

Higher codimension de Sitter branes

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We extend the arguments of Maldacena and Núñez to include higher codimension brane setups and derive a new no-go theorem. Specifically, we show that, under reasonable assumptions on the energy-momentum conservation and the bulk curvature, codimension-two branes fail to support stable de Sitter solutions. For codimensions higher than two embedded in a compact internal space, we show that negative tension sources would be required. This result places strong constraints on the viability of higher-dimensional braneworld models as a means to obtain de Sitter space within string theory.

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Introduction. A fundamental goal of string theory is to provide a UV-complete framework for cosmology. Given the observed accelerated cosmic expansion at late times [1–3], an obvious goal is to find a natural embedding of de Sitter space. However, finding stable de Sitter solutions within string compactifications has proven notoriously difficult. A variety of no-go theorems, beginning with the classical results of Maldacena and Núñez [4], demonstrate that, under broad assumptions such as the presence of a smooth compactification with controlled internal curvature, string theory struggles to accommodate de Sitter vacua. These results have been further reinforced by more recent swampland conjectures, including the de Sitter conjecture [5–7], which suggests that any consistent low-energy effective theory emerging from string theory should either avoid de Sitter space altogether or include instabilities leading to rapid decay. Of course, recent results from the DESI Collaboration [8] point toward a dynamical form for dark energy [9]. However, it is, perhaps, even more difficult to construct robust quintessence mechanisms within string theory [10–16]. See [17] for an explicit construction of inflation at early times and quintessence at late, albeit with a brane uplift.

Efforts to construct explicit de Sitter vacua have generally relied on introducing ingredients such as flux compactification and warped throats, with perturbative and nonperturbative corrections [18,19]. However, these scenarios have been subject to growing scrutiny, with concerns about their consistency within a fully UV-complete setup [20,21]. Given these challenges, alternative mechanisms for realizing de Sitter-like spacetimes in string theory have gained traction. This includes the so-called α' complete cosmologies, which attempt to go beyond perturbative string theory [22–27]. However, it is clear that they suffer from generic instabilities, as might have been expected from world-sheet no-go theorems [28].

Another interesting direction includes braneworld scenarios where our four-dimensional Universe arises as a dynamical brane embedded in a higher-dimensional bulk. Model building with codimension-one branes has enjoyed something of a renaissance of late especially in view of de Sitter phenomenology, most notably through the dark bubble and/or end-of-the-world branes [29–33]. However, here our interest lies in higher codimension brane setups, with two or more codimensions. Although these are much more difficult to study, the interplay between bulk curvature and localized energy-momentum sources can often lead to novel effects, such as the relaxation of the four-dimensional vacuum energy via self-tuning mechanisms [34–39]. Recent work has explored whether codimension-two setups can naturally lead to de Sitter solutions [40].

In this paper, we shall extend the Maldacena-Núñez argument to braneworld setups with higher codimension branes. By analyzing the structure of the Einstein equations in the presence of such branes, we derive a new no-go

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theorem that rules out de Sitter solutions for codimension-two branes under very mild assumptions. For higher codimensions beyond two, we find the need for negative effective brane tensions. Our work highlights the remaining obstacles in the search for a consistent string-theoretic realization of a de Sitter universe and, in doing so, builds upon earlier extensions of the Maldacena-Núñez theorem [41–46]. While these studies incorporated branes or considered energy conditions that can be expected to arise from low-energy string theory [47,48], they did not provide the general discussion of higher-codimensional branes developed here.¹ As a concrete application of our findings, we analyze in Supplemental Material [50] a well-studied six-dimensional supergravity model that was recently claimed to support de Sitter branes [40] (see also Refs. [51–65] therein). In this context, we demonstrate how our no-go result reproduces and strengthens earlier findings, including the vanishing of the on-brane curvature in the thin-brane limit [63]. We conclude with a discussion of implications, assumptions, and possible loopholes—particularly those related to finite thickness effects.

