

Minimal freeze-in dark matter: Reviving electroweak doublet dark matter with Boltzmann suppressed freeze-in

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Dark matter communicating with the Standard Model solely via electroweak interactions provides a compelling picture. However, thermal freeze-out of electroweak doublet dark matter is generically strongly excluded by direct detection. We show that $SU(2)_L$ doublet fermion dark matter evades direct detection if its mass exceeds 10^{10} GeV. If the neutral Dirac fermion is split into a pseudo-Dirac pair (via high dimension operator), this limit can be relaxed to 300 GeV. Provided the dark matter mass is above the reheat temperature of the Universe, the production rate never exceeds the Hubble rate in cases of interest; thus, the dark matter never thermalizes. We apply constraints from direct detection (e.g., LZ) and consider the discovery potential of Darwin. This scenario presents the most minimal model of freeze-in dark matter and is both elegant and highly predictive.

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I. INTRODUCTION

The most compelling dark matter models are those that only minimally extend the Standard Model. Three classic realizations are the Higgs portal, dark matter with electroweak interactions, and (nonthermal) gravitationally coupled dark matter. Notably, thermal freeze-out of dark matter via the Higgs or electroweak gauge bosons is generically excluded [1,2] (with some caveats, e.g., [3–5]). It was recently highlighted that Boltzmann suppressed freeze-in of scalar ϕ or fermion ψ dark matter via the Higgs portals $|\phi|^2|H|^2$ [6] or $\bar{\psi}\psi|H|^2$ [7] leads to viable models, evading experimental constraints. Here, we study Boltzmann-suppressed freeze-in of dark matter via electroweak gauge bosons, focusing on the realization in which dark matter is a fermion doublet of $SU(2)_L$ with an electromagnetically neutral component.

For dark matter to have electroweak interactions, it must be a nontrivial representation of $SU(2)_L$. While different representations have been considered in the literature (see, e.g., [8]), there are no known examples of fundamental matter fields transforming in any representation other than the singlet or fundamental. For this reason, we focus on electroweak doublet dark matter. In this case, the Standard Model electroweak bosons can be the sole mediators between dark matter and the Standard Model, making

the model highly predictive. Indeed, if the relic abundance of electroweak doublet dark matter is set by thermal freeze-out, its mass is required to be TeV scale (assuming standard cosmology [5]). This scenario is generically strongly excluded by direct detection [9,10]. As we show, direct detection limits require the mass of doublet dark matter to be very heavy.

We show that $SU(2)_L$ doublet fermion dark matter is a viable possibility if its mass is heavier than 10^{10} GeV. This mass bound can be relaxed to 300 GeV if the neutral dark matter state is pseudo-Dirac. In the pseudo-Dirac case, the tree-level spin-independent (SI) direct detection cross section can be forbidden; however, there is an irremovable loop-induced SI scattering cross section. We also require that the dark matter mass exceeds the reheat temperature of the Universe: $m_{\text{DM}} > T_{\text{rh}}$. In this case, the production rate typically never exceeds H , so the doublet never thermalizes.

Notably, while for freeze-out dark matter the mass of dark matter is bounded by partial-wave unitary constraints to be $\lesssim 100$ TeV [11], there is no such upper bound for freeze-in dark matter, and (much) heavier dark matter is perfectly consistent. Moreover, as we demonstrate, for the dark matter models studied here, the mass range 10^{10} – 10^{13} GeV has the potential to be discovered at forthcoming direct detection experiments.

II. ELECTROWEAK DOUBLET DARK MATTER

Nontrivial representations of $SU(2)_L$ are multi-component objects. Each component has a different electromagnetic (EM) charge after electroweak symmetry breaking (EWSB) given by $Q = T_3 + Y$. For an $SU(2)_L$

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doublet, these components are labeled by the third component of their isospin $T_3 = \pm \frac{1}{2}$. To avoid coupling to photons, the EM charge must satisfy $Q = 0$, which implies $T_3 = -Y$. Thus, for a doublet to have an electromagnetically neutral component after EWSB, the hypercharge of the doublet must be $Y_\chi = \pm \frac{1}{2}$.

We focus on fermion dark matter since scalar dark matter suffers from a fine-tuning “hierarchy” problem and also permits a model-independent renormalizable Higgs portal interaction, which makes it less predictive. For fermion dark matter, the minimal realization requires a pair of Weyl fermions χ_1 and χ_2 which are $SU(2)_L$ doublets (to avoid the Witten anomaly [12]). While the Weyl fermions could have differing hypercharges (or other quantum numbers) unless one adds additional states, anomaly cancelation constrains $Y[\chi_1]$ and $Y[\chi_2]$.

The minimal anomaly-free construction involves two left-handed (LH) doublets with opposite hypercharge $\pm \frac{1}{2}$. This model has a large tree-level vector coupling to the Z and thus leads to a large SI scattering cross section. Interestingly, if one introduces a modest mass splitting between the two EM neutral degrees of freedom, then the tree level SI cross section can be removed [3], which ameliorates these limits. We examine this variant following the minimal model.

Explicitly, in the minimal model, one supplements the Standard Model with two LH doublets of opposite hypercharge: $Y[\chi_1] = +\frac{1}{2}$ and $Y[\chi_2] = -\frac{1}{2}$. Each of the doublets can be written as follows with the EM charges shown as superscripts:

$$\chi_1 = \begin{pmatrix} \chi_1^+ \\ \chi_1^0 \end{pmatrix}, \quad \chi_2 = \begin{pmatrix} \chi_2^0 \\ \chi_2^- \end{pmatrix}, \quad (1)$$

and the Lagrangian is given by

$$\mathcal{L}_\chi = i\chi_1^\dagger \bar{\sigma}^\mu D_\mu \chi_1 + i\chi_2^\dagger \bar{\sigma}^\mu D_\mu \chi_2 - [M\epsilon^{ab}\chi_{1a}\chi_{2b} + \text{H.c.}], \quad (2)$$

where D_μ is the ordinary electroweak covariant derivative. Note that $\epsilon^{ab}\chi_{1a}\chi_{2b} = \chi_1^+\chi_2^- - \chi_1^0\chi_2^0$, since we have $\epsilon^{12} = -\epsilon^{21} = 1$. After EWSB, one can assemble the Weyl spinors into neutral Ψ^0 and charged $\Psi^\pm$ fields,

$$\Psi^0 \equiv \begin{pmatrix} \chi_2^0 \\ \bar{\chi}_1^0 \end{pmatrix}, \quad \Psi^+ \equiv \begin{pmatrix} \chi_1^+ \\ \bar{\chi}_2^- \end{pmatrix}. \quad (3)$$

The Lagrangian can be rewritten in terms of Ψ with $\mathcal{L}_\Psi = \mathcal{L}_{\text{int}} + \mathcal{L}_M$ with

$$\begin{aligned} \mathcal{L}_{\text{int}} = & \frac{g_2}{\cos\theta_W} Z_\mu \left[\left(\frac{1}{2} - \sin^2\theta_W \right) \bar{\Psi}^+ \gamma^\mu \Psi^+ + \frac{1}{2} \bar{\Psi}^0 \gamma^\mu \Psi^0 \right] \\ & + \frac{g_2}{\sqrt{2}} (\bar{\Psi}^+ \gamma^\mu \Psi^0 W_\mu^+ + \bar{\Psi}^0 \gamma^\mu \Psi^+ W_\mu^-) + e A_\mu \bar{\Psi}^+ \gamma^\mu \Psi^+. \end{aligned} \quad (4)$$

