Letter

Circular orbits and photon orbits at wormhole throats

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In this work we study timelike circular orbits and photon orbits at the throat of stationary and axisymmetric wormholes. We assume a global radial coordinate l connecting both sides, with twice differentiable metric components and vanishing first derivatives at the throat. With regard to the specific angular momentum ℓ of a test particle, we derive expressions for the boundaries of a spectrum of bound circular orbits. We further express these boundaries in terms of physical properties of the spacetime. With regard to the parameter space behavior, we found a phase transition occurring for wormholes with an ergoregion. Furthermore, the presented expressions could give constraints on wormhole properties, as they hint to possible instabilities, especially for fast rotating wormholes and wormholes with an ergoregion.

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Introduction. The theory of general relativity provides a framework for extraordinary concepts such as black holes. Black holes, once purely theoretical, are now firmly established through indirect and direct observations and are believed to dominate on larger scales as central astrophysical bodies. Besides black holes, also wormholes arose as one of the most prominent concepts. They describe a connection between two arbitrary distant regions of spacetime. In detail, this connection is referred to as the wormhole throat, and it can link regions within one universe or also link different ones. The traversability of such a connection requires a singularity-free and horizonless spacetime [1–7], which is usually linked to a negative energy density at the wormhole throat. An exotic form of matter has been proposed with such properties. However, there exist many alternative theories and extensions beyond general relativity offering alternative explanations [8–13]. To test such theories, it is crucial to identify observational signatures. In some astrophysical scenarios wormholes may mimic black hole features [14–22], while in others they exhibit distinctive phenomenology [23–27]. Identifying such differences is essential for future observations. In this work, we study null and timelike bound circular orbits at wormhole throats for a generalized class of wormholes. Our findings may give general hints on the stability of wormhole solutions, as for solutions with ergoregions, which are strengthened by other works [28,29] and furthermore be relevant also for observational signatures in the form of accretion disks, as these are usually modeled on the presence of stable circular orbits.

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Circular orbits at wormhole throats. We consider stationary, axisymmetric wormholes with local mirror symmetry at the throat. Both sides are connected by a global radial coordinate $l \in (-\infty, \infty)$, with $l = 0$ marking the throat. The metric components $g_{\mu\nu}$ are assumed $\mathcal{C}^2$ in l and, by symmetry, have vanishing first derivatives at the throat. While some rotating wormhole solutions fail to satisfy these conditions others do e.g. [30–33]. The generalized line element reads

$$ds^2 = g_{tt}dt^2 + g_{ll}dl^2 + g_{\theta\theta}d\theta^2 + g_{\varphi\varphi}d\varphi^2 + 2g_{t\varphi}dtd\varphi. \tag{1}$$

Circular motion of particles is characterized by a vanishing radial velocity component. For planar equatorial motion of a massive particle the Lagrangian $\mathcal{L}^*$ can then be written as

$$\mathcal{L}^* = g_{ll}\dot{l}^2 + g_{tt}\dot{t}^2 + g_{\varphi\varphi}\dot{\varphi}^2 + 2g_{t\varphi}\dot{t}\,\dot{\varphi} = -1, \tag{2}$$

where the dot marks the derivative with respect to the proper time τ. Due to the stationarity and axisymmetry of the spacetime, t and φ are cyclic coordinates, which leads to two constants of motion, namely the energy E and the angular momentum L of the particle. The derivatives $\dot{t}$ and $\dot{\varphi}$ can be expressed through E and L as

$$\dot{t} = \frac{g_{\varphi\varphi}E + g_{t\varphi}L}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}; \quad \dot{\varphi} = -\frac{g_{tt}L + g_{t\varphi}E}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}, \tag{3}$$

inserting these expressions into Eq. (2) leads to

$$g_{ll}\dot{l}^2 - \frac{1}{g_{t\varphi}^2 - g_{\varphi\varphi}g_{tt}}(g_{\varphi\varphi}E^2 + g_{tt}L^2 + 2g_{t\varphi}EL) + 1$$
$$= g_{ll}\dot{l}^2 + V_{\rm eff} = 0. \tag{4}$$

