

Muon beam dump experiments probe five-dimensional nature of $U(1)_{L_\mu-L_\tau}$

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We have investigated the prospects of probing the five-dimensional $U(1)_{L_\mu-L_\tau}$ interactions in present and future muon dump experiments, namely, NA64_μ, M³, MuSIC, and a future muon beam dump experiment. These experiments are classified into two categories: the first two can probe processes where feebly interacting massive particles go into invisible channels, while the latter two can probe processes where these states decay into muon pairs. These two types of experiments are complementary in that they allow exploration of different parameter regions of a model. In our scenario, the presence of multiple massive gauge bosons as Kaluza-Klein (KK) particles leads to an enhancement in the signal events compared to the corresponding four-dimensional scenario. In particular, the decay process into muon pairs enables mass reconstruction of the parent particle, making it possible to directly demonstrate the existence of multiple KK particles in at least some parameter regions. This can provide clear evidence that the origin of the $U(1)_{L_\mu-L_\tau}$ interaction lies in five dimensions. Furthermore, the muon ($g-2$) value, which is now consistent with the Standard Model, can be used to exclude specific parameter regions for new particles interacting with muons. We also carefully discuss the nontrivial effects arising from nonzero kinetic mixing.

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I. INTRODUCTION

In recent years, significant attention has been paid to light particles that feebly couple with the Standard Model (SM) sector, motivated by addressing the issues that cannot be solved within the SM, e.g., dark matter, neutrino profiles, and others; a recent review is available in Ref. [1]. The search for such particles is considered one of the most intriguing topics in particle phenomenology today.

When the mass of such particles is in the MeV range or GeV range but not particularly heavy, fixed-target beam-dump experiments are highly effective as a search method. Examples of such experiments conducted in the past include the following: CHARM [2], KEK [3], E141 [4], E137 [5], Orsay [6], E774 [7], LSND [8], NOMAD [9], and NA64 [10], which are classified according to the type of incident beam, i.e., protons [2,8,9] and electrons [3–7,10].

Generally, proton or electron beams are used in such experiments. However, recently, new types of beam dump experiments have been discussed, where the incident beam consists of particles other than protons and electrons.¹ Among these, one particularly interesting option is to use muons as the incident beam.² Various theoretical papers have already been published [39–55], and an experiment, NA64_μ, is already underway [56,57]. Naturally, these experiments are very useful to probe theoretical frameworks consisting of new feebly interacting light particles which couple to muons.

One such interesting model incorporates an additional $U(1)_{L_\mu-L_\tau}$ symmetry within the gauge sector [58–61]. This gauge extension is especially attractive as it is anomaly-free without introducing the right-handed neutrinos. This gauge extension has been widely employed to mediate light dark matter interactions [62–79], and to realize various leptogenesis mechanisms [80–84]. Furthermore, recent precise

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¹At the CERN Large Hadron Collider (LHC), a next-generation proton beam dump experiment, SHiP, is scheduled [11,12]. Also, the FASER experiment has been investigating new feebly interacting light particles with a displaced detector, as well as studying high-energy neutrinos [13,14]. See [15] for the Forward Physics Facility at the High-Luminosity LHC. Note that the bounds have been reported by the ATLAS [16–21] and LHCb [22,23] collaborations.

²Other such options are positrons [24–36], charged pions [37], and photons [36,38].

measurements of neutrino mass and mixing angles have enabled constraints to be placed on the $U(1)_{L_\mu-L_\tau}$ interaction through information from the active neutrino mass matrix (in particular, the minimal texture of it) [67,80,85,86]. Moreover, the information from recent precise measurements of N_{eff} will provide further information about very-light and feebly interacting gauge bosons [see previous works [87–90]]. Therefore, irrespective of the evolving status of the muon anomalous magnetic moment,³ particularly in light of recent QCD lattice determinations of the hadronic vacuum polarization contribution, which indicate no significant tension between the SM prediction and experimental measurements, the $U(1)_{L_\mu-L_\tau}$ gauged framework remains theoretically well motivated and phenomenologically testable across multiple experimental frontiers. It is important to emphasize that the $(g-2)_\mu$ observable constitutes only one among several experimental and cosmological probes of this symmetry.

In our previous study [92], we looked into a five-dimensional (5D) version of the basic $U(1)_{L_\mu-L_\tau}$ framework that added a single compact spatial dimension. The $U(1)_{L_\mu-L_\tau}$ gauge symmetry spreads out in the 5D bulk, introducing an infinite number of gauge bosons that are seen as Kaluza-Klein (KK) modes [93] from a four-dimensional (4D) point of view. In this method, the source of the MeV to GeV mass scale can be directly connected to the inverse size of the compact extra dimension under a twisted boundary condition, without the introduction of a new scalar boson.

This is one concrete example of the bigger framework for physics beyond the Standard Model, hidden spatial direction (or directions) in our Universe, commonly known as extra dimension(s). By utilizing the spatial structure of extra dimensions, it becomes possible to construct phenomenologically interesting models that are impossible to realize in four dimensions [see, e.g., the seminal works [94,95]]. Of course, by combining it with mechanisms that are also viable in four dimensions, a wider variety of discussions becomes possible. Furthermore (though beyond the scope of this paper's discussion), if matter coordination extends into the extra dimensions, there is also the possibility that experimentally verifiable differences in cosmic expansion may be observed compared to the case of four-dimensional spacetime in early cosmology [see, e.g., [96]].