Section III explains the coupling structure. Further, this construction permits a Dirac mass,

$$\mathcal{L}_M = -M\bar{\Psi}^0\Psi^0 - M\bar{\Psi}^+\Psi^+. \quad (5)$$

Thus, for fermion dark matter, the second Weyl field is not only needed for anomaly cancelation but provides the minimal manner of giving an appropriately large mass to the dark matter. At tree level, the charged and neutral Dirac fermions are degenerate. After EWSB, radiative corrections (for $M \gg m_Z$) imply [8]

$$\Delta \equiv m_{\Psi^+} - m_{\Psi^0} \approx 350 \text{ MeV}. \quad (6)$$

If the states carry a conserved charge (e.g., $\mathbb{Z}_2: \chi_i \rightarrow -\chi_i$), then Ψ^0 will be stable and is a dark matter candidate.

III. ELECTROWEAK DOUBLET Z COUPLINGS

As outlined above, after EWSB, one can combine the components of these fields into a neutral Dirac fermion Ψ^0 and a charged Dirac fermion $\Psi^\pm$. In this section, we derive the couplings of the new fermions following from their representations and hypercharge. For a Dirac fermion Ψ with electric charge Q and weak isospin eigenvalues T_3^L and T_3^R , for its left- and right-handed components, the vector and axial-vector couplings of these states to the Z are given by

$$\begin{aligned} \hat{g}_V &= \frac{e}{2 \sin\theta_W \cos\theta_W} (T_3^L + T_3^R - 2Q \sin^2\theta_W), \\ \hat{g}_A &= \frac{e}{2 \sin\theta_W \cos\theta_W} (T_3^L - T_3^R), \end{aligned} \quad (7)$$

with $e = g_2 \sin\theta_W$. We add a hat to the coupling to show that the coupling is specialized to the Z . This corresponds to an interaction structure of the form

$$\mathcal{L} \supset Z_\mu \bar{\Psi} \gamma^\mu (\hat{g}_V - \hat{g}_A \gamma^5) \Psi. \quad (8)$$

For the charged state Ψ^+ , one has $T_3^L = T_3^R = 1/2$ and $Q = +1$, giving

$$\begin{aligned} \hat{g}_V^{\Psi^+} &= \frac{e}{2 \sin\theta_W \cos\theta_W} (1 - 2 \sin^2\theta_W), \\ \hat{g}_A^{\Psi^+} &= 0. \end{aligned} \quad (9)$$

Similarly, for the neutral state Ψ^0 , its coupling to Z is given by

$$\hat{g}_V^{\Psi^0} = \frac{e}{2 \sin\theta_W \cos\theta_W}, \quad \hat{g}_A^{\Psi^0} = 0. \quad (10)$$

Thus, we observe that both the neutral and charged components have a nonzero tree-level vector coupling and a vanishing axial vector coupling.

Additionally, the quark couplings come from Eq. (7), with $T_3^R = 0$ for Standard Model fermions; thus,

$$\begin{aligned}\hat{g}_V^u &= \frac{e}{2 \sin \theta_W \cos \theta_W} \left(\frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right), \\ \hat{g}_A^u &= \frac{e}{4 \sin \theta_W \cos \theta_W}, \\ \hat{g}_V^d &= \frac{e}{2 \sin \theta_W \cos \theta_W} \left(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \right), \\ \hat{g}_A^d &= -\frac{e}{4 \sin \theta_W \cos \theta_W}.\end{aligned}\quad (11)$$

We take $\sin^2 \theta_W|_{\mu=m_Z} \approx 0.231$ here; this is the Z pole value. However, the parameter does not significantly change via running to intermediate scales. Evaluating $\sin^2 \theta_W(\mu) = g_Y^2(\mu)/(g_Y^2(\mu) + g_2^2(\mu))$ assuming Standard Model running, it is seen that $\sin^2 \theta_W \approx 0.36 \pm 0.06$ for $\mu \sim 10^{12 \pm 4}$ GeV [13]. Thus, the running of $\sin^2 \theta_W$ will not significantly impact our calculations. Similarly, all our calculations are at tree level. It is expected that results will only vary at the $\mathcal{O}(10\%)$ at greater precision.

IV. BOLTZMANN SUPPRESSED FREEZE-IN

Dark matter freeze-in [14,15] presents a distinctive alternative to the classic Weakly Interacting Massive Particle (WIMP) freeze-out picture. Boltzmann suppressed freeze-in, first outlined in Giudice–Kolb–Riotto [16], has recently received renewed attention, e.g., [6,17–27]. These scenarios consider the very conceivable possibility that the dark matter mass is higher than the maximum temperature of the thermal bath: $m_{\text{DM}} \gg T_{\text{rh}}$. In this case the dark matter production cross section is exponentially suppressed,

$$\langle \sigma v \rangle \propto \exp(-2m_{\text{DM}}/T). \quad (12)$$

The cross section for dark matter production via the process $q\bar{q}' \rightarrow \Psi^+\Psi^-, \bar{\Psi}^0\Psi^0, \bar{\Psi}^0\Psi^+$ involving an electro-weak boson (derived in Appendixes A and B) is given at leading order by

$$\sigma_{q\bar{q}' \rightarrow \Psi\Psi}(\beta) = \frac{\beta}{24\pi s} ((g_V^q)^2 + (g_A^q)^2)(g_V^\Psi)^2 \left(1 - \frac{\beta^2}{3}\right), \quad (13)$$

where $\beta(s) \equiv \sqrt{1 - 4(m_\Psi^2/s)}$. Note that the axial vector contribution occurs at $\mathcal{O}(\beta^3)$. The general form of $\sigma_{q\bar{q}' \rightarrow \Psi\Psi}$ is useful and holds for arbitrary q, q' initial states. This form can be specialized to different initial and final states by specifying the couplings.

Turning to the Boltzmann equation,

$$\dot{n}_\Psi + 3Hn_\Psi = \gamma(T). \quad (14)$$

Neglecting the quark masses, the reaction density γ for $q\bar{q}' \rightarrow \Psi^+\Psi^-, \bar{\Psi}^0\Psi^0, \bar{\Psi}^0\Psi^+$ (the latter arises from $u\bar{d}$ initial state, or similar, via W exchange) is given by

$$\gamma = \frac{T}{32\pi^4} \sum_{q,q'} \int_{4m_\Psi^2}^{\infty} ds [(s - 4m_\Psi^2) \sqrt{s} \sigma_{q\bar{q}' \rightarrow \Psi\Psi}] K_1\left(\frac{\sqrt{s}}{T}\right), \quad (15)$$

where K_1 is the modified Bessel function.

