Accelerated expansion of the Universe purely driven by scalar field fluctuations

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We show that scalar field fluctuations alone can drive cosmic acceleration, provided the Universe is spatially closed and the Compton wavelength of the field exceeds the radius of curvature. This mechanism may open new perspectives on inflation and dark energy, which could arise from a gas of sufficiently light bosons in a closed universe.

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I. INTRODUCTION

The current accelerated expansion of the Universe [1,2] is attributed to the presence of a negative-pressure component called dark energy that constitutes, approximately, 69% of the total energy density in the Universe. To date, the nature of this component is unknown. The simplest way to describe it is by means of a cosmological constant Λ, in which case the dark energy density remains constant in time, leading to the standard ΛCDM model (CDM stands for cold dark matter). Despite the success of this approach in terms of explaining the observational data, it requires a value of the dark energy density that is in blatant tension with the expected vacuum energy density in the framework of quantum field theory. This is the well-known cosmological constant problem [3]. Moreover, the Dark Energy Spectroscopic Instrument collaboration has recently reported evidence for a time-evolving dark energy density [4,5], motivating renewed interest in alternative scenarios such as quintessence models with minimal and nonminimal couplings to gravity [6–16] or dark energy–dark matter interactions [17–23].

Besides the late-time accelerated expansion associated with dark energy, the prevailing cosmological paradigm holds that the very early Universe underwent a period of quasiexponential expansion called inflation [24–26]. This primeval stage of cosmic acceleration can be driven by a scalar field (the inflaton) that slowly rolls down its potential, solving many of the issues of the standard big bang cosmology and providing accurate predictions about the CMB anisotropies and the large-scale structure in the Universe [27–29]. Nevertheless, it faces several problems related to the origin and naturalness of the initial state or its robustness against large inhomogeneities in both the scalar field and the metric (see, for instance, [30–33]).

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The phenomenology of both inflation and dynamical dark energy can be reproduced by means of a homogeneous scalar field that evolves under the influence of a potential. Typically, the shape of this potential and the initial conditions have to be carefully engineered in order to get a successful phase of accelerated expansion. For example, in the case of slow-roll inflation, the potential has to be sufficiently flat and the kinetic energy density of the field must remain subdominant. One may argue that this type of construction is contrived and completely different from the way that other well-known forms of energy in the universe are described. The question motivating this work is whether simple scalar field fluctuations about a minimum of the potential can induce cosmic acceleration. This kind of state seems more natural, as it closely parallels the CMB radiation. In this picture, the accelerated expansion of the Universe may simply be a consequence of a "Higgs boson background radiation," without the need to invoke a cosmological constant or a specially designed potential.

One may naïvely expect that such scalar radiation would exert positive pressure as in the case of the CMB, thereby precluding accelerated expansion. In this work, we will show that this is indeed the case in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe, but generally not in a closed one. More specifically, we will consider a well-motivated initial state for the field in the form of standard fluctuations in a closed universe and we will prove that the initial equation of state parameter can be smaller than $-1/3$ if the Compton wavelength of the bosons exceeds the initial scale factor. As we shall see, this unexpected behavior of radiation is a direct consequence of the nontrivial topology of spacetime. The compact topology of the universe plays the role of the physical boundaries in systems where the Casimir effect [34] takes place, sourcing additional contributions to the energy density and pressure of the fluctuations [35,36]. This has proven relevant in a variety of contexts, in particular in quantum cosmology. For example, back in 1984, Zeldovich and Starobinsky argued that such corrections could have a crucial impact on the quantum creation of universes with nontrivial spatial topologies [37] (see also [38] for a recent reexamination of this claim).

The Casimir effect in cosmological backgrounds has been studied by several authors over the years [39–64] (see [65] for a review) for scalar, spinor, and vector fields. The great majority of this work has focused on conformally coupled fields in the Einstein static universe, with the emphasis placed on the renormalization techniques required to regularize the formally divergent vacuum energy density. The possibility of accelerated expansion was typically not addressed, and in many cases effectively excluded by the assumption of conformal coupling. A notable exception is [52], where the authors pointed out that even massless fields minimally coupled to gravity can drive inflation. Although the analysis in that work was restricted to the Einstein static universe, it provided a generalization of earlier results to arbitrary curvature coupling. The present work is similar in spirit to that study in that it emphasizes the dynamical consequences of scalar field fluctuations beyond the conformally coupled case, but it is otherwise completely independent; in particular, all calculations presented here were carried out without prior knowledge of [52]. A crucial difference is that we derive the initial energy density and pressure of the fluctuations in a nonstatic universe, as well as the induced Hubble rate and the range of scalar field masses needed for cosmic acceleration. Furthermore, our approach is not restricted to vacuum fluctuations, but instead allows for a generic initial power spectrum.

The organization of the manuscript is the following. In Sec. II, we present the scalar field model under cosnideration. In Sec. III, we compute the initial energy density and pressure in a spatially flat FLRW universe for generic curvature coupling and power spectrum of the field, showing that accelerated expansion is impossible. In Sec. IV, we repeat the calculation for a closed universe and prove that cosmic acceleration is possible in this case. Finally, we present our conclusions in Sec. V.

II. THE MODEL

Consider a universe dominated by a real scalar field Φ with a vanishing potential energy density at its minimum, located at $\Phi = 0$. If the field is sitting at that minimum in the form of small fluctuations, $\Phi(t,\vec{x}) = 0 + \phi(t,\vec{x})$, the action can be expanded as

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi\phi^2 R\right], \tag{1}$$

where

$$V(\phi) = \frac{1}{2}M^2\phi^2 \tag{2}$$

and ξ is a possible nonminimal coupling to gravity.[1] We will assume that these scalar field fluctuations $\phi(t,\vec{x})$ with mass M induce, on average, a FLRW metric,

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + \Sigma^2(\chi)d\theta^2 + \Sigma^2(\chi)\sin^2\theta\, d\varphi^2], \tag{3}$$

with

$$\Sigma^2(\chi) = \begin{cases} \sinh^2\chi & \text{if } k = -1 \\ \chi^2 & \text{if } k = 0 \\ \sin^2\chi & \text{if } k = 1. \end{cases} \tag{4}$$

In other words, the metric appearing in the action is not an arbitrary background metric on top of which the fluctuations evolve, but the ensemble-averaged FLRW metric directly generated by them. The universe is completely dominated by the fluctuations. Here and henceforth, we will refer to ensemble-averaged Ricci scalar R, Hubble rate H, scale factor a, and curvature index k.

The equation of motion for the scalar field ϕ is

$$\ddot{\phi} - \frac{1}{a^2}\nabla^2\phi + 3H\dot{\phi} + (M^2 + \xi R)\phi = 0, \tag{5}$$

where dots denote derivatives with respect to cosmic time t and ∇^2 is the Laplacian associated with the comoving coordinates χ, θ, and φ.