On the other hand, phenomenologically, what deserves attention is the search for a method to directly distinguish between four-dimensional and five-dimensional physics. When all SM particles are propagating in the extra dimension(s), KK particles appear for all SM constituents (including those with strong interactions), dubbed the universal extra dimensions, making verification particularly

straightforward in the experiments at the LHC. However, such investigations have already been experimentally done, and no positive results have been obtained to date [97,98] [see also [99–101]]. On the other hand, if only new particles beyond the SM extend into the extra dimension(s), experimental constraints generally become weaker, making it possible to consider relatively light KK particles with sizes below the electroweak scale. In this work, we propose and systematically validate a strategy to search for the gauge boson(s) mediating such interactions, focusing on cases where these are light, couple predominantly to muons, and extend into extra dimensions. The analysis further elucidates the differences between four-dimensional and five-dimensional scenarios in this context. Actual calculations are performed using the aforementioned $U(1)_{L_\mu-L_\tau}$ interaction as an example, but the methodology is adaptable to other cases satisfying the aforementioned conditions.

In [92], as the first survey, we employed electron-neutrino elastic scattering (E ν ES), specifically the interaction $e\nu_X \rightarrow e\nu_X$ ($X: e, \bar{e}, \mu, \bar{\mu}$). To illustrate our methodology, we examined data from studies Borexino [102], TEXONO [103], and CHARM-II [104,105] and considered the future prospects in DUNE [106,107]. Note that Sec. I of our previous work [92] contains a list of other kinds of constraints on the 4D $U(1)_{L_\mu-L_\tau}$ scenario.

However, in the context of $U(1)_{L_\mu-L_\tau}$ gauge extension, the massive gauge bosons interact with electrons only through kinetic mixing terms [108]. So, through E ν ES, direct searches for muon interactions are not possible. The ongoing and proposed muon beam dump experiments, such as NA64 $_\mu$ [109], M³ [40], MuSIC [43,46], and a future muon beam dump experiment [45], provide an excellent avenue to directly probe such interactions for muons, even in scenarios with negligible kinetic mixing. These four experiments provide complementary results focusing on two decay processes of the massive gauge boson Z' , produced via bremsstrahlung from a muon beam: $Z' \rightarrow \text{invisible}$ (NA64 $_\mu$ and M³) and $Z' \rightarrow \mu^-\mu^+$ (MuSIC and a future muon beam dump experiment). In our framework, the existence of multiple Z' candidates leads to enhanced experimental sensitivity and a correspondingly broader scope for examining constraints and future prospects than those in the 4D counterpart. In particular, the decay of Z' into muon pairs is interesting because it allows for signal reconstruction, enabling direct confirmation of the existence of multiple vector bosons.

This paper is organized as follows. In Sec. II, we introduce our 5D $U(1)_{L_\mu-L_\tau}$ scenario. In Sec. III, we provide the formulas for the signal process via massive-vector bremsstrahlung in 4D and 5D. In Sec. IV, the details of the four muon beam dump experiments and how to estimate the number of signal events are explained. Section V provides numerical results and current status and prospects of our scenario in the future experiments, covering both cases where the kinetic mixing effect is zero and nonzero. Section VI

³The latest SM value is $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 38(63) \times 10^{-11}$, which is consistent within $\sim 0.6\sigma$ [91].

is devoted to providing a summary and discussion. In Appendix A, complementary plots are shown for the validity of our analysis. In Appendix B, we provide the justification of our application as a truncation of the summations of KK modes.

II. MODEL SPECIFICATIONS

This part gives a short summary of the five-dimensional $U(1)_{L_\mu-L_\tau}$ framework with an extra small spatial dimension. In this setup, all SM particles are stuck on a three-dimensional brane at $y = y_{\text{SM}}$. The $U(1)_{L_\mu-L_\tau}$ gauge boson and its KK excitations, on the other hand, can move through the five-dimensional bulk.

We use the coordinate $y \in [0, \pi R]$ to describe the extra spatial dimension, where R is the radius of compactification. It is assumed that the SM brane can be anywhere in the extra dimension, including the ends, and that its position is a free parameter. This generalization is given so that a more thorough phenomenological study can be done. This study focuses merely on the flat extradimensional case. We use twisted boundary conditions on the five-dimensional $U(1)_{L_\mu-L_\tau}$ gauge field: Neumann boundary conditions at $y = 0$ and Dirichlet boundary conditions at $y = \pi R$. As a result, there is no massless mode in the KK spectrum. Without using the Higgs mechanism or adding an extra singlet scalar field, the massless mode is taken away. The 5D $U(1)_{L_\mu-L_\tau}$ gauge coupling g'_{5D} is not dimensionless and has the mass dimension $-1/2$. In 4D effective theories, it is always accompanied by $\sqrt{\pi R}$ as the combination $g' = g'_{5D}/\sqrt{\pi R}$, and we can consider g' as a fundamental parameter.

A. Effective four-dimensional Lagrangian via KK decomposition

In this section, the theoretical framework for phenomenological calculations is introduced in terms of effective field theory to provide a common tool for treating the flat and warped backgrounds together.⁴ This paper focuses on the phenomenology of the flat case. The Lagrangian describing the effective 4D vectors' free part reads as

$$\begin{aligned} \mathcal{L}_{\text{eff}}^{(V\text{-free})} = \sum_n \left\{ -\frac{1}{4} \hat{Z}_{\mu\nu}^{(n)} \hat{Z}'^{(n)\mu\nu} + \frac{\epsilon_n}{2c_W} \hat{Z}_{\mu\nu}^{(n)} \hat{B}^{\mu\nu} \right. \\ \left. + \frac{1}{2} M_n^2 \hat{Z}_\mu^{(n)} \hat{Z}'^{(n)\mu} \right\} - \frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^3 W^{3\mu\nu}, \end{aligned} \quad (2.1)$$

⁴Details of the KK decompositions are available in Appendix A of Ref. [92]. For a string-theoretic realization of a bulk leptonic $U(1)$, see [110].