A no-go theorem. We consider a $D = d + n$ -dimensional spacetime with coordinates $X^A = (x^\mu, y^a)$, where x^μ are the d -dimensional coordinates for the external space and y^a describe the n -dimensional internal space. The metric ansatz is

$$ds^2 = e^{2\phi(y)} \bar{\gamma}_{\mu\nu}(x) dx^\mu dx^\nu + \hat{g}_{ab}(y) dy^a dy^b, \quad (1)$$

where $\bar{\gamma}_{\mu\nu}$ is assumed to be the metric for a maximally symmetric spacetime. The Ricci tensor components are explicitly computed as

$$R^\mu_\nu = e^{-2\phi} \bar{R}^\mu_\nu - \delta^\mu_\nu (\hat{\nabla}^2 \phi + d(\hat{\nabla} \phi)^2), \quad (2)$$

$$R^a_b = \hat{R}^a_b - d(\hat{\nabla}^a \hat{\nabla}_b \phi + \hat{\nabla}^a \phi \hat{\nabla}_b \phi). \quad (3)$$

From the Einstein equations, we have that

$$R^A_B = T^A_B - \frac{1}{D-2} T \delta^A_B, \quad (4)$$

from which we obtain the constraint $R^\mu_\mu = \tau$, where

$$\tau = \frac{1}{D-2} [(n-2)T^\mu_\mu - dT^a_a]. \quad (5)$$

Using (2), this yields

$$e^{-2\phi} \bar{R} - e^{-d\phi} \hat{\nabla}^2 e^{d\phi} = \tau. \quad (6)$$

¹Higher-codimensional branes were, however, hinted at as an interesting playground to circumvent existing no-go theorems in [49].

We now multiply through by $e^{d\phi}$ and integrate over a general section of the internal space, $\mathcal{M}$, implying

$$\bar{R}J(\mathcal{M}) - \int_{\partial\mathcal{M}} dS_{n-1} n_a \hat{\nabla}^a e^{d\phi} = K(\mathcal{M}), \quad (7)$$

where

$$J(\mathcal{M}) = \int_{\mathcal{M}} dV_n e^{(d-2)\phi}, \quad (8)$$

$$K(\mathcal{M}) = \int_{\mathcal{M}} dV_n e^{d\phi} \tau. \quad (9)$$

Here, $dV_n = \sqrt{\hat{g}} d^n y$ is the volume measure on the internal space section, $\mathcal{M}$, and dS_{n-1} the measure on its boundary $\partial\mathcal{M}$.

We now separate the full internal space $\mathcal{M}_{\text{tot}}$ into a compact region $\mathcal{M}_{\text{out}}$ containing no brane sources and compact regions $\mathcal{M}_i$ containing the i th brane source. It follows that

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_{\text{out}} \cup (\cup_i \mathcal{M}_i). \quad (10)$$

Of course, the full internal space is also compact, and we further assume that it has no overall boundary, $\partial\mathcal{M}_{\text{tot}} = 0$. We then see that

$$\partial\mathcal{M}_{\text{out}} = -\cup_i \partial\mathcal{M}_i. \quad (11)$$

Since

$$\bar{R}J(\mathcal{M}_{\text{out}}) - \int_{\partial\mathcal{M}_{\text{out}}} dS_{n-1} n_a \hat{\nabla}^a e^{d\phi} = K(\mathcal{M}_{\text{out}}), \quad (12)$$

$$\bar{R}J(\mathcal{M}_i) - \int_{\partial\mathcal{M}_i} dS_{n-1} n_a \hat{\nabla}^a e^{d\phi} = K(\mathcal{M}_i), \quad (13)$$

we infer that

$$\begin{aligned} \bar{R} \left[J(\mathcal{M}_{\text{out}}) + \sum_i J(\mathcal{M}_i) \right] \\ = K(\mathcal{M}_{\text{out}}) + \sum_i K(\mathcal{M}_i), \end{aligned} \quad (14)$$

or, equivalently,

$$\bar{R}J(\mathcal{M}_{\text{tot}}) = K(\mathcal{M}_{\text{out}}) + \sum_i K(\mathcal{M}_i). \quad (15)$$

Since $\mathcal{M}_{\text{tot}}$ is compact, it is clear that $J(\mathcal{M}_{\text{tot}}) > 0$ and finite.² Like Maldacena and Núñez [4], we also assume $\tau \leq 0$, albeit away from the brane sources, giving

²We stress that models with infinite bulk volume are not covered by our arguments.

