

Super-Heisenberg protocol for dark matter and high-frequency gravitational wave search

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(Received 30 April 2026; accepted 22 July 2026; published 12 August 2026)

We propose a quantum-enhanced sensing scheme for the detection of wavelike dark matter and high-frequency gravitational waves using two-dimensional ion crystals in a Penning trap. The protocol employs spin-motion squeezed states to improve the signal-to-noise ratio and enable a super-Heisenberg scaling with respect to the number of ions over a broad parameter range. We analyze the sensitivity of the protocol to representative wavelike dark matter candidates, including the axionlike particle and the dark photon, as well as to high-frequency gravitational waves, taking into account the decoherence and dephasing of the ion spins. Our results indicate that two-dimensional ion crystals and this new protocol provide a promising platform for probing previously unexplored parameter space in searches for light dark matter and high-frequency gravitational waves.

DOI: [10.1103/5821-4x1y](https://doi.org/10.1103/5821-4x1y)

I. INTRODUCTION

Quantum metrology provides a powerful framework for detecting extremely weak signals by exploiting nonclassical resources such as squeezing and entanglement. A variety of platforms have been explored for quantum sensing [1]. Among them, trapped-ion systems offer a particularly well-controlled setting, combining long coherence times with high-fidelity control of internal states and precise manipulation of collective motional modes [2,3]. In large ion crystals, for instance, spin-dependent optical dipole forces couple the collective spin with the motion [4], which has been exploited in recent experiments to demonstrate quantum-enhanced sensing of weak displacements and electric fields [5].

In conventional protocols with N qubits, the achievable precision is bounded by the standard quantum limit $N^{-1/2}$ for uncorrelated probes and can approach the Heisenberg limit N^{-1} when entangled states are employed [6]. While it has been recognized that nonlinear parameter encoding and many-body interactions can yield sensitivities that scale more favorably than the Heisenberg scaling, referred to as super-Heisenberg scaling [7–9], the interpretation of such

scaling depends sensitively on the definition of the resources being counted, requiring care in comparing different approaches [10]. Recently, it has been shown that suitably engineered spin-boson dynamics can give rise to a super-Heisenberg scaling of the quantum Fisher information [11,12].

Understanding the nature of dark matter is one of the most important open problems in modern physics. If dark matter is sufficiently light, it can behave as a coherently oscillating classical field in the Galactic halo [13,14]. Representative candidates for such wavelike dark matter include axionlike particles and dark photons. A variety of quantum sensing approaches have been developed to search for light dark matter, offering complementary strategies across different experimental platforms [15–22], including trapped-ion systems [5,23,24]. These oscillatory backgrounds can induce weak effective electric fields that drive collective motional modes of ions. Under resonant driving conditions, the resulting motional displacement can be mapped onto a measurable spin signals.

A similar detection principle also applies to high-frequency gravitational waves. While most searches have traditionally focused on the Hz–kHz frequency band [25,26], various scenarios predict signals at much higher frequencies [27]. This has stimulated the development of detector concepts that go beyond conventional interferometric approaches [20,28–30]. In ion traps, such high-frequency gravitational waves can couple to the ions and resonantly excite its oscillation modes [31,32].

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In this work, we study the application of a super-Heisenberg protocol to wavelike dark matter and high-frequency gravitational wave search by improving the protocol with two-dimensional ion crystals in Refs. [5,32]. Ions confined in a Penning trap self-organize into a planar crystal due to the balance between the trap potential and Coulomb repulsion, supporting collective vibrational modes. These modes can be resonantly excited by dark matter or gravitational waves. The resulting axial motion is transduced into a measurable collective-spin signal via optical dipole forces. We show that a protocol based on spin-motion squeezed states enables sensitivities that surpass the Heisenberg scaling, thereby enabling exploration of previously inaccessible regions of parameter space.

II. EXPERIMENTAL SETUP

In this section, we introduce the physical setup and outline the sensing protocol employed in this work. We begin in Sec. II A with a brief review of two-dimensional ion crystals in a Penning trap and the optical dipole forces which connect the spins and the motional degrees of freedom. In Sec. II B, we introduce the spin-motion squeezed state that serve as a key resource for quantum-enhanced sensing. Building on these elements, Sec. II C presents our protocol, which combines the spin-motion squeezed state with the idea of Refs. [5,32] to achieve enhanced sensitivity beyond the Heisenberg scaling.

A. Two-dimensional ion crystals

We consider a two-dimensional ion crystal confined in a Penning trap, where N ions are subject to a combination of a quadrupole electric field and a strong magnetic field [4,33]. In the rotating frame comoving with the ions, the effective trap potential is harmonic in both axial and radial directions, ensuring stable confinement under appropriate conditions on the rotation frequency. Multiple ions self-organize into a triangular Wigner crystal in the plane perpendicular to the magnetic field, with equilibrium positions determined by the balance between the trapping potential and Coulomb repulsion.

The axial motion of the ions is well described within the harmonic approximation [34,35], where the potential with respect to the axial motion

$$V_z = \frac{1}{2} m_{\text{ion}} \omega_z^2 \sum_{i=1}^N z_i^2 + \frac{1}{2} \sum_{i \neq j} \frac{\alpha_{\text{EM}}}{\sqrt{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^2 + (z_i - z_j)^2}} \quad (2.1)$$

$$= \frac{1}{2} m_{\text{ion}} \omega_z^2 \sum_i z_i^2 - \frac{\alpha_{\text{EM}}}{4} \sum_{i \neq j} \frac{(z_i - z_j)^2}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3} + \dots \quad (2.2)$$

gives rise to a set of collective phonon modes, with m_{ion} , ω_z , α_{EM} , and $(\boldsymbol{\rho}_i, z_i)$ denoting the ion mass, the angular

frequency of the single-ion axial mode, the fine-structure constant, and the position of the i th ion, respectively. The corresponding Hamiltonian can be diagonalized as $H_0 = \sum_k \omega_k \hat{a}_k^\dagger \hat{a}_k$, leading to a spectrum of axial vibrations often referred to as drumhead modes in the large-system limit,

$$\hat{z}_i = \sum_k \frac{b_{ik}}{\sqrt{2m_{\text{ion}}\omega_k}} (\hat{a}_k e^{-i\omega_k t} + \hat{a}_k^\dagger e^{i\omega_k t}), \quad (2.3)$$

where b_{ik} is the orthonormal matrix diagonalizing V_z , ω_k is the frequency for the k th mode, and $\hat{a}_k^{(\dagger)}$ is the ladder operator. One can find the explicit formulas of b_{ik} in the large-system limit in Ref. [32]. The center-of-mass mode, in which all ions oscillate in phase, has the highest frequency, while lower-frequency modes exhibit nontrivial spatial structures and may be nearly degenerate depending on the crystal geometry. In addition to motional degrees of freedom, each ion hosts a qubit system encoded in internal states, enabling coherent spin manipulation and state read-out. In this work, we focus on the use of $^9\text{Be}^+$ ions, but other species are available with the same protocol.

Spin-motion coupling is realized by optical dipole forces, generated by the interference of two laser beams applied to the ion crystal [4,5]. By appropriately choosing the laser polarizations, the differential ac Stark shift between the spin states can be canceled, leaving a spatially modulated, state-dependent force. In the Lamb-Dicke regime, this results in a force proportional to the axial displacement and oscillating at the beat frequency of the lasers. The resulting interaction couples the collective spin to the phonon modes and enables the transduction of motional excitations into measurable spin dynamics.

The spatially homogeneous optical dipole force predominantly couples to the center-of-mass mode, and one can engineer spatially inhomogeneous optical dipole force profiles by modulating the phase of the laser fields across the crystal [36]. By tuning the drive frequency to a target phonon mode and shaping the spatial dependence appropriately, the optical dipole force can selectively couple to specific collective modes, including nearly degenerate excitations. This tunable spin-phonon interaction

$$\hat{H}_{\text{ODF}} = \frac{g}{\sqrt{N}} (\hat{a} + \hat{a}^\dagger) \hat{J}_z, \quad (2.4)$$

where $\hat{a}^{(\dagger)}$ and $\hat{J}_z$ denote the ladder operator of the mode in focus and the z component of the total spin, respectively, provides a key resource for mode-selective sensing and for generating nonclassical states relevant to quantum-enhanced metrology.