where $n = 1, 2, 3, \dots$ discriminate massive KK states, $\hat{Z}_\mu^{(n)}$ denotes the 4D gauge eigenstate field of the n th KK excitation of the $U(1)_{L_\mu-L_\tau}$ gauge boson, and $\hat{B}_\mu$ and W_μ^3 are the gauge-eigenstate 4D gauge fields of the $U(1)_Y$ and the third component of $SU(2)_W$, respectively, where each field strength is defined as usual, $V_{\mu\nu} \leftrightarrow \partial_\mu V_\nu - \partial_\nu V_\mu$. Thus, for $\hat{Z}_\mu^{(n)}$, $\hat{B}_\mu$, and W_μ^3 , the respective field strengths are defined as follows:

$$\begin{aligned} \hat{Z}_{\mu\nu}^{(n)} &:= \partial_\mu \hat{Z}_\nu^{(n)} - \partial_\nu \hat{Z}_\mu^{(n)}, & \hat{B}_{\mu\nu} &:= \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu, \\ W_{\mu\nu}^3 &:= \partial_\mu W_\nu^3 - \partial_\nu W_\mu^3. \end{aligned} \quad (2.2)$$

ϵ_n is the effective kinetic mixing factor between $\hat{Z}_\mu^{(n)}$ and $\hat{B}_\mu$, which is defined as

$$\begin{aligned} \epsilon_n &= \epsilon_4 f_n, \\ f_n &:= f_V^{(n)}(y_{\text{SM}}) = \sqrt{2} \cos \left[\left(n - \frac{1}{2} \right) \frac{y_{\text{SM}}}{R} \right], \end{aligned} \quad (2.3)$$

where $f_V^{(n)}(y)$ is the n th KK mode wavefunction toward the flat y direction, and the effective dimensionless parameter ϵ_4 is defined as

$$\begin{aligned} \epsilon_4 &:= \epsilon_{5D}/\sqrt{\pi R} \\ [\epsilon_{5D} \text{ is a parameter in the 5D original action [92]].} \end{aligned} \quad (2.4)$$

M_n is the n th KK mass in the flat space and is given by

$$M_n := (2n-1)m_{\text{KK}}, \quad (n = 1, 2, 3, \dots); \quad m_{\text{KK}} := \frac{1}{2R}. \quad (2.5)$$

Note that ϵ_n and M_n take different forms in the cases of the flat and warped backgrounds, $v \simeq 246$ GeV is the Higgs vacuum expectation value, and c_W , s_W , g_2 , and g_1 are the cosine and the sine of the Weinberg angle, the $SU(2)_W$ gauge coupling, and the $U(1)_Y$ gauge coupling, respectively. Introducing the secondary basis,

$$\hat{B}_\mu = B_\mu + \sum_n \frac{\epsilon_n}{c_W} \tilde{Z}_\mu^{(n)}, \quad (2.6)$$

$$\hat{Z}_\mu^{(n)} = \tilde{Z}_\mu^{(n)}, \quad (2.7)$$

$$\begin{pmatrix} \tilde{Z}_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}, \quad (2.8)$$

the Lagrangian takes the following form:

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{(\text{V-free})} = & -\frac{1}{4} \sum_n \tilde{Z}'^{(n)}_{\mu\nu} \tilde{Z}^{(n)\mu\nu} - \frac{1}{4} \tilde{Z}_{\mu\nu} \tilde{Z}^{\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} \\
& + \frac{1}{2} \sum_n M_n^2 \tilde{Z}'^{(n)}_{\mu} \tilde{Z}^{(n)\mu} + \frac{1}{2} m_{Z,0}^2 \tilde{Z}_{\mu} \tilde{Z}^{\mu} \\
& + m_{Z,0}^2 t_W \sum_n \epsilon_n \tilde{Z}_{\mu} \tilde{Z}'^{(n)\mu} \\
& + \frac{1}{2} m_{Z,0}^2 t_W^2 \sum_{n,n'} \epsilon_n \epsilon_{n'} \tilde{Z}'^{(n)}_{\mu} \tilde{Z}'^{(n')\mu}, \quad (2.9)
\end{aligned}$$

where A_{μ} is the photon field, $A_{\mu\nu} := \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$, $t_W = s_W/c_W$, and $m_{Z,0}^2 := (g_1^2 + g_2^2)v^2/4$.⁵ Assuming that all of the dimensionless parameters $\{\epsilon_n\}_{n=1,2,3,\dots}$ are sufficiently small, the unitary transformation which diagonalizes the mass terms of Eq. (2.9) is approximately given by (up to $\mathcal{O}(\epsilon_n^2)$)

$$\tilde{Z}'^{(n)}_{\mu} = Z'^{(n)}_{\mu} - t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z_{\mu} + \mathcal{O}(\epsilon_n^2), \quad (2.11)$$

$$\tilde{Z}_{\mu} = Z_{\mu} + t_W \sum_n \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z'^{(n)}_{\mu} + \mathcal{O}(\epsilon_n^2), \quad (2.12)$$

where the diagonalized form in terms of the mass eigenstates is given as

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{(\text{V-free})} = & -\frac{1}{4} \sum_n Z'^{(n)}_{\mu\nu} Z'^{(n)\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} \\
& + \frac{1}{2} m_{Z,0}^2 Z_{\mu} Z^{\mu} + \frac{1}{2} \sum_n M_n^2 Z'^{(n)}_{\mu} Z'^{(n)\mu} + \mathcal{O}(\epsilon_n^2). \quad (2.13)
\end{aligned}$$