In the limit $m_\Psi \gg T$, the integral in Eq. (15) is dominated just above threshold for which $s \simeq 4m_\Psi^2$ and $\beta \ll 1$. Thus, the near-threshold form of the cross section is

$$\sigma_{q\bar{q}' \rightarrow \Psi\Psi}(s) \simeq \frac{1}{24\pi s} (C_\gamma^{qq'} + C_Z^{qq'} + C_W^{qq'}) \beta, \quad (16)$$

where we have collected the combination of couplings into

$$C_i^{qq'} \equiv ((g_V^q)^2 + (g_A^q)^2)(g_V^\Psi)^2. \quad (17)$$

In Appendixes B and C, we derive numerical values of C_Z , C_γ , and C_W for each case of interest.

Further, as derived in Appendix D, the reaction density [given Eq. (15)] evolves according to

$$\gamma(T) \simeq \sum_{q,q'} \left(\frac{C_\gamma^{qq'} + C_Z^{qq'} + C_W^{qq'}}{256\pi^4} \right) T^4 e^{-2m_\Psi/T}. \quad (18)$$

This can be specialized to the neutral or charged components by the appropriate choice of C_i .

We then use $\gamma(T)$ to calculate the dark matter freeze-in yields $Y \equiv n_\Psi/s$ arising from the Boltzmann equation,

$$\frac{dY}{dT} = -\frac{\gamma(T)}{sHT}, \quad (19)$$

where $s(T) = \frac{2\pi^2}{45} g_{\star s} T^3$ and $H(T) = \sqrt{\frac{\pi^2 g_{\star}}{90}} \frac{T^2}{M_{\text{P}}}$ in terms of the relativistic degrees of freedom $g_{\star}$, the effective entropy degrees of freedom $g_{\star s}$, and the Planck mass M_{P} . Substituting the reaction density from Eq. (18) gives

$$\frac{dY}{dT} = -\frac{135\sqrt{10}}{512\pi^7} \sum_{q,q'} \left(\frac{C_\gamma^{qq'} + C_Z^{qq'} + C_W^{qq'}}{g_{\star s} \sqrt{g_{\star}}} \right) \frac{M_{\text{P}}}{T^2} e^{-\frac{2m_\Psi}{T}}. \quad (20)$$

Assuming instantaneous reheating¹ to temperature T_{rh} , negligible initial Ψ abundances, and taking $m_\Psi \gg T_{\text{rh}}$, the freeze-in yield is

¹Boltzmann suppressed freeze-in of this model for noninstantaneous reheating is studied in the companion paper [28], as well as a discussion of gravitational production (which is found to be always subleading).

$$Y_{\text{FI}} = \frac{135\sqrt{10}}{512\pi^7} \sum_{q,q'} \frac{C_Y^{qq'} + C_Z^{qq'} + C_W^{qq'}}{g_{\star s} \sqrt{g_{\star}}} M_{\text{P}} \int_0^{T_{\text{rh}}} dT \frac{e^{-\frac{2m_{\Psi}}{T}}}{T^2}$$

$$\simeq \frac{135\sqrt{10}}{1024\pi^7} \sum_{q,q'} \frac{C_Y^{qq'} + C_Z^{qq'} + C_W^{qq'}}{g_{\star s} \sqrt{g_{\star}}} \frac{M_{\text{P}}}{m_{\Psi}} e^{-\frac{2m_{\Psi}}{T_{\text{rh}}}}. \quad (21)$$

Observe that the freeze-in yield scales as $Y_{\text{FI}} \propto \exp(-2m_{\Psi}/T_{\text{rh}})$, as expected in the Boltzmann suppressed regime. From Eq. (21), one can calculate the yield Y_{FI}^0 of Ψ^0 and $Y_{\text{FI}}^{\pm}$ of $\Psi^{\pm}$ by substituting the correct C_i .

Since $m_{\Psi^{\pm}} > m_{\Psi^0}$ [cf. Eq. (6)], the $\Psi^{\pm}$ will decay to Ψ^0 , thus contributing to the dark matter relic density. The $\Psi^{\pm} \rightarrow \Psi^0 \pi^{\pm}$ decay rate is [29] (cf. also [30])

$$\Gamma_{\Psi^{\pm}} \simeq \Gamma_{\pi^{\pm} \rightarrow \mu^{\pm} \nu_{\mu}} \frac{16\Delta^3}{m_{\pi} m_{\mu}^2} \sqrt{1 - \frac{m_{\pi}^2}{\Delta^2}} \left(1 - \frac{m_{\mu}^2}{m_{\pi}^2}\right)^{-2}.$$

The quantities above are all Standard Model and thus, we evaluate numerically to find $\Gamma_{\Psi^{\pm}} \approx 5 \times 10^{-14}$ GeV. Thus, the lifetime is $\tau \simeq 10^{-11}$ s, and for cosmological purposes, $\Psi^{\pm}$ decays rapidly. It follows that the dark matter relic abundance Y_{∞}^0 is the sum $Y_{\infty}^0 = Y_{\text{FI}}^0 + Y_{\text{FI}}^{\pm}$.

In Fig. 1, we show (black solid line) the relic abundance of $\text{SU}(2)_L$ doublet dark matter of mass m as a function of the reheat temperature T_{rh} (assuming instantaneous reheating). The black dashed line in Fig. 1 indicates $T_{\text{rh}} = m$, above which freeze-in is no longer Boltzmann suppressed. The red shaded “thermalization” region indicates parameter values for which the dark matter would enter equilibrium with the Standard Model thermal bath; we have demanded

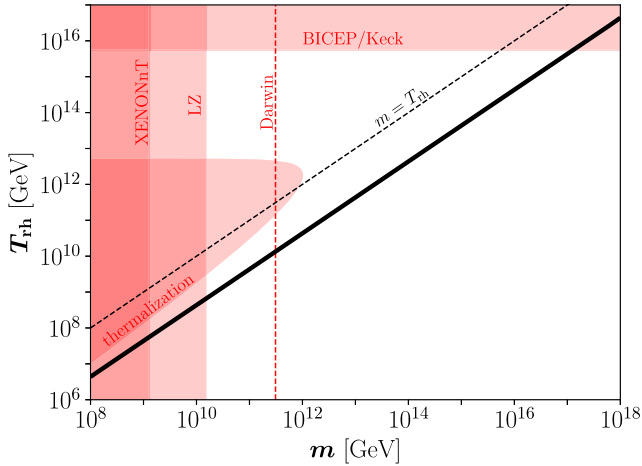


FIG. 1. The solid line is the reheat temperature T_{rh} which gives the observed dark matter relic density for Ψ^0 of mass m assuming instantaneous reheating. The vertical red areas mark exclusion from direct detection experiments and the projection by Darwin. The black dashed line indicates $T_{\text{rh}} = m$, above which freeze-in is no longer Boltzmann suppressed. The red shaded “thermalization” region indicates parameter values for which Ψ^0 would enter equilibrium with the Standard Model thermal bath; we have demanded

the production rate to be smaller than the Hubble expansion rate H , that is, $\gamma(T_{\text{rh}}) < n_{\text{eq}}(T_{\text{rh}})H(T_{\text{rh}})$, where n_{eq} is the dark matter equilibrium number density. The inflection in the boundary curve of the thermalization region in Fig. 1 corresponds to $m = T_{\text{rh}}$ which separates the relativistic and nonrelativistic regimes. Observe that thermalization does not constrain the solid line for the relevant mass range. In addition, we impose an upper bound on the reheating temperature $T_{\text{rh}} \lesssim 5.5 \times 10^{15}$ GeV; this comes from the BICEP/Keck limit on the tensor-to-scalar ratio.