$K(\mathcal{M}_{\text{out}}) \leq 0$. In the absence of the brane contributions given by $K(\mathcal{M}_i)$, we immediately recover the Maldacena-Núñez theorem, which yields $\bar{R} \leq 0$, ruling out de Sitter solutions on the external space. Here, we extend the theorem by including contributions from regions close to the brane sources. In these regions, we no longer make any assumptions about the sign of τ . However, we do assume conservation of the energy-momentum tensor. As we will discuss shortly, the energy-momentum tensor in these near-brane neighborhoods incorporates the effects of both bulk fields and regularized brane sources. We will later show that these near-brane contributions either diverge or vanish in the limit where any regulator is removed.

To explore the near-brane contributions more closely, we choose coordinates in a small neighborhood of the i th brane so that metric on the internal space is

$$\hat{g}_{ab}(y)dy^a dy^b = dr^2 + A^2(r)q_{ij}(\theta)d\theta^i d\theta^j. \quad (16)$$

Here, the brane is assumed to lie at $r = 0$, with r measuring the radial distance from the source. $A(r)$ measures the size of the compact $n - 1$ -dimensional space and vanishes as $r \rightarrow 0$ in agreement with having a higher codimensional defect. The compact space is typically a sphere with angular coordinates θ^i and metric $q_{ij}(\theta)$. For a sufficiently small neighborhood, we further assume that $\phi = \phi(r)$.

The value of ϕ at the brane is unphysical and can be scaled to any value we choose. This is because a shift $\phi \rightarrow \phi + c$ for some (possibly divergent) constant c can be absorbed by a constant rescaling of the metric $\bar{\gamma}_{\mu\nu} \rightarrow e^{-2c}\bar{\gamma}_{\mu\nu}$. Note that the physical brane curvature $e^{-2\phi(0)}\bar{R}$ is invariant under these shifts. Furthermore, since $\bar{R} \rightarrow e^{2c}\bar{R}$, $J \rightarrow e^{(d-2)c}J$, and $K \rightarrow e^{dc}K$, it follows that both sides of the curvature equation (14) pick up an overall factor of e^{dc} , which can be canceled. We will use this freedom to assume, without loss of generality, that $\phi(r)$ remains finite as $r \rightarrow 0^+$.

What about the energy-momentum tensor? As we alluded to earlier, in the neighborhood of the brane, there could be contributions from bulk fields as well as localized contributions from the brane itself. For higher codimension branes, the latter are typically regularized in some way. This requirement arises because the gravitational field generally diverges at the position of a higher-codimensional object, much like the Coulomb potential diverges at the position of a point charge. Such divergences are not a problem *per se*, but they must be treated in a manner that ensures observable quantities such as the on-brane curvature remain finite and can be computed reliably. Regularizing the brane by assigning it a finite transverse width is particularly convenient, as it allows one to extract the matching conditions for the bulk fields. This approach has been studied extensively for codimension-two branes, where we may consider a codimension-one brane of fixed radius and subsequently takes the radius to be small [57–59,62,63].

More generally, we assume that the (regularized) energy-momentum tensor has the following nonvanishing components:

$$T_B^A = (T_\nu^\mu(r), T_r^r(r), T_j^i(r)) \quad (17)$$

in the neighborhood of the brane, depending only on the radial coordinate. By integrating the regularized source over this small neighborhood, we obtain the effective energy-momentum of the source:

$$T_{\text{eff } B}^A = \int_{\mathcal{M}_i} d^n y \sqrt{\hat{g}} T_B^A, \quad (18)$$

which must be finite for a successful regularization. Henceforth, we take $\mathcal{M}_i$ to be the region defined by $0 \leq r \leq \epsilon$, for some arbitrarily small $\epsilon > 0$. It follows that

$$T_{\text{eff } B}^A = \Omega_{n-1} \int_0^\epsilon dr A^{n-1} T_B^A, \quad (19)$$

where $\Omega_{n-1} = \int d^{n-1}\theta \sqrt{q}$ is the volume of the compact angular directions.