In the above expression, Z_{μ} and $Z'^{(n)}_{\mu}$ are the Z boson and the n th $U(1)_{L_{\mu}-L_{\tau}}$ gauge boson in the mass eigenbasis, respectively. We summarize the relations between the gauge eigenstates and the mass eigenstates below;

$$\begin{aligned}
\hat{B}_{\mu} = & s_W Z_{\mu} + c_W A_{\mu} + \sum_n \left[\frac{\epsilon_n}{c_W} + s_W t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} \right] Z'^{(n)}_{\mu} \\
& + \mathcal{O}(\epsilon_n^2), \quad (2.14)
\end{aligned}$$

$$\begin{aligned}
W_{\mu}^3 = & c_W Z_{\mu} - s_W A_{\mu} + c_W t_W \sum_n \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z'^{(n)}_{\mu} \\
& + \mathcal{O}(\epsilon_n^2), \quad (2.15)
\end{aligned}$$

$$\hat{Z}'^{(n)}_{\mu} = \tilde{Z}'^{(n)}_{\mu} = Z'^{(n)}_{\mu} - t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z_{\mu} + \mathcal{O}(\epsilon_n^2). \quad (2.16)$$

In terms of the mass eigenstates, the effective Lagrangian describing the interaction terms takes the following form;

$$\begin{aligned}
\mathcal{L}_{\text{eff,int}} = & \sum_{a=e,\mu,\tau} \left\{ \bar{l}_R^a i \gamma^{\mu} \partial_{\mu} l_R^a + g_1 s_W \bar{l}_R^a \gamma^{\mu} l_R^a Z_{\mu} + e \bar{l}_R^a \gamma^{\mu} l_R^a A_{\mu} + \bar{l}_R^a \gamma^{\mu} g_1 \sum_n \left[\frac{\epsilon_n}{c_W} + s_W t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} \right] Z'^{(n)}_{\mu} l_R^a \right. \\
& + \bar{l}_R^a \gamma^{\mu} g' Q_a \sum_n f_n \left[Z'^{(n)}_{\mu} - t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z_{\mu} \right] l_R^a + \bar{\nu}_L^a i \gamma^{\mu} \partial_{\mu} \nu_L^a + \bar{l}_L^a i \gamma^{\mu} \partial_{\mu} l_L^a + \frac{g_2}{\sqrt{2}} (\bar{\nu}_L^a \gamma^{\mu} l_L^a W_{\mu}^+ + \bar{l}_L^a \gamma^{\mu} \nu_L^a W_{\mu}^-) \\
& + \bar{\nu}_L^a \gamma^{\mu} \left\{ \frac{1}{2} \left[(g_2 c_W + g_1 s_W) Z_{\mu} + \sum_n \left((g_2 c_W + g_1 s_W) t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} + g_1 \frac{\epsilon_n}{c_W} \right) Z'^{(n)}_{\mu} \right] \right. \\
& + g' Q_a \sum_n f_n \left(Z'^{(n)}_{\mu} - t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z_{\mu} \right) \left. \right\} \nu_L^a + \bar{l}_L^a \gamma^{\mu} \left\{ e A_{\mu} + \frac{1}{2} \left[(-g_2 c_W + g_1 s_W) Z_{\mu} \right. \right. \\
& + \sum_n \left((-g_2 c_W + g_1 s_W) t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} + g_1 \frac{\epsilon_n}{c_W} \right) Z'^{(n)}_{\mu} \left. \right] + g' Q_a \sum_n f_n \left(Z'^{(n)}_{\mu} - t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} Z_{\mu} \right) \left. \right\} l_L^a \left. \right\} + \mathcal{O}(\epsilon_n^2), \quad (2.17)
\end{aligned}$$

where we introduced the shorthand notation for the $U(1)_{L_{\mu}-L_{\tau}}$ charges:

$$Q_a = \begin{cases} 0 & \text{for } a = e, \\ +1 & \text{for } a = \mu, \\ -1 & \text{for } a = \tau. \end{cases} \quad (2.18)$$

$l_{R,L}^a$ and ν_L^a ($a = e, \mu, \tau$) represent the charged leptons and neutrinos with definite chiralities in 4D.

⁵If we take into account the second-order perturbation for the eigenvalues, the unperturbed mass eigenvalues ($m_{Z,0}^2$ and M_n^2) are slightly shifted as follows:

$$m_{Z,0}^2 \rightarrow m_{Z,0}^2 \left[1 - t_W^2 m_{Z,0}^2 \sum_n \frac{\epsilon_n^2}{M_n^2 - m_{Z,0}^2} \right], \quad M_n^2 \rightarrow M_n^2 \left[1 + t_W^2 \frac{\epsilon_n^2 m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} \right]. \quad (2.10)$$

In this paper, we do not take into account the mass shift at the second order of ϵ_n , in particular, $m_{Z,0} \simeq m_Z$.

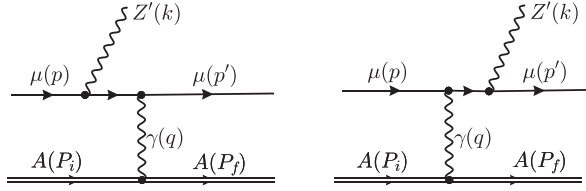


FIG. 1. The lowest-order $2 \rightarrow 3$ production process is given by $\mu(p) + A(P_i) \rightarrow \mu(p') + A(P_f) + Z'(k)$, where p and P_i denote the initial four-momenta of the incoming muon and target nucleus, respectively, and p' and P_f are the final-state four-momenta of the outgoing muon and recoiling nucleus. The outgoing Z' boson carries momentum k , while $q = P_i - P_f$ represents the momentum of the intermediate virtual photon exchanged in the process. Here, A , γ , μ , and Z' denote the target nucleus, photon, muon, and the new gauge boson, respectively.

