Letter

Cosmological discrete self-similarity in primordial black hole formation

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We demonstrate that discrete self-similarity (DSS), originally discovered in the asymptotically flat collapse of a massless scalar field, survives in primordial black hole (PBH) formation within an expanding cosmological background. Using fully relativistic simulations of massless scalar-field collapse in a Friedmann-Lemaître-Robertson-Walker universe, we resolve the near-critical regime and find clear log-periodic oscillations in the PBH mass-scaling relation. This result provides a proof of principle that PBHs may retain information about the critical behavior in their formation realized in the early Universe: PBH formation in scalar-field kination exhibits DSS, while radiation-fluid collapse is characterized by continuous self-similarity. The resulting mass scaling imprints characteristic log-periodic modulations in an illustrative PBH mass spectrum, which may survive when PBH formation is dominated by a sufficiently narrow range of scales, with possible consequences for PBH phenomenology. Our results show that PBHs may encode not only the amplitude of primordial perturbations, but also the nature of the matter sector and the dynamical state of the Universe at horizon reentry.

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Introduction. Primordial black holes (PBHs) [1–4] provide a sensitive probe of the early universe and physics at energy scales inaccessible to laboratories. In many scenarios PBHs form from the collapse of large-amplitude perturbations shortly after horizon reentry, leading to a critical regime where the black-hole mass follows a universal scaling law

$$M_{\rm PBH} \propto (p - p_{\rm th})^{\gamma},$$

where p is a control parameter of the initial data, $p_{\rm th}$ denotes its critical value separating dispersion from collapse, and γ is a universal critical exponent. Because the pre-BBN expansion history is poorly constrained, PBHs may probe not only the amplitude of primordial perturbations, but also the matter model and dynamical phase of the Universe at horizon reentry [5].

Critical phenomena in gravitational collapse were discovered by Choptuik in an asymptotically flat spacetime for a massless scalar field [6], revealing both the scaling exponent γ and a discretely self-similar (DSS) critical solution characterized by "echoes" periodic in logarithmic scale. In that setting, the mass scaling is more precisely described by [7,8]

$$\ln M_{\rm PBH} = \gamma \ln(p - p_{\rm th}) + f[\ln(p - p_{\rm th})], \tag{1}$$

where f is a periodic function encoding the DSS of the critical solution. We denote by $P_{\ln}$ the period of f in its argument $\ln(p - p_{\rm th})$, so that $f(x + P_{\ln}) = f(x)$. In standard DSS collapse one may equivalently characterize self-similarity by an echoing period Δ in logarithmic self-similar time, with $P_{\ln} = \Delta/(2\gamma)$ [7]. The above log-periodic feature is not seen in the critical behavior caused by a continuously self-similar (CSS) solution observed in the perfect fluid system with a linear equation of state including radiation fluid. For a timelike scalar-field gradient, a massless scalar admits a local perfect-fluid representation with effective equation of state $w = 1$, which underlies our construction of superhorizon initial data below. Nevertheless, its nonlinear collapse dynamics is not equivalent to that of a perfect stiff fluid. This distinction is essential here: the DSS behavior studied below is a property of the massless scalar-field system and probes the field-theoretic nature of the dominant component, not only the background equation of state. See Brady *et al.* [9].

In the cosmological context relevant for PBH formation, critical collapse takes place within an expanding background that introduces a physical scale through the Hubble horizon, thereby breaking exact scale invariance. This naturally raises the question of whether the DSS critical solution survives in such an environment. Scalar-field kination is not an *ad hoc* alternative to radiation domination, but a well-studied postinflationary phase arising in quintessential inflation, runaway or nonoscillatory inflation, moduli-dominated histories, and scalar-field reheating scenarios [10–13]. PBH formation during kination has recently attracted interest because the nonstandard expansion history can modify PBH abundances, the required small-scale primordial power, and induced gravitational-wave signals [14–16], while nonlinear scalar inhomogeneities during kination have been studied with numerical relativity [17]. This setting provides a proof of principle that PBHs can probe different forms of critical behavior in the early Universe: radiation-fluid collapse exhibits CSS, whereas massless scalar-field kination exhibits DSS. In this Letter we demonstrate that DSS persists in cosmological massless scalar-field collapse, and clarify how the near-critical self-similar solution emerges within an expanding background.