Circular motion requires the conditions $V_{\rm eff} = 0$ and $\partial_l V_{\rm eff} = 0$. This system of two equations can be solved for E and L,

$$g_{\varphi\varphi}E^2 + g_{tt}L^2 + 2g_{t\varphi}EL - (g_{t\varphi}^2 - g_{\varphi\varphi}g_{tt}) = 0, \tag{5}$$

$$E^2\partial_l g_{\varphi\varphi} + L^2\partial_l g_{tt} + 2EL\partial_l g_{t\varphi} - \partial_l(g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}) = 0. \tag{6}$$

In view of a simple notation, from now on every quantity will denote its value at the throat. Since the first derivatives of the metric components vanish at the wormhole throat, Eq. (6) is always trivially satisfied. As a consequence the system of equations is underdetermined, and a spectrum of solutions for E and L is possible. To analyze the spectrum, Eq. (5) can be rearranged for the energy,

$$\frac{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}{g_{tt}\ell^2 + 2g_{t\varphi}\ell + g_{\varphi\varphi}} = E^2, \tag{7}$$

where $\ell = \frac{L}{E}$ is the specific angular momentum of the particle. It follows immediately that $E^2 > 0$ is the boundary for the existence of circular orbits. Furthermore, for a normalized mass energy, bound circular orbits require for the energy $E \leq 1$. Combining these boundaries, leads to a possible spectrum of solutions for ℓ, for which bound circular orbits exist at the wormhole throat. Thus, the term in Eq. (7) can be written in an inequality for the parameter space of ℓ for which orbits exist,

$$0 < \frac{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}{g_{tt}\ell^2 + 2g_{t\varphi}\ell + g_{\varphi\varphi}} \leq 1. \tag{8}$$

Solving for the limits of this inequality gives the limiting values for ℓ, which define the shape of the parameter space. The numerator in the expression corresponds to the negative determinant of the two-dimensional induced metric for g_{ij} with $(i,j) = t, \varphi$, which is always positive. As a consequence the denominator must strictly be positive in order to satisfy the lower bound of the inequality. Thus, the roots of the denominator give the boundary for the existence of circular orbits,

$$\ell_\pm = \frac{-g_{t\varphi} \pm \sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}}{g_{tt}} \coloneqq \ell_E^\pm, \tag{9}$$

where we define the boundary of existence for circular orbits as $\ell_E^\pm$. In order to determine the region for the boundedness of such orbits, the expression (8) can be solved for the equality in the upper limit,

$$\ell_\pm = \frac{-g_{t\varphi} \pm \sqrt{(g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi})(1 + g_{tt})}}{g_{tt}} \coloneqq \ell_B^\pm, \tag{10}$$

where we define $\ell_B^\pm$ as the boundary for closed circular orbits. For both limits, we assumed that $g_{tt} \neq 0$ at the throat. In the special case of $g_{tt} = 0$ the boundaries are given by $\ell_E = -\frac{g_{\varphi\varphi}}{2g_{t\varphi}}$ and $\ell_B = \frac{g_{t\varphi}^2 - g_{\varphi\varphi}}{2g_{t\varphi}}$. Furthermore, it follows from Eq. (10) that $g_{tt} > -1$ must hold for the existence of bound orbits. To determine which of the boundaries is the lower bound and which is the upper bound, the differences $\ell_E^+ - \ell_E^-$ and $\ell_B^+ - \ell_B^-$ can be analyzed,

$$\ell_E^+ - \ell_E^- = 2\frac{\sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}}{g_{tt}};$$
$$\ell_B^+ - \ell_B^- = 2\frac{\sqrt{(g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi})(1 + g_{tt})}}{g_{tt}}. \tag{11}$$

Since the inequalities are quadratic expressions of ℓ, they possess an extremum between the boundaries. With consideration of the sign of the inequalities, the second derivative of the inequalities with respect to ℓ determines then if the allowed values for ℓ lie in between the boundaries or outside of them. Furthermore, the relation between the boundaries of existence and the boundaries for bound circular orbits can be analyzed by taking the differences $\ell_E^+ - \ell_B^+$ and $\ell_E^- - \ell_B^-$,

$$\ell_E^\pm - \ell_B^\pm = \frac{\pm\sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}} \mp \sqrt{(g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi})(1 + g_{tt})}}{g_{tt}}, \tag{12}$$

The second derivatives of the inequalities (with respect to ℓ) and each of the defined differences depend all on the sign of g_{tt} at the throat. We differentiate here now between both cases $g_{tt} < 0$ and $g_{tt} > 0$ (which usually corresponds to an ergoregion) and analyze their implications.