III. CROSS SECTION FORMULA FOR LIGHT Z' BREMSSTRAHLUNG

A. 4D $U(1)_{L_\mu-L_\tau}$ case

In beam-dump experiments for light new particle searches, the significant process is the initial and final-state radiation (bremsstrahlung) off an accelerated charged lepton, which bombards an atom of a fixed target. For experiments involving an incident muon-beam, the relevant process and the kinematics are given by

$$\mu(p) + A(P_i) \rightarrow \mu(p') + A(P_f) + Z'(k), \quad (3.1)$$

where the corresponding Feynman diagrams are shown in Fig. 1. A denotes an atom of the target, and $p, P_i = (M_i, \mathbf{0})$, p', P_f and k are the four momenta in the lab frame, where M_i is the mass of the target atom which is stationary in the lab frame.

The cross section of the Z' -boson bremsstrahlung was formulated in [111] for the dark photon, based on the discussion on the axion-bremsstrahlung off an electron beam [112]. If the initial charged lepton (muon) is sufficiently accelerated, the Weizsäcker-Williams (WW) approximation is applicable [111–116]; see also [44,117–119] for detailed verifications/application limits of this approximation for bremsstrahlung.⁶ The WW approximation transforms the full $2 \rightarrow 3$ process [see Eq. (3.1) and

⁶It is also known as the method of virtual quanta, a semi-classical technique developed to simplify phase space integration in processes involving high-energy charged particles. It effectively models the interaction of a relativistic charged particle with an external electromagnetic field as a flux of equivalent real photons. Initially designed for bremsstrahlung calculations and later applied in fixed-target and beam-dump studies. Furthermore, an investigation into the nonzero momentum transfer in the bremsstrahlung from the proton is found in [120] [see also [121–123]].

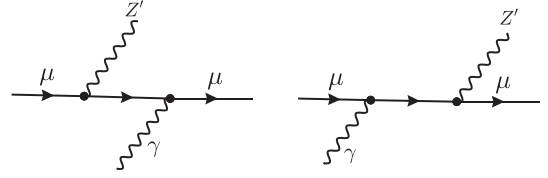


FIG. 2. Feynman diagram illustrating a $2 \rightarrow 2$ scattering process mediated by the 4-D gauge boson Z' associated with the $U(1)_{L_\mu-L_\tau}$ symmetry.

Fig. 1] into a simpler $2 \rightarrow 2$ scattering process (Fig. 2) by factoring out the photon flux via the target atom.⁷

More precisely, this approximation is valid when the beam energy is much larger than both the muon mass m_μ and the mass of the new vector boson $m_{Z'}$. Under these conditions, the differential cross section for the full $2 \rightarrow 3$ process ($d\sigma_{2 \rightarrow 3}^{Z'}$) is factorized and can be written as

$$\frac{1}{E_0^2 x} \frac{d\sigma_{2 \rightarrow 3}^{Z'}}{dx d \cos \theta_{Z'}} = \frac{8\alpha^2 g'^2 Q_\mu^2 \chi_{Z'} \beta_{Z'}}{4\pi} \left[\frac{1-x+\frac{x^2}{2}}{u^2} + \frac{(1-x)^2 m_{Z'}^2}{u^4} \left(m_{Z'}^2 - \frac{ux}{1-x} \right) \right], \quad (3.2)$$

where E_0 and $\theta_{Z'}$ are the energy of the incoming single muon beam and the emission angle of the outgoing Z' boson (taken as relative to the beam axis) in the lab frame. α is the electromagnetic fine-structure constant, g' is the 4D $U(1)_{L_\mu-L_\tau}$ gauge coupling, and Q_μ is the muon charge of the $U(1)_{L_\mu-L_\tau}$ gauge interaction ($Q_\mu = +1$). The quantity $\chi_{Z'}$ denotes the effective virtual photon flux, whose details will be provided in the next paragraph. The variable

$$x := \frac{E_{Z'}}{E_0} \quad (3.3)$$

is the fraction of energy carried by Z' , where $E_{Z'}$ and E_0 denote the energies of Z' and the initial muon, respectively. u is a kinematic function that depends on both x and $\theta_{Z'}$. Several relevant kinematic quantities are defined as follows:

$$\beta_{Z'} := \sqrt{1 - \frac{m_{Z'}^2}{E_0^2}}, \quad (3.4)$$

$$q := P_i - P_f, \quad (3.5)$$

$$t := -q^2, \quad (3.6)$$

⁷Note that all of the Feynman diagrams in this manuscript are drawn with FeynGame 3.0 [124–126].

$$u = u(x, \theta_{Z'}) := E_0^2 \theta_{Z'}^2 x + m_{Z'}^2 \left(\frac{1-x}{x} \right) + m_\mu^2 x, \quad (3.7)$$

where $\beta_{Z'}$ is the boost factor of the outgoing Z' in the lab frame, q represents the four momentum of the virtual photon, respectively. Note that, in the numerator of Eq. (3.2), except in $u(x, \theta_{Z'})$, we ignored the terms proportional to m_μ^2 . Also, the form of Eq. (3.2) agrees with that of Eq. (A12) of [111], up to the difference in the types of the $U(1)$ interactions.