We now want to estimate the near-brane contribution to our formula (14) for the brane curvature. To this end, we exploit the energy-conservation equation

$$\partial_r T_r^r + \frac{A'}{A} [(n-1)T_r^r - T_i^i] + \phi'(dT_r^r - T_\mu^\mu) = 0. \quad (20)$$

This allows us to eliminate the angular contribution T_i^i in our expression for $K(\mathcal{M}_i)$. In particular, we find that

$$\tau = \frac{1}{D-2} \left[\left(n-2 + \frac{d\phi' A}{A'} \right) T_\mu^\mu - \frac{dA}{A'} \partial_r T_r^r - d \left(n + \frac{d\phi' A}{A'} \right) T_r^r \right]. \quad (21)$$

Plugging this into our expression (9) for $K(\mathcal{M}_i)$, we obtain

$$K(\mathcal{M}_i) = \frac{e^{d\phi(0)}}{D-2} [(n-2)T_{\text{eff } \mu}^\mu - \Omega_{n-1}(\Delta_1(\mathcal{M}_i) + \Delta_2(\mathcal{M}_i))], \quad (22)$$

where we define

$$\Delta_1(\mathcal{M}_i) = \int_0^\epsilon dr A^{n-1} \omega(r) T_\mu^\mu, \quad (23)$$

$$\Delta_2(\mathcal{M}_i) = de^{-d\phi(0)} \int_0^\epsilon dr \frac{\partial_r (A^n e^{d\phi} T_r^r)}{A'}, \quad (24)$$

and a weighting function

$$\omega(r) = (n-2)[1 - e^{d(\phi(r)-\phi(0))}] - e^{d(\phi(r)-\phi(0))} d\phi' \frac{A}{A'}. \quad (25)$$

Near the regularized brane, we take $A \approx A_0^{(\epsilon)} r^\alpha$ and $\phi \approx \phi_0^{(\epsilon)} + \phi_1^{(\epsilon)} r^\beta$, where $\alpha, \beta > 0$ and the constant coefficients depending on the regularization scale. $\phi_0^{(\epsilon)}$ is assumed to remain finite and nonvanishing as we remove the regulator, $\epsilon \rightarrow 0^+$. As explained earlier, this assumption is carried out without loss of generality thanks to the freedom to shift, $\phi \rightarrow \phi + c$, for any constant c . We impose even weaker constraints on $\phi_1^{(\epsilon)}$ and $A_0^{(\epsilon)} > 0$, requiring that $A_0^{(\epsilon)} \epsilon^{\alpha-1}$ remains finite and that $\phi_1^{(\epsilon)} \epsilon^\beta \rightarrow 0$ as $\epsilon \rightarrow 0^+$. The first condition ensures that the gradient $A'(\epsilon)$ remains finite in the limit $\epsilon \rightarrow 0^+$ (in accordance with having a genuine codimension- n defect), while the second guarantees that $\phi(\epsilon) \rightarrow \phi_0$.

We now estimate the weighting function

$$\omega(r) \approx -\left(n-2+\frac{\beta}{\alpha}\right) d\phi_1^{(\epsilon)} r^\beta, \quad (26)$$

which is vanishingly small in the limit that $\epsilon \rightarrow 0^+$ for $0 \leq r \leq \epsilon$. Although T_μ^μ can and does diverge in this limit, it does so while guaranteeing that $T_{\text{eff}}^\mu{}_\mu = \Omega_{n-1} \int_0^\epsilon dr A^{n-1} T_\mu^\mu$ remains finite. The presence of a vanishingly small weighting function in Δ_1 will generically³ force the integral to vanish as we remove the regulator. We conclude that $\Delta_1 \rightarrow 0$ as $\epsilon \rightarrow 0^+$.