Setup: Cosmological scalar-field collapse. We perform fully relativistic numerical simulations of spherically symmetric collapse using a previously validated numerical code based on the extended Misner-Sharp formalism [18,19]. The spacetime metric is written in comoving coordinates to a perfect fluid adapted to spherical symmetry,

$$ds^2 = -e^{2\phi}dt^2 + e^{\lambda}dA^2 + R^2 d\Omega^2,$$

where $\phi(t,A)$ is the lapse function, $\lambda(t,A)$ encodes the radial metric component, $R(t,A)$ is the areal radius, and $d\Omega^2$ is the metric on the unit sphere. This formulation leads to a coupled system of evolution and constraint equations for the metric and matter variables, which we solve numerically (see Supplemental Material [20] for details).

Initial conditions are constructed using the gradient-expansion formalism [21,22], which provides a systematic description of superhorizon perturbations in an expanding universe. In this framework, all initial inhomogeneities are encoded in a single curvature perturbation function $K(A)$, from which the metric and matter variables are consistently determined order by order in spatial gradients. To leading nontrivial order, the dynamical variables admit expansions of the form

$$R = aA(1+\epsilon^2\tilde{R}[K]), \qquad \rho_\psi = \rho_b(1+\epsilon^2\tilde{\rho}_\psi[K]),$$

where ϵ is the gradient-expansion parameter, a is the scale factor, and ρ_b denotes the background energy density. This construction yields consistent initial data well outside the horizon, which are subsequently evolved through horizon reentry.

We consider a real, massless scalar field ψ. We use the gradient expansion and the initial curvature profile $K(A)$ to construct the associated initial scalar-field energy density $\rho_\psi(t_0,A)$. The scalar-field variables are then initialized as

$$\psi(t_0,A) = \partial_A\psi(t_0,A) = 0, \qquad \Pi(t_0,A) = \sqrt{2\rho_\psi(t_0,A)},$$

where $\Pi \equiv e^{-\phi}\dot{\psi}$. With this choice the scalar field is initially comoving with the coordinates, $\nabla_\mu\psi \propto \delta^t_\mu$, so that the timelike-gradient condition required for the initial perfect-fluid identification is satisfied by construction, while no restriction is imposed on the subsequent dynamical evolution of the scalar field.

As initial conditions, we consider two families of curvature profiles $K(A)$: a Gaussian profile and a piecewise rational profile, given respectively by

$$K_G(A) = pe^{-(A/A_m)^2},$$

and

$$K_P(A) = \frac{p[1-(3x^2-2x^3)]}{1+(A/A_m)^2},$$

where

$$x(A) = \begin{cases} 0, & A < A_m, \\ \frac{A-A_m}{4A_m}, & A_m < A < 5A_m, \\ 1, & A > 5A_m. \end{cases}$$

In the above expressions, A_m sets the characteristic comoving scale of the perturbation and is chosen so that the corresponding physical scale is much larger than the initial cosmological horizon, $R_H \equiv H^{-1}$.

The formation of a PBH is identified through the appearance of a future outer trapping horizon (see Supplemental Material [20]). Specifically, we monitor the expansions of outgoing and ingoing radial null geodesics, Θ_+ and Θ_-, and locate the first radius at which $\Theta_+ = 0$ while $\Theta_- < 0$. The corresponding Misner-Sharp mass evaluated at the apparent horizon defines the PBH mass M_{PBH}.

For comparison with the PBH literature, we also characterize the initial data by the maximum compaction function, $\mathcal{C}_{max}$, evaluated on our Misner-Sharp slicing (comoving to a fiducial radiation fluid),

$$\mathcal{C}(t,A) \equiv \frac{2[M(t,A)-M_b(t,A)]}{R(t,A)},$$

where $M(t,A)$ is the Misner-Sharp mass and $M_b(t,A)$ denotes the corresponding background mass.