The energy-momentum tensor, computed as $T_{\mu\nu} = -(2/\sqrt{-g})\delta S_\phi/\delta g^{\mu\nu}$ [with S_ϕ given by the terms in (1) which depend explicitly on ϕ and its derivatives], reads

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right] + \xi(G_{\mu\nu} + g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\phi^2, \tag{6}$$

where $G_{\mu\nu}$ is the Einstein tensor, ∇_μ denotes the covariant derivative, and the box operator is defined as $\Box = -\partial_t^2 + (1/a^2)\nabla^2 - 3H\partial_t$. The energy density and pressure, computed as $\rho = T_{00}$ and $p = g^{ij}T_{ij}/3$, respectively, can be shown to be

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}(\vec{\nabla}\phi)^2 + V(\phi) + \xi\left[-\frac{2}{a^2}(\vec{\nabla}\phi)^2 - \frac{2}{a^2}\phi\nabla^2\phi + 3\left(H^2 + \frac{k}{a^2}\right)\phi^2 + 6H\phi\dot{\phi}\right], \tag{7}$$

[1]As we will see, cosmic acceleration in a closed universe will be possible even for minimal coupling ($\xi = 0$).

$$
\begin{aligned}
p = &\frac{1}{2}\dot{\phi}^2 - \frac{1}{6a^2}(\vec{\nabla}\phi)^2 - V(\phi) \\
&+ \xi\left\{-2\dot{\phi}^2 + \frac{4}{3a^2}(\vec{\nabla}\phi)^2 - \frac{2}{3a^2}\phi\nabla^2\phi + 2\phi V'(\phi)\right. \\
&\left.+ \left[H^2 + \frac{k}{a^2} + \left(2\xi - \frac{1}{3}\right)R\right]\phi^2 + 2H\phi\dot{\phi}\right\}. \qquad (8)
\end{aligned}
$$

If we introduce conformal time (defined as $d\eta = dt/a$) and the rescaled field

$$\psi(\eta, \vec{x}) = a(\eta)\phi(\eta, \vec{x}), \qquad (9)$$

the equation of motion (5) can be rewritten as

$$\psi'' - \nabla^2\psi + a^2 M_{\mathrm{eff}}^2 \psi = 0, \qquad (10)$$

where primes indicate derivatives with respect to conformal time η and the time-dependent effective mass $M_{\mathrm{eff}}(\eta)$ is given by

$$M_{\mathrm{eff}}^2(\eta) = M^2 + \left(\xi - \frac{1}{6}\right)R(\eta) + \frac{k}{a^2(\eta)}. \qquad (11)$$

In terms of the field ψ, the energy density and pressure become

$$
\begin{aligned}
\rho = &\frac{1}{2a^4}(\psi')^2 + \frac{1-4\xi}{2a^4}(\vec{\nabla}\psi)^2 \\
&+ \frac{1}{2a^2}\left[M^2 + (1-6\xi)H^2 + 6\xi\frac{k}{a^2}\right]\psi^2 \\
&- \frac{2\xi}{a^4}\psi\nabla^2\psi - \frac{H}{a^3}(1-6\xi)\psi\psi', \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
p = &\frac{1-4\xi}{2a^4}(\psi')^2 - \frac{1-8\xi}{6a^4}(\vec{\nabla}\psi)^2 + \frac{1}{2a^2}\left[(4\xi-1)M^2\right. \\
&\left.+ (1-6\xi)H^2 + 2\xi\frac{k}{a^2} + \left(4\xi - \frac{2}{3}\right)\xi R\right]\psi^2 \\
&- \frac{2\xi}{3a^4}\psi\nabla^2\psi - \frac{H}{a^3}(1-6\xi)\psi\psi'. \qquad (13)
\end{aligned}
$$

Given the initial conditions for ψ, and in particular its power spectrum, one can use the last two expressions to compute the ensemble-averaged energy density and pressure and determine whether the universe is accelerating at the initial time. According to the acceleration equation, we will have

$$\frac{\ddot{a}_i}{a_i} = -\frac{4\pi G}{3}(\langle\rho_i\rangle + 3\langle p_i\rangle), \qquad (14)$$

where the subscript i denotes evaluation at the initial time. In the following Sections, we will determine whether $\langle\rho_i\rangle + 3\langle p_i\rangle < 0$, in which case the universe will undergo accelerated expansion.

III. SPATIALLY FLAT UNIVERSE

For $k = 0$, we self-consistently choose the initial conditions

$$\psi(\eta_i, \vec{x}) = \int \frac{d^3q}{\sqrt{2(2\pi)^3\omega_{\vec{q}}}}(\alpha_{\vec{q}}e^{i\vec{q}\cdot\vec{x}} + \alpha_{\vec{q}}^* e^{-i\vec{q}\cdot\vec{x}}), \qquad (15)$$

$$\psi'(\eta_i, \vec{x}) = -i\int d^3q\sqrt{\frac{\omega_{\vec{q}}}{2(2\pi)^3}}(\alpha_{\vec{q}}e^{i\vec{q}\cdot\vec{x}} - \alpha_{\vec{q}}^* e^{-i\vec{q}\cdot\vec{x}}), \qquad (16)$$

which are well motivated by the equation of motion (10). Indeed, at the quantum level, these conditions minimize the expectation value of the Hamiltonian in the initial vacuum state (see, for example, chapter 6 in [66]). In these expressions, the comoving angular frequency is given by

$$\omega_{\vec{q}}^2 = \vec{q}^2 + a_i^2 M_{\mathrm{eff}}^2(\eta_i). \qquad (17)$$

On the other hand, the real and imaginary parts of the complex coefficients $\alpha_{\vec{q}}$ will be taken to be independent random variables with zero mean and variance $\langle|\alpha_{\vec{q}}|^2\rangle/2$. One can use this information to find the initial ensemble averages of ψ^2, $(\psi')^2$, $(\vec{\nabla}\psi)^2$, $\psi\nabla^2\psi$, and $\psi\psi'$, which are needed to compute the averages of (12) and (13),

$$\langle\psi^2\rangle = \frac{1}{(2\pi)^3}\int d^3q\frac{\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}, \qquad (18)$$

$$\langle(\psi')^2\rangle = \frac{1}{(2\pi)^3}\int d^3q\,\omega_{\vec{q}}\langle|\alpha_{\vec{q}}|^2\rangle, \qquad (19)$$

$$\langle(\vec{\nabla}\psi)^2\rangle = \frac{1}{(2\pi)^3}\int d^3q\frac{\vec{q}^2\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}} = -\langle\psi\nabla^2\psi\rangle, \qquad (20)$$