If $g_{tt} < 0$, then the expressions in (11) give $\ell_E^+ < \ell_E^-$ and $\ell_B^+ < \ell_B^-$ as the numerator is always positive. Furthermore, the second derivative of (9) is negative, which implies that the extremum between the roots is a maximum. As a consequence the allowed region for ℓ lies in between ℓ_E^+ and ℓ_E^-. The second derivative of (10) is positive and the allowed region for ℓ lies also in between ℓ_B^+ and ℓ_B^-. The difference in (12) for $\ell_E^+ - \ell_B^+$ is negative as the numerator is positive, due to $(1+g_{tt}) \in (0,1)$. Following the same reasoning, the difference for $\ell_E^- - \ell_B^-$ is always positive. We have therefore $\ell_E^+ < \ell_E^-$ and $\ell_B^- < \ell_E^-$. Combining all inequalities leads to $\ell_E^+ < \ell_B^+ < \ell_B^- < \ell_E^-$. The parameter space for bound circular orbits is thus given by $\ell \in [\ell_B^+, \ell_B^-]$.

If $g_{tt} > 0$, then the expressions in (11) give $\ell_E^- < \ell_E^+$ and $\ell_B^- < \ell_B^+$. The second derivative of (9) is positive and the extremum between the roots is a minimum. The region for allowed values of ℓ lies outside the interval $[\ell_E^-, \ell_E^+]$. The second derivative of (10) is negative and the extremum is a maximum, thus the allowed region of ℓ for bound orbits lies also outside the interval (ℓ_B^-, ℓ_B^+). The differences in (12) lead to $\ell_E^+ < \ell_B^+$ and $\ell_B^- < \ell_E^-$, since now $(1+g_{tt}) > 1$. Bringing all inequalities together leads to $\ell_B^- < \ell_E^- < \ell_E^+ < \ell_B^+$. The parameter space for bound circular orbits is thus given by $\ell \in (-\infty, \ell_B^-] \vee \ell \in [\ell_B^+, \infty)$. Table I gives a compact overview.

Stability of orbits. Considering now the stability of the orbits, stable circular motion requires the extremum of V_{eff} to be a minimum and thus the inequality $\partial_l^2 V_{\mathrm{eff}} \geq 0$ has to be satisfied at the throat,

$$E^2 \partial_l^2 g_{\varphi\varphi} + L^2 \partial_l^2 g_{tt} + 2EL \partial_l^2 g_{t\varphi} - \partial_l^2 (g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi}) \leq 0. \tag{13}$$

Rearranging this inequality in terms of ℓ and furthermore inserting the expression (7) for the squared energy E^2 leads to

$$\frac{\ell^2 \partial_l^2 g_{tt} + 2\ell \partial_l^2 g_{t\varphi} + \partial_l^2 g_{\varphi\varphi}}{(\ell^2 g_{tt} + 2\ell g_{t\varphi} + g_{\varphi\varphi})} - \frac{\partial_l^2 (g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi})}{g_{t\varphi^2} - g_{tt} g_{\varphi\varphi}} \leq 0. \tag{14}$$