Critical scaling and discrete self-similarity. We first determine the collapse threshold for each family of initial data. In terms of the tuning parameter p, the threshold value p_{th} is obtained by bisection, separating dispersal from PBH formation with absolute precision $|p-p_{th}| \sim 10^{-8}$. For the Gaussian and piecewise rational curvature profiles we find

$$p^G_{th} = 0.089\,573\,044,$$
$$p^P_{th} = 0.046\,397\,240.$$

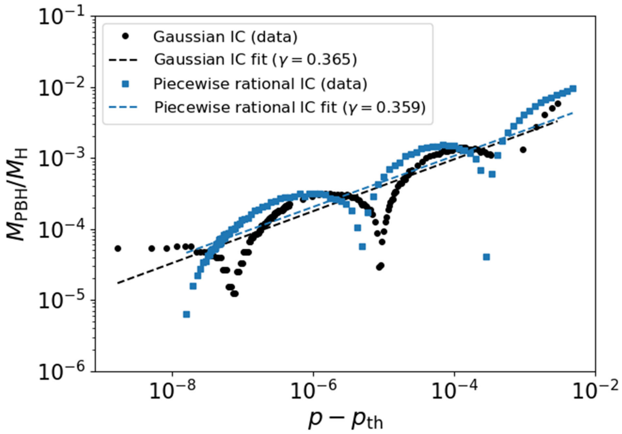

FIG. 1. PBH mass scaling for Gaussian (black) and piecewise rational (blue) initial data, with the PBH mass normalized by the horizon mass at horizon entry. Both profiles follow the same power-law envelope with superimposed log-periodic oscillations, consistent with a common DSS period.

The corresponding threshold values expressed in terms of the maximum of the compaction function are

$$\mathcal{C}^{\rm G}_{\rm max,th} = 0.559\,548\,79, \qquad \mathcal{C}^{\rm P}_{\rm max,th} = 0.535\,547\,84.$$

Having determined the collapse threshold, we probe the near-critical regime by tuning p slightly above $p_{\rm th}$ and measuring the resulting PBH masses. The corresponding mass-scaling relations for both families of initial data are shown in Fig. 1. In both cases the black-hole mass follows a power-law envelope in $(p - p_{\rm th})$ with superimposed log-periodic oscillations characteristic of DSS critical collapse, consistent with the approximate repetition of suitably rescaled near-critical field profiles in self-similar variables (see Supplemental Material [20]). The DSS modulation makes $M_{\rm PBH}$ nonmonotonic in p and produces deep minima at specific phases of the echoing cycle. Resolving their detailed behavior would require higher resolution near these phases; whether the minima remain finite or approach zero is left for future investigation [23]. Previous studies of asymptotically flat scalar-field collapse modeled this oscillatory modulation as sinusoidal [24]. In our simulations, however, the downward excursions appear sharper than the upward peaks. While mild systematic deviations from a sinusoidal form can also be observed in asymptotically flat collapse [24], the substantially larger number of simulations performed in this work ($\sim 10^3$ for each initial condition) allows for a more detailed sampling of the near-critical regime. This extended dataset suggests somewhat more complex oscillatory structure: a simple sinusoidal model captures the overall modulation but does not fully reproduce the detailed shape of the residuals (see Supplemental Material [20]).

Following Eq. (1), we extract the critical exponent γ and the logarithmic oscillation period $P_{\ln}$ by fitting the mass-scaling relation. To determine the oscillation period independently, we perform a Lomb-Scargle analysis of the residuals after subtracting the power-law envelope (see Supplemental Material [20]). This procedure yields

$$\gamma_{\rm G} = 0.365 \pm 0.017, \qquad P_{\ln,\rm G} = 4.67 \pm 0.10,$$

while for piecewise rational initial data we find

$$\gamma_{\rm P} = 0.359 \pm 0.019, \qquad P_{\ln,\rm P} = 4.74 \pm 0.14.$$

For comparison, in the asymptotically flat collapse of a massless scalar field the DSS critical solution has $\gamma_{\rm AF} \simeq 0.374$ and $P_{\ln,\rm AF} \simeq 4.60$ [6,7]. In practice, extracting γ and $P_{\ln}$ requires sampling extremely close to criticality. In our numerical simulations—which evolve the collapse on a cosmological background—we were able to reach $(p - p_{\rm th}) \sim 10^{-8}$, corresponding to a finite dynamically accessible near-critical mass range. This level of fine-tuning of the initial-data parameter is comparable to that achieved in the original asymptotically flat studies of Choptuik as well as in many subsequent numerical investigations of critical behavior. The resolved range therefore allows a meaningful comparison with previous numerical results, although its finite extent does not establish conclusively that the fully asymptotic limit has been reached. The fitted parameters should therefore be regarded as effective values extracted over the available window. Nevertheless, both families of initial data yield mutually consistent results, and the measured γ and $P_{\ln}$ remain within 1σ of the asymptotically flat benchmark, supporting the interpretation that the local near-critical dynamics approaches the standard DSS intermediate attractor. Note also that the measured values for γ are clearly distinct from the CSS value ~ 0.94 [9].