V. DIRECT DETECTION

We next apply experimental limits from XENONnT [31] and LZ [10]. The bounds are linearly extrapolated from those presented in [10,31]. This extrapolation is generally considered valid since heavy dark matter with sufficiently large cross sections will interact with direct detection experiments, however, the higher mass implies that fewer states are needed to comprise the relic density. Thus, the dark matter flux is diminished, leading to a linear fall off in the constraints. This can be seen in LZ’s dedicated heavy dark matter analysis [32]. The spin-independent cross section for Ψ^0 scattering on an individual nucleon $N = p, n$ is [33]

$$\sigma_{\text{SI}}^{(N)} = \frac{\mu_N^2}{\pi} \left(\frac{g_V^{\Psi^0} g_V^N}{m_Z^2} \right)^2 \sim \begin{cases} 4.7 \times 10^{-40} \text{ cm}^2 & N = n, \\ 2.6 \times 10^{-42} \text{ cm}^2 & N = p, \end{cases} \quad (22)$$

where $g_V^p = 2g_V^u + g_V^d$ and $g_V^n = g_V^u + 2g_V^d$, for g_V^u, g_V^d, g_V^{Ψ} in Sec. III. The 2025 LZ limit is [10]

$$\sigma_{\text{SI}}^{(n,p)}|_{m_{\text{DM}} \gg m_Z} < 3 \times 10^{-47} \text{ cm}^2 \left(\frac{m_{\text{DM}}}{\text{TeV}} \right), \quad (23)$$

implying a lower mass bound $m_{\text{DM}} > 1.4 \times 10^{10}$ GeV.

In Fig. 2, we show the SI limits on electroweak doublet dark matter. For instantaneous reheating, the dark matter mass uniquely determines the required T_{rh} , and these are shown as contours. We also show the cosmological limits from BICEP/Keck [34] that constrain T_{rh} , and this restricts $m \lesssim 10^{17}$ GeV. We find that indirect detection does not constrain the parameter space of interest; see Appendix E. These mass bounds are also marked on Fig. 1 as shaded areas.

It is interesting to consider the discovery reach of the future Darwin experiment [35]. Darwin probes deep into the available parameter space, although there remains a window between Darwin’s SI reach and the BICEP/Keck limit. However, the parameter space below Darwin’s reach lies in the neutrino fog² [35,36] and thus will require novel experimental advancements to completely exclude these

²For $m \gg \text{TeV}$, the recoil spectrum is mass independent (see, e.g., [36]). Since the dark matter flux scales as $1/m$, maintaining a fixed event rate relative to the (mass-independent) neutrino background requires $\sigma_{\text{SI}} \propto m$, leading to a linear rise in the neutrino fog (i.e., running parallel to the dark matter constraint).

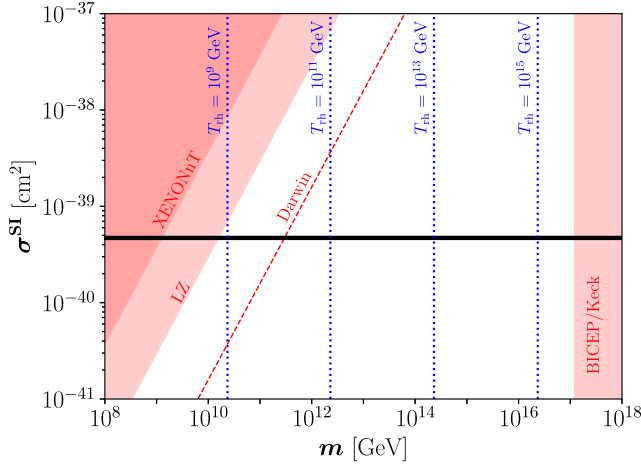


FIG. 2. Constraints on minimal freeze-in dark matter. We apply the spin-independent limits from XENONnT [31] and LZ [10]. We also show the anticipated discovery reach of the proposed Darwin experiment [35]. The parameter space below the “Darwin” line lies within the neutrino fog. For instantaneous reheating, the dark matter mass uniquely determines the required T_{rh} , and we plot some characteristic contours. We show the cosmological limits (assuming single field inflationary reheating) from BICEP/Keck [34] which constrain T_{rh} .

models. More optimistically, the Darwin experiment can discover electroweak doublet dark matter with mass in the range 10^{10} – 10^{13} GeV.

VI. PSEUDO-DIRAC CASE

Interestingly, higher-dimensional operators, generated by high-scale physics, may alter the expectations for direct detection. In the construction above, the neutral Dirac fermion $\Psi^0 \equiv (\chi_2^0, \bar{\chi}_1^0)$ has a tree-level vector coupling to the Z, which leads to a large SI scattering rate. The tree-level Z coupling can be removed by splitting the neutral Dirac fermion into two Majorana states (a pseudo-Dirac pair). This can be achieved via a small Majorana mass for the neutral components [3]. A clean realization of this is via dimension-5 operators involving the Standard Model Higgs schematically of the form $\frac{1}{\Lambda}(\chi_1 H)^2$, where Λ is the mass scale of new physics. This is reminiscent of the Weinberg operator $(LH)^2$.

At low energy, one may parametrize the splitting via

$$\mathcal{L} \supset -M\chi_1^0\chi_2^0 - \frac{\delta_1}{2}\chi_1^0\chi_1^0 - \frac{\delta_2}{2}\chi_2^0\chi_2^0 + \text{H.c.}, \quad (24)$$

where M is the Dirac mass from Eq. (5), and $\delta_{1,2} \ll M$ are the induced Majorana masses. Then, the physical states are

$$\begin{aligned} \Psi_1 &\simeq \frac{1}{\sqrt{2}}(\Psi^0 + (\Psi^0)^c), \\ \Psi_2 &\simeq \frac{-i}{\sqrt{2}}(\Psi^0 - (\Psi^0)^c), \end{aligned} \quad (25)$$

with split masses $m_{1,2} \simeq M \mp \frac{1}{2}\delta$ for $\delta \equiv \delta_1 + \delta_2$. Being Majorana fermions, the vector current vanishes, but an off-diagonal Z coupling remains $\bar{\Psi}_2\gamma^\mu\Psi_1$. While inelastic scatterings $\Psi_1 N \rightarrow \Psi_2 N$ can be possible, for $\delta \gg O(100)$ keV, this is forbidden.

In the absence of inelastic scattering, the leading direct detection cross section arises from EW loop-induced operators [37]. The characteristic size for such loop-induced scattering cross sections is 10^{-47} cm², roughly a factor $O(10^{-7})$ below the tree-level case. The reduced SI cross section significantly weakens the direct detection limits.³ Compared to direct detection limits, for $\sigma_{\text{SI}} \sim 10^{-47}$ cm², the limit on the dark matter mass is $m \gtrsim 330$ GeV with Darwin probing up to 6 TeV. This requires a low T_{rh} and allows an interesting alternative at the EW scale. Current collider and indirect detection bounds do not further constrain this model [38]; indeed, it has been suggested the 14 year Fermi data may mildly favor TeV-scale doublet dark matter [39]. Such a model is discoverable at the forthcoming CTAO-North experiment [40].