This represents again a quadratic inequality for the specific angular momentum ℓ, where the boundaries of stability are given by solving for the equality. For the sake of more compact expressions we define here $D = g_{t\varphi}^2 - g_{tt} g_{\varphi\varphi}$, solving the inequality for the boundaries gives then

$$\Rightarrow \ell_\pm = \frac{D\partial_l^2 g_{t\varphi} - g_{t\varphi}\partial_l^2 D \pm \sqrt{(g_{t\varphi}\partial_l^2 D - D\partial_l^2 g_{t\varphi})^2 - (g_{tt}\partial_l^2 D - D\partial_l^2 g_{tt})(g_{\varphi\varphi}\partial_l^2 D - D\partial_l^2 g_{\varphi\varphi})}}{g_{tt}\partial_l^2 D - D\partial_l^2 g_{tt}} \coloneqq \ell_S^\pm, \tag{15}$$

where we define the boundaries of the stability for circular orbits as $\ell_S^\pm$. Stable orbits only exist if the expression under the square root is not negative, which gives a necessary condition for stability,

$$(g_{t\varphi}\partial_l^2 D - D\partial_l^2 g_{t\varphi})^2 - (g_{tt}\partial_l^2 D - D\partial_l^2 g_{tt})(g_{\varphi\varphi}\partial_l^2 D - D\partial_l^2 g_{\varphi\varphi}) \geq 0. \tag{16}$$

In order to analyze the relation between the boundaries, we conduct the same procedure as for the limits on existence and boundedness in Sec. II. The difference $\ell_S^+ - \ell_S^-$ and the second derivative with respect to ℓ of the inequality in (14) are given by

$$\ell_S^+ - \ell_S^- = 2\frac{\sqrt{(g_{t\varphi}\partial_l^2 D - D\partial_l^2 g_{t\varphi})^2 - (g_{tt}\partial_l^2 D - D\partial_l^2 g_{tt})(g_{\varphi\varphi}\partial_l^2 D - D\partial_l^2 g_{\varphi\varphi})}}{g_{tt}\partial_l^2 D - D\partial_l^2 g_{tt}}, \tag{17}$$

$$\partial_\ell^2 (\ell^2 (D\partial_l^2 g_{tt} - g_{tt}\partial_l^2 D) + 2\ell (D\partial_l^2 g_{t\varphi} - g_{t\varphi}\partial_l^2 D) + D\partial_l^2 g_{\varphi\varphi} - g_{\varphi\varphi}\partial_l^2 D) = 2(D\partial_l^2 g_{tt} - g_{tt}\partial_l^2 D). \tag{18}$$

It follows from these expressions, that the stability behavior is determined by the sign of $D\partial_l^2 g_{tt} - g_{tt}\partial_l^2 D$ at the throat. In general it is not feasible to evaluate this expression, since it depends not only on the sign of g_{tt} and $\partial_l^2 g_{tt}$, but also on the other metric components and their second derivatives, as well as on their relationship between each other. However, if the expression is positive at the throat, then $\ell_S^+ < \ell_S^-$ and the extremum between the roots $\ell_S^\pm$ is a minimum, the region of stable orbits is then given by $\ell \in [\ell_S^+, \ell_S^-]$. If the expression is negative at the throat, then $\ell_S^- < \ell_S^+$ and the extremum between the roots is a maximum, the region of stable orbits is then given by $\ell \in (-\infty, \ell_S^-] \vee \ell \in [\ell_S^+, \infty)$.

Photon orbits at wormhole throats. Photon orbits describe circular orbits for photons or in general for massless particles. We follow here the same procedure as in Sec. II and start with the Lagrangian $\mathcal{L}^*$ for massless

TABLE I. Relationship between the boundaries of existence and boundedness for circular orbits at the throat with respect to ℓ. A different sign of g_{tt} at the throat leads to a phase transition in the parameter space. It should be noted that bound orbits require furthermore $g_{tt} > -1$ as a necessary condition in case of $g_{tt} < 0$.