The second integral, Δ_2 , requires a little more work. To estimate the contribution of T_r^r , we make use of the Einstein equation:

$$T_r^r = \frac{1}{2} \left[(n-1)(n-2) \left(\frac{A'}{A} \right)^2 - \frac{R(q)}{A^2} - \bar{R} e^{-2\phi_0} + 2(n-1) d\phi' \frac{A'}{A} + d(d-1) \phi'^2 \right], \quad (27)$$

where $R(q) > 0$ is the Ricci curvature of the compact space with metric q_{ij} . It follows that, in a neighborhood of the brane,

$$T_r^r \approx \sum_i \lambda_i r^{-2\gamma_i} \approx \frac{1}{2} \left[(n-1)(n-2) \alpha^2 r^{-2} - \frac{R(q)}{(A_0^{(\epsilon)})^2} r^{-2\alpha} - \bar{R} e^{-2\phi_0} + 2(n-1) d\alpha \beta \phi_1 r^{\beta-2} + d(d-1) \beta^2 \phi_1^2 r^{2\beta-2} \right]. \quad (28)$$

This yields

³This will certainly be true if the regulated source T_μ^μ does not flip sign for $0 \leq r \leq \epsilon$, as can be easily proved using the triangle inequality.

$$\Delta_2(\mathcal{M}_i) \approx (A_0^{(\epsilon)})^{n-1} d \sum_i \lambda_i s_i \left[\frac{r^{\rho_i}}{\rho_i} \right]_0^\epsilon \quad (29)$$

with $s_i = n - 2\gamma_i/\alpha$ and $\rho_i = \alpha(s_i - 1) + 1$. Note that this result has a divergence at the origin if and only if $\rho_i \leq 0$ and $\lambda_i s_i \neq 0$ for any i . This divergence corresponds to a bad regulator and is there even for finite brane thickness. We, therefore, restrict attention to the complement, corresponding to good regulators where there is no divergence at the origin at finite thickness. In that case, the contribution at the origin vanishes, and we are left with

$$\Delta_2(\mathcal{M}_i) \approx (A_0^{(\epsilon)})^{n-1} d \sum_i \lambda_i s_i \frac{\epsilon^{\rho_i}}{\rho_i} = \frac{dW(\epsilon)}{2\alpha^n} \epsilon^{n-2}, \quad (30)$$

where

$$W(\epsilon) = [A'(\epsilon)]^{n-1} \left[\frac{(n-1)(n-2)\alpha^2(\alpha n-2)}{\alpha(n-1)-1} - \frac{\alpha n}{\alpha(n-1)+1} \bar{R} e^{-2\phi_0} \epsilon^2 + \frac{2d(n-1)\alpha\beta(\alpha n+\beta-2)}{\alpha(n-1)+\beta-1} [\phi(\epsilon)-\phi_0] + d(d-1) \frac{\beta^2(\alpha n+2\beta-2)}{\alpha(n-1)+2\beta-1} [\phi(\epsilon)-\phi_0]^2 \right] - [A'(\epsilon)]^{n-3} R(q) \frac{(n-2)\alpha^3}{\alpha(n-3)+1}. \quad (31)$$

As we remove the regulator, $\epsilon \rightarrow 0^+$, recall that we required that $\phi(\epsilon) \rightarrow \phi_0$ and that $A'(\epsilon)$ remains finite. For $n=2$, we see that the first and last terms vanish identically in our expression for $W(\epsilon)$, with the remaining terms vanishing in the limit $\epsilon \rightarrow 0^+$. Meanwhile, for $n \geq 3$, $W(\epsilon)$ remains finite. It follows that, for $n \geq 2$, the right-hand side of (30) vanishes as $\epsilon \rightarrow 0^+$.

We, therefore, have two possibilities: As we remove the regulator, either Δ_2 diverges or it vanishes. In fact, this outcome might have been anticipated: Δ_2 can be understood as capturing brane contributions from internal directions transverse to the brane—such as radial and angular directions. In the regulated system, these contributions can be finite and nonzero. However, as the regulator is removed, symmetry considerations suggest that such off-brane contributions must either vanish or diverge. For well-defined regulators, they vanish.