B. Spin-motion squeezed state

The Heisenberg limit sets the ultimate bound on the precision of parameter estimation when the parameter is

encoded via a collective unitary evolution acting on N quantum resources, as opposed to independent or separable operations. In the protocol introduced below, both the N spins and the motional degrees of freedom are utilized, thereby circumventing the assumptions underlying the Heisenberg scaling, where the spin-motion squeezed state

$$|\text{init}\rangle = \sum_{n=0}^J c_{2n} |-J + 2n, 0\rangle + \sum_{n=1}^{\infty} c_{2n+2J} |J, 2n\rangle \quad (2.5)$$

plays a central role. Here, the state $|m, n\rangle$ denotes the simultaneous eigenstate of the operators $\hat{J}_z$ and $\hat{a}^\dagger \hat{a}$ with eigenvalues m and n , respectively, and $J = N/2$ is the maximum of m .

We prepare this state by following Ref. [11]. We start from the state $|-J, 0\rangle$, and the motional squeezed state

$$|\psi\rangle = \sum_{n=0}^{\infty} a_{2n} |-J, 2n\rangle, \quad a_{2n} = \frac{\sqrt{(2n)!}}{2^n n!} z^{n/2} (1-z)^{1/4} e^{-in\phi} \quad (2.6)$$

with $z = \tanh^2 r$ is generated using two sideband transitions and laser cooling [37]. Next, the Tavis-Cummings Hamiltonian

$$\hat{H}_{\text{TC}}(t) = \zeta(t) \hat{J}_z + \frac{\lambda(t)}{\sqrt{N}} (\hat{J}_+ \hat{a} + \hat{J}_- \hat{a}^\dagger) \quad (2.7)$$

is performed to $|\psi\rangle$ adiabatically. Here, $\hat{J}_\pm = \sum_i \hat{\sigma}_i^\pm = \sum_i (\hat{\sigma}_i^x \pm i \hat{\sigma}_i^y)/2$. At the initial time t_i , the detuning $\zeta(t)$ is set to satisfy $|\zeta(t_i)| \gg \lambda(t_i)$ and $\zeta(t_i) < 0$, such that the state $|-J, 2n\rangle$ are eigenstates of the Tavis-Cummings Hamiltonian, $\hat{H}_{\text{TC}}(t_i) |-J, 2n\rangle \simeq -J\zeta(t_i) |-J, 2n\rangle$. The detuning function is then adiabatically varied to a final value satisfying $\zeta(t_f) \gg \lambda(t_f)$ and $\zeta(t_f) > 0$. Provided that the adiabatic condition holds throughout the evolution, the system remains in the instantaneous eigenstates of the Hamiltonian and the conserved operator $\hat{N} = \hat{a}^\dagger \hat{a} + \hat{J}_z + J$. Each initial state $|-J, 2n\rangle$ is adiabatically mapped to the final state $|-J + 2n, 0\rangle$ for $n \leq J$ and $|J, 2n - 2J\rangle$ for $n > J$. Consequently, the spin-motion squeezed state (2.5) is obtained with $c_{2n} = a_{2n} \exp[-i \int_{t_i}^{t_f} dt E_{2n}(t)]$.

C. Protocol

In this subsection, we demonstrate that phonon-mode excitations can be detected by following the protocol of Ref. [5], with the initial state replaced by the spin-motion squeezed state in Eq. (2.5), enabling high-sensitivity detection of external fields such as wavelike dark matter and gravitational waves. We assume an external wave that resonantly couples to a phonon mode of the ion crystal, described by the interaction Hamiltonian

$$\hat{H}_{\text{int}} = \alpha \hat{a} + \alpha^* \hat{a}^\dagger, \quad (2.8)$$

where α denotes the amplitude of the target field.

First, we prepare the spin-motion squeezed state (2.5). After the spin rotation by $\hat{U}_y(\pi/2) = \exp(-i \frac{\pi}{2} \hat{J}_y)$, the optical dipole force operation (2.4) is performed during the time τ . We hold the system during the time $T - 2\tau$ and change the state by applying the inverse optical dipole force operation. Then, the spin rotation by $\hat{U}_x(\vartheta) = \exp(-i \vartheta \hat{J}_x)$ yields the final state

$$|\text{fin}\rangle = \hat{U}_x(\vartheta) e^{-i\tau(\hat{H}_{\text{ODF}} + \hat{H}_{\text{int}})} \hat{U}_x(\pi) e^{-i(T-2\tau)\hat{H}_{\text{int}}} \times e^{-i\tau(\hat{H}_{\text{ODF}} + \hat{H}_{\text{int}})} \hat{U}_y(\pi/2) |\text{init}\rangle \quad (2.9)$$

$$= [\hat{U}_x(\vartheta) \hat{U}_z(\varphi) \hat{U}_y(\pi/2)] \otimes e^{-iT\hat{H}_{\text{int}}} |\text{init}\rangle \quad (2.10)$$

with

$$\varphi = \frac{2g\tau(T-\tau)}{\sqrt{N}} (\text{Im } \alpha). \quad (2.11)$$

The rotation angle ϑ is determined to minimize the variance of $\hat{J}_z$, $\text{var}(\hat{J}_z) = \langle \hat{J}_z^2 \rangle - \langle \hat{J}_z \rangle^2$. Finally, the z -component of the total spin $\hat{J}_z$ is measured by using the spin-dependent global resonance fluorescence method [38,39].

The expectation value of $\hat{J}_z$ is given by

$$\langle \hat{J}_z \rangle \simeq \varphi \sin \vartheta \left[\sum_{n=0}^J |c_{2n}|^2 (-J + 2n) + J \sum_{n=1}^{\infty} |c_{2n+2J}|^2 \right] \quad (2.12)$$

$$= -\varphi \sin \vartheta \left[J - \frac{z}{1-z} + \frac{2}{\sqrt{\pi}} \frac{z^{J+1}}{1-z} \times \frac{\Gamma(J+3/2)}{\Gamma(J+2)} {}_2F_1(1/2, J, J+2; z) \right] \quad (2.13)$$

up to the linear order of φ . We compare this result with the case in which the ground state $|-J, 0\rangle$ is used in place of $|\text{init}\rangle$, for which $\langle \hat{J}_z \rangle_0 \simeq \varphi J$ [32]. The absolute value of the ratio $\langle \hat{J}_z \rangle / \langle \hat{J}_z \rangle_0$ is plotted in Fig. 1 for $r = 1.0$ (blue), $r = 1.5$ (green), and $r = 2.0$ (red), where, for the purpose of illustration, the angle ϑ is fixed to $\pi/2$. The scaling of the expectation value with respect to $N = 2J$ is improved for sufficiently large N , and the ratio approaches unity as $N \rightarrow \infty$. Although the values are smaller than unity, this $\mathcal{O}(1)$ suppression is compensated in the sensitivity by the variance of $\hat{J}_z$.