Survival of DSS in an expanding universe. The persistence of DSS in a cosmological background can be understood from the hierarchy of scales that develops near the black-hole threshold. Although the Friedmann-Lemaître-Robertson-Walker geometry (FLRW) introduces the Hubble radius H^{-1} as a natural large-scale reference, the region that undergoes critical collapse becomes progressively smaller as the threshold is approached. In particular, the characteristic size of the collapsing core scales as $(p - p_{\rm th})^{\gamma}$, and can therefore be made arbitrarily small compared to the cosmological horizon.

As a consequence, sufficiently close to criticality the dynamics in the central region is governed by local nonlinear gravitational collapse, while the cosmological expansion primarily affects only the large-scale exterior and its matching to the FLRW background. The near-central evolution thus approaches the same universal DSS critical solution known in asymptotically flat spacetime. Our results indicate that discrete self-similarity survives in an expanding universe as a manifestation of local critical dynamics, with cosmological effects entering through the global structure of the spacetime rather than altering the core collapse itself.

Implications for primordial black-hole mass functions. The presence of DSS can leave characteristic imprints on the PBH mass function. The log-periodic modulation in Eq. (1) modifies the mapping between the collapse parameter and the resulting PBH mass through a modulation of the Jacobian relating p and $M_{\rm PBH}$.

At a given horizon mass, the PBH mass function may be written as

$$\psi(\ln M_{\rm PBH}) \equiv \frac{dn_{\rm PBH}}{d\ln M_{\rm PBH}} \propto \sum_i P(\delta_H)\left|\frac{d\delta_H}{d\ln M_{\rm PBH}}\right|_{\delta_{H,i}}, \quad (2)$$

where the sum runs over all supercritical solutions $\delta_{H,i} > \delta_{H,\rm th}$ satisfying the mass-scaling relation for a fixed $M_{\rm PBH}$, $\delta_H = C_{\rm max}$ and $P(\delta_H)$ is the distribution of the maximum of the compaction function. We work at fixed horizon mass, corresponding to an infinitely narrow distribution of formation scales. Even in this limit, critical scaling maps the continuous distribution of supercritical amplitudes into an extended PBH mass function, producing the minimum width familiar from standard critical-collapse calculations [25,26]. Using δ_H as our control parameter,

$$\ln M_{\rm PBH} = \gamma \ln(\delta_H - \delta_{H,\rm th}) + f[\ln(\delta_H - \delta_{H,\rm th})],$$

so that

$$\frac{d\ln M_{\rm PBH}}{dx} = \gamma + f'(x), \qquad x = \ln(\delta_H - \delta_{H,\rm th}),$$

and the Jacobian acquires a log-periodic modulation.

We model the oscillatory contribution phenomenologically using a periodic function motivated by the numerical residuals (see Supplemental Material [20] for details),

$$f(x) = C + A\ln\left[\epsilon + \left|\cos\left(\pi\frac{x}{P_{\ln}} + \phi\right)\right|\right], \quad (3)$$

where $P_{\ln}$ is fixed independently from the residual analysis, $C \simeq 0$ and $A \simeq 0.3$ from a least-square fitting, and ϵ is a small positive regularization parameter.

The DSS-induced modulation redistributes statistical weight in mass space, generating characteristic shifts and localized enhancements whose spacing is set by the universal DSS period. In particular, since the mass function involves the inverse of $\gamma + f'(x)$, the modulation can lead to sharp, resonancelike enhancements when $\gamma + f'(x)$ becomes small, corresponding to a local compression of the mapping between δ_H and $M_{\rm PBH}$. These features are smoothed by the finite width of $P(\delta_H)$ and by the regularization parameter ϵ, but can still produce pronounced distortions in the mass spectrum. Figure 2 illustrates a representative realization of this behavior, where spikes are generated by the nonmonotonicity of $M_{\rm PBH}(\delta_H)$. Here $P(\delta_H)$ is a zero-mean Gaussian distribution with standard deviation σ, and the horizon mass is held fixed. The figure is therefore a diagnostic construction that isolates the DSS imprint of the collapse dynamics, rather than a mass function derived from a finite-width primordial power spectrum. A realistic calculation additionally requires an integration over formation scales. For a narrow primordial spectrum, PBH production is localized around a small range of horizon masses, and the corresponding DSS patterns remain approximately aligned in logarithmic mass, allowing the modulation to survive. For a broad spectrum, contributions from widely separated horizon masses superpose with displaced peaks and troughs and progressively average out. Since the characteristic DSS spacing is $\Delta\ln M_{\rm PBH} \simeq \gamma P_{\ln} \simeq 1.7$, integration over several such intervals is expected to strongly suppress the signal. The detailed survival depends on the smoothing prescription and will be studied quantitatively in a companion work in preparation [27].