	Existence	Boundedness	Parameter space
$g_{tt} < 0$	$\ell_E^+ < \ell_E^-$	$\ell_B^+ < \ell_B^-$	$\ell \in [\ell_B^+, \ell_B^-]$
$g_{tt} > 0$	$\ell_E^- < \ell_E^+$	$\ell_B^- < \ell_B^+$	$\ell \in (-\infty, \ell_B^-] \vee \ell \in [\ell_B^+, \infty)$

particles. In case of planar motion within the equatorial plane it is now given by

$$\mathcal{L}^* = g_{ll}\dot{l}^2 + g_{tt}\dot{t}^2 + g_{\varphi\varphi}\dot{\varphi}^2 + 2g_{t\varphi}\dot{t}\,\dot{\varphi} = 0. \tag{19}$$

This leads to a slightly different radial effective potential $V_{\rm eff}^P = V_{\rm eff} - 1$. The impact parameter $\ell = \frac{L}{E}$ of photons corresponding to the photon orbits is determined by the roots of the radial effective potential $V_{\rm eff}^P$, which gives rise to $g_{tt}\ell^2 + 2\ell g_{t\varphi} + g_{\varphi\varphi} = 0$. This equation is identical to the equation for the limits of existence of circular orbits for particles with mass. The impact parameters of the photon orbits, $\ell_P^\pm$, are thus identical to the boundaries of existence of circular orbits for particles with mass, $\ell_P^\pm = \ell_E^\pm$. For the static limit, $g_{t\varphi} = 0$, the expressions for the impact parameter at the throat are identical to the ones presented in [34]. The second derivative of $V_{\rm eff}^P$ determines the stability of the photon orbits, for stable orbits it cannot be negative,

$$\ell_\pm = \frac{-\partial_l^2 g_{t\varphi} \pm \sqrt{(\partial_l^2 g_{t\varphi})^2 - \partial_l^2 g_{tt}\partial_l^2 g_{\varphi\varphi}}}{\partial_l^2 g_{tt}} \coloneqq P_S^\pm, \tag{20}$$

where we define $P_S^\pm$ as the critical value for the stability of the photon orbits. From $P_S^\pm$ it follows directly, that the stability behavior is determined by the second derivative of g_{tt} at the throat. If $\partial_l^2 g_{tt} > 0$, then $P_S^+ > P_S^-$ and the photon orbits are stable if $\ell_P^\pm \in [P_S^-, P_S^+]$. If $\partial_l^2 g_{tt} < 0$, then $P_S^+ < P_S^-$ and the stable region lies outside the interval (P_S^+, P_S^-), stability requires therefore $\ell_P^\pm \notin (P_S^+, P_S^-)$.

Stable photon orbits could hint to instabilities of the spacetime [35,36]. The general stability of the wormhole spacetime may thus possibly be analyzed by evaluating the expressions for stable and unstable photon orbits. The conditions for unstable orbits correspond to an interchanged relation of $\ell_P^\pm$ with regard to the stability regions bounded by $P_S^\pm$. For both cases with regard to the sign of $\partial_l^2 g_{tt}$ the inequalities are the same. Writing them out by inserting the expressions for $\ell_P^\pm$ and $P_S^\pm$ leads to the following constraints on the metric functions,

$$\frac{-g_{t\varphi} \pm \sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}}{g_{tt}} < \frac{-\partial_l^2 g_{t\varphi} - \sqrt{(\partial_l^2 g_{t\varphi})^2 - \partial_l^2 g_{tt}\partial_l^2 g_{\varphi\varphi}}}{\partial_l^2 g_{tt}}, \tag{21}$$

$$\vee\, \frac{-g_{t\varphi} \pm \sqrt{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}}{g_{tt}} > \frac{-\partial_l^2 g_{t\varphi} + \sqrt{(\partial_l^2 g_{t\varphi})^2 - \partial_l^2 g_{tt}\partial_l^2 g_{\varphi\varphi}}}{\partial_l^2 g_{tt}}, \tag{22}$$

where for $\partial_l^2 g_{tt} > 0$ either the upper inequality or the lower inequality must be satisfied for unstable photon orbits and for $\partial_l^2 g_{tt} < 0$ both inequalities have to be satisfied.