VII. CONCLUDING REMARKS

The idea of dark matter communicating with the Standard Model only via electroweak interactions presents a compelling picture. We have shown that within the framework of Boltzmann suppressed freeze-in, electroweak doublet dark matter is consistent with current searches. We highlight that this model is arguably⁴ the most minimal dark matter freeze-in scenario imaginable in terms of field content introducing only a single new Weyl fermion pair and nothing else. Due to this minimality, the scenario presented is extremely predictive. The predictive strength of this setting is partly due to our (not unreasonable) assumption of efficient inflationary reheating. For inefficient reheating or nonstandard cosmology (different from matterlike prior to reheating; e.g., kination domination, cf. [41]), then the maximum temperature of the Universe T_{max} and the equation of state of the early Universe ω will typically impact the freeze-in calculations (cf. [41,42]). Moreover, it is interesting to consider freeze-in of $SU(2)_L$ representations other than the doublet (for instance, the triplet or quintuplet representations). We study the impact of both the particle content and cosmological variations and how they alter constraints and detection prospects in a companion paper [28].

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³Since the mass splitting is generated by EWSB, freeze-in is unaffected because with $m, T_{\text{rh}} \gg m_Z$ the mass splitting vanishes.

⁴Gravitationally coupled scalars Φ may seem minimal, but sub- M_{P} mass scalars must add new states to avoid fine-tuning. They also worsen the hierarchy problem of the Higgs due to $|H|^2|\Phi|^2$.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FREEZE-IN PRODUCTION VIA PHOTON AND Z BOSON

In the main text, we showed that the correct dark matter relic density could be reproduced for electroweak doublet fermion dark matter while avoiding direct detection constraints for heavy dark matter within the Boltzmann suppressed freeze-in framework. In the following, we provide details that support the main text, specifically:

- (1) A calculation of $q\bar{q} \rightarrow \Psi^+\Psi^-, \bar{\Psi}^0\Psi^0$ cross sections.
- (2) A computation of the numerical value of the coupling combination C_Z and C_γ .
- (3) A calculation of $q\bar{q}' \rightarrow \Psi^\pm\Psi^0, \bar{\Psi}^0\Psi^0$ cross sections via $W^\pm$.
- (4) A derivation of the reaction density $\gamma(T)$.
- (5) A discussion of indirect detection prospects.

In this appendix, we derive the production cross section $q\bar{q} \rightarrow \Psi^+\Psi^-, \bar{\Psi}^0\Psi^0$. As we calculate the freeze-in production in the regime $T \gg m_Z$, it can be considered more

intuitive to work in the (χ_1, χ_2) , corresponding to the symmetric phase on the basis (B, W_3) . However, the results of the calculations are basis independent, and since at late time, after EWSB, the Ψ^0 states are identified as dark matter, we find it simpler to always work in the $(\Psi^0, \Psi^\pm)$ basis. In what follows, we neglect the quark masses and work in the limit $m_Z^2 \ll m_\Psi^2, s$.

The process $q(p_1)\bar{q}(p_2) \rightarrow \Psi^+(k_1)\Psi^-(k_2)$ leading to the production of a pair of fermions with mass m_Ψ and momenta k_1, k_2 from two quarks with momenta p_1, p_2 via a vector boson arises from the interaction Lagrangian,

$$\mathcal{L}_{\text{int}} = V_\mu [\bar{q}\gamma^\mu (g_V^q - g_A^q\gamma^5)q + \bar{\Psi}\gamma^\mu (g_V^\Psi - g_A^\Psi\gamma^5)\Psi], \quad (\text{A1})$$

with generic vector and axial-vector couplings $g_V^{q,\Psi}, g_A^{q,\Psi}$ given in Eq. (7). Neglecting the mediator mass, so the propagator is $-i/s$, the tree-level amplitude is

$$\mathcal{M} = \frac{-i}{s} [\bar{v}(p_2)\gamma^\mu (g_V^q - g_A^q\gamma^5)u(p_1)] \times [\bar{u}(k_1)\gamma_\mu (g_V^\Psi - g_A^\Psi\gamma^5)v(k_2)]. \quad (\text{A2})$$

Squaring the matrix element and summing over final-state spins and colors and averaging over initial-state spins and colors, it follows that $|\overline{\mathcal{M}}|^2 \equiv \frac{1}{(2 \times 2)N_c^2} \sum_{\text{spins, colors}} |\mathcal{M}|^2 = \frac{1}{4N_c} \sum_{\text{spins}} |\mathcal{M}|^2 = \frac{1}{12} \text{Tr}[\cdots] \frac{1}{s} \text{Tr}[\cdots]$, which leads to

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{1}{12s^2} \text{Tr}[\not{p}_2\gamma^\mu (g_V^q - g_A^q\gamma^5)\not{p}_1\gamma^\nu (g_V^q - g_A^q\gamma^5)] \text{Tr}[(k_1 + m_\Psi)\gamma_\mu (g_V^\Psi - g_A^\Psi\gamma^5)(k_2 - m_\Psi)\gamma_\nu (g_V^\Psi - g_A^\Psi\gamma^5)] \\ &= \frac{1}{12s^2} [4((g_V^q)^2 + (g_A^q)^2)(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1 \cdot p_2)] \\ &\quad \times [4((g_V^\Psi)^2 + (g_A^\Psi)^2)(k_1^\mu k_2^\nu + k_1^\nu k_2^\mu - g^{\mu\nu} (k_1 \cdot k_2 - m_\Psi^2)) - 4((g_V^\Psi)^2 - (g_A^\Psi)^2)m_\Psi^2 g^{\mu\nu}] \\ &= \frac{2}{3s^2} [((g_V^q)^2 + (g_A^q)^2)((g_V^\Psi)^2(t^2 + u^2 + 2m_\Psi^2 s) + (g_A^\Psi)^2(t^2 + u^2 - 2m_\Psi^2 s)) + 4g_V^q g_A^q g_V^\Psi g_A^\Psi s(t - u)]. \end{aligned} \quad (\text{A3})$$

In terms of the Mandelstam invariants,

$$s = (p_1 + p_2)^2, \quad t = (p_1 - k_1)^2, \quad u = (p_1 - k_2)^2, \quad (\text{A4})$$

with $s + t + u = 2m_\Psi^2$ for massless initial-state quarks. Defining

$$\beta \equiv \sqrt{1 - \frac{4m_\Psi^2}{s}}, \quad (\text{A5})$$

and ϑ as the angle between $\vec{p}_1$ and $\vec{k}_1$, one has

$$t = m_\Psi^2 - \frac{s}{2}(1 - \beta \cos \vartheta), \quad u = m_\Psi^2 - \frac{s}{2}(1 + \beta \cos \vartheta). \quad (\text{A6})$$

The differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{k}|}{|\vec{p}|} |\overline{\mathcal{M}}|^2 = \frac{\beta}{64\pi^2 s} |\overline{\mathcal{M}}|^2. \quad (\text{A7})$$