The photon flux $\chi_{Z'}$ appearing in Eq. (3.2) is determined by the electric form factor $G_2(t)$ as [111,115]

$$\chi_{Z'} := \int_{t_{\min}}^{t_{\max}} dt \frac{t - t_{\min}}{t^2} G_2(t), \quad (3.8)$$

with

$$t_{\min} = \left(\frac{m_{Z'}^2}{2E_0} \right)^2, \quad t_{\max} = m_{Z'}^2. \quad (3.9)$$

The magnetic form factor $G_1(t)$ is neglected since its contribution is small in the regime of interest [111]. The full electric form factor $G_2(t)$ includes both elastic and inelastic components. The elastic contribution is given by

$$G_{2,\text{el}}(t) = \left(\frac{a^2 t}{1 + a^2 t} \right)^2 \left(\frac{1}{1 + \frac{t}{d}} \right)^2 Z^2, \quad (3.10)$$

with $a = 111Z^{-1/3}/m_e$ and $d = 0.164 \text{ GeV}^2 A^{-2/3}$. m_e represents the electron mass, and Z and A denote the atomic number and the atomic mass number of the target material, respectively.⁸ The inelastic part is expressed as

$$G_{2,\text{in}}(t) = \left(\frac{a'^2 t}{1 + a'^2 t} \right)^2 \left(\frac{1 + \frac{t}{4m_p^2} (\mu_p^2 - 1)}{(1 + \frac{t}{0.71 \text{ GeV}^2})^4} \right)^2 Z, \quad (3.11)$$

with $a' = 773Z^{-2/3}/m_e$, where m_p is the proton mass and $\mu_p = 2.79$ is the magnetic moment of the proton (in nuclear magneton). The total electric form factor is given by

$$G_2(t) = G_{2,\text{el}}(t) + G_{2,\text{in}}(t). \quad (3.12)$$

Owing to $t_{\max} = m_{Z'}^2$ in Eq. (3.9), for heavier Z' , the associated momentum transfer to the nucleus t becomes

⁸The choice of target material varies across the considered experiments. For the NA64 _{μ} experiment, the target is lead with atomic number $Z = 82$ and mass number $A = 207.2$. In the MuSIC setup, gold is used as the target material, characterized by $Z = 79$ and $A = 196.97$. The M³ experiment employs tungsten, with $Z = 74$ and $A = 183.84$. Finally, for the proposed future muon beam dump experiment, the target is water, with an effective average atomic number $Z = 10$ and mass number $A = 18$ [127].

larger, causing the recoil to probe progressively shorter-distance nuclear structures. For such energetic domains, additional contributions can arise via the photon exchange in the deep-inelastic scattering (DIS) regime. A simple but reasonable criterion can take

$$t_{\min} \gtrsim 1 \text{ GeV}^2 \quad (\text{as an estimation of the DIS-active domain}) [53] \text{ [see also [111]].} \quad (3.13)$$

Also, we comment on the properties of the photon flux $\chi_{Z'}$ in Eq. (3.8): (i) for a larger E_0 (with focusing on a specific $m_{Z'}$), $t_{\min}$ is smaller, and eventually $\chi_{Z'}$ becomes larger; (ii) for a significantly large $m_{Z'}$ (with focusing on a specific E_0), $t_{\min}$ becomes larger and $\chi_{Z'}$ becomes very small.

Integrating $\cos \theta_{Z'}$ from $\theta_{Z'\min} = 0$ to $\theta_{Z'\max}$, the differential cross section with respect to the energy fraction x takes the simplified form⁹:

$$\frac{d\sigma_{2 \rightarrow 3}^{Z'}}{dx} = \frac{8\alpha^2 g^2 Q_\mu^2 \chi_{Z'}}{4\pi} \beta_{Z'} \left(m_{Z'}^2 \frac{1-x}{x} + m_\mu^2 x \right)^{-1} \left(1 - x + \frac{x^2}{3} \right), \quad (3.14)$$

where $\theta_{Z'\max}$ was taken to be sufficiently large, say, as $\theta_{Z'\max} > \sim \mathcal{O}(10^{-3})$. So, the result is saturated and is insensitive to the value of $\theta_{Z'\max}$. This is consistent with Eq. (5) in [45], and also Eq. (A14) in [111] (after accounting for the difference in the types of the $U(1)$ interactions). The minimum and maximum of x are kinematically determined as [44]

$$x_{\min} = \frac{m_{Z'}}{E_0}, \quad x_{\max} = 1 - \frac{m_\mu}{E_0}. \quad (3.15)$$

In the absence of an appreciable kinetic mixing effect, the decay width of a static Z' boson into a charged lepton-antilepton pair ($l\bar{l}$, with $l = \mu, \tau$) is given by [45]

$$\Gamma(Z' \rightarrow l\bar{l}) = \frac{g^2 Q_l^2}{12\pi} m_{Z'} \left(1 + \frac{2m_l^2}{m_{Z'}^2} \right) \sqrt{1 - \frac{4m_l^2}{m_{Z'}^2}}, \quad (3.16)$$

while the corresponding decay width into a neutrino-antineutrino pair of the second and third generations (ν_μ, ν_τ) is given by [87]

$$\Gamma(Z' \rightarrow \nu_\alpha \bar{\nu}_\alpha) = \frac{g^2 Q_{\nu_\alpha}^2}{24\pi} m_{Z'}. \quad (3.17)$$

Here, m_l denotes the mass of the muon or tau lepton, and the above expressions are written under the assumption that no additional contributions from kinetic mixing are present.

⁹The integration over the angle is performed by substituting the variable u with $\cos \theta_{Z'}$. For details, refer to Appendix A in [111].

