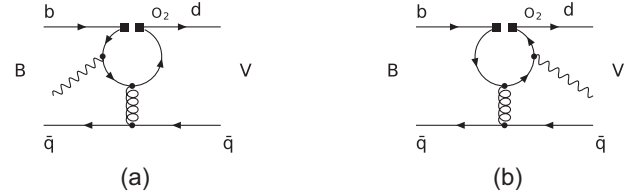


FIG. 4. Diagrams in which the operator O_2 is inserted in the quark loop, and a photon is emitted from the internal loop quark line.

where m_i is the mass of the quark in the inner quark loop, and here $i = u$ and c quarks. With the function I^ν given in Eq. (46), the contributions of the diagrams (a)–(d) in Fig. 3 are calculated to be

$$\begin{aligned} \mathcal{M}_{1i}^{(a)} &= \frac{1}{2} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i Q_q \int_0^\infty k_T dk_T \int_{x_1^a}^{x_1^a} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^a) C_2(t_8^a) \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_q + k_z) (E_Q + m_Q) x_1 x_2 m_0 [\phi_V^a(x_2, b_2) - \phi_V^v(x_2, b_2)] \\ &\times \left[G\left(m_i^2, -\frac{m_B^2}{4}, t_8^a\right) - \frac{2}{3} \right] K(k) J_0(k_T b_1) H_8^a(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^a) - S_V(t_8^a)] S_t(x_1), \end{aligned} \quad (48)$$

$$\begin{aligned} \mathcal{M}_{1i}^{(b)} &= \frac{1}{2} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i Q_q \int_0^\infty k_T dk_T \int_{x_1^b}^{x_1^b} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^b) C_2(t_8^b) \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_q - k_z) (E_Q + m_Q) x_2^2 m_0 [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] \\ &\times \left[G\left(m_i^2, -\frac{m_B^2}{4}, t_8^b\right) - \frac{2}{3} \right] K(k) J_0(k_T b_1) H_8^b(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^b) - S_V(t_8^b)] S_t(x_2), \end{aligned} \quad (49)$$

$$\begin{aligned}\mathcal{M}_{1i}^{(c)} &= \frac{1}{2} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^c) C_2(t_8^c) \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_q + k_z) (E_Q + m_Q) x_2 m_0 [\phi_V^a(x_2, b_2) + \phi_V^v(x_2, b_2)] \\ &\times \left[G\left(m_i^2, -\frac{m_B^2}{4}, t_8^c\right) - \frac{2}{3} \right] K(k) J_0(k_T b_1) H_8^c(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^c) - S_V(t_8^c)] S_t(x_1),\end{aligned}\quad (50)$$

$$\begin{aligned}\mathcal{M}_{1i}^{S(d)} &= \frac{1}{2} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^d) C_2(t_8^d) \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_Q + m_Q) x_2 \left[G\left(m_i^2, -\frac{m_B^2}{4}, t_8^d\right) - \frac{2}{3} \right] \{m_0 [\phi_V^a(x_2, b_2) \\ &\times (E_q(x_2 - 1) - k_z(x_2 + 1)) + 3\phi_V^v(x_2, b_2)(E_q(x_2 + 1) - k_z(x_2 - 1))] \\ &+ m_B \phi_V^T(x_2, b_2)(3E_q + k_z)\} K(k) J_0(k_T b_1) H_8^d(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^d) - S_V(t_8^d)] S_t(x_2),\end{aligned}\quad (51)$$

$$\mathcal{M}_{1i}^{P(a)} = \mathcal{M}_{1i}^{S(a)}, \quad \mathcal{M}_{1i}^{P(b)} = -\mathcal{M}_{1i}^{S(b)}, \quad \mathcal{M}_{1i}^{P(c)} = -\mathcal{M}_{1i}^{S(c)}, \quad (52)$$

$$\begin{aligned}\mathcal{M}_{1i}^{P(d)} &= -\frac{1}{2} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i Q_q \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^{1/\Lambda} b_1 db_1 b_2 db_2 \alpha_s(t_8^d) C_2(t_8^d) \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_Q + m_Q) x_2 \left[G\left(m_i^2, -\frac{m_B^2}{4}, t_8^d\right) - \frac{2}{3} \right] \{m_0 [3\phi_V^a(x_2, b_2) \\ &\times (E_q(x_2 + 1) - k_z(x_2 - 1)) + \phi_V^v(x_2, b_2)(E_q(x_2 - 1) - k_z(x_2 + 1))] \\ &+ m_B \phi_V^T(x_2, b_2)(3E_q + k_z)\} K(k) J_0(k_T b_1) H_8^d(x_1, x_2, b_1, b_2) \exp[-S_B(t_8^d) - S_V(t_8^d)] S_t(x_2),\end{aligned}\quad (53)$$

where the subscript i indicates the flavor of the inner quarks propagating in the quark loop.

The amplitudes of $B^+ \rightarrow \rho^+ \gamma$, $B^0 \rightarrow \rho^0 \gamma$ and $B^0 \rightarrow \omega \gamma$ contributed by the inner quark-loop diagrams are

$$\begin{aligned}M(B^+ \rightarrow \rho^+ \gamma)_{1i}^j &= M_{1i}^{j(a)}(Q_b) + M_{1i}^{j(b)}(Q_d) \\ &+ M_{1i}^{j(c)}(Q_u) + M_{1i}^{j(d)}(Q_u),\end{aligned}\quad (54)$$

$$\begin{aligned}M(B^0 \rightarrow \rho^0 \gamma)_{1i}^j &= -\frac{1}{\sqrt{2}} [M_{1i}^{j(a)}(Q_b) + M_{1i}^{j(b)}(Q_d) \\ &+ M_{1i}^{j(c)}(Q_d) + M_{1i}^{j(d)}(Q_d)],\end{aligned}\quad (55)$$

$$\begin{aligned}M(B^0 \rightarrow \omega \gamma)_{1i}^j &= \frac{1}{\sqrt{2}} [M_{1i}^{j(a)}(Q_b) + M_{1i}^{j(b)}(Q_d) \\ &+ M_{1i}^{j(c)}(Q_d) + M_{1i}^{j(d)}(Q_d)].\end{aligned}\quad (56)$$

B. Contributions of internal-loop-quark-line emission

The effective interaction term for $b \rightarrow d$ transition with one photon and one gluon emitted from the quark loop can be calculated in the $\overline{\text{MS}}$ scheme, which is

$$I = \bar{d} \gamma^\rho (1 - \gamma_5) T^a b I_{\mu\nu\rho} \epsilon_\gamma^\mu \epsilon_g^\nu, \quad (57)$$

with

$$\begin{aligned}I_{\mu\nu\rho} &= A_4 [(q \cdot k) \epsilon_{\mu\nu\rho\sigma} (q - k)^\sigma + \epsilon_{\nu\rho\sigma\tau} q^\sigma k^\tau k_\mu \\ &- \epsilon_{\mu\rho\sigma\tau} q^\sigma k^\tau q_\nu] + A_5 [\epsilon_{\mu\rho\sigma\tau} q^\sigma k^\tau k_\nu \\ &- k^2 \epsilon_{\mu\nu\rho\sigma} q^\sigma],\end{aligned}\quad (58)$$

where q is the momentum of the photon, k the momentum of the gluon, and

$$\begin{aligned}A_4 &= \frac{4ieg}{3\pi^2} \int_0^1 dx \int_0^{1-x} dy \\ &\times \frac{xy}{x(1-x)k^2 + 2xyqk - m_i^2 + i\epsilon},\end{aligned}\quad (59)$$

$$\begin{aligned}A_5 &= -\frac{4ieg}{3\pi^2} \int_0^1 dx \int_0^{1-x} dy \\ &\times \frac{x(1-x)}{x(1-x)k^2 + 2xyqk - m_i^2 + i\epsilon}.\end{aligned}\quad (60)$$