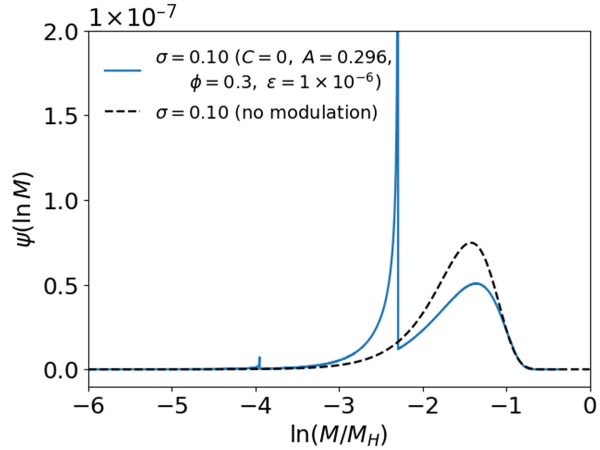

FIG. 2. Illustrative PBH mass functions derived from the critical scaling relation. The dashed curve corresponds to the case without DSS modulation, while the solid curve includes a log-periodic modulation of the form $f(x)$ described in the text. We assume a Gaussian distribution $P(\delta_H)$. The horizon mass is held fixed, so the width of the mass function is generated by critical scaling rather than by a finite-width primordial power spectrum. The DSS modulation redistributes statistical weight in mass space, producing not only shifts and distortions of the dominant peak but also spikes due to the nonmonotonicity of $M_{\rm PBH}(\delta_H)$.

Conclusions. In this Letter we have demonstrated that discrete self-similarity (DSS), originally discovered in the asymptotically flat collapse of a massless scalar field, also emerges in primordial black-hole (PBH) formation within an expanding cosmological background. Fully relativistic simulations reveal clear log-periodic modulations in the PBH mass-scaling relation, providing direct evidence for a DSS critical solution in the cosmological setting.

The critical exponents and logarithmic periods extracted from two distinct families of initial data are mutually consistent and close to the asymptotically flat benchmarks, supporting the interpretation that DSS is a local property of critical collapse, largely insensitive to the global cosmological expansion. The extended dataset also reveals departures from a simple sinusoidal modulation. Although the two initial-data families yield mutually consistent logarithmic periods, differences in the fitted phase and in the detailed manner in which the DSS modulation is sampled suggest that a sufficiently resolved PBH mass pattern could, in principle, retain some information about the primordial perturbation profile. Establishing this connection quantitatively requires a broader survey of initial data.

The DSS modulation can produce characteristic log-periodic structure in the PBH mass function, suggesting that, when this structure survives the averaging over formation scales, PBHs may probe not only primordial fluctuation amplitudes but also the critical behavior realized in the early Universe. Exact DSS relies on the scale-free massless-scalar system. In realistic kination scenarios, a subdominant potential, radiation component, or scalar-field decay introduces additional scales; if these effects remain negligible over the near-critical region, the DSS solution may persist as an intermediate attractor over a finite range, with the modulation eventually truncated or smoothed.