The single-measurement sensitivity $\Delta\varphi$ is defined by

$$\frac{|\varphi|}{\Delta\varphi} = \frac{|\langle \hat{J}_z \rangle|}{\sqrt{\text{var}(\hat{J}_z)}} \simeq \frac{|\langle \hat{J}_z \rangle|}{\sqrt{\langle \hat{J}_z^2 \rangle}}, \quad (2.14)$$

where the expectation value of $\hat{J}_z^2$ is given by

$$\begin{aligned} \langle \hat{J}_z^2 \rangle \simeq & \frac{1}{2} \sum_{n=0}^J |c_{2n}|^2 (J(J+1) - (-J+2n)^2) + \frac{J}{2} \sum_{n=1}^{\infty} |c_{2n+2J}|^2 \\ & + \frac{1}{2} \text{Re} \left[e^{-2i\vartheta} \sum_{n=0}^J c_{2n+2}^* c_{2n} \sqrt{(2J-2n)(2J-2n-1)(2n+1)(2n+2)} \right], \end{aligned} \quad (2.15)$$

neglecting higher-order contributions in φ . To gain qualitative insight into the N -dependence of $\text{var}(\hat{J}_z)$, we consider a lower bound of the variance,

$$\begin{aligned} \text{var}(\hat{J}_z)_{\text{low}} = & \frac{1}{2} \sum_{n=0}^J |c_{2n}|^2 (J(J+1) - (-J+2n)^2) + \frac{J}{2} \sum_{n=1}^{\infty} |c_{2n+2J}|^2 \\ & - \frac{1}{2} \sum_{n=0}^J |c_{2n+2} c_{2n}| \sqrt{(2J-2n)(2J-2n-1)(2n+1)(2n+2)}, \end{aligned} \quad (2.16)$$

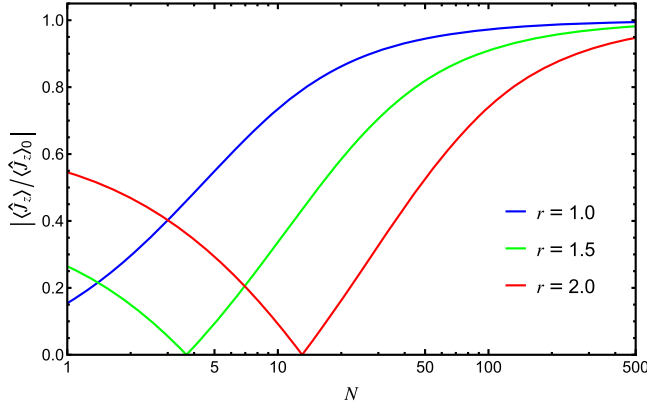


FIG. 1. Absolute values of the ratio $\langle \hat{J}_z \rangle / \langle \hat{J}_z \rangle_0$ as functions of N for $r = 1.0$ (blue), $r = 1.5$ (green), and $r = 2.0$ (red), with $\vartheta = \pi/2$. Here, $\langle \hat{J}_z \rangle$ and $\langle \hat{J}_z \rangle_0$ denote the expectation values obtained when the protocol is initialized in the spin-motion squeezed state and in the ground state, respectively.

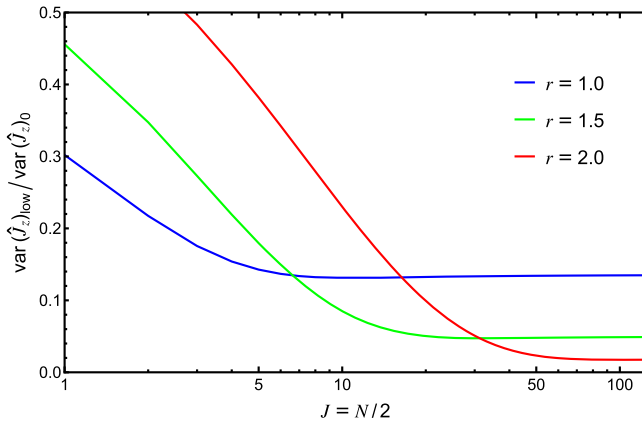


FIG. 2. Ratios $\text{var}(\hat{J}_z)_{\text{low}} / \text{var}(\hat{J}_z)_0$ as functions of N for $r = 1.0$ (blue), $r = 1.5$ (green), and $r = 2.0$ (red), with $\vartheta = \pi/2$.

by comparison with the variance in the ground-state case, $\text{var}(\hat{J}_z)_0 = J/2$. The ratio $\text{var}(\hat{J}_z)_{\text{low}} / \text{var}(\hat{J}_z)_0$ is plotted in Fig. 2 for $r = 1.0$ (blue), $r = 1.5$ (green), and $r = 2.0$ (red). Although this estimate underestimates the exact values, it captures the qualitative behavior observed in the numerical results presented in Sec. IV. The adiabatically prepared spin-motion squeezed state reduces the relevant spin variance through squeezing. As N increases, the protocol becomes advantageous in terms of the variance of $\hat{J}_z$, which determines the statistical uncertainty of the measurement, while the ratio exhibits saturation as $N \rightarrow \infty$.

III. PHYSICS TARGETS

In this section, we review the target signals considered in this work, namely, wavelike dark matter and high-frequency gravitational waves. We focus on representative candidates for wavelike dark matter, including axionlike particles and dark photons, which can induce weak electric fields that resonantly excite the center-of-mass mode of the ion crystal. We also discuss high-frequency gravitational waves and their coupling to the crystal through both indirect and direct mechanisms. The former arises from graviton-photon conversion in the presence of a background magnetic field and can likewise excite the center-of-mass mode. In contrast, the direct coupling allows for the excitation of the parity-odd modes of the ion crystal. This feature is unique to gravitational waves and enables their discrimination from wavelike dark matter signals.

A. Dark matter

Dark matter with a mass $m_{\text{DM}} = \mathcal{O}(\text{neV})$ behaves as a classical wave, since the occupation number within a volume of order the de Broglie wavelength is much larger than unity. Furthermore, the dark matter distribution is effectively uniform over this length scale, which is typically

much larger than the size of an ion trap system. Consequently, the spatial variation of the classical field can be neglected. For the case of wavelike dark matter, the dark matter field Φ can be expressed as

$$\Phi(\mathbf{x}, t) = \Phi_0 \cos(m_{\text{DM}}t - \phi). \quad (3.1)$$

The phase ϕ of the field is randomly distributed but remains approximately constant over the coherence time $T_{\text{DM}} = 2\pi/m_{\text{DM}}v_{\text{DM}}^2 \simeq 0.4\text{s} \times (10\text{ neV}/m_{\text{DM}})$, where $v_{\text{DM}} \sim 10^{-3}$ [40,41] is the relative speed of the dark matter. Since dark matter is nonrelativistic, $v_{\text{DM}} \ll 1$, the amplitude Φ_0 is related to the local dark matter density as $\rho_{\text{DM}} = m_{\text{DM}}^2 \Phi_0^2/2 \sim 0.45\text{ GeV cm}^{-3}$ [42,43]. We focus on two benchmark models, the axionlike particle and the dark photon, which can induce a classical electric field that resonantly excites the center-of-mass mode ($k=1$) of the ion crystal.

The axionlike particle $\Phi = a$ is a pseudoscalar field which couples with photons as

$$\mathcal{L} = -\frac{1}{4}g_{a\gamma}aF^{\mu\nu}\tilde{F}_{\mu\nu}, \quad (3.2)$$

where $F^{\mu\nu}$ is the electromagnetic field tensor and $\tilde{F}^{\mu\nu}$ is its dual. Under a static magnetic field B_z of radius R , the axion field is converted into an electric field

$$E_z = \epsilon_a \sqrt{2\rho_{\text{DM}}} \sin(m_{\text{DM}}t - \phi) \quad (3.3)$$

along the magnetic field, where

$$\epsilon_a = \frac{B_z}{2} \frac{g_{a\gamma}}{m_{\text{DM}}} (m_{\text{DM}}R)^2 \left[\left(\log \frac{m_{\text{DM}}R}{2} + \gamma - \frac{1}{2} \right)^2 + \left(\frac{\pi}{2} \right)^2 \right]^{1/2} \quad (3.4)$$

is the conversion efficiency for $m_{\text{DM}}R \ll 1$ with $\gamma = 0.5772\dots$ being the Euler-Mascheroni constant [44]. When the frequency ω_z matches the dark matter mass m_{DM} , this induced electric field excites the center-of-mass mode of the ion crystal via the interaction Hamiltonian (2.8) with

$$\alpha = -ie^{-i\phi} \frac{e\epsilon_a}{2} \sqrt{\frac{N\rho_{\text{DM}}}{m_{\text{DM}}\omega_z}} \quad (3.5)$$

under the rotating wave approximation.