With the effective interaction given in Eq. (57), the amplitudes contributed by the diagrams (a) and (b) shown

in Fig. 4 can be calculated, and the results are

$$\begin{aligned} \mathcal{M}_{2i}^S = & \frac{16}{3} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^1 dy_1 dy_2 \int_0^{1/\Lambda} b_1 db_1 \alpha_s(t_{2i}) C_2(t_{2i}) \\ & \times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_Q + m_Q) \frac{x_2}{y_1 y_2 x_2 m_B^2 - m_i^2} \{ -y_1(1-y_1)x_2(E_q - k_z)m_0 \\ & \times [\phi_V^a(x_2, b_1) + \phi_V^v(x_2, b_1)] - y_1 y_2 [m_0((E_q + k_z)\phi_V^a(x_2, b_1) + (k_z(1+2x_2) \\ & + E_q(1-2x_2))\phi_V^v(x_2, b_1)) + m_B(k_z - E_q)\phi_V^T(x_2, b_1)] \} K(k) J_0(k_T b_1) \\ & \times H_{2i}^{(e)}(x_1, x_2, y_1, y_2, b_1) \exp[-S_B(t_{2i}) - S_V(t_{2i})], \end{aligned} \quad (61)$$

$$\begin{aligned} \mathcal{M}_{2i}^P = & -\frac{16}{3} \frac{em_B^4 f_B G_F}{\sqrt{27}} \eta_i \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^1 dx_2 \int_0^1 dy_1 dy_2 \int_0^{1/\Lambda} b_1 db_1 \alpha_s(t_{2i}) C_2(t_{2i}) \\ & \times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) (E_Q + m_Q) \frac{x_2}{y_1 y_2 x_2 m_B^2 - m_i^2} \{ y_1(1-y_1)x_2(E_q - k_z)m_0 \\ & \times [\phi_V^a(x_2, b_1) + \phi_V^v(x_2, b_1)] + y_1 y_2 [m_0((E_q + k_z)\phi_V^v(x_2, b_1) + (k_z(1+2x_2) \\ & + E_q(1-2x_2))\phi_V^a(x_2, b_1)) + m_B(k_z - E_q)\phi_V^T(x_2, b_1)] \} K(k) J_0(k_T b_1) \\ & \times H_{2i}^{(e)}(x_1, x_2, y_1, y_2, b_1) \exp[-S_B(t_{2i}) - S_V(t_{2i})], \end{aligned} \quad (62)$$

where

$$x_{yi} = x_1 x_2 m_B^2 - \frac{y_2 x_2 m_B^2}{1 - y_1} + \frac{m_i^2}{y_1(1 - y_1)}, \quad (63)$$

and the hard function and hard scale are

$$\begin{aligned} H_{2i}^{(e)} = & [\theta(x_{yi}) K_0(\sqrt{x_{yi}} b_1) + \theta(-x_{yi}) \frac{i\pi}{2} \\ & \times H_0^{(1)}(\sqrt{-x_{yi}} b_1)] - K_0(\sqrt{x_1 x_2} m_B b_1), \end{aligned} \quad (64)$$

$$t_{2i} = \max \left(\sqrt{x_1 x_2} m_B, \sqrt{|x_{yi}|}, \frac{1}{b_1} \right), \quad (65)$$

respectively. The decay amplitudes for each decay modes are

$$\begin{aligned} M(B^+ \rightarrow \rho^+ \gamma)_{2i}^j &= M_{2i}^j, \\ M(B^0 \rightarrow \rho^0 \gamma)_{2i}^j &= -\frac{M_{2i}^j}{\sqrt{2}}, \\ M(B^0 \rightarrow \omega \gamma)_{2i}^j &= \frac{M_{2i}^j}{\sqrt{2}}. \end{aligned} \quad (66)$$

IV. THE ANNIHILATION DIAGRAM CONTRIBUTIONS

In this section we consider the contributions of the annihilation diagrams, where the operators of tree-level and QCD penguins are inserted with all kinds of possibilities.

A. Tree annihilation

The annihilation diagrams with the insertion of the tree-level operators are shown in Fig. 5. For the convenience of expressing the decay amplitude relevant to each diagram, we define the combinations of the tree-level Wilson coefficients as

$$a_1(t) = C_1(t) + \frac{C_2(t)}{3}, \quad a_2(t) = C_2(t) + \frac{C_1(t)}{3}, \quad (67)$$

then the decay amplitude for each diagram can be given as

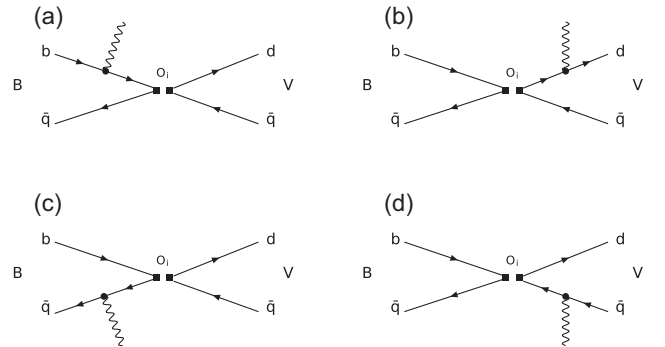


FIG. 5. Annihilation diagrams in which the operators O_1 , O_2 are inserted.

$$\begin{aligned}\mathcal{M}_{A_k}^{S(a)} &= \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^{1/\Lambda} b_1 db_1 \\ &\times \eta_u \cdot \mathcal{C}a_k(t_A^a) Q_b \exp[-S_B(t_A^a)] S_t(x_1) \\ &\times [4m_B m_0 (E_q + k_z) (E_Q + m_Q)] \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ &\times K(k) J_0(k_T b_1) K_0(\sqrt{1+x_1} b_1),\end{aligned}\quad (68)$$

$$\mathcal{M}_{A_k}^{P(a)} = \mathcal{M}_{A_k}^{S(a)}, \quad (69)$$

$$\begin{aligned}\mathcal{M}_{A_k}^{S(b)} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_u \cdot \mathcal{C}a_k(t_A^b) Q_q \\ &\times \exp[-S_V(t_A^b)] S_t(x_2) \phi_M(x_2) m_0 \\ &\times [x_2 \phi_V^a(x_2) - (x_2 - 2) \phi_V^v(x_2)] \\ &\times \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{1-x_2} b_2) \right),\end{aligned}\quad (70)$$

$$\begin{aligned}\mathcal{M}_{A_k}^{P(b)} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_u \cdot \mathcal{C}a_k(t_A^b) Q_q \\ &\times \exp[-S_V(t_A^b)] S_t(x_2) \phi_M(x_2) m_0 \\ &\times [(x_2 - 2) \phi_V^a(x_2) - x_2 \phi_V^v(x_2)] \\ &\times \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{1-x_2} b_2) \right),\end{aligned}\quad (71)$$

$$\begin{aligned}\mathcal{M}_{A_k}^{S(c)} &= \int_0^\infty k_T dk_T \int_{x_1^d}^{x_1^u} dx_1 \int_0^{1/\Lambda} b_1 db_1 \\ &\times \eta_u \cdot \mathcal{C}a_k(t_A^c) Q_{q'} \exp[-S_B(t_A^c)] S_t(x_1) \\ &\times [4m_B m_0 (E_q + k_z) (E_Q + m_Q)] \\ &\times \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) \\ &\times K(k) J_0(k_T b_1) K_0(\sqrt{x_1} b_1),\end{aligned}\quad (72)$$

$$\mathcal{M}_{A_k}^{P(c)} = -\mathcal{M}_{A_k}^{S(c)}, \quad (73)$$

$$\begin{aligned}\mathcal{M}_{A_k}^{S(d)} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_u \cdot \mathcal{C}a_k(t_A^d) Q_{q''} \\ &\times \exp[-S_V(t_A^d)] S_t(x_2) \phi_M(x_2) m_0 \\ &\times [(x_2 - 1) \phi_V^a(x_2) + (x_2 + 1) \phi_V^v(x_2)] \\ &\times \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{x_2} b_2) \right),\end{aligned}\quad (74)$$

$$\begin{aligned}\mathcal{M}_{A_k}^{P(d)} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_u \cdot \mathcal{C}a_k(t_A^d) Q_{q''} \\ &\times \exp[-S_V(t_A^d)] S_t(x_2) \phi_M(x_2) (-m_0) \\ &\times [(x_2 + 1) \phi_V^a(x_2) + (x_2 - 1) \phi_V^v(x_2)] \\ &\times \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{x_2} b_2) \right)\end{aligned}\quad (75)$$

where $k = 1, 2$ and $\mathcal{C}$ is

$$\mathcal{C} = \frac{\pi f_V}{8m_B \sqrt{6}} \frac{em_B^2 f_B G_F}{\sqrt{27}}.$$

The amplitudes for all the decay modes contributed by the annihilation diagrams with tree-level operator insertion are

$$\begin{aligned}M(B^+ \rightarrow \rho^+ \gamma)_A^j &= M_{A_2}^{j(a)}(Q_b) + M_{A_2}^{j(b)}(Q_d) \\ &+ M_{A_2}^{j(c)}(Q_u) + M_{A_2}^{j(d)}(Q_u),\end{aligned}\quad (76)$$

$$\begin{aligned}M(B^0 \rightarrow \rho^0 \gamma)_A^j &= \frac{1}{\sqrt{2}} [M_{A_1}^{j(a)}(Q_b) + M_{A_1}^{j(b)}(Q_u) \\ &+ M_{A_1}^{j(c)}(Q_d) + M_{A_1}^{j(d)}(Q_u)],\end{aligned}\quad (77)$$

$$\begin{aligned}M(B^0 \rightarrow \omega \gamma)_A^j &= \frac{1}{\sqrt{2}} [M_{A_1}^{j(a)}(Q_b) + M_{A_1}^{j(b)}(Q_u) \\ &+ M_{A_1}^{j(c)}(Q_d) + M_{A_1}^{j(d)}(Q_u)].\end{aligned}\quad (78)$$

B. QCD penguin annihilation

The diagrams with the insertion of the penguin operators are depicted in Figs. 6 and 7.