We can also recast the differential cross section in Eq. (3.2) in terms of the boost factor γ and pseudorapidity η of the Z' in the lab frame, using the relations [111]¹⁰:

$$\theta_{Z'} = \sin^{-1}(\text{sech } \eta), \quad x = \frac{\gamma m_{Z'}}{E_0}. \quad (3.18)$$

The integration range for pseudorapidity is set as

$$-\ln \left[\tan \left(\frac{\theta_{Z'}^{\max}}{2} \right) \right] \lesssim \eta \lesssim +\infty, \\ 1 \lesssim \gamma \lesssim \left(1 - \frac{m_\mu}{E_0} \right) \frac{E_0}{m_{Z'}}. \quad (3.19)$$

Finally, we will mention another possible amplitude for the signal process $\mu^- A \rightarrow \mu^- A Z'$. If Z' is attached to quarks via the nonzero kinetic mixing, Z' can be emitted from a proton of the target, and the doubly virtual Compton scattering (VVCS) off the hadronic state is not always suppressed [128]. In the case of the dark photon search in a tantalum-targeted electron beam dump experiment, the authors of [128] showed that the VVCS signal process provides an $\sim 10\%$ extra contribution if the mass of the dark photon is sufficiently lighter than the initial beam energy. Of course, when the kinetic mixing is set to zero, this contribution *vanishes*. However, if we consider the case where the kinetic mixing is nonzero, we should bear this in mind.¹¹

B. Extra-dimensional $U(1)_{L_\mu-L_\tau}$ case with kinetic mixing

As introduced in Sec. II, the (minimal) five-dimensional $U(1)_{L_\mu-L_\tau}$ scenario includes multiple Z' bosons as KK vector bosons as $Z'^{(n)}$ ($n = 1, 2, 3, \dots$). As we will see

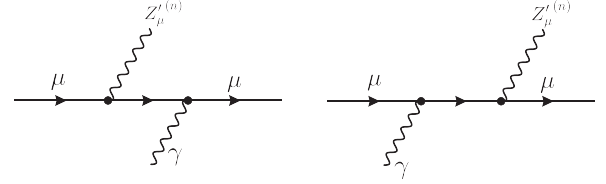


FIG. 3. Feynman diagram illustrating a $2 \rightarrow 2$ scattering process mediated by the extra-dimensional gauge boson $Z'^{(n)}$ associated with the $U(1)_{L_\mu-L_\tau}$ symmetry.

later, their decay widths are proportional to their physical masses, and higher modes rapidly decay compared with lower modes. So, the interference effects between different KK modes (as intermediate states of the full chain of $\mu^- A \rightarrow \mu^- A (Z'^{(n)})^* \rightarrow \mu^- A f \bar{f}$; f is a $U(1)_{L_\mu-L_\tau}$ -charged SM fermion) are negligible. Further, the narrow decay widths of these KK modes as compared to the mass difference of two adjacent states imply that the resonances do not overlap, and that the cross section can be accurately estimated by considering one (on shell) KK mode at a time, which decays to $f \bar{f}$. Therefore, we need only consider the cross sections of bremsstrahlung for each KK vector boson separately.

Under the above approximation, the calculation of each cross section is very straightforward. In the following, we derive a more complete expression that does not neglect the term proportional to the muon mass in the numerator, which we will use in our analysis. Using the WW approximation introduced in Sec. III A, the differential cross section of the $2 \rightarrow 3$ process involving the n th KK mode $Z'^{(n)}$ involving the final state can be calculated in terms of the energy fraction x and the angle $\theta_{Z'^{(n)}}$ (see Fig. 3) with the help of FeynCalc 10.1.0 [129–132] as

$$\frac{1}{E_0^2 x} \frac{d\sigma_{2 \rightarrow 3}^{Z'^{(n)}}}{dx d\cos \theta_{Z'^{(n)}}} = \frac{\alpha^2 \chi_{Z'^{(n)}} \beta_{Z'^{(n)}}}{\pi M_n^2 u^4} \left\{ \left(C_V^{(n)} \right)^2 M_n^2 [2M_n^4 (1-x)^2 - 4m_\mu^2 u x (1-x) + 4m_\mu^4 x^2 (1-x) \right. \\ \left. + u^2 (2+x(x-2)) + 2M_n^2 (1-x) (-u x + m_\mu^2 (2+x(x-2))) \right] \\ \left. + \left(C_A^{(n)} \right)^2 [2M_n^6 (1-x)^2 + 2m_\mu^2 u^2 x^2 + M_n^2 (8m_\mu^2 u x (1-x) - 8m_\mu^4 x^2 (1-x) + u^2 (2+x(x-2))) \right. \right. \\ \left. \left. + 2M_n^4 (1-x) (-u x + m_\mu^2 (-4+x(x+4))) \right] \right\}, \quad (3.20)$$

which results in the following form after the integration over $\cos \theta_{Z'^{(n)}}$:

¹⁰Note that the following expression of the pseudorapidity is equivalent to the definition $\eta = -\ln[\tan(\frac{\theta}{2})]$, where θ is in the principal value range.

¹¹In Ref. [128], another possibility of the signal process for the final state $e^- A \rightarrow e^- A (l^- l^+)$ (with l being a charged lepton) was examined through a Bethe-Heitler trident production with a spacelike Z' . The authors of [128] concluded that this is negligible.

$$\frac{d\sigma_{2\rightarrow 3}^{Z'(n)}}{dx} = \frac{\alpha^2 \chi_{Z'(n)} \beta_{Z'(n)} x}{3\pi M_n^2 (M_n^2(1-x) + m_\mu^2 x^2)^2} \left\{ \left(C_V^{(n)} \right)^2 M_n^2 [m_\mu^2 x^2 (x(3x-4) + 4) - 2M_n^2 (x-1)(x(x-3) + 3)] \right. \\ \left. + \left(C_A^{(n)} \right)^2 [6m_\mu^4 x^4 - 2M_n^4 (x-1)(x(x-3) + 3) + m_\mu^2 M_n^2 x^2 (x-4)(3x-4)] \right\}. \quad (3.21)$$