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Data availability. The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

[1] Y. B. Zel'dovich and I. D. Novikov, The hypothesis of cores retarded during expansion and the hot cosmological model, Sov. Astron. **10**, 602 (1967).
[2] S. Hawking, Gravitationally collapsed objects of very low mass, Mon. Not. R. Astron. Soc. **152**, 75 (1971).
[3] B. J. Carr and S. W. Hawking, Black holes in the early universe, Mon. Not. R. Astron. Soc. **168**, 399 (1974).
[4] *Primordial Black Holes*, edited by C. Byrnes, G. Franciolini, T. Harada, P. Pani, and M. Sasaki, Springer Series in Astrophysics and Cosmology (Springer, New York, 2025).
[5] R. Allahverdi *et al.*, The first three seconds: A review of possible expansion histories of the early universe, Open J. Astrophys. **4**, 1 (2021).
[6] M. W. Choptuik, Universality and scaling in gravitational collapse of a massless scalar field, Phys. Rev. Lett. **70**, 9 (1993).
[7] S. Hod and T. Piran, Fine structure of Choptuik's mass scaling relation, Phys. Rev. D **55**, 440 (1996).
[8] C. Gundlach, Understanding critical collapse of a scalar field, Phys. Rev. D **55**, 695 (1997).
[9] P. R. Brady, M. W. Choptuik, C. Gundlach, and D. W. Neilsen, Black hole threshold solutions in stiff fluid collapse, Classical Quantum Gravity **19**, 6359 (2002).
[10] B. Spokoiny, Deflationary universe scenario, Phys. Lett. B **315**, 40 (1993).
[11] M. Joyce, Electroweak baryogenesis and the expansion rate of the universe, Phys. Rev. D **55**, 1875 (1997).
[12] P. J. E. Peebles and A. Vilenkin, Quintessential inflation, Phys. Rev. D **59**, 063505 (1999).
[13] Y. Gouttenoire, G. Servant, and P. Simakachorn, Kination cosmology from scalar fields and gravitational-wave signatures, arXiv:2111.01150.
[14] S. Bhattacharya, S. Mohanty, and P. Parashari, Primordial black holes and gravitational waves in nonstandard cosmologies, Phys. Rev. D **102**, 043522 (2020).
[15] I. Dalianis and G. P. Kodaxis, Reheating in runaway inflation models via the evaporation of mini primordial black holes, Universe **8**, 31 (2022).
[16] K. Harigaya, K. Inomata, and T. Terada, Induced gravitational waves with kination era for recent pulsar timing array signals, Phys. Rev. D **108**, 123538 (2023).
[17] C. Cheng, P. Giannadakis, L. Heurtier, and E. A. Lim, Nonlinear dynamics and primordial black hole formation during kination, J. Cosmol. Astropart. Phys. 07 (2026) 048.
[18] E. Milligan, L. E. Padilla, D. J. Mulryne, and J. C. Hidalgo, Primordial black hole formation in a scalar field dominated universe, J. Cosmol. Astropart. Phys. 10 (2025) 025.
[19] L. E. Padilla, E. Milligan, D. J. Mulryne, and J. C. Hidalgo, Primordial black hole formation in a scalar field dominated universe: Investigation of the critical nature of the collapse, J. Cosmol. Astropart. Phys. 04 (2026) 049.
[20] See Supplemental Material at http://link.aps.org/supplemental/10.1103/58r5-8zng for details of the evolution equations, horizon identification, near-critical self-similar profiles, residual and Lomb-Scargle analyses, phenomenological fitting, and numerical convergence.
[21] M. Shibata and M. Sasaki, Black hole formation in the Friedmann universe: Formulation and computation in numerical relativity, Phys. Rev. D **60**, 084002 (1999).

[22] T. Harada, C.-M. Yoo, T. Nakama, and Y. Koga, Cosmological long-wavelength solutions and primordial black hole formation, Phys. Rev. D **91**, 084057 (2015).

[23] The masses reported here are formation masses. Postformation accretion may increase these masses and smooth deep minima in the final PBH mass function, but it does not change their origin in the DSS modulation of the near-critical collapse process.

[24] O. Rinne, Type II critical collapse on a single fixed grid: A gauge-driven ingoing boundary method, Gen. Relativ. Gravit. **52**, 117 (2020).

[25] A. M. Green and A. R. Liddle, Critical collapse and the primordial black hole initial mass function, Phys. Rev. D **60**, 063509 (1999).

[26] A. D. Gow, C. T. Byrnes, and A. Hall, An accurate model for the primordial black hole mass distribution from a peak in the power spectrum, Phys. Rev. D **105**, 023503 (2022).

[27] L. E. Padilla, T. Harada, and H. Iizuka, Discrete self-similarity imprints on primordial-black-hole mass functions (to be published).