We define the combinations of the Wilson coefficients of QCD penguin operators as

$$\begin{aligned}a_3(t) &= C_3(t) + \frac{C_4(t)}{3}, & a_4(t) &= C_4(t) + \frac{C_3(t)}{3}, \\ a_5(t) &= C_5(t) + \frac{C_6(t)}{3}, & a_6(t) &= C_6(t) + \frac{C_5(t)}{3}.\end{aligned}\quad (79)$$

The amplitudes for the insertion of $(V-A)(V-A)$ type of operators in Fig. 6 are

$$\begin{aligned}\mathcal{M}_{A_{1k}}^{S(a)-} &= \int_{x_1^d}^{x_1^u} dx_1 \int_0^{1/\Lambda} b_1 db_1 \int_0^\infty k_T dk_T \eta_t \cdot \mathcal{C}Q_b a_k(t_{A_1}^a) \exp[-S_B(t_{A_1}^a)] S_t(x_1) [4m_B m_0 \\ &\times (E_q + k_z) (E_Q + m_Q)] \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) K(k) J_0(k_T b_1) K_0(\sqrt{1+x_1} b_1),\end{aligned}\quad (80)$$

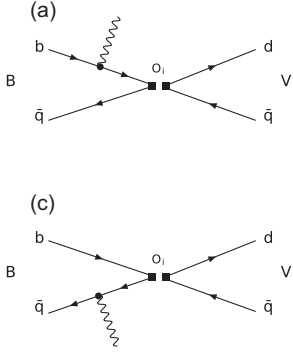


FIG. 6. Annihilation diagrams in which the QCD penguin operators are inserted.

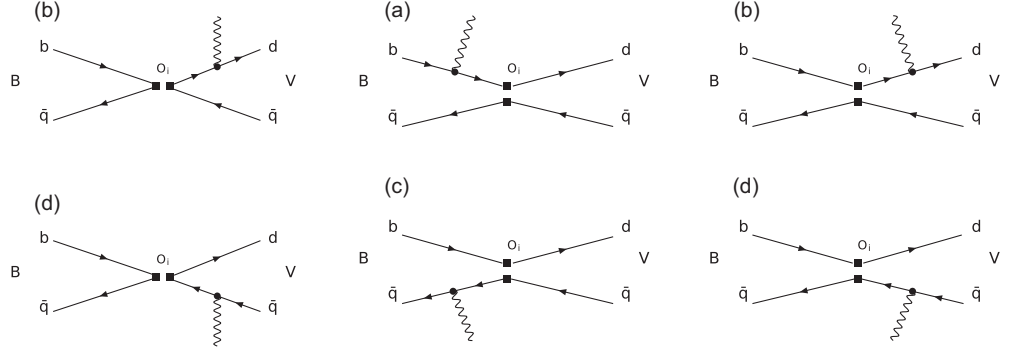


FIG. 7. The other type of annihilation diagrams with operator insertion.

$$\mathcal{M}_{A1_k}^{P(a)-} = \mathcal{M}_{A1_k}^{S(a)-}, \quad (81)$$

$$\begin{aligned} \mathcal{M}_{A1_k}^{S(b)-} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_t \cdot \mathcal{C} Q_q a_k(t_{A1}^b) \exp[-S_V(t_{A1}^b)] S_t(x_2) \\ &\times m_0 [x_2 \phi_V^a(x_2) - (x_2 - 2) \phi_V^v(x_2)] \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{1 - x_2} b_2) \right), \end{aligned} \quad (82)$$

$$\begin{aligned} \mathcal{M}_{A1_k}^{P(b)-} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_t \cdot \mathcal{C} Q_q a_k(t_{A1}^b) \exp[-S_V(t_{A1}^b)] S_t(x_2) \\ &\times m_0 [(x_2 - 2) \phi_V^a(x_2) - x_2 \phi_V^v(x_2)] \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{1 - x_2} b_2) \right), \end{aligned} \quad (83)$$

$$\begin{aligned} \mathcal{M}_{A1_k}^{S(c)-} &= \int_{x_1^d}^{x_1^u} dx_1 \int_0^{1/\Lambda} b_1 db_1 \int_0^\infty k_T dk_T \eta_t \cdot \mathcal{C} Q_{q'} a_k(t_{A1}^c) \exp[-S_B(t_{A1}^c)] S_t(x_1) [4m_B m_0 \\ &\times (E_q + k_z)(E_Q + m_Q)] \left(\frac{m_B}{2} + \frac{k_T^2}{2x_1^2 m_B} \right) K(k) J_0(k_T b_1) K_0(\sqrt{x_1} b_1), \end{aligned} \quad (84)$$

$$\mathcal{M}_{A1_k}^{P(c)-} = -\mathcal{M}_{A1_k}^{S(c)-}, \quad (85)$$

$$\begin{aligned} \mathcal{M}_{A1_k}^{S(d)-} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_t \cdot \mathcal{C} Q_{q''} a_k(t_{A1}^d) \exp[-S_V(t_{A1}^d)] S_t(x_2) \\ &\times m_0 [(x_2 - 1) \phi_V^a(x_2) + (x_2 + 1) \phi_V^v(x_2)] \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{x_2} b_2) \right), \end{aligned} \quad (86)$$

$$\begin{aligned} \mathcal{M}_{A1_k}^{P(d)-} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_t \cdot \mathcal{C} Q_{q''} a_k(t_{A1}^d) \exp[-S_V(t_{A1}^d)] S_t(x_2) \\ &\times (-m_0) [(x_2 + 1) \phi_V^a(x_2) + (x_2 - 1) \phi_V^v(x_2)] \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{x_2} b_2) \right), \end{aligned} \quad (87)$$

where k in the subscript runs from 3 to 6. The diagrams in Fig. 6 only contribute to the neutral decay modes. Then the contributions of $(V - A)(V - A)$ type operators to the amplitudes of the decay modes $B^0 \rightarrow \rho^0 \gamma$ and $B^0 \rightarrow \omega \gamma$ are

$$M(B^0 \rightarrow \rho^0 \gamma)_{A1}^{j-} = \frac{1}{\sqrt{2}} [(M_{A1_3}^{j(b)-}(Q_u) - M_{A1_3}^{j(b)-}(Q_d)) + (M_{A1_3}^{j(d)-}(Q_u) - M_{A1_3}^{j(d)-}(Q_d))], \quad (88)$$

$$M(B^0 \rightarrow \omega \gamma)_{A1}^{j-} = \frac{1}{\sqrt{2}} [2M_{A1_3}^{j(a)-}(Q_b) + \{M_{A1_3}^{j(b)-}(Q_u) + M_{A1_3}^{j(b)-}(Q_d)\} + 2M_{A1_3}^{j(c)-}(Q_d) + \{M_{A1_3}^{j(d)-}(Q_u) + M_{A1_3}^{j(d)-}(Q_d)\}]. \quad (89)$$