$$\langle\psi\psi'\rangle = 0. \qquad (21)$$

The reader is referred to the Appendix for details on the derivation of these expressions. Substituting them into (12) and (13), we get

$$
\begin{aligned}
\langle\rho_i\rangle = &\frac{1}{(2\pi)^3 a_i^4}\int d^3q\omega_{\vec{q}}\langle|\alpha_{\vec{q}}|^2\rangle \\
&+ \frac{1}{2(2\pi)^3 a_i^2}(1-6\xi)\left(H_i^2 + \frac{R_i}{6}\right)\int d^3q\frac{\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}, \qquad (22)
\end{aligned}
$$

$$
\begin{aligned}
\langle p_i\rangle = &\frac{1}{3(2\pi)^3 a_i^4}\int d^3q\frac{\vec{q}^2\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}} \\
&+ \frac{1}{2(2\pi)^3 a_i^2}(1-6\xi)\left(H_i^2 - \frac{R_i}{6}\right)\int d^3q\frac{\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}. \qquad (23)
\end{aligned}
$$

For standard vacuum fluctuations we have $\langle|\alpha_{\vec{q}}|^2\rangle = 1/2$, and for thermal fluctuations at temperature T,

$$\langle|\alpha_{\vec{q}}|^2\rangle = \frac{1}{e^{\omega_{\vec{q}}/\tilde{T}} - 1}, \tag{24}$$

with $\tilde{T} = a_i T$. In any case, it is clear that the usual expressions for Minkowski space are recovered if the second terms on the right-hand sides vanish. This happens in a static universe ($H_i = 0$, $R_i = 0$) or if the field is conformally coupled to gravity ($\xi = 1/6$). Otherwise, corrections to those expressions exist in an expanding universe, even for minimal coupling ($\xi = 0$). Can the second term in (23) make the pressure sufficiently negative to cause accelerated expansion? To answer this question, note first that the second terms on the right-hand sides depend implicitly on the left-hand sides. Using

$$H_i^2 + \frac{R_i}{6} = 4\pi G(\langle\rho_i\rangle - \langle p_i\rangle) \tag{25}$$

and

$$H_i^2 - \frac{R_i}{6} = \frac{4\pi G}{3}(\langle\rho_i\rangle + 3\langle p_i\rangle), \tag{26}$$

and defining the integrals

$$\rho_0 = \frac{1}{(2\pi)^3 a_i^4}\int d^3q\, \omega_{\vec{q}}\langle|\alpha_{\vec{q}}|^2\rangle, \tag{27}$$

$$p_0 = \frac{1}{3(2\pi)^3 a_i^4}\int d^3q \frac{\vec{q}^2\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}, \tag{28}$$

$$I = \frac{1}{(2\pi)^3 a_i^2}\int d^3q \frac{\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}, \tag{29}$$

$$x = 2\pi I G(1 - 6\xi), \tag{30}$$

one can solve for $\langle\rho_i\rangle$ and $\langle p_i\rangle$ in (22) and (23) and rewrite them in the compact form

$$\langle\rho_i\rangle = \frac{3(1-x)\rho_0 - 3xp_0}{4x^2 - 6x + 3}, \tag{31}$$

$$\langle p_i\rangle = \frac{3(1-x)p_0 + x\rho_0}{4x^2 - 6x + 3}. \tag{32}$$

One can finally use these two expressions to show that cosmic acceleration cannot occur. Taking into account that the polynomial $4x^2 - 6x + 3$ is positive for any x, we first note that $\langle\rho_i\rangle + 3\langle p_i\rangle < 0$ requires

$$\rho_0 < (4x - 3)p_0. \tag{33}$$

However, since we are considering a spatially flat universe, Friedmann's equation imposes the additional condition $\langle\rho_i\rangle > 0$, which implies

$$(1 - x)\rho_0 > xp_0. \tag{34}$$

Since both ρ_0 and p_0 are strictly positive, the last inequality can only be satisfied for $x < 1$. Otherwise, the left-hand side would be negative and the right-hand side would be positive. Now, if we multiply both sides of (33) by $1 - x$, we get

$$\begin{aligned}(1-x)\rho_0 &< (-4x^2 + 7x - 3)p_0 \\ &= -(4x^2 - 6x + 3)p_0 + xp_0.\end{aligned} \tag{35}$$

Since $4x^2 - 6x + 3$ is positive, the right-hand side is smaller than xp_0. Therefore, (33) and (34) are clearly incompatible, meaning that $\langle\rho_i\rangle + 3\langle p_i\rangle < 0$ and $\langle\rho_i\rangle > 0$ cannot be satisfied simultaneously. Note that the incompatibility holds for any curvature coupling ξ and any choice of spectrum $\langle|\alpha_{\vec{q}}|^2\rangle$.

It should be stressed that this proof relies on the fact that ρ_0 and p_0 are positive, as follows from (27) and (28). However, this property may not hold for quantum vacuum fluctuations. In this case, the aforementioned expressions are formally divergent and one has to apply a renormalization procedure, after which the resulting finite quantities are not guaranteed to remain positive. Therefore, the problem of quantum vacuum fluctuations requires additional care and is left for future work.

Even if accelerated expansion is not possible in the flat universe (with the caveat above), it is interesting to see how the usual $\langle\rho\rangle \propto T^4$ law in the case of a thermal spectrum is modified in the high-temperature limit, defined as $T \gg M_{\mathrm{eff}}(\eta_i)$. This is shown in Fig. 1. In this limit, for $\xi = 0$, we have $x \sim (T/M_p)^2$, with M_p the Planck mass. If $x \ll 1$ ($T \ll M_p$), one can easily check from (31) and (32) that the deviations from the standard ρ_0 and p_0 are very small. However, at temperatures closer to the Planck mass, the corrections are significant.