Substituting Eq. (A3) and integrating the azimuthal angle to get a factor 2π gives

$$\begin{aligned} \frac{d\sigma}{d\cos \vartheta} &= \frac{\beta}{32\pi s} \frac{1}{3} \{ ((g_V^q)^2 + (g_A^q)^2) [(g_V^\Psi)^2(1 + \cos^2 \vartheta) \\ &\quad + (1 - \beta^2)\sin^2 \vartheta] + (g_A^\Psi)^2 \beta^2 (1 + \cos^2 \vartheta) \\ &\quad + 4g_V^q g_A^q g_V^\Psi g_A^\Psi \beta \cos \vartheta \}, \end{aligned}$$

where we used $m_\Psi^2 = \frac{s}{4}(1 - \beta^2)$, and we have integrated the azimuthal angle over 2π . Integrating the differential cross

section over $\cos \vartheta \in [-1, 1]$ leads to the cross section

$$\sigma_{q\bar{q} \rightarrow \Psi\Psi}(\beta) = \frac{\beta}{24\pi s} ((g_V^q)^2 + (g_A^q)^2) \left[(g_V^\Psi)^2 \left(1 - \frac{\beta^2}{3}\right) + (g_A^\Psi)^2 \frac{2\beta^2}{3} \right]. \quad (\text{A8})$$

Note that when integrating over $\cos \vartheta$ the term proportional to $g_V^q g_A^q g_V^\Psi g_A^\Psi$ vanishes. Thus, for the vectorlike model (with $g_V \neq 0$ and $g_A = 0$) to leading order in β ,

$$\sigma_{q\bar{q} \rightarrow \Psi\Psi}^{(Z)}(\beta) \simeq \frac{\beta}{24\pi s} ((g_V^q)^2 + (g_A^q)^2)(g_V^\Psi)^2. \quad (\text{A9})$$

Additionally, the quark couplings come from Eq. (11), and numerically, these are approximately

$$\hat{g}_V^\mu \approx 0.07, \quad \hat{g}_A^\mu \approx 0.19, \quad \hat{g}_V^d \approx -0.13, \quad \hat{g}_A^d \approx -0.19, \quad (\text{A10})$$

where we use $\sin^2 \theta_W \approx 0.231$ to obtain the numerical values.

It remains to state explicitly the Z -mediated production cross section for the vectorlike case, which is given by

$$\begin{aligned} \sigma_{q\bar{q} \rightarrow \bar{\Psi}^0 \Psi^0}^{(Z)} &\simeq \frac{\pi \alpha^2}{24 \sin^4 \theta_W \cos^4 \theta_W s} \frac{\beta}{s} \\ &\times [(T_3^q - 2Q_q \sin^2 \theta_W)^2 + (T_3^q)^2], \\ \sigma_{q\bar{q} \rightarrow \Psi^+ \Psi^-}^{(Z)} &\simeq \frac{\pi \alpha^2}{24 \sin^4 \theta_W \cos^4 \theta_W s} \frac{\beta}{s} [(T_3^q - 2Q_q \sin^2 \theta_W)^2 \\ &+ (T_3^q)^2][(1 - 2\sin^2 \theta_W)^2]. \end{aligned} \quad (\text{A11})$$

For $T \ll m_\Psi$, production is dominated near threshold; thus, $s \simeq 4m_\Psi^2$ and $\beta \ll 1$.

The process $q\bar{q} \rightarrow \bar{\Psi}^0 \Psi^0$ is only mediated by the Z boson, and in this case, we use $g_{A,V}^\Psi = g_{A,V}^0$ in Eq. (A9). For the process $q\bar{q} \rightarrow \Psi^+ \Psi^-$, there is both Z -mediated production (for which we use $g_{A,V}^\Psi = g_{A,V}^\pm$) and photon-mediated production contribution. We can specialize the general formula of Eq. (A9) to the photon case by taking $g_A^q = g_A^\Psi = 0$ with $g_V^q = eQ_q$, and $g_V^\Psi = e$ (since $Q_\Psi = 1$), leading to

$$\sigma_{q\bar{q} \rightarrow \Psi^+ \Psi^-}^{(\gamma)}(s) = \frac{2\pi \alpha^2 Q_q^2}{3s} \beta \left(1 - \frac{\beta^2}{3}\right). \quad (\text{A12})$$

Note that in the relativistic limit $\beta \rightarrow 1$, this gives the standard Drell–Yan form.

It is interesting to consider the ratio of photon to Z channels for the vectorlike model; when referring to the Z couplings, we add a hat ($\hat{g}$) to indicate this and avoid

confusion with the γ couplings (which we immediately write in terms of Q). Recall from Eqs. (9) and (10) that $\hat{g}_V^{\Psi^\pm} = \frac{e}{2 \sin \theta_W \cos \theta_W} (1 - 2 \sin^2 \theta_W)$ and $\hat{g}_A^{\Psi^\pm} = 0$; the ratio is

$$\begin{aligned} \frac{\sigma_{q\bar{q} \rightarrow \Psi^+ \Psi^-}^{(Z)}}{\sigma_{q\bar{q} \rightarrow \Psi^+ \Psi^-}^{(\gamma)}} \Big|_{T \ll m_\Psi} &\simeq \frac{(\hat{g}_V^q)^2 + (\hat{g}_A^q)^2}{4e^2 Q_q^2 \sin^2 \theta_W \cos^2 \theta_W} (1 - 2 \sin^2 \theta_W)^2 \\ &\approx \begin{cases} 0.37 & q = u, \\ 1.9 & q = d. \end{cases} \end{aligned} \quad (\text{A13})$$

APPENDIX B: NUMERICAL VALUE OF THE COUPLING COMBINATION

In this appendix, we calculate the value of the combination of couplings $\mathcal{C}$ defined in Eq. (17). Recall that we define [compare to Eq. (A9)] the $\mathcal{C}$ implicitly via

$$\sigma(s) \simeq \frac{\beta}{24\pi s} (\mathcal{C}_\gamma^{qq'} + \mathcal{C}_Z^{qq'} + \mathcal{C}_W^{qq'}), \quad (\text{B1})$$

with

$$\mathcal{C}_i \equiv ((g_V^q)^2 + (g_A^q)^2)(g_V^\Psi)^2. \quad (\text{B2})$$

Evaluating the relevant combination of couplings for the neutral C_Z^0 and charged $C_Z^\pm$ components, one has

$$\begin{aligned} C_Z^0 &= \frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W} ((\hat{g}_V^q)^2 + (\hat{g}_A^q)^2), \\ C_Z^\pm &= \frac{e^2}{4 \sin^2 \theta_W \cos^2 \theta_W} ((\hat{g}_V^q)^2 + (\hat{g}_A^q)^2) (1 - 2 \sin^2 \theta_W)^2. \end{aligned} \quad (\text{B3})$$

Then, evaluating $\mathcal{C}$, we find for the Z channel

$$\begin{aligned} C_Z^{0,q} &\approx \begin{cases} 5.5 \times 10^{-3} & q = u \\ 7.1 \times 10^{-3} & q = d \end{cases}, \\ C_Z^{\pm,q} &\approx \begin{cases} 1.6 \times 10^{-3} & q = u \\ 2.1 \times 10^{-3} & q = d \end{cases}, \end{aligned} \quad (\text{B4})$$

and for the charged component photon channel, one has

$$C_\gamma^{\pm,q} = \begin{cases} (2e^2/3)^2 \approx 4.3 \times 10^{-3} & q = u \\ (e^2/3)^2 \approx 1.1 \times 10^{-3} & q = d \end{cases}. \quad (\text{B5})$$