The dark photon is a massive vector boson associated with an extra U(1) gauge group besides the Standard Model. It has very weak interactions with the Standard Model particles via the kinetic mixing with the photon,

$$\mathcal{L} = \frac{1}{2}\epsilon F^{\mu\nu}F'_{\mu\nu}, \quad (3.6)$$

where $F'^{\mu\nu}$ is the field strength tensor of the dark photon field. The electric field component of the dark photon dark

matter is given by $\mathbf{E} = \mathbf{E}_0 \sin(m_{\text{DM}}t - \phi)$, with the amplitude related to the local dark matter density, $\rho_{\text{DM}} = \mathbf{E}_0^2/2$. We focus on the z -component

$$E_z = \epsilon_{\text{DP}} \sqrt{2\rho_{\text{DM}}} \sin(m_{\text{DM}}t - \phi) \quad (3.7)$$

with $\epsilon_{\text{DP}} \simeq \epsilon \cos \theta (m_{\text{DM}}R)^2$ [45]. The polarization angle θ is assumed to be randomly distributed. The interaction of the dark photon dark matter is described by Eqs. (2.8) and (3.5) with replacement of ϵ_a with ϵ_{DP} .

B. Gravitational waves

To correctly describe the effect of gravitational waves on the ion crystal, it is essential to adopt an appropriate coordinate system. The most convenient choice is the proper detector frame, whose origin follows the geodesic of the center of mass of the ion crystal [46,47]. In this frame, the origin is chosen to coincide with the freely falling center of the crystal, such that the coordinate values faithfully represent the positions of individual ions. Within this formulation, both the center-of-mass mode ($k=1$) and the parity-odd modes ($k=2, 3$) can be resonantly excited by gravitational waves.

We consider gravitational waves propagating on a Minkowski background, with the metric expressed as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric and $h_{\mu\nu}$ denotes the transverse-traceless perturbation. We model the gravitational wave as a plane wave,

$$h_{ij}(\mathbf{x}, t) = h_+ e_{ij}^+ \cos(\omega t - \mathbf{k} \cdot \mathbf{x} + \phi_+) + h_\times e_{ij}^\times \cos(\omega t - \mathbf{k} \cdot \mathbf{x} + \phi_\times), \quad (3.8)$$

where $\omega = 2\pi f$ is the angular frequency, $\mathbf{k}$ is the wavevector, and $h_+, h_\times$ together with $\phi_+, \phi_\times$ denotes the amplitudes and phases of the two polarization modes, respectively. The corresponding polarization tensors are given by

$$e_{ij}^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos^2\theta & 0 & -\sin\theta \cos\theta \\ 0 & -1 & 0 \\ -\sin\theta \cos\theta & 0 & \sin^2\theta \end{pmatrix}, \quad e_{ij}^\times = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \cos\theta & 0 \\ \cos\theta & 0 & -\sin\theta \\ 0 & -\sin\theta & 0 \end{pmatrix}, \quad (3.9)$$

where the $+$ polarization corresponds to a deformation along the y -direction and θ denotes the propagation angle in the xz plane. Hereafter, we assume an unpolarized gravitational wave, such that $h_+ = h_\times \equiv h_0$ and $\phi_+ = \phi_\times \equiv \phi$, and randomness of the polarization θ and the phase ϕ for simplicity.

Gravitational waves are known to convert into electromagnetic waves in the presence of a background magnetic

field. The center-of-mass mode can be excited indirectly via the generated electromagnetic field. This graviton-photon conversion can be described by an effective interaction of the form

$$\mathcal{L} = j_{\text{eff}}^\mu A_\mu, \quad j_{\text{eff}}^\mu = \partial_\nu (h_\lambda^\mu \bar{F}^{\lambda\nu} - h_\lambda^\nu \bar{F}^{\lambda\mu}) - \frac{1}{2} (\partial_\nu h_\lambda^\lambda) \bar{F}^{\nu\mu}, \quad (3.10)$$

where $\bar{F}^{\mu\nu} = \partial^\mu \bar{A}^\nu - \partial^\nu \bar{A}^\mu$ denotes the field strength tensor associated with the background electromagnetic field $\bar{A}^\mu$ [48]. The metric $h_{\mu\nu}$ in the effective current is evaluated in Fermi normal coordinates [48]. Note that the induced electric field is independent of the details of the ion trap system apart from the boundary effects. We assume that the electrodes and other metallic shielding are placed outside the magnetic field, which allows us to neglect the boundary effects. Under the long-wavelength approximation $\omega R \ll 1$, this interaction induces an electric field along the axial direction of the form

$$E_z \simeq \frac{h_0}{\sqrt{2}} B_z (\omega R)^2 f(L/2R) \sin^2 \theta \cos(\omega t + \phi), \quad (3.11)$$

where B_z is a cylindrically symmetric background magnetic field of radius R and height L , and $f(x) = \log(x + \sqrt{1+x^2})$ is determined by the magnetic field geometry [31]. The electric-field contribution is negligible compared with the magnetic-field contribution, since the trapping electric field is $\sim 1 \text{ V/cm} \sim 10^{-4} \text{ eV}^2$, whereas the magnetic field is $\sim 1 \text{ T} \sim 10^2 \text{ eV}^2$.

In the nonrelativistic limit, the interaction between the gravitational field and an ion is described by $\frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \simeq \frac{1}{2} T^{00} \delta g_{00}$, where $T^{\mu\nu}$ is the energy-momentum tensor of the ion, and $\delta g_{\mu\nu}$ is the metric perturbation. In the proper detector frame, this leads to the interaction Hamiltonian $H = \frac{1}{2} m_{\text{ion}} R_{0k0l} x^k x^l$, where $R_{\mu\nu\rho\sigma}$ is the

Riemann tensor evaluated at the origin $x^i = 0$ [49]. Here, the transverse-traceless gauge is employed to calculate R_{0k0l} , because the Riemann tensor is gauge invariant at linear order in the metric perturbation. We obtain the effective interaction Hamiltonian describing the coupling between a plane gravitational wave and the ion crystal as Eq. (2.8) with

$$\alpha \simeq -\frac{\sqrt{N m_{\text{ion}} \omega_2^3 R_{\text{cry}}}}{8 \zeta_{11} \sqrt{\zeta_{11}^2 - 1}} h_0 \sin \theta (\cos \theta - i) e^{i\phi}, \quad (3.12)$$

where $\zeta_{11} = 1.841\dots$ is the first zero of the first Bessel function and R_{cry} denotes the radius of the ion crystal, for the parity-odd mode $\hat{a} = (\hat{a}_2 + i\hat{a}_3)/\sqrt{2}$ [32]. Henceforth, we consider monochromatic gravitational waves for simplicity.

IV. SENSITIVITY

In this section, we present results obtained from numerical calculations of the time evolution governed by the Tavis-Cummings Hamiltonian. For the adiabatic transition employed in the following simulations, the parameters are chosen as $\zeta(t) = \zeta_0 \sin(\gamma t/2)$, $\lambda(t) = \lambda_0 \cos^2(\gamma t/2)$, $\zeta_0/2\pi = 2.5 \text{ kHz}$, $\lambda_0/2\pi = 1.5 \text{ kHz}$, $-t_i = t_f = \pi/\gamma$, and $\gamma/2\pi = 2.75 \text{ kHz}$. The Hilbert space is truncated by imposing $|c_{2n+2J}| = 10^{-3}$. The duration of the protocol is $T = 0.04 \text{ s} < T_{\text{DM}} (m_{\text{DM}} = 10 \text{ neV})$, the optical dipole force coupling is $g/2\pi = 3.9 \text{ kHz}$, and the duration of a single optical dipole force operation is $\tau = 0.3 \text{ ms}$. In this setup, the rotation angle satisfies $\vartheta \simeq \pi/2$, such that $\sin \vartheta \simeq 1$.