The contributions of operators of $(V - A)(V + A)$ type to the amplitudes are tagged by a symbol “+” in the superscript, which are

$$\begin{aligned}
 M_{A1}^{S(a)+}(Q_b) &= M_{A1}^{S(a)-}(Q_b), \\
 M_{A1}^{P(a)+}(Q_b) &= M_{A1}^{P(a)-}(Q_b), \\
 M_{A1}^{S(b)+}(Q_q) &= M_{A1}^{S(b)-}(Q_q), \\
 M_{A1}^{P(b)+}(Q_q) &= -M_{A1}^{P(b)-}(Q_q), \\
 M_{A1}^{S(c)+}(Q_{q'}) &= M_{A1}^{S(c)-}(Q_{q'}), \\
 M_{A1}^{P(c)+}(Q_{q'}) &= M_{A1}^{P(c)-}(Q_{q'}), \\
 M_{A1}^{S(d)+}(Q_{q''}) &= M_{A1}^{S(d)-}(Q_{q''}), \\
 M_{A1}^{P(d)+}(Q_{q''}) &= -M_{A1}^{P(d)-}(Q_{q''}), \quad (90)
 \end{aligned}$$

and the contributions of $(V - A)(V + A)$ operators to the physical amplitudes are

$$\begin{aligned}
 M(B^0 \rightarrow \rho^0 \gamma)_{A1}^{j+} &= \frac{1}{\sqrt{2}} [\{M_{A1_5}^{j(b)+}(Q_u) - M_{A1_5}^{j(b)+}(Q_d)\} \\
 &\quad + \{M_{A1_5}^{j(d)+}(Q_u) - M_{A1_5}^{j(d)+}(Q_d)\}], \quad (91)
 \end{aligned}$$

$$\begin{aligned}
 M(B^0 \rightarrow \omega \gamma)_{A1}^{j+} &= \frac{1}{\sqrt{2}} [2M_{A1_5}^{j(a)+}(Q_b) + \{M_{A1_5}^{j(b)+}(Q_u) \\
 &\quad + M_{A1_5}^{j(b)+}(Q_d)\} + 2M_{A1_5}^{j(c)+}(Q_d) \\
 &\quad + \{M_{A1_5}^{j(d)+}(Q_u) + M_{A1_5}^{j(d)+}(Q_d)\}]. \quad (92)
 \end{aligned}$$

For the contributions of the annihilation diagrams shown in Fig. 7 with $(V - A)(V - A)$ insertion, the results $M_{A2}^{(S,P)-}$ are the same as $M_{A1}^{(S,P)-}$, and the amplitudes for the decay modes are

$$M(B^+ \rightarrow \rho^+ \gamma)_{A2}^{j-} = M_{A2_4}^{j(a)-}(Q_b) + M_{A2_4}^{j(b)-}(Q_d) + M_{A2_4}^{j(c)-}(Q_u) + M_{A2_4}^{j(d)-}(Q_u), \quad (93)$$

$$M(B^0 \rightarrow \rho^0 \gamma)_{A2}^{j-} = -\frac{1}{\sqrt{2}} [M_{A2_4}^{j(a)-}(Q_b) + M_{A2_4}^{j(b)-}(Q_d) + M_{A2_4}^{j(c)-}(Q_d) + M_{A2_4}^{j(d)-}(Q_d)], \quad (94)$$

$$M(B^0 \rightarrow \omega \gamma)_{A2}^{j-} = \frac{1}{\sqrt{2}} [M_{A2_4}^{j(a)-}(Q_b) + M_{A2_4}^{j(b)-}(Q_d) + M_{A2_4}^{j(c)-}(Q_d) + M_{A2_4}^{j(d)-}(Q_d)]. \quad (95)$$

While for the insertion of $(V - A)(V + A)$ operators, the results are

$$\mathcal{M}_{A2_k}^{S(a)+} = \mathcal{M}_{A2_k}^{P(a)+} = 0, \quad (96)$$

$$\begin{aligned}
 \mathcal{M}_{A2_k}^{S(b)+} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_i \mathcal{C}_{ann} Q_d a_k(t_{A2}^b) \\
 &\quad \times \exp[-S_V(t_{A2}^b)] S_i(x_2) [-2m_B \phi_V^T(x_2)] \\
 &\quad \times \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{1-x_2} b_2) \right), \quad (97)
 \end{aligned}$$

$$\mathcal{M}_{A2_k}^{P(b)+} = -\mathcal{M}_{A2_k}^{S(b)+}, \quad (98)$$

$$\mathcal{M}_{A2_k}^{S(c)+} = \mathcal{M}_{A2_k}^{P(c)+} = 0, \quad (99)$$

$$\begin{aligned}
 \mathcal{M}_{A2_k}^{S(d)+} &= \int_0^1 dx_2 \int_0^{1/\Lambda} b_2 db_2 \eta_i \mathcal{C}_{ann} Q_{q'} a_k(t_{A2}^d) \\
 &\quad \times \exp[-S_V(t_{A2}^d)] S_i(x_2) [2m_B \phi_V^T(x_2)] \\
 &\quad \times \left(-i \frac{\pi}{2} H_0^{(1)}(\sqrt{x_2} b_2) \right), \quad (100)
 \end{aligned}$$

$$\mathcal{M}_{A2_k}^{P(d)+} = -\mathcal{M}_{A2_k}^{S(d)+}, \quad (101)$$

where

$$\mathcal{C}_{ann} = \frac{em_B^2 f_B G_F}{\sqrt{27}}.$$

The amplitudes corresponding to the annihilation diagrams in Fig. 7 with $(V - A)(V + A)$ operator insertion are

$$M(B^+ \rightarrow \rho^+ \gamma)_{A2}^{j+} = M_{A2_6}^{j(a)+}(Q_b) + M_{A2_6}^{j(b)+}(Q_d) + M_{A2_6}^{j(c)+}(Q_u) + M_{A2_6}^{j(d)+}(Q_u), \quad (102)$$

$$M(B^0 \rightarrow \rho^0 \gamma)_{A2}^{j+} = -\frac{1}{\sqrt{2}} [M_{A2_6}^{j(a)+}(Q_b) + M_{A2_6}^{j(b)+}(Q_d) + M_{A2_6}^{j(c)+}(Q_d) + M_{A2_6}^{j(d)+}(Q_d)], \quad (103)$$

$$M(B^0 \rightarrow \omega\gamma)_{A2}^{j+} = \frac{1}{\sqrt{2}} [M_{A2_6}^{j(a)+}(Q_b) + M_{A2_6}^{j(b)+}(Q_d) + M_{A2_6}^{j(c)+}(Q_d) + M_{A2_6}^{j(d)+}(Q_d)]. \quad (104)$$

Finally the total amplitudes of all the decay modes for $j = S, P$ are

$$M^j(B^+ \rightarrow \rho^+\gamma) = M(B^+ \rightarrow \rho^+\gamma)_{7\gamma}^j + M(B^+ \rightarrow \rho^+\gamma)_{8g}^j + M(B^+ \rightarrow \rho^+\gamma)_{1i}^j + M(B^+ \rightarrow \rho^+\gamma)_{2i}^j + M(B^+ \rightarrow \rho^+\gamma)_A^j + M(B^+ \rightarrow \rho^+\gamma)_{A2}^{j-} + M(B^+ \rightarrow \rho^+\gamma)_{A2}^{j+}, \quad (105)$$

$$M^j(B^0 \rightarrow \rho^0\gamma) = M(B^0 \rightarrow \rho^0\gamma)_{7\gamma}^j + M(B^0 \rightarrow \rho^0\gamma)_{8g}^j + M(B^0 \rightarrow \rho^0\gamma)_{1i}^j + M(B^0 \rightarrow \rho^0\gamma)_{2i}^j + M(B^0 \rightarrow \rho^0\gamma)_A^j + M(B^0 \rightarrow \rho^0\gamma)_{A1}^{j-} + M(B^0 \rightarrow \rho^0\gamma)_{A1}^{j+} + M(B^0 \rightarrow \rho^0\gamma)_{A2}^{j-} + M(B^0 \rightarrow \rho^0\gamma)_{A2}^{j+}, \quad (106)$$

$$M^j(B^0 \rightarrow \omega\gamma) = M(B^0 \rightarrow \omega\gamma)_{7\gamma}^j + M(B^0 \rightarrow \omega\gamma)_{8g}^j + M(B^0 \rightarrow \omega\gamma)_{1i}^j + M(B^0 \rightarrow \omega\gamma)_{2i}^j + M(B^0 \rightarrow \omega\gamma)_A^j + M(B^0 \rightarrow \omega\gamma)_{A1}^{j-} + M(B^0 \rightarrow \omega\gamma)_{A1}^{j+} + M(B^0 \rightarrow \omega\gamma)_{A2}^{j-} + M(B^0 \rightarrow \omega\gamma)_{A2}^{j+}. \quad (107)$$

V. THE CONTRIBUTION OF SOFT TRANSITION FORM FACTORS

Only when the scale for the hard function H is high enough, can the factorization formula given in Eq. (10) holds. Therefore, the perturbative calculations in PQCD approach explained in Secs. II, III, and IV must be restricted in the regions with the scale $\mu > \mu_c$, where μ_c is the infrared momentum cutoff scale which serves to separate the hard and soft interactions. It is reasonable to choose μ_c around 1 GeV in QCD. For the interactions in the hard regime $\mu = t_i > \mu_c$, the contributions to the decay amplitudes are calculated with PQCD approach, here t_i 's stand for the hard scales chosen for each diagram relevant to the hard transition process. While for the contributions with the scale $\mu < \mu_c$, we can introduce soft form factors to absorb the soft contribution.