If there is a divergence in Δ_2 , there is also a divergence in the brane curvature, via Eq. (14), so we dismiss this as unphysical. In contrast, if there is no divergence, we have $\Delta_1, \Delta_2 \rightarrow 0$ as $\epsilon \rightarrow 0^+$ and so

$$K(\mathcal{M}_i) \approx \frac{e^{d\phi_0}}{D-2} (n-2) T_{\text{eff}}^\mu{}_\mu. \quad (32)$$

For higher codimensions ($n \geq 3$), the brane sources give a positive contribution to the brane curvature if and only if $T_{\text{eff}}^\mu > 0$. For effective brane tensions $T_{\text{eff}}^\nu = -\sigma\delta^\nu_\mu$, the tension must be negative $\sigma < 0$. Actually, negative tension objects exist in string theory in the form of Op planes [66], although they correspond to fixed points of an orientifold symmetry and are not dynamical objects. Moreover, it has recently been argued in [48] that Op planes are not suited to generate de Sitter solutions when the tadpole cancellation condition is taken into account. More speculatively, negative tension branes have also been proposed as formal, supersymmetric objects that arise via T duality on a timelike circle [67,68]. For codimension-two branes ($n = 2$), we see that (32) vanishes and fails to give the positive contribution required to support a positive curvature on the brane. In this case, we might hope that subleading contributions come in with the right sign to support de Sitter solutions. However, it is clear that these contributions are vanishingly small in the limit where $\epsilon \rightarrow 0^+$.

This completes our no-go theorem, ruling out the possibility of de Sitter vacua on codimension-two branes. For branes of higher codimension, the presence of negative tension objects appears to be necessary. An obvious loophole, however, is to retain the regulator and allow for branes of finite thickness. Strictly speaking, such objects are no longer truly of codimension two or higher. To see how finite-thickness branes might give rise to de Sitter solutions, let us return to Eq. (30) and assume $\rho_i > 0$ and $\lambda_i s_i \neq 0$ while keeping ϵ finite. It is straightforward to see that, when $\lambda_i s_i < 0$, Δ_2 acquires a finite negative value. Through Eq. (14), this contributes positively to the brane curvature, thereby opening up the possibility of de Sitter solutions on branes of finite thickness.

Conclusions. In this work, we have extended the Maldacena-Nuñez no-go theorem to higher codimension brane setups and derived robust constraints on the existence of de Sitter vacua in such models. Our analysis rests on several mild but well-motivated assumptions: a compact internal space, supergravity in the bulk (i.e., $\tau \leq 0$), finite on-brane curvature, and the presence of genuine codimension- n sources—i.e., delta-function-like defects with no finite width. We have also worked entirely within the framework of standard general relativity.

Under these assumptions, we showed that codimension-two branes cannot support de Sitter curvature, while higher

codimension branes require sources with negative effective tension. These can, in principle, be realized by orientifold planes in string theory [66], although such objects are nondynamical and additional stability constraints need to be taken into account [48]. This result places strong constraints on braneworld approaches to de Sitter model building and highlights the difficulty of obtaining positive cosmological constant solutions in higher-dimensional theories with localized sources.

Nevertheless, an intriguing loophole emerges when the assumption of infinitesimal brane thickness is relaxed. Finite-thickness effects can be systematically incorporated via induced gravity terms on the brane, as shown in [69,70]. These corrections modify the junction conditions, introducing new contributions that depend explicitly on the brane curvature. Within our formalism, such effects correspond to curvature depending contributions to T^μ_μ . It is notable that an induced Einstein Hilbert term on the brane mimics negative tension when the brane curvature is positive.

However, for higher codimension defects, these induced gravity terms are often accompanied by ghostlike instabilities [71,72]. That said, the universality of these ghost modes has been disputed, at least for large enough brane tensions [38,73,74]. This suggests that finite brane thickness, encoded via induced curvature corrections, presents a promising direction for future investigation. While not guaranteed to circumvent our no-go theorem, this loophole provides a technically well-defined and physically motivated framework to explore whether regulated sources can support metastable de Sitter solutions. A detailed study of such configurations, particularly their stability and embedding in string compactifications, may shed new light on the possibility of realizing de Sitter space in higher-dimensional braneworld scenarios.

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