We first examine the expectation value and variance of $\hat{J}_z$. Figure 3 shows these quantities in comparison with the case where the protocol is initialized in the ground state, for which $\langle \hat{J}_z \rangle_0 = N/2$ and $\text{var}(\hat{J}_z)_0 = N/4$. We plot $-\langle \hat{J}_z \rangle$ because the response is negative. In the left panel, the analytical result based on Eq. (2.13) is shown by the dashed

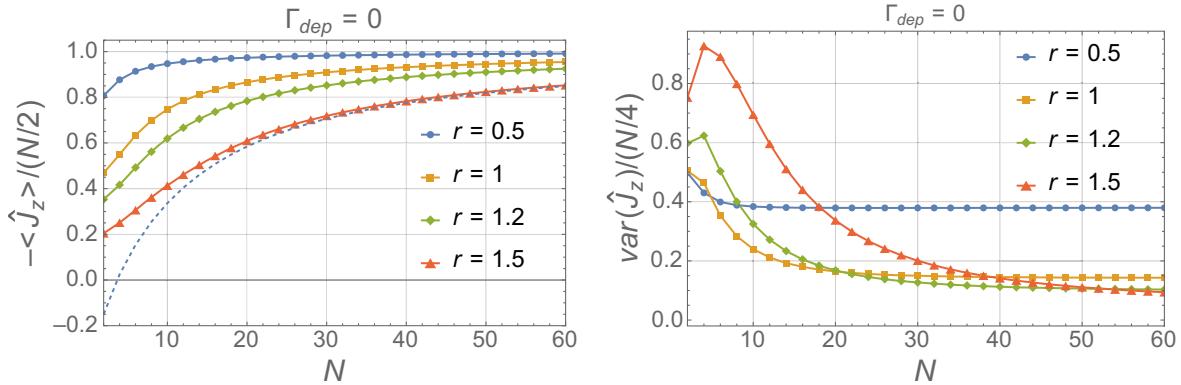


FIG. 3. Numerical results of the expectation value (left) and the variance (right) of $\hat{J}_z$ as a function of N for $r = 0.5$ (blue), $r = 1.0$ (orange), $r = 1.2$ (green), and $r = 1.5$ (red). The results are compared with those obtained when the protocol is initialized in the ground state instead of the spin-motion squeezed state. The dashed curve in the left panel shows the result obtained from Eq. (2.13) for $r = 1.5$.

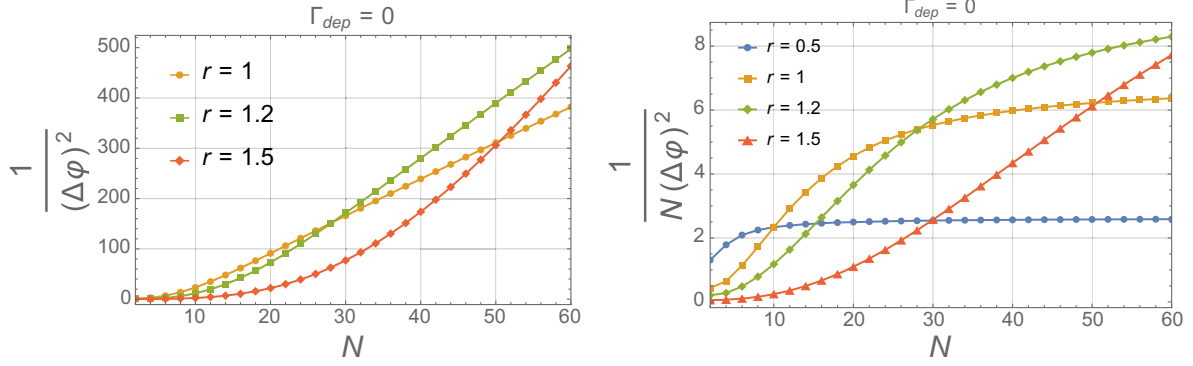


FIG. 4. Sensitivities $(\Delta\phi)^{-2}$ as a function of N for $r = 0.5$ (blue), $r = 1.0$ (orange), $r = 1.2$ (green), and $r = 1.5$ (red). The right panel shows the sensitivity relative to that obtained with a nonsqueezed initial state; values greater than unity indicate an enhancement due to the spin-motion squeezed state.

curve. The deviation between analytical and numerical results at small N arises from incomplete adiabaticity due to insufficient level crossings, which breaks the conservation of $\hat{N}$ and is not captured by Eq. (2.5). This discrepancy becomes negligible when the transition time is increased by an order of magnitude. Both panels reproduce the qualitative behavior discussed in Sec. II C for sufficiently large N .

Next, the sensitivity $\Delta\phi$, defined in Eq. (2.14), is shown in Fig. 4 for several values of r . The right panel compares the sensitivity with that obtained when the protocol is initialized in the ground state. For sufficiently large N , the inverse sensitivity normalized by the ground-state result exceeds unity, indicating an improvement over the non-squeezed case.

Figure 5 shows the sensitivity in comparison with the Heisenberg scaling $(\Delta\phi)^2 \propto N^{-2}$. A positive slope in this figure indicates scaling faster than N^2 , corresponding to super-Heisenberg behavior. We observe such scaling over a

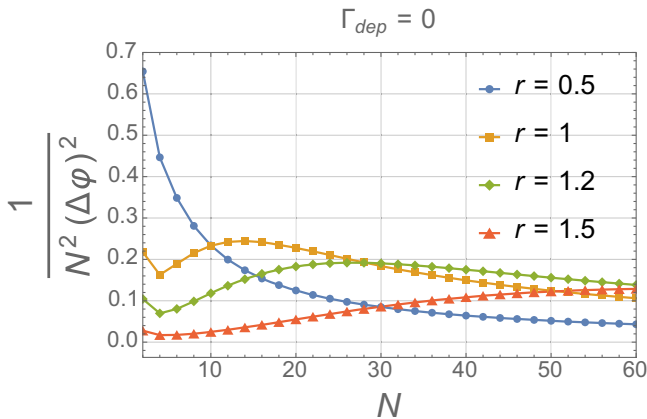


FIG. 5. Sensitivities $(N\Delta\phi)^{-2}$ as a function of N for $r = 0.5$ (blue), $r = 1.0$ (orange), $r = 1.2$ (green), and $r = 1.5$ (red). On the plateau, the sensitivity follows the Heisenberg scaling $(\Delta\phi)^2 \propto N^{-2}$, whereas in the regime where the curves increase, it exhibits super-Heisenberg scaling $\propto N^{-2k}$ with $k > 1$.

range of N that depends on the choice of r , and larger values of r extend this regime to higher N .

Figure 6 shows the dependence of the sensitivity $(\Delta\phi)^{-2}$ on the squeezing parameter r . A peak structure is observed, whose location depends on N , indicating that the optimal value of r should be tuned for a given system size.

In a realistic setup, spin dephasing during the adiabatic transition must be taken into account. This effect is incorporated by solving the master equation for the density matrix

$$\frac{\partial \hat{\rho}}{\partial t} = -i[\hat{H}_{\text{TC}}, \hat{\rho}] + \Gamma_{\text{dep}} \left(\hat{J}_z \hat{\rho} \hat{J}_z - \frac{1}{2} \{ \hat{J}_z^2, \hat{\rho} \} \right). \quad (4.1)$$

The numerical results are shown in Fig. 7. In the left panel, the sensitivity follows Heisenberg scaling $(\Delta\phi)^2 \propto N^{-2}$ on a plateau and exhibits super-Heisenberg scaling $\propto N^{-2k}$ with $k > 1$ in the regime where the curves increase. We find that the super-Heisenberg region is reduced by dephasing, and the sensitivity degrades exponentially with increasing Γ_{dep} . Decoherence during the optical dipole force operations is also included via an exponential suppression factor $e^{-2\Gamma_{\text{dec}}\tau}$.