As a final comment, let us note that the results shown here are valid under the assumption of small scalar field fluctuations (that is, the assumption of a quadratic potential), which may or may not be consistent with Planckian temperatures depending on the mass of the field and the curvature of the full potential at its minimum. Let $W(\Phi)$ be the full potential and consider the next few terms in the Taylor expansion around the minimum at $\Phi = 0$,

$$W(\phi) \approx \frac{1}{2}M^2\phi^2 + \frac{1}{6}A\phi^3 + \frac{1}{24}B\phi^4, \tag{36}$$

where A and B are the third and fourth derivatives of the potential at that point, respectively. On average, the second term on the right-hand side vanishes, so the quadratic

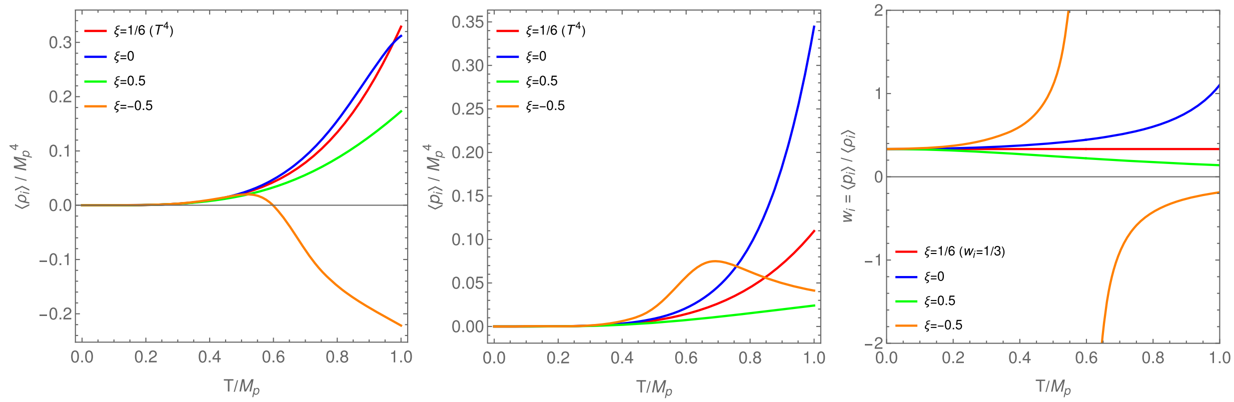

FIG. 1. Thermal energy density (left), pressure (center), and equation of state parameter $w_i = \langle p_i \rangle / \langle \rho_i \rangle$ (right) in the high-temperature limit for several values of the nonminimal coupling parameter ξ. The usual $\langle \rho \rangle \propto T^4$ law (with $w = 1/3$) is recovered for the conformal coupling $\xi = 1/6$. For other values of ξ (even for $\xi = 0$, which corresponds to the field being minimally coupled to gravity), this law gets corrections.

approximation will break down when the the ensemble average of the third term becomes comparable to that of the first term. In other words, the quadratic approximation should remain valid for temperatures such that

$$M^2 \langle \phi^2 \rangle \gg \frac{|B|}{12} \langle \phi^4 \rangle. \tag{37}$$

In the high-temperature limit, we have $\langle \phi^2 \rangle \propto T^2$ and $\langle \phi^4 \rangle \propto T^4$. Dropping numerical prefactors, we conclude that

$$T \ll \frac{M}{\sqrt{|B|}} \tag{38}$$

is needed in order for the quadratic approximation to hold. Therefore, Planckian temperatures are consistent with "small fluctuations" of the scalar field if

$$|B| \ll \left(\frac{M}{M_p} \right)^2. \tag{39}$$

IV. SPATIALLY CLOSED UNIVERSE

In the case $k = 1$, we can choose initial conditions analogous to the ones used in the previous Section by expanding the field ψ in terms of hyperspherical harmonics [39] (see also chapter 5 in [67]),

$$\psi(\eta_i, \vec{x}) = \sum_J \frac{1}{\sqrt{2\omega_n}} [\alpha_n Y_J(\vec{x}) + \alpha_n^* Y_J^*(\vec{x})], \tag{40}$$

$$\psi'(\eta_i, \vec{x}) = -i \sum_J \sqrt{\frac{\omega_n}{2}} [\alpha_n Y_J(\vec{x}) - \alpha_n^* Y_J^*(\vec{x})], \tag{41}$$

where

$$\omega_n^2 = n(n+2) + a_i^2 M_{\mathrm{eff}}^2(\eta_i) \tag{42}$$

and the index J is used for the three discrete indices $n = 0, 1, \ldots;\ l = 0, 1, \ldots, n$, and $m = -l, -l+1, \ldots, l$. In a closed universe, the hyperspherical harmonic $Y_J(\vec{x})$ is a solution of

$$\nabla^2 Y_J(\vec{x}) = -n(n+2) Y_J(\vec{x}) \tag{43}$$

and is given by

$$Y_J(\vec{x}) = \Pi_{nl}^{(+)}(\chi) y_{lm}(\theta, \varphi), \tag{44}$$

with $y_{lm}(\theta, \varphi)$ the usual spherical harmonics and

$$\Pi_{nl}^{(+)}(\chi) = \left[\frac{\pi n^2 (1 - n^2) \cdots (l^2 - n^2)}{2} \right]^{-1/2} (i \sin\chi)^l \times \frac{d^{1+l} \cos(n\chi)}{d(\cos\chi)^{1+l}}, \tag{45}$$

which can also be expressed in terms of Gegenbauer polynomials [44].

Proceeding as in the previous Section, one finds

$$\langle \psi^2 \rangle = \frac{1}{2\pi^2} \sum_{n=0}^{\infty} \frac{(n+1)^2 \langle |\alpha_n|^2 \rangle}{\omega_n}, \tag{46}$$

$$\langle (\psi')^2 \rangle = \frac{1}{2\pi^2} \sum_{n=0}^{\infty} (n+1)^2 \omega_n \langle |\alpha_n|^2 \rangle, \tag{47}$$

$$\langle (\vec{\nabla}\psi)^2 \rangle = \frac{1}{2\pi^2} \sum_{n=0}^{\infty} \frac{n(n+2)(n+1)^2 \langle |\alpha_n|^2 \rangle}{\omega_n} = -\langle \psi \nabla^2 \psi \rangle, \tag{48}$$

$$\langle \psi \psi' \rangle = 0. \tag{49}$$

Substituting (46)–(49) into (12) and (13), we get

$$\langle\rho_i\rangle = \frac{1}{2\pi^2 a_i^4}\sum_{n=0}^{\infty}(n+1)^2\omega_n\langle|\alpha_n|^2\rangle + \frac{1-6\xi}{4\pi^2 a_i^2}\left(H_i^2+\frac{R_i}{6}-\frac{1}{a_i^2}\right)\sum_{n=0}^{\infty}\frac{(n+1)^2\langle|\alpha_n|^2\rangle}{\omega_n}, \tag{50}$$

$$\langle p_i\rangle = \frac{1}{2\pi^2 a_i^4}\sum_{n=0}^{\infty}\frac{n(n+2)(n+1)^2\langle|\alpha_n|^2\rangle}{3\omega_n} + \frac{1}{4\pi^2 a_i^2}\left[(1-6\xi)\left(H_i^2-\frac{R_i}{6}\right)+\frac{1-2\xi}{a_i^2}\right]\sum_{n=0}^{\infty}\frac{(n+1)^2\langle|\alpha_n|^2\rangle}{\omega_n}. \tag{51}$$