The transition form factors relevant to the radiative decays of B meson can be defined by the matrix element of the tensor current [27]

$$\begin{aligned} & \langle V(p_1) | \bar{q} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | \bar{B}(p_B) \rangle \\ &= 2i\epsilon_{\mu\nu\alpha\beta} \epsilon_V^{\nu*} p_B^\alpha p_1^\beta T_1(q^2) + [\epsilon_V^*(m_B^2 - m_V^2) \\ & \quad - (q \cdot \epsilon_V^*)(p_1 + p_B)_\mu] T_2(q^2) + (q \cdot \epsilon_V^*) \\ & \quad \times \left[q_\mu - \frac{q^2}{m_B^2 - m_V^2} (p_1 + p_B)_\mu \right] T_3(q^2), \end{aligned} \quad (108)$$

where $T_{1,2,3}(q^2)$ are the transition form factors corresponding to the tensor current, and there is the relation $T_1(0) = T_2(0)$. With the form factors defined by the tensor current, the matrix element induced by the operator $O_{7\gamma}$ can be derived as

$$\begin{aligned} \langle V\gamma | Q_{7\gamma} | \bar{B}(p_B) \rangle &= \frac{ie}{4\pi^2} m_b [(\epsilon_\gamma^* \cdot \epsilon_V^*)(m_B^2 - m_V^2) \\ & \quad \times T_2(0) + 2i\epsilon_{\mu\nu\alpha\beta} \epsilon_\gamma^{\mu*} \epsilon_V^{\nu*} p_\gamma^\alpha p_V^\beta T_1(0)]. \end{aligned} \quad (109)$$

There is actually only one independent form factor that contributes, because of the relation $T_1(0) = T_2(0)$. The form factors for $B \rightarrow \rho$ and $B \rightarrow \omega$ can be separated into hard and soft form factors as the follows

$$\begin{aligned} T_{1\rho}(0) &= h_\rho + \xi_\rho, \\ T_{1\omega}(0) &= h_\omega + \xi_\omega, \end{aligned} \quad (110)$$

where $h_{\rho,\omega}$ denote the hard form factors, which can be calculated with PQCD approach. The hard contributions to the decay amplitudes have been given in Sec. II, which are corresponding to the diagrams in Fig. 1. $\xi_{\rho,\omega}$ are the soft form factors, which are the nonperturbative quantities and absorb the soft contributions. The soft form factors are treated as input parameters in this work.

One can obtain the contributions of the soft form factors to $M^{S,P}$ by using the definition of $M^{S,P}$ in Eq. (6) and Eqs. (109) and (110) by only considering soft part

$$\begin{aligned} M_{\text{soft}}^S &= \frac{G_F}{\sqrt{2}} (-V_{tb}^* V_{td}) \frac{e}{4\pi^2} m_b (m_B^2 - m_V^2) C_{7\gamma}(\mu_c) \xi_{\rho,\omega}(0), \\ M_{\text{soft}}^P &= \frac{G_F}{\sqrt{2}} (-V_{tb}^* V_{td}) \frac{e}{2\pi^2} m_b p_V \cdot p_\gamma C_{7\gamma}(\mu_c) \xi_{\rho,\omega}(0). \end{aligned} \quad (111)$$

Then the amplitude $M^{S,P}$ calculated by PQCD should be replaced by

$$M^{S,P} \rightarrow M^{S,P} + M_{\text{soft}}^{S,P}. \quad (112)$$

VI. NUMERICAL RESULT AND DISCUSSION

For the numerical treatment, the input parameters used in this work are collected in Table I. The CKM parameters in Wolfenstein parametrization are λ , A , ρ and η . The definition $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$ are usually used. The CKM parameters, meson masses and life times can be found in PDG [16]. The decay constant f_B is the result calculated in the relativistic potential model from Ref. [24], and the other decay constants of the light vector mesons are kept the same value as Ref. [11].

The decay width and branching ratio are calculated according to the following formulas

$$\Gamma = \frac{|M^S|^2 + |M^P|^2}{8\pi M_B}, \quad Br = \frac{\tau_B}{\hbar} \Gamma, \quad (113)$$

and the branching ratios calculated in this work are defined as the average of the decay mode and its charge conjugate

$$Br(B^\pm \rightarrow \rho^\pm \gamma) = \frac{1}{2} [Br(B^+ \rightarrow \rho^+ \gamma) + Br(B^- \rightarrow \rho^- \gamma)], \quad (114)$$

$$Br(B^0 \rightarrow \rho^0 \gamma) = \frac{1}{2} [Br(B^0 \rightarrow \rho^0 \gamma) + Br(\bar{B}^0 \rightarrow \rho^0 \gamma)]. \quad (115)$$

The direct CP asymmetry parameters for the charged and neutral decay modes are defined by

$$A_{cp}(B^\pm \rightarrow \rho^\pm \gamma) = \frac{\Gamma(B^- \rightarrow \rho^- \gamma) - \Gamma(B^+ \rightarrow \rho^+ \gamma)}{\Gamma(B^- \rightarrow \rho^- \gamma) + \Gamma(B^+ \rightarrow \rho^+ \gamma)}, \quad (116)$$

and

TABLE II. The form factors.

	PQCD	LCSR
ξ_ρ	$0.10^{+0.02}_{-0.02}$	...
ξ_ω	$0.07^{+0.01}_{-0.02}$	...
h_ρ	$0.18^{+0.01}_{-0.01}$	...
h_ω	$0.17^{+0.01}_{-0.01}$	...
$T_{1\rho}(0)$	$0.28^{+0.03}_{-0.03}$	0.267 ± 0.021
$T_{1\omega}(0)$	$0.24^{+0.03}_{-0.03}$	0.242 ± 0.022

$$A_{cp}(B^0 \rightarrow \rho^0(\omega)\gamma) = \frac{\Gamma(\bar{B}^0 \rightarrow \rho^0(\omega)\gamma) - \Gamma(B^0 \rightarrow \rho^0(\omega)\gamma)}{\Gamma(\bar{B}^0 \rightarrow \rho^0(\omega)\gamma) + \Gamma(B^0 \rightarrow \rho^0(\omega)\gamma)}. \quad (117)$$

Using the input parameters given in Table I, we calculate the branching ratios and CP violations of the decays $B \rightarrow \rho(\omega)\gamma$ with the soft form factors as unknown parameters, and confront these quantities to experimental data. By fitting the experimental data for the branching ratios and CP violation, we obtain the values of the soft form factors, which are shown in Table II. The errors mainly come from the uncertainties of the parameters in wave functions of B meson, light mesons and the errors of the experimental data. We also calculate the hard form factors h_ρ and h_ω in PQCD by restricting the momentum scale to be $\mu > \mu_c$, which are also shown in Table II. The total form factors $T_{1\rho}(0)$ and $T_{1\omega}(0)$ can be obtained by summing the hard and soft form factors. The comparison of the total form factors obtained in this work and that from LCSR [28] can be found in the last two rows in Table II, which shows the consistence of the fitted results of the soft form factors in the modified PQCD approach and the form factors calculated from LCSR method.

With the soft form factors presented in Table II, the amplitudes M^S and M^P for each decay mode can be calculated, which are shown in Table III. It is shown that

TABLE I. Summary of the input parameters.

CKM parameters and QCD constant				
λ	A	$\bar{\rho}$	$\bar{\eta}$	$\Lambda_{\overline{MS}}^{(f=4)}$
0.22501 ± 0.00068	$0.826^{0.016}_{-0.015}$	0.1591 ± 0.0094	$0.3523^{+0.0073}_{-0.0071}$	250 MeV
Masses				
M_W	M_B	M_ρ	M_ω	m_c
80.41 GeV	5.28 GeV	0.78 GeV	0.78 GeV	1.5 MeV
Meson decay constants				
f_B	f_ρ	f_ρ^T	f_ω	f_ω^T
210 MeV	220 MeV	160 MeV	195 MeV	160 MeV
B meson lifetime			μ_c	
τ_{B^0}	$\tau_{B^\pm}$		1 GeV	
1.517 ps	1.638 ps			

TABLE III. The amplitudes of individual Feynman diagrams and soft contributions ($\times 10^{-9}$).

decay mode		$O7\gamma$	$O8g$	NLO	ALL	ξ_{soft}
M^S	$B^+ \rightarrow \rho^+\gamma$	$5.00 + 2.40i$	$0.08 + 0.17i$	$0.67 - 2.86i$	$5.76 - 0.27i$	$-1.56 + 3.22i$
	$B^0 \rightarrow \rho^0\gamma$	$-3.54 - 1.71i$	$-0.12 - 0.02i$	$-0.26 - 0.23i$	$-3.91 - 1.96i$	$1.10 - 2.28i$
	$B^0 \rightarrow \omega\gamma$	$3.27 + 1.58i$	$0.12 + 0.02i$	$-0.20 + 0.03i$	$3.19 + 1.63i$	$-1.10 + 2.28i$
M^P	$B^+ \rightarrow \rho^+\gamma$	$-5.00 - 2.42i$	$-0.10 - 0.17i$	$-1.84 + 2.34i$	$-6.94 - 0.25i$	$1.56 - 3.22i$
	$B^0 \rightarrow \rho^0\gamma$	$3.54 + 1.74i$	$0.11 + 0.02i$	$-0.03 - 0.03i$	$3.62 + 1.70i$	$-1.10 + 2.28i$
	$B^0 \rightarrow \omega\gamma$	$-3.27 - 1.58i$	$-0.11 - 0.02i$	$0.04 + 0.03i$	$-3.34 - 1.57i$	$1.10 - 2.28i$

TABLE IV. Branching ratios and CP asymmetries with asymmetric errors (double-line display).