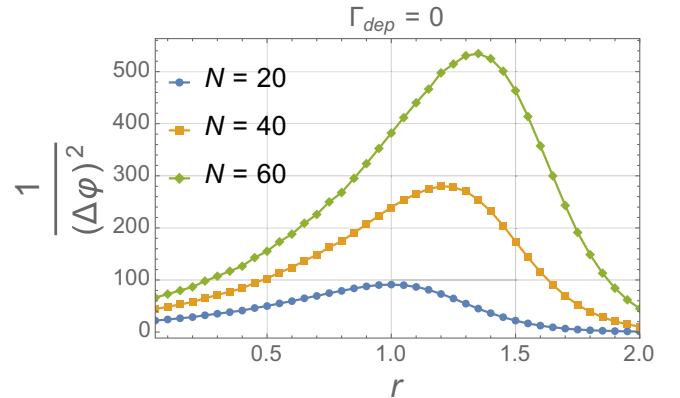


FIG. 6. Sensitivities $(\Delta\phi)^{-2}$ as a function of the squeezing parameter r for $N = 20$ (blue), $N = 40$ (orange), and $N = 60$ (green).

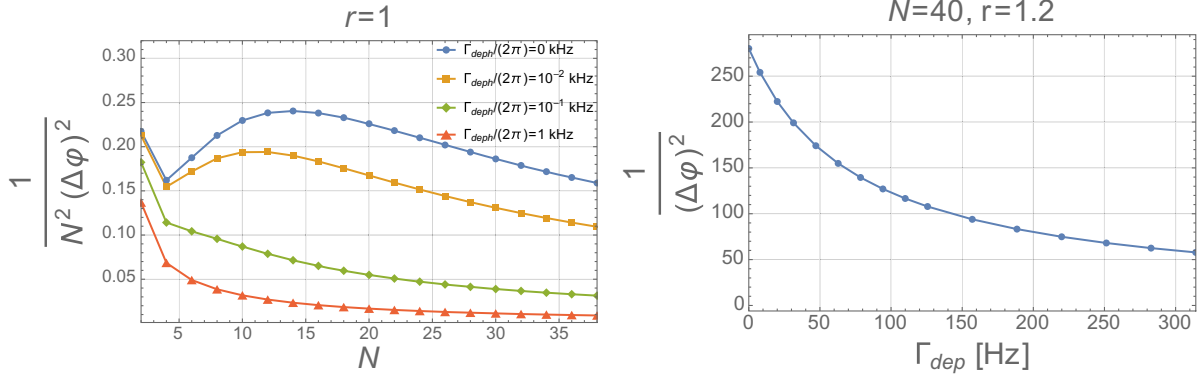


FIG. 7. Sensitivities $(N\Delta\phi)^{-2}$ as functions of N (left) for $\Gamma_{\text{dep}}/2\pi = 0$ (blue), 0.01 kHz (orange), 0.1 kHz (green), 1 kHz (red) with $r = 1$, and as a function of Γ_{dep} (right) for $N = 40$ and $r = 1.2$.

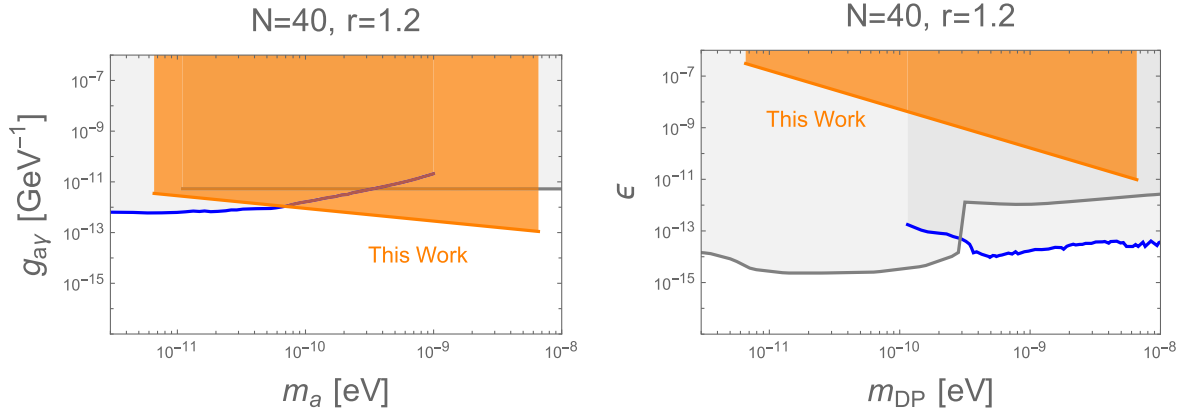


FIG. 8. Sensitivity to the axion-photon coupling (left) and the kinetic mixing of the dark photon (right) as functions of the dark matter mass, $m_{\text{DM}} = m_a$ or m_{DP} , obtained by scanning over the mass range during a one-year measurement. The blue and gray regions in the left panel are excluded by the M82 [51] and by the magnetic white dwarfs [52], respectively. The blue and gray regions in the right panel are constrained by the spacecraft, Parker Solar Probe [53], and by the heating of cosmic gases [54], respectively.

applied to $(\Delta\phi)^{-2}$ [5]. In the following estimates, we assume $\Gamma_{\text{dep}} = \Gamma_{\text{dec}} = 100$ Hz and neglect laser detuning and thermal noise for simplicity. We note that thermal noise slightly affects the effective super-Heisenberg regime for heating rates of $\mathcal{O}(1)$ ms⁻¹ [11].

Finally, we estimate the sensitivities to dark matter and gravitational waves. The sensitivities at 68% confidence level (corresponding to Eq. (2.14)) to the axion-photon coupling $g_{a\gamma}$ and the kinetic mixing parameter of the dark photon ϵ are given by

$$g_{a\gamma} \simeq 1.2 \times 10^{-14} \text{ GeV}^{-1} \times \left(\frac{\Delta\phi}{0.0081} \right) \left(\frac{m_{\text{ion}}}{8.3 \text{ GeV}} \right)^{1/2} \left(\frac{m_{\text{DM}}}{1 \text{ neV}} \right)^{-1/2} \left(\frac{B_z}{1 \text{ T}} \right)^{-1} \left(\frac{R}{3 \text{ m}} \right)^{-2} \left(\frac{T_{\text{tot}}}{1 \text{ day}} \right)^{-1/2} \quad (4.2)$$

and

$$\epsilon = 1.2 \times 10^{-11} \times \left(\frac{\Delta\phi}{0.0081} \right) \left(\frac{m_{\text{ion}}}{8.3 \text{ GeV}} \right)^{1/2} \left(\frac{m_{\text{DM}}}{1 \text{ neV}} \right)^{-3/2} \left(\frac{R}{3 \text{ m}} \right)^{-2} \left(\frac{T_{\text{tot}}}{1 \text{ day}} \right)^{-1/2}, \quad (4.3)$$

respectively, for $N = 40$ and $r = 1.2$.¹ We assume that the measurement is repeated $N_{\text{rep}} = T_{\text{tot}}/(T + 2\pi/\gamma)$ times. Figure 8 shows the sensitivities obtained by scanning over the dark matter mass during a one-year measurement. The number of bins is taken to be $N_{\text{bin}} = Q \log(f_{\text{max}}/f_{\text{min}})$ with $Q \sim 10^4$, $f_{\text{max}} = 10$ MHz, and $f_{\text{min}} = 10$ kHz [5]. Also shown are existing

¹The sensitivity for the dark photon improves by $\sim 10^4$ under free boundary conditions.