Now we use

$$H_i^2+\frac{R_i}{6} = 4\pi G(\langle\rho_i\rangle - \langle p_i\rangle) - \frac{1}{a_i^2} \tag{52}$$

and

$$H_i^2-\frac{R_i}{6} = \frac{4\pi G}{3}(\langle\rho_i\rangle + 3\langle p_i\rangle) - \frac{1}{a_i^2}, \tag{53}$$

and define

$$\rho_0 = \frac{1}{2\pi^2 a_i^4}\sum_{n=0}^{\infty}(n+1)^2\omega_n\langle|\alpha_n|^2\rangle, \tag{54}$$

$$p_0 = \frac{1}{2\pi^2 a_i^4}\sum_{n=0}^{\infty}\frac{n(n+2)(n+1)^2\langle|\alpha_n|^2\rangle}{3\omega_n}, \tag{55}$$

$$I = \frac{1}{2\pi^2 a_i^2}\sum_{n=0}^{\infty}\frac{(n+1)^2\langle|\alpha_n|^2\rangle}{\omega_n}, \tag{56}$$

$$x = 2\pi I G(1-6\xi). \tag{57}$$

Inserting these expressions into (50) and (51) and solving for $\langle\rho_i\rangle$ and $\langle p_i\rangle$, we find

$$\langle\rho_i\rangle = \frac{3(1-x)\rho_0 - 3xp_0}{4x^2-6x+3} + \frac{3I}{a_i^2}\frac{(1-8\xi)x+6\xi-1}{4x^2-6x+3}, \tag{58}$$

$$\langle p_i\rangle = \frac{3(1-x)p_0 + x\rho_0}{4x^2-6x+3} - \frac{I}{a_i^2}\frac{x-6\xi}{4x^2-6x+3}. \tag{59}$$

Due to the compact topology of spacetime, note that the ensemble-averaged energy density and pressure in a spatially closed universe get an additional term with respect to the flat case. Note also that the extra term in (59) can contribute negatively to the pressure, in particular for minimal coupling ($\xi = 0$).

A. Sanity check: Conformal coupling

As a sanity check, we may consider the case of conformal coupling, which is well known in the literature. Setting $\xi = 1/6$ ($x = 0$), one gets

$$\langle\rho_i\rangle = \rho_0 \tag{60}$$

and

$$\begin{aligned}\langle p_i\rangle &= p_0 + \frac{I}{3a_i^2}\\ &= \frac{1}{6\pi^2 a_i^4}\sum_{n=0}^{\infty}\frac{(n+1)^2\langle|\alpha_n|^2\rangle}{\omega_n}(\omega_n^2 - a_i^2M^2),\end{aligned} \tag{61}$$

where in the last line we have used $\omega_n^2 = (n+1)^2 + a_i^2M^2$, which can be easily shown using (11) and (42). These results agree with the ones reported in [44] for vacuum fluctuations. It should be emphasized that the pressure is manifestly positive in this case, so fluctuations of a conformally coupled scalar cannot produce cosmic acceleration. We also note that the equation of state parameter is $1/3$ if the field is massless, as expected.

B. Accelerated expansion: Minimal coupling

Now we will show that the following two conditions can be satisfied simultaneously for other couplings:

(i) $\langle\rho_i\rangle + 3\langle p_i\rangle < 0$.

(ii) $H_i^2 = \frac{8\pi G}{3}\langle\rho_i\rangle - \frac{1}{a_i^2} > 0$.

This will imply that a spatially closed universe dominated by scalar field fluctuations can undergo accelerated expansion. In order to exemplify this, let us focus on the minimal coupling case $\xi = 0$ for simplicity. From (58) and (59) it follows that $\langle\rho_i\rangle + 3\langle p_i\rangle < 0$ if

$$\tilde{I} > \frac{\tilde{\rho}_0 + 3\tilde{p}_0}{1+\frac{8\pi}{\tilde{a}_i^2}\tilde{p}_0}, \tag{62}$$

where we have defined the dimensionless variables $\tilde{I} = a_i^2 I$, $\tilde{\rho}_0 = a_i^4\rho_0$, $\tilde{p}_0 = a_i^4 p_0$, and $\tilde{a}_i = a_i/L_p$, with L_p the Planck length. Note that

$$\tilde{I} = \frac{\langle|\alpha_0|^2\rangle}{2\pi^2\omega_0} + \frac{1}{2\pi^2}\sum_{n=1}^{\infty}\frac{(n+1)^2\langle|\alpha_n|^2\rangle}{\omega_n}, \tag{63}$$

so (62) can be easily satisfied if ω_0 is sufficiently small. Indeed, in the limit $\omega_0 \to 0$, we see that $\tilde{I} \to \infty$ while $\tilde{\rho}_0$ and $\tilde{p}_0$ approach finite values. An important assumption we are making here is that the infinite sums are convergent and positive. This naturally happens for a thermal spectrum of the form (24), but not for vacuum fluctuations.[2] With this in mind, let us further assume that ω_0 is so small that $\tilde{I} \gg \tilde{\rho}_0, \tilde{p}_0$. In this case, condition (i) above, or, equivalently, Eq. (62), is clearly satisfied. In order for condition (ii) to be satisfied as well, one can use (58) to show that $x > 3/2$ is needed. In terms of the dimensionless variables, x is given by

$$x = \frac{2\pi \tilde{I}}{\tilde{a}_i^2}. \tag{64}$$

Therefore, even if $\tilde{I}$ is huge, x can be order 1 if $\tilde{a}_i$ is big. The range $x \in (3/2, \infty)$ can be translated into a range for ω_0: $\omega_0 \in (0, \Omega_0)$. For instance, consider the thermal spectrum (24), and suppose that (63) is dominated by the first term on the right-hand side. Assume also that we choose $\tilde{T} \gg \omega_0$, so that $\langle |\alpha_0|^2 \rangle \approx \tilde{T}/\omega_0$. Then, using (64), we see that $x > 3/2$ means

$$\omega_0 < \sqrt{\frac{2\tilde{T}}{3\pi \tilde{a}_i^2}} \equiv \Omega_0. \tag{65}$$

Once $\tilde{a}_i$ and the parameters characterizing the spectrum ($\tilde{T}$ in the thermal case) are chosen, we can find a continuum of configurations that allow for accelerated expansion as we vary ω_0 from 0 to Ω_0. A concrete example is the following: if the initial scale factor is $\tilde{a}_i = 100$, for a thermal state at temperature $\tilde{T} = 0.1$, ω_0 in the range (0, 0.015) produces accelerated expansion (and the Hubble rate is real). For instance, for $\omega_0 = 0.0012$, we get the equation of state parameter $w_i = \langle p_i \rangle / \langle \rho_i \rangle \approx -0.61$ and the Hubble rate $H_i \approx \pm 0.0039 M_p$.