Mode	LO	+NLO	+NLO + ξ_V	Data [16]
$\text{Br}(B^+ \rightarrow \rho^+\gamma) (\times 10^{-6})$	1.31	1.38	$1.34^{+0.12}_{-0.05}$	1.29 ± 0.20
$\text{Br}(B^0 \rightarrow \rho^0\gamma) (\times 10^{-6})$	0.56	0.58	$0.81^{+0.02}_{-0.01}$	0.82 ± 0.13
$\text{Br}(B^0 \rightarrow \omega\gamma) (\times 10^{-6})$	0.48	0.50	$0.60^{+0.13}_{-0.23}$	0.44 ± 0.17
$A_{CP}(B^+ \rightarrow \rho^+\gamma)$	0.127	0.108	$-0.058^{+0.031}_{-0.022}$	-0.08 ± 0.15
$A_{CP}(B^0 \rightarrow \rho^0\gamma)$	0.064	0.045	$0.037^{+0.001}_{-0.001}$	...
$A_{CP}(B^0 \rightarrow \omega\gamma)$	-0.062	-0.083	$-0.063^{+0.009}_{-0.026}$	...

the contributions of $O_{7\gamma}$ are overwhelmingly dominant in all the contributions. The NLO contribution from quark loop and annihilation diagrams in $B^+ \rightarrow \rho^+\gamma$ is larger than that in other decay modes. The column “All” denotes the contributions of all the leading and next-to-leading order diagrams except the contribution of the soft form factor. The column “ ξ_{soft} ” means the contributions of the soft form factor. This table shows that the relative contribution of the soft form factor respect to hard contribution in the neutral decay mode is larger than that in the charged mode.

The theoretically calculated results of the branching ratios and CP violations for the decay modes considered in this work are shown in Table IV, where “LO” denotes the leading-order contributions corresponding to the operators $O_{7\gamma}$ and O_{8g} , “+NLO” the results including the contributions up to the next-to-leading order diagrams (the quark-loop and annihilation diagrams), and “+NLO+ ξ_V ” the total contributions including both the LO, NLO and the soft form factor contributions. The comparison of LO and NLO shows that the differences of these two columns are very small, which means the contributions of NLO diagrams are not large. The difference is smaller than a few percent, which indicates that the perturbative calculation is indeed effective in the modified PQCD approach. The contribution of the soft form factor slightly lower the branching ratio of $B^+ \rightarrow \rho^+\gamma$ decay mode, and affect the CP violation largely. With the soft form factors given in Table II, the theoretical outputs of both the branching ratios and CP violations are well consistent with the experimental data.

A few remarks are in the following.

- (1) An interesting question is how the physical results depend the value of the cutoff scale parameter μ_c . We calculate the perturbative contributions and fit the

soft form factors by varying the value of μ_c from 0.8 to 2.0 GeV. The form factors, the branching ratios and CP violations with different values of μ_c are presented in Table V, which shows that the fitted form factors and branching ratios are only slightly changed around $\mu_c = 1$ GeV. The last row of Table V is for the relevant values of χ^2 in each fit. The value of χ^2 for $\mu_c = 1$ GeV is the smallest, which indicates that $\mu_c = 1$ GeV gives the best fit. In addition, if the cutoff scale is taken to be a value large than 1 GeV, much more perturbative contributions are cut off, while for values smaller than 1 GeV, more nonperturbative contributions would be included in the perturbative calculation. Therefore $\mu_c = 1$ GeV is a favorable choice, and varying μ_c slightly around 1 GeV will not change the physical results too much.

- (2) The soft form factors are the unknown nonperturbative parameters for the decays we study in this work, which cannot be calculated reliably with perturbative method in QCD, and can only be treated as input parameters in this work. The soft form factors can be obtained in principle by nonperturbative methods, such as lattice QCD (LQCD), LCSR, or directly constrained from experimental data. Because there are already enough relatively precise experimental data at preset, so we choose to obtain the soft form factors directly by fitting the experimental data. The key point is that the number of the input soft form factors are less than the final physical results, i.e., the branching ratios and CP violations. For example, there is only one soft form factor for $B \rightarrow \rho$ transition, ξ_ρ , however, there are

TABLE V. The μ_c -dependence of the hard form factors $h_{\rho,\omega}$, the fitted soft form factors $\xi_{\rho,\omega}$, the branching ratios and CP violations, and the relevant values of χ^2 in the fit. $\text{Br}(\rho^+, \rho^0, \omega)$ and $\text{CP}_{\rho^+, \rho^0, \omega}$ denote the branching ratios and direct CP violations of $B \rightarrow \rho^+ \gamma, \rho^0 \gamma$ and $\omega \gamma$ decays, respectively.

μ_c (GeV)	$\mu_c = 0.8$	$\mu_c = 0.9$	$\mu_c = 1.0$	$\mu_c = 1.1$	$\mu_c = 1.2$	$\mu_c = 1.5$	$\mu_c = 2.0$
h_ρ	0.224	0.201	0.18	0.163	0.147	0.112	0.076
ξ_ρ	0.046	0.062	0.10	0.128	0.138	0.174	0.188
h_ω	0.206	0.185	0.17	0.151	0.138	0.106	0.072
ξ_ω	0.029	0.049	0.07	0.08	0.071	0.106	0.125
$T_{1\rho}$	0.27	0.263	0.28	0.29	0.285	0.286	0.264
$T_{1\omega}$	0.235	0.234	0.24	0.231	0.209	0.212	0.197
$\text{Br}(\rho^+) (\times 10^{-6})$	2.03	1.58	1.34	1.26	1.15	1.18	1.18
$\text{Br}(\rho^0) (\times 10^{-6})$	0.99	0.82	0.81	0.83	0.89	0.89	0.88
$\text{Br}(\omega) (\times 10^{-6})$	0.82	0.68	0.60	0.63	0.44	0.43	0.44
CP_{ρ^+}	0.026	0.007	-0.058	-0.107	-0.135	-0.17	-0.137
CP_{ρ^0}	0.031	0.036	0.037	0.036	0.034	0.028	0.022
CP_{ω^0}	-0.069	-0.071	-0.063	-0.079	-0.066	-0.037	-0.016
χ^2	20.435	3.673	0.225	1.309	0.866	0.930	0.785

four theoretical outputs waiting to compare with experimental data including two branching ratios and two CP violation parameters for $B^+ \rightarrow \rho^+ \gamma$ and $B^0 \rightarrow \rho^0 \gamma$. So there is a certain bit of predictive power in this calculation. We indeed predict the direct CP violations in $B^0 \rightarrow \rho^0 \gamma$ and $B^0 \rightarrow \omega \gamma$ decays in this work, for which there are no measured results in experiment at present. Our method presents a framework to understand the experimental data of the radiative decays of B meson based on the PQCD.

- (3) The sum of the fitted soft form factor and the calculated hard part results in the total form factor, which is shown to be consistent with the transition form factor obtained by LCSR in [28]. This implies that the soft form factors as input parameters in this work are reasonable in QCD. However, the total form factors obtained by LCSR cannot be simply used in our approach. The total form factors should be separated as soft and hard parts. The scale of the hard part should be evaluated with the scale of the Wilson coefficients together, and only the soft part is set at the scale μ_c . Note that, if treat the form factor totally as a whole nonperturbative input parameters, smaller effective values for the form factors have to be used to make the theoretical results consistent with experimental data [9].
- (4) The uncertainties in Ref. [11] mainly come from the errors of the CKM parameters, the uncertainties of the parameters in B and light meson wave functions. Both of these uncertainties used in Ref. [11] are large. At present, the CKM parameters are extremely more precise than two decades ago. The wave function of B meson is phenomenologically selected in Ref. [11]. The parameter in it can only be estimated to be in a large range, which causes large

uncertainty. But the B meson wave function used in this work is obtained by solving the bound-state equation in the relativistic potential model for $b\bar{q}$ system. The masses of both the ground and excited states should be consistent with experimental data, so the parameters in the potential model receive strict restrictions, which results in the parameters for the B meson wave function with small uncertainty in the potential model. Therefore, the uncertainties of the results in this work are significantly reduced.