Boundaries of the parameter space in terms of physical quantities. Considering an asymptotically flat wormhole, we can conduct a deeper investigation on the calculated boundaries of the parameter domain. We assume, that the metric can be written in standard form without loss of generalization as

$$ds^2 = -Ndt^2 + Rdl^2 + Td\theta^2 + K(d\varphi - \omega dt)^2, \tag{23}$$

where all metric functions possess axial symmetry and are stationary. The function ω characterizes the spacetime rotation and is strictly increasing in terms of the angular momentum J of the wormhole. At the wormhole throat it corresponds to the angular frequency of the wormhole. Due to the asymptotic flatness we have $R \to 1$ and $\omega \to 0$ for $|l| \to \infty$, which also leads to $N \to 1$ for $|l| \to \infty$. The metric functions T and K are monotonically increasing for larger values of $|l|$, with $T, N \to l^2$ in the equatorial plane for $|l| \gg 0$. Expressing the metric components through the metric functions gives for the boundaries $\ell_E^\pm$ and $\ell_B^\pm$

$$\ell_E^\pm = \frac{K\omega \pm \sqrt{NK}}{Kw^2 - N} = \ell_P^\pm;\;\; \ell_B^\pm = \frac{K\omega \pm \sqrt{NK(1 + K\omega^2 - N)}}{Kw^2 - N}. \tag{24}$$

To incorporate physical properties of the spacetime into these expressions, the metric function K can be expressed through the circumference of the wormhole throat C_T as $K = \frac{C_T^2}{4\pi^2}$. Furthermore, if there is no ergoregion present around the wormhole, the metric function N can be expressed in terms of the gravitational redshift z, which a distant observer would measure for signals coming from a static source at the throat,

$$N = \frac{4\pi^2 + C_T^2\omega^2(1+z)^2}{4\pi^2(1+z)^2}. \tag{25}$$

In order to investigate how the parameter space for bound circular orbits varies with respect to the physical properties of the spacetime, the interval between the boundaries $\ell_B^\pm$ can be analyzed. The interval size $\mathcal{B}$ is given by the absolute value of the difference between the boundaries. In case of no ergoregion $\mathcal{B}$ is identical to the parameter space volume for bound circular orbits and it is fully expressible through C_T, ω and z,

$$\mathcal{B} = |\ell_B^\pm - \ell_B^\mp| = 2\frac{\sqrt{NK(1+K\omega^2-N)}}{|K\omega^2 - N|}$$
$$\Leftrightarrow \mathcal{B}(C_T,\omega,z) = \frac{C_T}{2\pi^2}\sqrt{(4\pi^2 + C_T^2\omega^2(1+z)^2)(z^2+2z)}. \tag{26}$$

It follows immediately, that the parameter space for bound circular orbits is a monotonically increasing function of C_T, ω and z. Thus, greater values for each of these properties correspond also to a greater parameter space domain for bound circular orbits. Particle orbits at the throat are therefore more likely to occur for faster rotating wormholes, as well as for larger ones, and as the dependence on the redshift factor indicates also for wormholes with higher energy-density. Analogously, the impact parameter of the photon orbits can be expressed through C_T, ω and z,

$$\ell_P^\pm(C_T,\omega,z) = -\frac{C_T(1+z)\left(\omega \pm \sqrt{4\pi^2 + C_T^2\omega^2(1+z)^2}\right)}{4\pi^2} \tag{27}$$

Regarding now the case of a wormhole with an ergoregion, the metric function N cannot be expressed solely through C_T, ω and z. A source located at the wormhole throat cannot be static and must be in motion, which leads to inseparable expressions for the redshift z with regard to N. Trying to incorporate the redshift factor in an expression for $\mathcal{B}$ would thus only complicate the expression. For the analysis of wormholes with an ergoregion we therefore express $\mathcal{B}$ as a function of C_T, ω and N,

$$\mathcal{B}(C_T,\omega,N) = 2C_T\frac{\sqrt{4\pi^2N(1-N) + NC_T^2\omega^2}}{C_T^2\omega^2 - 4\pi^2 N}, \tag{28}$$