A question that may arise at this point is the following. We have

$$\omega_0^2 = a_i^2 \left(M^2 - \frac{R_i}{6} + \frac{1}{a_i^2} \right), \tag{66}$$

where

$$R_i = 8\pi G (\langle \rho_i \rangle - 3 \langle p_i \rangle) \tag{67}$$

is the initial Ricci scalar. Consider the specific example indicated above. Clearly, R_i depends implicitly on temperature, so ω_0 and T are not independent variables. How do we know that $\omega_0 = 0.0012$ is compatible with $T = \tilde{T}/a_i = 10^{-3} M_p$? Since ω_0 also depends on the bare mass of the scalar field, M, one can always make ω_0 and T compatible by choosing M appropriately.

In conclusion, accelerated expansion can be achieved for a specific range of scalar field masses. This range, as well as the corresponding Hubble rate and equation of state parameter, can be easily deduced when $\tilde{I} \gg \tilde{\rho}_0, \tilde{p}_0$. In this limit, (58) and (59) reduce to

$$\langle \rho_i \rangle \approx \frac{3x(x-1)}{2\pi G a_i^2 (4x^2 - 6x + 3)}, \tag{68}$$

$$\langle p_i \rangle \approx -\frac{x^2}{2\pi G a_i^2 (4x^2 - 6x + 3)}. \tag{69}$$

It immediately follows that the equation of state parameter is

$$w_i \approx -\frac{x}{3(x-1)}. \tag{70}$$

If $x \to \infty$, then $w_i \to -1/3$, and if $x \to 3/2$, $w_i \to -1$. Now, combining (68) with Friedmann's equation, we get

$$H_i \approx \pm \frac{1}{a_i} \sqrt{\frac{2x-3}{4x^2 - 6x + 3}}. \tag{71}$$

Note that $H_i \to 0$ in the two limiting cases $x \to \infty$ and $x \to 3/2$. However, $\dot{H}_i$ is different in each case. Inserting (68) and (69) into

$$\dot{H}_i = -4\pi G (\langle p_i \rangle + \langle \rho_i \rangle) + \frac{1}{a_i^2}, \tag{72}$$

we find

$$\dot{H}_i \approx \frac{3}{a_i^2 (4x^2 - 6x + 3)}. \tag{73}$$

This implies that $\dot{H}_i \to 0$ as $x \to \infty$, but $\dot{H}_i \to 1/a_i^2$ as $x \to 3/2$. Therefore, we may refer to $x \to \infty$ as the "static universe limit." In the other limit, $x \to 3/2$, the universe will start out with $H_i = 0$ and $w_i = -1$, and will subsequently inflate. This is reminiscent of a bounce or a quantum-driven onset of inflation [68,69].

Finally, we can insert (68) and (69) into (67) and (66) to get

$$M \approx \frac{1}{a_i} \sqrt{\frac{4x-3}{4x^2 - 6x + 3}}. \tag{74}$$

Here, we also made use of the fact that $\omega_0 \ll 1$. Taking the $x \to \infty$ and $x \to 3/2$ limits, we obtain the range of scalar field masses for which the initial state undergoes accelerated expansion,

$$M \in (0, 1/a_i). \tag{75}$$

[2] Similarly to the spatially flat case, the sums (54), (55), and (56) may not remain positive after the renormalization procedure and the conclusions extracted from this analysis could change.

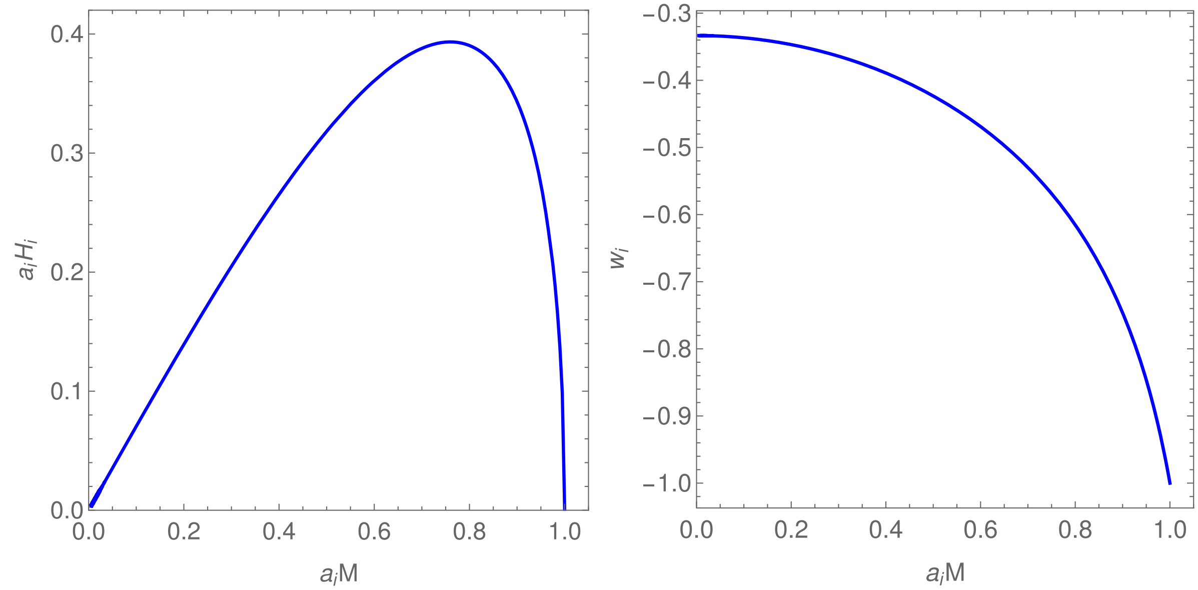

FIG. 2. Hubble rate (left) and equation of state parameter (right) corresponding to the different scalar field masses that allow for accelerated expansion.

This is perfectly consistent with a claim made by Hawking and Ellis at the end of section IV.3 in [70], where it is shown that the energy-momentum tensor of a scalar field minimally coupled to gravity globally satisfies the strong energy condition in a region of size L if $L \gtrsim 1/M$. In other words, the convergence of timelike geodesics should be unaffected over distances greater than the Compton wavelength of the field. This is precisely what we have obtained: accelerated expansion (that is, violation of the strong energy condition) takes place if the radius of curvature, a_i, is smaller than $1/M$.