In Ref. [11], the end-point behavior of the B meson wave function as $x \rightarrow 0$ is chosen to be $\sim x^2$ (x is the longitudinal momentum fraction of the light quark in B meson), which helps strongly suppress the infrared end-point contribution in addition to the suppression effect of the Sudakov factor. With the two-fold suppression to the soft contribution, the perturbative contribution is overwhelmingly dominant in the calculation of Ref. [11], so the factorization works well even without the cutoff of the scale below 1 GeV. But for the B meson wave function obtained from the potential model used in this work, the end-point behavior is $\sim \sqrt{x}$ as $x \rightarrow 0$, from which the suppression effect to the infrared contribution at the end point region is greatly weakened. Large part of the calculated transition form factor falls in the nonperturbative region (about 40%, see Ref. [17]). With the infrared cutoff scale introduced, the nonperturbative contributions existing in the perturbative calculation are removed and the relevant contribution is replaced by the soft form factor, which makes the applicability of the perturbative calculation being improved and in general the total form factor obtained here is smaller than that of Ref. [11]. Therefore, using the B meson wave function obtained from the potential model changes the contribution structure and causes a large physical effect.

VII. SEEKING MASSLESS DARK PHOTONS IN THE DECAYS OF BOTTOM MESON

In the SM electromagnetic interaction is described by the $U(1)_{\text{em}}$ symmetry group, which is the consequence of the spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry, where L denotes the left-handed states of the fermion doublets, and Y the hypercharge of the particles. In principle, SM can be extended by introducing an extra $U(1)$ symmetry group [29–31], which governs a sector of matter of darkness [32]. The symmetry of the extra $U(1)$ group can be unbroken, then a massless $U(1)$ gauge boson will be generated, which can be called the dark photon [33,34]. The only connection between the dark sector and the matter particles in SM is through the dark photon. The effective Lagrangian describing the interaction between the SM fermion and the massless dark photon is [33,35]

$$\mathcal{L}_{\text{NP}} = \frac{1}{\Lambda_{\text{NP}}^2} (C_{jk}^D \bar{q}_j \sigma^{\mu\nu} d_k H + C_{jk}^U \bar{q}_j \sigma^{\mu\nu} u_k \tilde{H} + \text{H.c.}) \bar{F}_{\mu\nu}, \quad (118)$$

where Λ_{NP} denotes the large mass scale for the new physics of dark sector, $C_{jk}^{U,D}$'s the couplings of the quark fields and the Higgs scalar, $\bar{q}_j$ the left-handed quark doublet of

$SU(2)_L$ group, d_k (u_k) the right-handed down (up)-type quark singlet, H the SM Higgs doublet, and $\tilde{H} = i\sigma_2 H^*$, where σ_2 is the second Pauli matrix. $\bar{F}_{\mu\nu} = \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$ is the dark photon strength tensor. The indices $i, j, k = 1, 2, 3$.

After electroweak symmetry breaking, the terms for the $b \rightarrow d$ FCNC current interaction with the massless dark photon can be extracted from the $\mathcal{C}_{jk}^D$ sector

$$\mathcal{L}_{db\gamma'} = \bar{d}(\mathbb{C} + \gamma_5 \mathbb{C}_5) \sigma^{\mu\nu} b \bar{F}_{\mu\nu} + \text{H.c.}, \quad (119)$$

where

$$\mathbb{C} = \Lambda_{\text{NP}}^{-2} (C_{13}^D + C_{31}^{D*}) v / \sqrt{8} \quad (120)$$

and

$$\mathbb{C}_5 = \Lambda_{\text{NP}}^{-2} (C_{13}^D - C_{31}^{D*}) v / \sqrt{8}. \quad (121)$$

Using the effective Lagrangian presented in Eq. (119) to calculate the decay amplitude, the matrix elements $\langle V(P_1) | \bar{q} \sigma_{\mu\nu} b | \bar{B}(P_B) \rangle$ and $\langle V(P_1) | \bar{q} \sigma_{\mu\nu} \gamma_5 b | \bar{B}(P_B) \rangle$ should be treated. Three form factors can be defined for these two matrix elements

$$\langle V(P_1) | \bar{q} \sigma_{\mu\nu} b | \bar{B}(P_B) \rangle = \left(-i\epsilon_{\mu\nu\alpha\beta} \epsilon_V^{\alpha*} P_1^\beta \bar{T}_1(q^2) - i\epsilon_{\mu\nu\alpha\beta} \epsilon_V^{\alpha*} P_B^\beta \bar{T}_2(q^2) - i\bar{T}_3(q^2) \frac{P_B \cdot \epsilon_V^*}{P_B \cdot P_1} \cdot \epsilon_{\mu\nu\alpha\beta} P_1^\alpha P_B^\beta \right), \quad (122)$$

$$\langle V(P_1) | \bar{q} \sigma_{\mu\nu} \gamma_5 b | \bar{B}(P_B) \rangle = \left((P_1^\mu P_B^\nu - P_B^\mu P_1^\nu) \frac{P_B \cdot \epsilon_V^*}{P_B \cdot P_1} \bar{T}_3(q^2) + (\epsilon_V^{\mu*} P_B^\nu - P_B^\mu \epsilon_V^{\nu*}) \bar{T}_2(q^2) + (\epsilon_V^{\mu*} P_1^\nu - P_1^\mu \epsilon_V^{\nu*}) \bar{T}_1(q^2) \right), \quad (123)$$

here $q^\nu = P_B^\nu - P_1^\nu$. Then for $\langle V(P_1) | \bar{q} \sigma_{\mu\nu} q^\nu b | \bar{B}(P_B) \rangle$ and $\langle V(P_1) | \bar{q} \sigma^{\mu\nu} q_\nu \gamma_5 b | \bar{B}(P_B) \rangle$, we have

$$\begin{aligned} \langle V(P_1) | \bar{q} \sigma_{\mu\nu} q^\nu b | \bar{B}(P_B) \rangle &= 2i\epsilon_{\mu\nu\alpha\beta} \epsilon_V^{\mu*} \epsilon_V^{\nu} P_B^\alpha P_1^\beta T_1(q^2), \\ \langle V(P_1) | \bar{q} \sigma^{\mu\nu} q_\nu \gamma_5 b | \bar{B}(P_B) \rangle &= [\epsilon_V^{\mu*} (m_B^2 - m_V^2) - (q \cdot \epsilon_V^*)(p_1 + p_B)_\mu] T_2(q^2) + (q \cdot \epsilon_V^*) \left[q_\mu - \frac{q^2}{m_B^2 - m_V^2} (p_1 + p_B)_\mu \right] T_3(q^2), \end{aligned} \quad (124)$$

where the relations between $T_i(q^2)$ and $\bar{T}_i(q^2)$ are

$$\begin{aligned} T_1(q^2) &= \frac{1}{2} [\bar{T}_1(q^2) + \bar{T}_2(q^2)], \\ T_2(q^2) &= \frac{1}{2} [\bar{T}_1(q^2) + \bar{T}_2(q^2) + \frac{q^2}{m_B^2 - m_V^2} (\bar{T}_2(q^2) - \bar{T}_1(q^2))], \\ T_3(q^2) &= \frac{1}{2} \left[\bar{T}_1(q^2) - \bar{T}_2(q^2) - \frac{m_B^2 - m_V^2}{P_B \cdot P_1} \bar{T}_3(q^2) \right], \end{aligned}$$

Note that the form factors $T_i(q^2)$'s are the same as that in Eq. (108). For the massless dark photon we have $\epsilon_V^* \cdot q = 0$ and $q^2 = 0$, then the amplitudes M^S and M^P for $B \rightarrow V\gamma'$ decay can be calculated to be

$$\begin{aligned}\mathcal{M}'^S &= 2(m_B^2 - m_V^2)T_2(0)\mathbb{C}_5, \\ \mathcal{M}'^P &= 4P_V \cdot P_\gamma T_1(0)\mathbb{C},\end{aligned}\quad (125)$$

and the branching ratio is

$$Br(B \rightarrow V\gamma') = \frac{\tau_B T_1(0)^2 (m_B^2 - m_V^2)^2}{2\pi m_B} (|\mathbb{C}|^2 + |\mathbb{C}_5|^2). \quad (126)$$