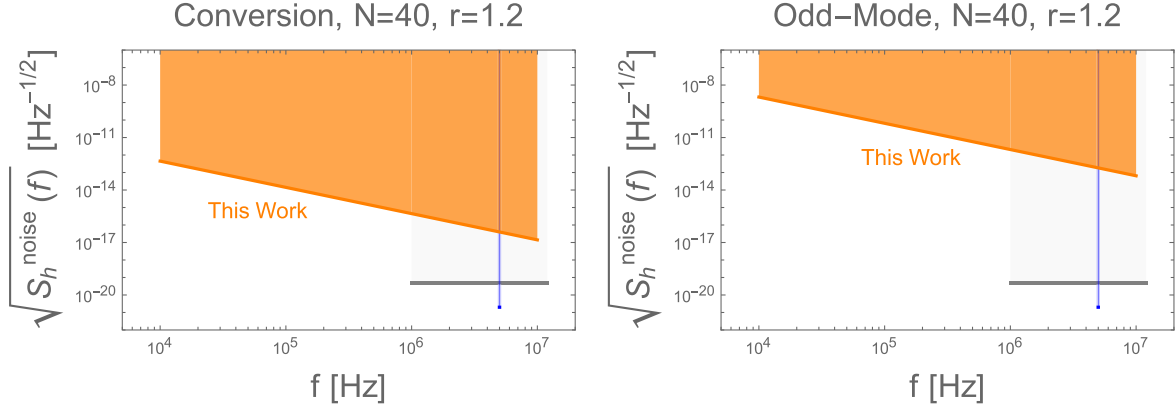


FIG. 9. Sensitivities to the noise-equivalent spectral density of gravitational waves as functions of frequency, obtained by scanning over the frequency range during a one-year observation, via the center-of-mass mode (left) and the parity-odd modes (right). The gray band and the blue line indicate existing experimental constraints from the Fermilab Holometer [29] and the Bulk Acoustic Wave experiments [28], respectively.

experimental constraints [50]. This setup can potentially exclude the currently allowed parameter regions, and in particular, for axionlike particles, it enables access to previously unexplored regions even for relatively small N and r . Larger values of N and r further improve the sensitivity.

The sensitivities at 68% confidence level to the gravitational wave strain are given by

$$h_0(f) \simeq 1.6 \times 10^{-16} \times \left(\frac{\Delta\varphi}{0.0081} \right) \left(\frac{m_{\text{ion}}}{8.3 \text{ GeV}} \right)^{1/2} \left(\frac{f}{1 \text{ MHz}} \right)^{-3/2} \times \left(\frac{R}{3 \text{ m}} \right)^{-2} \left(\frac{B_z}{1 \text{ T}} \right)^{-1} \left(\frac{T_{\text{tot}}}{1 \text{ day}} \right)^{-1/2} \quad (4.4)$$

via graviton-photon conversion, where $L = 1 \text{ m}$ and

$$h_0(f) \simeq 7.5 \times 10^{-13} \times \left(\frac{\Delta\varphi}{0.0081} \right) \left(\frac{m_{\text{ion}}}{8.3 \text{ GeV}} \right)^{-1/2} \times \left(\frac{f}{1 \text{ MHz}} \right)^{-3/2} \left(\frac{R_{\text{cry}}}{0.1 \text{ mm}} \right)^{-1} \left(\frac{T_{\text{tot}}}{1 \text{ day}} \right)^{-1/2} \quad (4.5)$$

via the parity-odd mode. Using the same parameters as in the dark matter case, the sensitivity to the power spectrum density $S_h(f) = Th_0^2(f)$ at 68% confidence level, i.e., the noise-equivalent spectral density [27], is shown in Fig. 9. The sensitivities are obtained by scanning over frequency during a one-year observation with the same binning as in the dark matter analysis. The ion crystal enables searches in frequency ranges that have not been explored by existing experiments, and larger values of N and r can achieve sensitivities comparable to or exceeding current limits.

V. CONCLUSION

We investigate a quantum-enhanced scheme for the detection of wavelike dark matter and high-frequency

gravitational waves using two-dimensional ion crystals. We propose a detection protocol initialized in a spin-motion squeezed state, adiabatically prepared via motional squeezing and the Tavis-Cummings Hamiltonian, in which weak external forces are converted into measurable collective-spin signals. We show that this protocol improves the signal-to-noise ratio and enables super-Heisenberg scaling with respect to the number of ions. We also estimate the corresponding sensitivities to wave-like dark matter candidates, such as the axionlike particle and the dark photon, as well as to high-frequency gravitational waves.

At the same time, the enhancement is sensitive to experimental imperfections. In particular, we find that dephasing reduces the parameter region in which super-Heisenberg scaling appears, while decoherence during the optical dipole force operations degrades the overall sensitivity. Additional effects neglected in the present analysis, such as finite temperature, frequency detuning errors, and imperfect adiabaticity, should be incorporated in a more complete assessment of experimental performance. Furthermore, our analysis focuses on the regime $N < 100$ and $r < 2$ due to computational limitations, whereas larger values are advantageous for probing weaker interactions. Extending this framework to other ion-trap architectures is an interesting direction for future work.

ACKNOWLEDGMENTS

This work was supported by JSPS KAKENHI Grants No. JP23KJ2173 (W. N.) and No. JP24KJ1157 (R. T.).