In order to better visualize our results, we provide in Fig. 2 parametric plots of (70), (71), and (74) showing what the induced Hubble rate and equation of state parameter are depending on the scalar field mass. It is interesting to note that there is a special mass M_* for which the expansion rate is maximal. The value of x that maximizes the positive solution in (71) is $x_* = (3 + \sqrt{3})/2$, corresponding to

$$a_i M_* = \frac{1}{3^{1/4}} \approx 0.76. \tag{76}$$

For this special mass, the Hubble rate and equation of state parameter are

$$H_* = \frac{M_*}{\sqrt{2+\sqrt{3}}} \approx 0.52 M_* \tag{77}$$

and

$$w_* = -\frac{1}{\sqrt{3}} \approx -0.58. \tag{78}$$

V. CONCLUSIONS

In most models of inflation and dark energy, cosmic acceleration is driven by a homogeneous scalar field that evolves under the influence of a specially designed potential. Motivated by the search for a more natural mechanism, in this work we have proposed the possibility that accelerated expansion be caused instead by simple scalar field fluctuations about a minimum of the potential. In this picture, the Universe is dominated by a gas of bosons forming a scalar field background radiation analogous to the CMB.

We first addressed this possibility in a spatially flat FLRW universe, showing that the well-motivated initial state given by (15) and (16) cannot induce accelerated expansion, regardless of the curvature coupling ξ and the form of the scalar field spectrum $\langle|\alpha_n|^2\rangle$. We also showed how the usual $\langle\rho\rangle \propto T^4$ scaling is modified for thermal, nonconformal fields in the high-temperature limit. Even for minimal coupling, this law gets significant corrections for temperatures close to the Planck mass.

We then turned to the case of a spatially closed FLRW universe. We demonstrated that the initial state given by (40) and (41), naturally motivated by the equation of motion, is capable of inducing accelerated expansion. This effect is a direct consequence of the compact topology of the closed universe, as evidenced by the extra terms appearing in the expressions for the ensemble-averaged energy density and pressure relative to the flat case. Therefore, we may well interpret this topology-sensitive mechanism as a cosmological Casimir effect.

Accelerated expansion can take place even for minimal coupling, provided the effective mass of the field is sufficiently small. In this case, reality of the Hubble rate requires that the Compton wavelength of the bosons (defined as the inverse of the bare mass) be greater than the scale factor. Under these conditions, the topological corrections make the pressure sufficiently negative, leading to repulsive gravity. The Hubble rate induced by such a state ranges from zero to roughly half the bare mass of the scalar field. Interestingly, when the Compton wavelength matches the curvature radius (that is, the scale factor), the Hubble rate vanishes and the equation of state parameter is -1, resembling a cosmic bounce or a quantum-driven onset of inflation.

Although our results are expressed in terms of an arbitrary scalar field spectrum $\langle|\alpha_n|^2\rangle$, special care must be taken when vacuum fluctuations are considered. In this case, the finite sums obtained after renormalization are not guaranteed to be positive and the conclusions would need to be reexamined. A careful treatment of renormalization effects is deferred to upcoming publications. Future work will also be aimed at computing the time evolution of the initial states considered in this study and assessing the viability of the fluctuation-driven acceleration as a model to describe inflation and dark energy in our Universe.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: COMPUTATION OF ENSEMBLE AVERAGES

In this Appendix, we will give the necessary details to derive the ensemble averages (18)–(21), corresponding to a spatially flat universe. In the case of a closed universe, the Laplacian has a discrete spectrum labeled by n, with eigenvalues $n(n+2)$ and degeneracy $(n+1)^2$. Therefore, one can obtain the closed universe averages (46)–(49) from the flat case by making these two replacements,

$$\vec{q}^2 \to n(n+2), \tag{A1}$$

$$\frac{1}{(2\pi)^3}\int d^3q \to \frac{1}{2\pi^2}\sum_{n=0}^{\infty}(n+1)^2. \tag{A2}$$

Let $f_\psi(\vec{q})$ be the Fourier transform of $\psi(\eta_i,\vec{x})$, namely,

$$\psi(\eta_i,\vec{x}) = \int d^3q f_\psi(\vec{q})e^{i\vec{q}\cdot\vec{x}}. \tag{A3}$$

Similarly, $f_\pi(\vec{q})$ will denote the Fourier transform of $\psi'(\eta_i,\vec{x})$,

$$\psi'(\eta_i,\vec{x}) = \int d^3q f_\pi(\vec{q})e^{i\vec{q}\cdot\vec{x}}. \tag{A4}$$

Comparison of these two expressions with (15) and (16) yields

$$f_\psi(\vec{q}) = \frac{1}{\sqrt{2(2\pi)^3\omega_{\vec{q}}}}(\alpha_{\vec{q}}+\alpha^*_{\vec{q}}), \tag{A5}$$

$$f_\pi(\vec{q}) = -i\sqrt{\frac{\omega_{\vec{q}}}{2(2\pi)^3}}(\alpha_{\vec{q}}-\alpha^*_{\vec{q}}). \tag{A6}$$

As pointed out in the main text, the real and imaginary parts of $\alpha_{\vec{q}}$ ($A_{\vec{q}}$ and $B_{\vec{q}}$, respectively) are independent random variables with vanishing expectation value and variance $\langle|\alpha_{\vec{q}}|^2\rangle/2$, meaning that

$$\langle A_{\vec{q}}\rangle = \langle B_{\vec{p}}\rangle = 0, \tag{A7}$$

$$\langle A_{\vec{q}}B_{\vec{p}}\rangle = 0, \tag{A8}$$

$$\langle A_{\vec{q}}A_{\vec{p}}\rangle = \langle B_{\vec{q}}B_{\vec{p}}\rangle = \frac{\langle|\alpha_{\vec{q}}|^2\rangle}{2}\delta^{(3)}(\vec{q}-\vec{p}). \tag{A9}$$

One can use these expressions to find $\langle f_\psi(\vec{q})f_\psi(\vec{p})\rangle$, $\langle f_\pi(\vec{q})f_\pi(\vec{p})\rangle$ and $\langle f_\psi(\vec{q})f_\pi(\vec{p})\rangle$. After some straightforward algebra, we get

$$\langle f_\psi(\vec{q})f_\psi(\vec{p})\rangle = \frac{\langle|\alpha_{\vec{q}}|^2\rangle}{(2\pi)^3\sqrt{\omega_{\vec{q}}\omega_{\vec{p}}}}\delta^{(3)}(\vec{q}+\vec{p}), \tag{A10}$$

$$\langle f_\pi(\vec{q})f_\pi(\vec{p})\rangle = \frac{\sqrt{\omega_{\vec{q}}\omega_{\vec{p}}}}{(2\pi)^3}\langle|\alpha_{\vec{q}}|^2\rangle\delta^{(3)}(\vec{q}+\vec{p}), \tag{A11}$$

$$\langle f_\psi(\vec{q})f_\pi(\vec{p})\rangle = 0. \tag{A12}$$

With this information, we can now compute the averages (18)–(21):