APPENDIX C: FREEZE-IN PRODUCTION VIA W BOSON

In addition to the Z - and γ -mediated channels $q\bar{q} \rightarrow \Psi^+ \Psi^-$ and $q\bar{q} \rightarrow \bar{\Psi}^0 \Psi^0$, freeze-in for the electroweak

doublet model can also proceed via “coproduction” channels of the form

$$\begin{aligned} u(p_1)\bar{d}(p_2) &\rightarrow W^+ \rightarrow \Psi^+(k_1)\Psi^0(k_2), \\ d(p_1)\bar{u}(p_2) &\rightarrow W^- \rightarrow \Psi^-(k_1)\bar{\Psi}^0(k_2), \end{aligned} \quad (\text{C1})$$

and analogously for heavier quarks. The dominant charged-current coproduction channels are those proportional to the largest CKM elements, specifically $u\bar{d}$, $c\bar{s}$, $t\bar{b}$, and conjugates. Other channels are CKM suppressed and lead to at most $\mathcal{O}(10\%)$ corrections. The relevant interactions are

$$\mathcal{L} \supset \frac{g_2}{\sqrt{2}} W_\mu^+ (\bar{u}\gamma^\mu P_L V_{ud}d) + \frac{g_2}{\sqrt{2}} W_\mu^+ (\bar{\Psi}^+\gamma^\mu P_L \Psi^0) + \text{H.c.}, \quad (\text{C2})$$

where $P_L = (1 - \gamma^5)/2$, and $V_{ud} \approx 0.97$ is the CKM matrix element. The tree-level $u\bar{d} \rightarrow \Psi^+\Psi^0$ matrix element is

$$\begin{aligned} \mathcal{M}_W &= \frac{-i}{s} \left(\frac{g_2}{\sqrt{2}} V_{ud} \right) \left(\frac{g_2}{\sqrt{2}} \right) [\bar{v}(p_2)\gamma^\mu P_L u(p_1)] \\ &\quad \times [\bar{u}(k_1)\gamma_\mu P_L v(k_2)] \\ &= \frac{-i}{s} [\bar{v}(p_2)\gamma^\mu (g_V^{ud,W} - g_A^{ud,W}\gamma^5)u(p_1)] \\ &\quad \times [\bar{u}(k_1)\gamma_\mu (g_V^{\Psi,W} - g_A^{\Psi,W}\gamma^5)v(k_2)], \end{aligned} \quad (\text{C3})$$

where in the latter equation, we have matched the conventions of Eq. (A1), by identifying the effective vector and axial couplings,

$$g_V^{ud,W} = g_A^{ud,W} = \frac{g_2}{2\sqrt{2}} V_{ud}, \quad g_V^{\Psi,W} = g_A^{\Psi,W} = \frac{g_2}{2\sqrt{2}}. \quad (\text{C4})$$

Since $m_{\Psi^+} = m_{\Psi^0}$ [cf. Eq. (6)], the kinematics are the same as the diagonal case (up to couplings); thus, we reuse the general result of Eq. (A8),

$$\begin{aligned} \sigma_{q\bar{q}' \rightarrow \Psi\Psi}(\beta) &= \frac{\beta}{24\pi s} \left((g_V^q)^2 + (g_A^q)^2 \right) \\ &\quad \times \left[(g_V^\Psi)^2 \left(1 - \frac{\beta^2}{3} \right) + (g_A^\Psi)^2 \frac{2\beta^2}{3} \right]. \end{aligned} \quad (\text{C5})$$

Substituting Eq. (C4) gives the coproduction cross sections,

$$\sigma_{u\bar{d} \rightarrow \Psi^+\Psi^0}^{(W)}(s) = \sigma_{d\bar{u} \rightarrow \Psi^-\bar{\Psi}^0}^{(W)}(s) = \frac{\beta}{24\pi s} \frac{g_2^4}{32} |V_{ud}|^2 \left(1 + \frac{\beta^2}{3} \right). \quad (\text{C6})$$

Taking the limit $\beta \ll 1$, we write the cross section in the form

$$\sigma_{d\bar{u} \rightarrow \Psi^-\bar{\Psi}^0}^{(W)}(s) \Big|_{\beta \ll 1} \simeq \frac{\beta}{24\pi s} C_W^{qq'}. \quad (\text{C7})$$

Noting that $C_W^{u\bar{d}} = C_W^{d\bar{u}}$, we drop the bars in the superscripts, and for the first generation,

$$C_W^{ud} = \frac{g_2^4}{32} |V_{ud}|^2 = \frac{1}{32} \frac{e^4}{\sin^4 \theta_W} |V_{ud}|^2 = \frac{\pi^2 \alpha^2}{2 \sin^4 \theta_W} |V_{ud}|^2. \quad (\text{C8})$$

For general up-type and down-type flavors (u_i, d_j) , the replacement is $|V_{ud}|^2 \rightarrow |V_{ij}|^2$ to get the general $C_W^{u_i d_j} \propto |V_{ij}|^2$. Using the values for the CKM matrix elements [13],

$$|V_{ud}| \approx 0.97, \quad |V_{cs}| \approx 0.98, \quad |V_{tb}| \approx 1.0, \quad (\text{C9})$$

numerically, the leading channels corresponding to C_W values of

$$C_W^{ud} \approx 5.4 \times 10^{-3}, \quad C_W^{cs} \approx 5.4 \times 10^{-3}, \quad C_W^{tb} \approx 5.6 \times 10^{-3}. \quad (\text{C10})$$

Comparing with Eqs. (B4) and (B5), we see that these are comparable to the Z and γ mediator production. Accordingly, this leads to an $\mathcal{O}(1)$ enhancement compared to just including the neutral channel alone.

APPENDIX D: DERIVATION OF THE REACTION DENSITY $\gamma(T)$

In this appendix, we derive the reaction density $\gamma(T)$. Specifically, we show explicitly how Eq. (18) follows from Eqs. (15) and (16). Since production is dominated near threshold in the Boltzmann suppressed freeze-in limit $T \ll m$, we can parametrize the center of mass energy as $s \simeq 4m_\Psi^2 + \varepsilon$ with $\varepsilon \ll 4m_\Psi^2$. It follows that

$$\beta \simeq \sqrt{\frac{\varepsilon}{4m_\Psi^2}} \ll 1. \quad (\text{D1})$$

The product $(s - 4m_\Psi^2)\sqrt{s}\sigma(s)$ which appears in the square brackets of Eq. (15) can be approximated near threshold as

$$(s - 4m_\Psi^2)\sqrt{s}\sigma(s) \simeq \sum_{q,q'} \left(\frac{C_\gamma^{qq'} + C_Z^{qq'} + C_W^{qq'}}{96\pi m_\Psi^2} \right) \varepsilon^{3/2}. \quad (\text{D2})$$

Since $s \sim m_\Psi \gg T$, we can use the large value approximation of the Bessel function $K_1(z) \simeq \sqrt{\frac{\pi}{2z}} e^{-z}$ for $z \gg 1$, and working to first order in ε , we obtain

$$K_1\left(\frac{\sqrt{s}}{T}\right) \simeq \sqrt{\frac{T\pi}{4m_\Psi}} e^{-\frac{2m_\Psi}{T}} e^{-\frac{\varepsilon}{4m_\Psi T}}. \quad (\text{D3})$$