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

- [1] C. L. Degen, F. Reinhard, and P. Cappellaro, Quantum sensing, *Rev. Mod. Phys.* **89**, 035002 (2017).
- [2] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Quantum dynamics of single trapped ions, *Rev. Mod. Phys.* **75**, 281 (2003).
- [3] R. Blatt and D. Wineland, Entangled states of trapped atomic ions, *Nature (London)* **453**, 1008 (2008).
- [4] J. W. Britton, B. C. Sawyer, A. C. Keith, C.-C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Engineered two-dimensional Ising interactions in a trapped-ion quantum simulator with hundreds of spins, *Nature (London)* **484**, 489 (2012).
- [5] K. A. Gilmore, M. Affolter, R. J. Lewis-Swan, D. Barberena, E. Jordan, A. M. Rey, and J. J. Bollinger, Quantum-enhanced sensing of displacements and electric fields with two-dimensional trapped-ion crystals, *Science* **373**, 673 (2021).
- [6] S. L. Braunstein and C. M. Caves, Statistical distance and the geometry of quantum states, *Phys. Rev. Lett.* **72**, 3439 (1994).
- [7] S. Boixo, S. T. Flammia, C. M. Caves, and J. Geremia, Generalized limits for single-parameter quantum estimation, *Phys. Rev. Lett.* **98**, 090401 (2007).
- [8] S. Boixo, A. Datta, M. J. Davis, S. T. Flammia, A. Shaji, and C. M. Caves, Quantum metrology: Dynamics versus entanglement, *Phys. Rev. Lett.* **101**, 040403 (2008).
- [9] M. Napolitano, M. Koschorreck, B. Dubost, N. Behbood, R. J. Sewell, and M. W. Mitchell, Interaction-based quantum metrology showing scaling beyond the Heisenberg limit, *Nature (London)* **471**, 486 (2011).
- [10] M. M. Rams, P. Sierant, O. Dutta, P. Horodecki, and J. Zakrzewski, At the limits of criticality-based quantum metrology: Apparent super-Heisenberg scaling revisited, *Phys. Rev. X* **8**, 021022 (2018).
- [11] V. P. Pavlov and P. A. Ivanov, Super-Heisenberg scaling of quantum Fisher information using spin-motion states, *Phys. Rev. A* **111**, 062426 (2025).
- [12] V. P. Pavlov, B. S. Nikolova, and P. A. Ivanov, Adiabatic Ramsey interferometry for measuring weak nonlinearities with super-Heisenberg precision, *arXiv:2603.29574*.
- [13] P. Arias, D. Cadamuro, M. Goodsell, J. Jaeckel, J. Redondo, and A. Ringwald, WISPy cold dark matter, *J. Cosmol. Astropart. Phys.* **06** (2012) 013.
- [14] P. W. Graham and S. Rajendran, New observables for direct detection of axion dark matter, *Phys. Rev. D* **88**, 035023 (2013).
- [15] A. V. Dixit, S. Chakram, K. He, A. Agrawal, R. K. Naik, D. I. Schuster, and A. Chou, Searching for dark matter with a superconducting qubit, *Phys. Rev. Lett.* **126**, 141302 (2021).
- [16] S. Chigusa, M. Hazumi, E. D. Herbschleb, Y. Matsuzaki, N. Mizuochi, and K. Nakayama, Nuclear spin metrology with nitrogen vacancy center in diamond for axion dark matter detection, *Phys. Rev. D* **111**, 075028 (2025).
- [17] S. Chen, H. Fukuda, T. Inada, T. Moroi, T. Nitta, and T. Sicanugrist, Quantum enhancement in dark matter detection with quantum computation, *Phys. Rev. Lett.* **133**, 021801 (2024).
- [18] M. Filzinger, A. R. Caddell, D. Jani, M. Steinell, L. Giani, N. Huntemann, and B. M. Roberts, Ultralight dark matter search with space-time separated atomic clocks and cavities, *Phys. Rev. Lett.* **134**, 031001 (2025).
- [19] M. Jiang, T. Hong, D. Hu, Y. Chen, F. Yang, T. Hu *et al.*, Long-baseline quantum sensor network as dark matter haloscope, *Nat. Commun.* **15**, 3331 (2024).
- [20] C. F. A. Baynham *et al.*, A prototype differential atom interferometer for fundamental physics, *Nature (London)* **654**, 622 (2026).
- [21] S. Chigusa, T. Kasamaki, T. Kusano, T. Moroi, K. Nakayama, N. Ozawa, Y. Takahashi, A. Umemoto, and A. Vutha, Detecting dark matter using optically trapped Rydberg atom tweezer arrays, *Phys. Rev. Lett.* **136**, 151801 (2026).
- [22] R. Kang, Q. Hu, X. Cai, W. Yu, J. Zhou, X. Rong, and J. Du, Scalable architecture for dark photon searches: Superconducting-qubit proof of principle, *Phys. Rev. Lett.* **135**, 181004 (2025).
- [23] A. Ito, R. Kitano, W. Nakano, and R. Takai, Quantum entanglement of ions for light dark matter detection, *J. High Energy Phys.* **02** (2024) 124.
- [24] L. Badurina, D. Blas, J. Ellis, and S. A. R. Ellis, Ultralight dark matter detection with trapped-ion interferometry, *Phys. Rev. D* **113**, 092004 (2026).
- [25] KAGRA, LIGO Scientific, and Virgo collaborations, Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA, *Living Rev. Relativity* **19**, 1 (2016).
- [26] LIGO Scientific and Virgo Collaborations, Observation of gravitational waves from a binary black hole merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
- [27] N. Aggarwal, O. D. Aguiar, D. Blas, A. Bauswein, G. Cella, S. Clesse *et al.*, Challenges and opportunities of gravitational-wave searches above 10 kHz, *Living Rev. Relativity* **28**, 10 (2025).
- [28] M. Goryachev and M. E. Tobar, Gravitational wave detection with high frequency phonon trapping acoustic cavities, *Phys. Rev. D* **90**, 102005 (2014); **108**, 129901 (E) (2023).
- [29] A. S. Chou, R. Gustafson, C. Hogan, B. Kamai, O. Kwon, R. Lanza *et al.*, MHz gravitational wave constraints with decameter Michelson interferometers, *Phys. Rev. D* **95**, 063002 (2017).
- [30] D. Carney, G. Higgins, G. Marocco, and M. Wentzel, Superconducting levitated detector of gravitational waves, *Phys. Rev. Lett.* **134**, 181402 (2025).
- [31] R. Takai, Probing high-frequency gravitational waves with entangled vibrational qubits in linear Paul traps, *Phys. Rev. D* **113**, 082002 (2026).
- [32] A. Ito, R. Kitano, W. Nakano, and R. Takai, Quantum sensing of high-frequency gravitational waves with ion crystals, *J. Cosmol. Astropart. Phys.* **05** (2026) 039.
- [33] B. C. Sawyer, J. W. Britton, A. C. Keith, C. C. J. Wang, J. K. Freericks, H. Uys, M. J. Biercuk, and J. J. Bollinger, Spectroscopy and thermometry of drumhead modes in a mesoscopic trapped-ion crystal using entanglement, *Phys. Rev. Lett.* **108**, 213003 (2012).
- [34] X.-P. Huang, J. J. Bollinger, T. B. Mitchell, W. M. Itano, and D. H. E. Dubin, Precise control of the global rotation of strongly coupled ion plasmas in a penning trap, *Phys. Plasmas* **5**, 1656 (1998).

- [35] D. Porras and J. I. Cirac, Quantum manipulation of trapped ions in two dimensional Coulomb crystals, *Phys. Rev. Lett.* **96**, 250501 (2006).
- [36] A. M. Polloreno, A. M. Rey, and J. J. Bollinger, Individual qubit addressing of rotating ion crystals in a Penning trap, *Phys. Rev. Res.* **4**, 033076 (2022).
- [37] D. Kienzler, H.-Y. Lo, B. Keitch, L. de Clercq, F. Leupold, F. Lindenfelser *et al.*, Quantum harmonic oscillator state synthesis by reservoir engineering, *Science* **347**, 1261033 (2015).
- [38] O. Hosten, R. Krishnakumar, N. J. Engelsen, and M. A. Kasevich, Quantum phase magnification, *Science* **352**, 1552 (2016).
- [39] R. J. Lewis-Swan, D. Barberena, J. A. Muniz, J. R. Cline, D. Young, J. K. Thompson, and A. M. Rey, Protocol for precise field sensing in the optical domain with cold atoms in a cavity, *Phys. Rev. Lett.* **124**, 193602 (2020).
- [40] P. J. McMillan and J. J. Binney, The uncertainty in galactic parameters, *Mon. Not. R. Astron. Soc.* **402**, 934 (2010).
- [41] J. Bovy, D. W. Hogg, and H.-W. Rix, Galactic masers and the Milky Way circular velocity, *Astrophys. J.* **704**, 1704 (2009).
- [42] P. F. de Salas and A. Widmark, Dark matter local density determination: Recent observations and future prospects, *Rep. Prog. Phys.* **84**, 104901 (2021).
- [43] L. Söding, R. L. Bartel, and P. Mertsch, Local dark matter density from gaia dr3 k-dwarfs using Gaussian processes, *Mon. Not. R. Astron. Soc.* **542**, 2987 (2025).
- [44] J. Ouellet and Z. Bogorad, Solutions to axion electrodynamics in various geometries, *Phys. Rev. D* **99**, 055010 (2019).
- [45] S. Chaudhuri, P. W. Graham, K. Irwin, J. Mardon, S. Rajendran, and Y. Zhao, Radio for hidden-photon dark matter detection, *Phys. Rev. D* **92**, 075012 (2015).
- [46] F. K. Manasse and C. W. Misner, Fermi normal coordinates and some basic concepts in differential geometry, *J. Math. Phys. (N.Y.)* **4**, 735 (1963).
- [47] W.-T. Ni and M. Zimmermann, Inertial and gravitational effects in the proper reference frame of an accelerated, rotating observer, *Phys. Rev. D* **17**, 1473 (1978).
- [48] W. Ratzinger, S. Schenk, and P. Schwaller, A coordinate-independent formalism for detecting high-frequency gravitational waves, *J. High Energy Phys.* **08** (2024) 195.
- [49] A. Ito and J. Soda, A formalism for magnon gravitational wave detectors, *Eur. Phys. J. C* **80**, 545 (2020).
- [50] C. O'Hare, cajohare/AxionLimits: AxionLimits, <https://cajohare.github.io/AxionLimits/> (2020).
- [51] O. Ning and B. R. Safdi, Leading axion-photon sensitivity with NuSTAR observations of M82 and M87, *Phys. Rev. Lett.* **134**, 171003 (2025).
- [52] C. Dessert, D. Dunsy, and B. R. Safdi, Upper limit on the axion-photon coupling from magnetic white dwarf polarization, *Phys. Rev. D* **105**, 103034 (2022).
- [53] H. An, S. Ge, J. Liu, and M. Liu, In Situ measurements of dark photon dark matter using parker solar probe: Going beyond the radio window, *Phys. Rev. Lett.* **134**, 171001 (2025).
- [54] S. D. McDermott and S. J. Witte, Cosmological evolution of light dark photon dark matter, *Phys. Rev. D* **101**, 063030 (2020).