(i) $\langle\psi^2\rangle$: According to the convolution theorem, the Fourier transform of ψ^2 is

$$f_{\psi^2}(\vec{p}) = \int d^3q f_\psi(\vec{q})f_\psi(\vec{p}-\vec{q}). \tag{A13}$$

Therefore,

$$\langle\psi^2\rangle = \int d^3p \int d^3q \langle f_\psi(\vec{q}) f_\psi(\vec{p}-\vec{q})\rangle e^{i\vec{p}\cdot\vec{x}}. \tag{A14}$$

Using (A10) and computing the d^3p integral, we get

$$\langle\psi^2\rangle = \frac{1}{(2\pi)^3}\int d^3q \frac{\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}. \tag{A15}$$

(ii) $\langle(\psi')^2\rangle$: Applying exactly the same procedure with f_π instead of f_ψ, we get

$$\langle(\psi')^2\rangle = \frac{1}{(2\pi)^3}\int d^3q\,\omega_{\vec{q}}\langle|\alpha_{\vec{q}}|^2\rangle. \tag{A16}$$

(iii) $\langle(\vec{\nabla}\psi)^2\rangle$: The square of the gradient of ψ is given by

$$(\vec{\nabla}\psi)^2 = a^2 g^{ij}\partial_i\psi\partial_j\psi = (\partial_\chi\psi)^2 + \frac{(\partial_\theta\psi)^2}{\Sigma^2(\chi)} + \frac{(\partial_\varphi\psi)^2}{\Sigma^2(\chi)\sin^2\theta}. \tag{A17}$$

The Fourier transform of $\partial_i\psi$ can be easily deduced by taking the derivative of (A3) with respect to x^i:

$$f_{\psi_{,i}}(\vec{p}) = i p_i f_\psi(\vec{p}). \tag{A18}$$

Now, by the convolution theorem, the Fourier transform of $(\partial_i\psi)^2$ is

$$\begin{aligned} f_{\psi^2_{,i}}(\vec{p}) &= \int d^3q f_{\psi_{,i}}(\vec{q}) f_{\psi_{,i}}(\vec{p}-\vec{q}) \\ &= \int d^3q\, q_i(q_i - p_i) f_\psi(\vec{q}) f_\psi(\vec{p}-\vec{q}), \end{aligned} \tag{A19}$$

so

$$\langle(\partial_i\psi)^2\rangle = \int d^3p \int d^3q\, q_i(q_i-p_i) \times \langle f_\psi(\vec{q}) f_\psi(\vec{p}-\vec{q})\rangle e^{i\vec{p}\cdot\vec{x}}. \tag{A20}$$

Using (A10) and computing the d^3p integral, we get

$$\langle(\partial_i\psi)^2\rangle = \frac{1}{(2\pi)^3}\int d^3q \frac{q_i^2\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}. \tag{A21}$$

Finally, inserting this into (A17) and taking into account that $\vec{q}^2 = a^2 g^{ij} q_i q_j$, we get

$$\langle(\vec{\nabla}\psi)^2\rangle = \frac{1}{(2\pi)^3}\int d^3q \frac{\vec{q}^2\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}. \tag{A22}$$

(iv) $\langle\psi\nabla^2\psi\rangle$: The Laplacian of ψ is given by

$$\begin{aligned} \nabla^2\psi &= \frac{a^2}{\sqrt{\gamma}}\partial_i(\sqrt{\gamma}g^{ij}\partial_j\psi) = \partial_\chi^2\psi + \frac{\partial_\theta^2\psi}{\Sigma^2(\chi)} \\ &\quad + \frac{\partial_\varphi^2\psi}{\Sigma^2(\chi)\sin^2\theta} + \frac{2\Sigma'(\chi)}{\Sigma(\chi)}\partial_\chi\psi + \frac{\partial_\theta\psi}{\Sigma^2(\chi)\tan\theta}, \end{aligned} \tag{A23}$$

where γ denotes the determinant of the spatial part of the metric. The Fourier transform of $\partial_i^2\psi$ can be easily deduced by taking the second derivative of (A3) with respect to x^i:

$$f_{\psi_{,ii}}(\vec{p}) = -p_i^2 f_\psi(\vec{p}). \tag{A24}$$

Now, by the convolution theorem, the Fourier transform of $\psi\partial_i^2\psi$ is

$$\begin{aligned} f_{\psi\psi_{,ii}}(\vec{p}) &= \int d^3q f_\psi(\vec{q}) f_{\psi_{,ii}}(\vec{p}-\vec{q}) \\ &= -\int d^3q (p_i - q_i)^2 f_\psi(\vec{q}) f_\psi(\vec{p}-\vec{q}). \end{aligned} \tag{A25}$$

Therefore,

$$\langle\psi\partial_i^2\psi\rangle = -\int d^3p \int d^3q (p_i-q_i)^2 \times \langle f_\psi(\vec{q}) f_\psi(\vec{p}-\vec{q})\rangle e^{i\vec{p}\cdot\vec{x}}, \tag{A26}$$

which yields

$$\langle\psi\partial_i^2\psi\rangle = -\frac{1}{(2\pi)^3}\int d^3q \frac{q_i^2\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}} \tag{A27}$$

once we use (A10) and perform the d^3p integral.

Regarding the $\psi\partial_i\psi$ terms, using (A18) and the convolution theorem, we get

$$\begin{aligned} f_{\psi\psi_{,i}}(\vec{p}) &= \int d^3q f_\psi(\vec{q}) f_{\psi_{,i}}(\vec{p}-\vec{q}) \\ &= i\int d^3q (p_i - q_i) f_\psi(\vec{q}) f_\psi(\vec{p}-\vec{q}). \end{aligned} \tag{A28}$$

Then, the average $\langle\psi\partial_i\psi\rangle$ is

$$\langle\psi\partial_i\psi\rangle = -i\int d^3q \frac{q_i\langle|\alpha_{\vec{q}}|^2\rangle}{\omega_{\vec{q}}}. \tag{A29}$$

Putting everything together and taking into account that $\vec{q}^2 = a^2 g^{ij} q_i q_j$, we obtain

$$\langle \psi \nabla^2 \psi \rangle = -\frac{1}{(2\pi)^3} \int d^3 q \frac{\vec{q}^2 \langle |\alpha_{\vec{q}}|^2 \rangle}{\omega_{\vec{q}}}. \tag{A30}$$

(v) $\langle \psi \psi' \rangle$: Application of the convolution theorem and (A12) immediately yields

$$\langle \psi \psi' \rangle = \int d^3 p \int d^3 q \langle f_\psi(\vec{q}) f_\pi(\vec{p} - \vec{q}) \rangle e^{i\vec{p}\cdot\vec{x}} = 0. \tag{A31}$$

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