where the denominator is always positive since $g_{tt} > 0$. Due to the transition of the parameter space behavior for orbits if $g_{tt} > 0$ at the throat, the interval size $\mathcal{B}$ now represents the size of the region outside of the parameter domain for bound circular orbits. Considering a variation of the angular frequency for fixed C_T and N, the interval size $\mathcal{B}$ is shrinking for faster rotating wormholes, with $\mathcal{B}(\omega) = \mathcal{O}(\frac{1}{\omega})$. Thus, for an increasing ω the interval size approaches asymptotically zero and the parameter space for bound circular orbits approaches asymptotically $\ell \in (-\infty, \infty)$. This could be interpreted as a hint to possible instabilities of wormholes with ergoregions and faster rotating wormholes, as more and more mass could accumulate at the throat. Fixing ω and N and increasing the throat circumference C_T leads also to a decrease of $\mathcal{B}$ as the numerator gets smaller and the denominator larger. For growing C_T it approaches asymptotically $2\frac{\sqrt{N}}{\omega}$. General statements on the behavior of $\mathcal{B}$ with regard to N are difficult, since N is coupled to the other quantities by $\frac{C_T\omega^2}{4\pi^2} > N$, which must be satisfied in case of an ergoregion. Regarding the impact parameters of photon orbits, they can be analogously written as

$$\ell_P^\pm(C_T,\omega,N) = \frac{C_T^2\omega \pm 2\pi C_T\sqrt{N}}{C_T^2\omega^2 - 4\pi^2 N}. \tag{29}$$

Considering now the stability boundaries of the timelike and photon orbits, $\ell_S^\pm$ and $P_S^\pm$, the expressions are given in terms of the metric functions as

$$\ell_S^\pm = \frac{K^2\omega\partial_l^2 N - NK^2\partial_l^2\omega \pm \sqrt{K^4\omega^2(\partial_l^2 N)^2 + K^2N^2\partial_l^2 N\partial_l^2 K}}{K^2\omega^2\partial_l^2 N - N^2\partial_l^2 K - 2\omega NK^2\partial_l^2\omega}, \tag{30}$$

$$P_S^\pm = \frac{K\partial_l^2\omega + \omega\partial_l^2 K \pm \sqrt{K^2(\partial_l^2\omega)^2 + \partial_l^2 N\partial_l^2 K}}{2K\omega\partial_l^2\omega + \omega^2\partial_l^2 K - \partial_l^2 N}. \tag{31}$$

Due to the complexity of these expressions and the various conditions between the metric functions and their derivatives, it is not feasible to derive general statements out of these expressions. However, in the static limit the expressions simplify and it can be shown for a wormhole with unstable photon rings that $|\ell_S^\pm| < |\ell_E^\pm|$ and $|\ell_B^\pm| < |\ell_E^\pm|$ always hold, which guarantees the existence of circular orbits. If furthermore $|\ell_B^\pm| < |\ell_S^\pm| < |\ell_E^\pm|$, then all of the circular orbits are also stable. This constraint can be expressed through the metric functions as the inequality $0 < \partial_l^2 N/N + (\partial_l^2 K/K)(N-1)$.

Conclusion and outlook. We derived general expressions for stationary and axisymmetric wormholes, that determine the existence of circular timelike and photon orbits at the throat. Assuming a local mirror symmetry and twice differentiable metric components at the throat, we obtained the limits of existence $\ell_E^\pm$ and boundedness $\ell_B^\pm$ of such orbits with respect to the angular momentum of the particle. Furthermore, we also obtained expressions for the limits of stability of such timelike and photon orbits.

We found, that the sign of g_{tt} at the throat governs the parameter space behavior. For wormholes with no ergoregion ($g_{tt} < 1$) the parameter spectrum for bound orbits lies within finite bounds, for wormholes with an ergoregion the spectrum diverges. With the additional assumption of asymptotical flatness, we expressed the bounds of the parameter domain as functions of physical properties, namely the throat circumference, the angular frequency of the wormhole and the redshift. We analyzed how the bounds are influenced by variation of these properties and found that larger values enlarge the parameter domain for circular orbits at the throat.

Some of these results could be interpreted as hints to instabilities of specific wormhole solutions, since especially for wormholes with an ergoregion the parameter domain diverges. This could potentially narrow down the properties any physically feasible wormhole spacetime should exhibit. Moreover, stable circular orbits could lead to the formation of matter accumulations and accretion disks surrounding the throat, giving rise to a luminous central region which could be potentially significant for observational properties.

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