The form factor $T_1(0)$ is obtained in this work, and presented in Table II. For the parameters $\mathbb{C}$ and $\mathbb{C}_5$, we take

$$|\mathbb{C}|^2 + |\mathbb{C}_5|^2 < \frac{3.6 \times 10^{-18}}{\text{GeV}^2}, \quad (127)$$

which is obtained by using the branching ratio $Br(b \rightarrow q\gamma') < 1.0 \times 10^{-5}$ (See Fig. 5 of Ref. [34]), and using Eq. (61) of [34]:

$$Br(b \rightarrow q\gamma') = 12Br^{\text{exp}}(B \rightarrow X_c \bar{\nu} e) / [G_F^2 |V_{cb}|^2 m_b^2 f_1(m_c^2/m_b^2)] \cdot (1/(\Lambda_L^{bq})^2 + 1/(\Lambda_R^{bq})^2) \quad (128)$$

with $Br^{\text{exp}}(B \rightarrow X_c \bar{\nu} e) = 0.108$ listed in PDG [16], $f_1(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$ and utilizing the relation $1/(\Lambda_L^{bq})^2 + 1/(\Lambda_R^{bq})^2 = 8|\mathbb{C}|^2 + 8|\mathbb{C}_5|^2$.

With Eq. (126), the upper limit in (127), and using the transition form factors $T_{1\rho}$, $T_{1\omega}$ presented in Table II for $T_1(0)$, we arrive at

$$\begin{aligned}Br(B^+ \rightarrow \rho^+ \gamma') &< 1.55 \times 10^{-5}, \\ Br(B^0 \rightarrow \rho^0 \gamma') &< 1.44 \times 10^{-5}, \\ Br(B^0 \rightarrow \omega \gamma') &< 1.06 \times 10^{-5}.\end{aligned}\quad (129)$$

It is interesting for experiments of Belle II and LHCb to perform the corresponding detections, which can set constraint on the combination of the new physics parameters $|\mathbb{C}|^2 + |\mathbb{C}_5|^2$ on the experimental side.

VIII. SUMMARY

We study the radiative decays of $B \rightarrow \rho\gamma$ and $B \rightarrow \omega\gamma$ in the modified PQCD approach. The branching ratios and CP violations are calculated with the transverse momenta of quark and gluons included. Sudakov factor is also included in the calculation, which helps to suppress the long-distance contributions. An infrared momentum cutoff scale μ_c is introduced to separate the soft and hard contributions. For the contributions with the interaction scale $\mu > \mu_c$, the amplitude can be calculated in the perturbative method. While, for the contributions with the scale $\mu < \mu_c$, soft form factors are introduced to collect the soft contributions. By fitting to the experimental data, we obtain the reasonable values for the soft form factors. The sum of the soft and hard form factors is consistent with the total form factor calculated with LCSR method. We also estimate the upper limit of the branching ratios of $B \rightarrow \rho$

and $B \rightarrow \omega$ transitions with the radiation of the massless dark photon by using the transition form factors obtained in the modified PQCD approach. The upper limits obtained in this work can be helpful for detecting the existence of dark photon in B decays.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: THRESHOLD AND SUDAKOV FACTOR

The functions S_B and S_V in Eqs. (17), (18), etc. are given in the following:

$$S_B(\mu) = s(x, b, m_B) - \frac{1}{\beta_1} \ln \frac{\ln(\mu/\Lambda_{\text{QCD}})}{\ln(1/(b\Lambda_{\text{QCD}}))}, \quad (A1)$$

$$\begin{aligned}S_V(\mu) &= s(x, b, m_B) + s(1-x, b, m_B) \\ &\quad - \frac{1}{\beta_1} \ln \frac{\ln(\mu/\Lambda_{\text{QCD}})}{\ln(1/(b\Lambda_{\text{QCD}}))},\end{aligned}\quad (A2)$$

where the exponential $\exp -s(x, b, Q)$ is the Sudakov factor. The function $s(x, b, Q)$ up to next-to-leading order has been given in Ref. [14], which is

$$\begin{aligned}
s(x, b, Q) = & \frac{A^{(1)}}{2\beta_1} \hat{q} \ln\left(\frac{\hat{q}}{\hat{b}}\right) - \frac{A^{(1)}}{2\beta_1} (\hat{q} - \hat{b}) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{\hat{b}} - 1\right) - \left[\frac{A^{(2)}}{4\beta_1^2} - \frac{A^{(1)}}{4\beta_1} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right)\right] \ln\left(\frac{\hat{q}}{\hat{b}}\right) \\
& + \frac{A^{(1)}\beta_2}{4\beta_1^3} \hat{q} \left[\frac{\ln(2\hat{q}) + 1}{\hat{q}} - \frac{\ln(2\hat{b}) + 1}{\hat{b}}\right] + \frac{A^{(1)}\beta_2}{8\beta_1^3} [\ln^2(2\hat{q}) - \ln^2(2\hat{b})] \\
& + \frac{A^{(1)}\beta_2}{8\beta_1^3} \ln\left(\frac{e^{2\gamma_E-1}}{2}\right) \left[\frac{\ln(2\hat{q}) + 1}{\hat{q}} - \frac{\ln(2\hat{b}) + 1}{\hat{b}}\right] - \frac{A^{(2)}\beta_2}{16\beta_1^4} \left[\frac{2\ln(2\hat{q}) + 3}{\hat{q}} - \frac{2\ln(2\hat{b}) + 3}{\hat{b}}\right] \\
& - \frac{A^{(1)}\beta_2}{16\beta_1^4} \frac{\hat{q} - \hat{b}}{\hat{b}^2} [2\ln(2\hat{b}) + 1] + \frac{A^{(2)}\beta_2^2}{432\beta_1^6} \frac{\hat{q} - \hat{b}}{\hat{b}^3} [9\ln^2(2\hat{b}) + 6\ln(2\hat{b}) + 2] \\
& + \frac{A^{(2)}\beta_2^2}{1728\beta_1^6} \left[\frac{18\ln^2(2\hat{q}) + 30\ln(2\hat{q}) + 19}{\hat{q}^2} - \frac{18\ln^2(2\hat{b}) + 30\ln(2\hat{b}) + 19}{\hat{b}^2}\right], \tag{A3}
\end{aligned}$$

where

$$\hat{q} = \ln\left(\frac{xQ_m}{\sqrt{2}\Lambda}\right), \quad \hat{b} = \ln\left(\frac{1}{b\Lambda}\right), \tag{A4}$$

$$\begin{aligned}
\beta_1 &= \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24}, \\
A^{(1)} &= \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln\left(\frac{e^{\gamma_E}}{2}\right), \tag{A5}
\end{aligned}$$

with γ_E being the Euler constant, and $\gamma_E = 0.57721$.

The threshold factor $S_t(x)$ can be parametrized as [23,26]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)} [x(1-x)]^c, \tag{A6}$$

where c is determined to be 0.3.

APPENDIX B: WAVE FUNCTION

The distribution amplitudes for the vector meson ρ can be expressed in terms of the Gegenbauer polynomials as [25]

$$\begin{aligned}
\phi_\rho^T(x) &= \frac{3f_\rho^T}{\sqrt{6}} x(1-x)[1 + 0.2C_2^{3/2}(t)], \tag{B1} \\
\phi_\rho^v(x) &= \frac{f_\rho}{2\sqrt{6}} \left[\frac{3}{4}(1+t^2) + 0.24(3t^2-1) \right. \\
&\quad \left. + 0.12(3-30t+35t') \right], \tag{B2}
\end{aligned}$$

$$\phi_\rho^a(x) = \frac{3f_\rho}{4\sqrt{6}} t[1 + 0.93(10x^2 - 10x + 1)], \tag{B3}$$

and

$$\phi_V^T(x) = \frac{f_V^T}{2\sqrt{2N_c}} \phi_\perp, \tag{B4}$$

$$\phi_V^v(x) = \frac{f_V}{2\sqrt{2N_c}} g_\perp^{(v)}, \tag{B5}$$

$$\phi_V^a(x) = \frac{f_V}{8\sqrt{2N_c}} \frac{d}{dx} g_\perp^{(a)}, \tag{B6}$$

with the normalization condition

$$\int_0^1 dx \phi_i(x) = 1, \tag{B7}$$

where $\phi_i = \{\phi_\perp, g_\perp^{(v)}, g_\perp^{(a)}\}$, and

$$C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1). \tag{B8}$$

For the distribution amplitudes of ω , we use the same functions as that of ρ meson because of the similarity of these two vector mesons.

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