Thus, we can rewrite the reaction density as an integral over ε as follows:

$$\gamma(T) \simeq \sum_{q,q'} \frac{T}{32\pi^4} \left(\frac{C_\gamma^{qq'} + C_Z^{qq'} + C_W^{qq'}}{96\pi m_\Psi^2} \right) \sqrt{\frac{\pi T}{4m_\Psi}} e^{-\frac{2m_\Psi}{T}} \times \int_0^\infty d\varepsilon \varepsilon^{3/2} e^{-\frac{\varepsilon}{4m_\Psi T}}. \quad (\text{D4})$$

The ε integral evaluates to $24\sqrt{\pi} m_\Psi^{5/2} T^{5/2}$, and thus, we obtain the reaction for $m_\Psi \gg T$,

$$\gamma(T) \simeq \sum_{q,q'} \left(\frac{C_\gamma^{qq'} + C_Z^{qq'} + C_W^{qq'}}{256\pi^4} \right) T^4 e^{-2m_\Psi/T}. \quad (\text{D5})$$

This can be specialized to the neutral or charged components by the appropriate choice of $\mathcal{C}$.

APPENDIX E: INDIRECT DETECTION

In regions of high density, such as the Galactic Center, the dark matter Ψ^0 can undergo pair annihilation into Standard Model particles at a late time, while this can potentially lead to indirect detection signals. In the high mass limit $m_\Psi \gg m_t$, the leading annihilation route is into W^+W^- via a t -channel diagram. The Ψ^0 are nonrelativistic at late time, with velocities of the order of $\sim 10^{-3}$ in virialized environments such as galactic halos. As a result, the long range exchange of gauge bosons between Ψ^0 states induces an attractive potential and enhances the tree-level Ψ^0 pair-annihilation cross sections due to the Sommerfeld effect; see, e.g., [43,44].

For brevity, we restrict our attention to the minimal model. Defining the two-body state,

$$\psi_i = \{\bar{\Psi}^0 \Psi^0, \Psi^+ \Psi^-\}, \quad (\text{E1})$$

the form of the interaction potential V_{ij} corresponding to the Lagrangian of Eq. (4) is given by

$$V_{ij}(r) \simeq \begin{pmatrix} 0 & -\frac{1}{r} \alpha_2 e^{-m_W r} \\ -\frac{1}{r} \alpha_2 e^{-m_W r} & 2\Delta - \frac{1}{r} \alpha_{\text{em}} - \frac{\alpha_2 (1 - 2\sin^2 \theta_W)^2}{4r \cos^2 \theta_W} e^{-m_Z r} \end{pmatrix}, \quad (\text{E2})$$

where $\alpha_2 \equiv g^2/4\pi$. The Sommerfeld factors s_i can be determined by solving the coupled nonrelativistic s -wave Schrödinger equations for $\psi_i(r)$ with the boundary conditions $\psi_i(\infty) = \{e^{ikr}, 0\}$, using the variable phase method outlined in, e.g., [45]. This leads to the Sommerfeld enhanced s -wave cross sections,

$$\langle \sigma v \rangle = \sum_{i,j} \sum_{G,G'} s_i \Gamma_{ij}^{GG'} s_j^*, \quad (\text{E3})$$

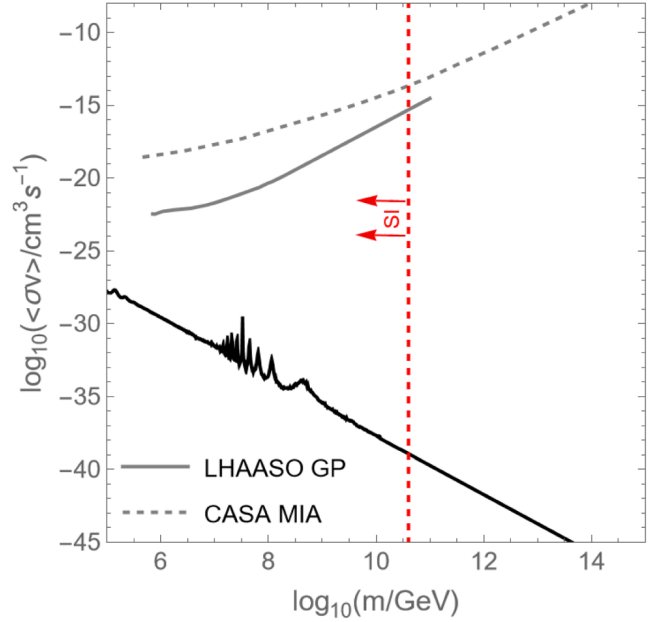


FIG. 3. Indirect detection bounds on the vectorlike model. The total annihilation cross section (for $v = 10^{-3}$) is shown as the black solid curve. We also present the leading constraints, namely, LHAASO galactic plane bounds [46,47] and CASA-MIA [48], assuming 100% annihilations to W^+W^- . The red dotted line marks the mass below which the (SI) scattering cross section is excluded by direct detection. Note the main sensitivity of indirect detection experiments lies within the mass range that is already excluded by direct detection limits.

where $\Gamma_{ij}^{GG'}$ denotes the tree-level annihilation rates to gauge bosons G, G' and can be written as

$$\begin{aligned} \Gamma_{ij}^{WW} &\simeq \frac{\pi \alpha_2^2}{16M^2} \begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 4 \end{pmatrix}, \\ \Gamma_{ij}^{ZZ} &\simeq \frac{\pi \alpha_2^2 (1 - 2\sin^2 \theta_W)^4}{16M^2 \cos^4 \theta_W} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\ \Gamma_{ij}^{\gamma Z} &\simeq \frac{\pi \alpha_{\text{em}} \alpha_2 (1 - 2\sin^2 \theta_W)^2}{M^2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\ \Gamma_{ij}^{\gamma\gamma} &\simeq \frac{\pi \alpha_{\text{em}}^2}{M^2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}. \end{aligned} \quad (\text{E4})$$

We use Eqs. (E3) and (E4) to calculate the Sommerfeld enhanced cross section by summing over annihilation channels of dark matter into gauge boson pairs,

$$GG' = \{W^+W^-, ZZ, \gamma Z, \gamma\gamma\}. \quad (\text{E5})$$

The total annihilation cross section $\bar{\Psi}^0 \Psi^0 \rightarrow GG'$ is shown as the black solid curve in Fig. 3. We note that the dominant annihilation channel is W^+W^- with other channels contributing at subleading order; thus,

$$\langle\sigma v\rangle_{\text{tot}} \approx \langle\sigma v\rangle_{WW}. \quad (\text{E6})$$

One can compare the annihilation cross section with experimental bounds. Figure 3 shows the leading indirect detection constraints: showing constraints [46] (“Max” limits) derived from LHAASO’s galactic plane continuum γ -rays [47] and γ -ray limits based on CASA-MIA [48] (also

cf. [49]) We apply limits assuming 100% annihilation to W^+W^- pairs, since this matches the full annihilation spectrum to good approximation. Inspecting Fig. 3, we conclude that even the strongest indirect detection limits due to LHAASO are not constraining over the parameter space of interest (i.e., over the parameter space not already excluded by direct detection).

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