The coefficients $C_V^{(n)}$ and $C_A^{(n)}$ represent the vector and axial-vector parts of the interaction, defined as

$$C_V^{(n)} := \frac{1}{2} \left[\frac{3}{2} g_1 \frac{\epsilon_n}{c_W} + \frac{1}{2} (3g_1 s_W - g_2 c_W) t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} \right] \\ + f_n g' Q_\mu, \quad (3.22)$$

$$C_A^{(n)} := \frac{1}{4} \left[g_1 \frac{\epsilon_n}{c_W} + (g_1 s_W + g_2 c_W) t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} \right]. \quad (3.23)$$

Here, there are two differences between the formula for the 4D $U(1)_{L_\mu-L_\tau}$ Z' in Eq. (3.14) and one of the 5D ones in Eq. (3.21). One is apparent; the gauge-boson mass $m_{Z'}$ should be replaced with the corresponding KK mass M_n .¹² The other one is more nontrivial. Due to the kinetic mixing effect, the axial-vector coupling is present, and the form of the differential cross section becomes more involved. We point out that, if we take the limits $C_A^{(n)} \rightarrow 0$ (realized by $\epsilon_n \rightarrow 0$) and $m_\mu \rightarrow 0$, Eqs. (3.20) and (3.21) are reduced to the 4D forms in Eqs. (3.2) and (3.14) (up to the factor f_n), respectively. Refer to Sec. II A for the parameters appearing in Eqs. (3.22) and (3.23).

The decay width of a static KK mode $Z_\mu^{l(n)}$ into a charged lepton pair ($l\bar{l}$, with $l = e, \mu, \tau$) is given by

$$\Gamma(Z_\mu^{l(n)} \rightarrow l\bar{l}) = \frac{M_n}{24\pi} \sqrt{1 - \frac{4m_l^2}{M_n^2}} \left[\left(C_{Ll}^{(n)2} + C_{Rl}^{(n)2} \right) \right. \\ \left. + \frac{m_l^2}{M_n^2} \left(6C_{Ll}^{(n)} C_{Rl}^{(n)} - C_{Ll}^{(n)2} - C_{Rl}^{(n)2} \right) \right], \quad (3.24)$$

while the corresponding decay width into neutrino pairs ($\nu_\alpha \bar{\nu}_\alpha$, with $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$) is

$$\Gamma(Z_\mu^{l(n)} \rightarrow \nu_\alpha \bar{\nu}_\alpha) = \frac{M_n}{24\pi} \left(C_{L\nu_\alpha}^{(n)} \right)^2. \quad (3.25)$$

The effective couplings are given by the chiral decomposition (in the 4D perspective):

$$C_{Rl}^{(n)} = g_1 \left(\frac{\epsilon_n}{c_W} + s_W t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} \right) + f_n g' Q_l, \quad (3.26)$$

¹²Here, we should be careful about the dependence of $m_{Z'}$ in $\beta_{Z'}$, $\chi_{Z'}$, also in $x_{\min}$ and $x_{\max}$.

$$C_{Ll}^{(n)} = \frac{1}{2} \left[(-g_2 c_W + g_1 s_W) t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} + g_1 \frac{\epsilon_n}{c_W} \right] + f_n g' Q_l, \quad (3.27)$$

$$C_{L\nu_\alpha}^{(n)} = \frac{1}{2} \left[(g_2 c_W + g_1 s_W) t_W \frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2} + g_1 \frac{\epsilon_n}{c_W} \right] + f_n g' Q_{\nu_\alpha}. \quad (3.28)$$

It is straightforward to verify that, in the limit $\epsilon_n \rightarrow 0$ and $f_n \rightarrow 1$, the above expressions reduce to the standard four-dimensional results of Eqs. (3.16) and (3.17).

IV. EXPERIMENTAL SETUPS AND SIGNAL ESTIMATIONS

We focus on the following muon beam dump experiments: NA64 μ , M³, MuSIC, and a future muon beam dump, where the first two and the last two experiments investigate the decay channels $Z' \rightarrow$ invisible and $Z' \rightarrow \mu^- \mu^+$, respectively. In what follows, one by one, we describe the methods for connecting the cross section formula for the emission of Z' or $Z'^{(n)}$ ($n = 1, 2, 3, \dots$), as discussed in Sec. III, to the number of events in the experiment.

A. NA64 μ : Beam-dump experiment at CERN

NA64 μ is a fixed-target beam-dump experiment at CERN, employing a 160 GeV muon beam from the Super Proton Synchrotron [133]. Its primary goal is to search for invisible decays of a light vector boson Z' , produced via a bremsstrahlung process,

$$\mu^- A(\text{Pb}) \rightarrow \mu^- A(\text{Pb}) Z', \quad Z' \rightarrow \text{invisible},$$

where the target is made of lead (Pb) [57]. Note that, for the 4D $U(1)_{L_\mu-L_\tau}$ scenario, the invisible final state dominantly consists of $Z' \rightarrow \nu_\mu \bar{\nu}_\mu$ and $Z' \rightarrow \nu_\tau \bar{\nu}_\tau$. In the presence of nonvanishing kinetic mixing, the channel $Z' \rightarrow \nu_e \bar{\nu}_e$ may also be relevant. For later convenience, we introduce the notation $\nu\bar{\nu} := \sum_{\alpha=e,\mu,\tau} \nu_\alpha \bar{\nu}_\alpha$.

The experimental signature is characterized by a single scattered muon carrying less than 80 GeV of energy, with no accompanying activity in the downstream detectors. In the 2022 run, the experiment collected a dataset corresponding to $(1.98 \pm 0.02) \times 10^{10}$ muons on target (N_{MOT}), and the future upgrades are expected to achieve 10^{13} muons on target [57].

In the 4D framework, the expected number of signal events corresponding to invisible decays of the form $Z' \rightarrow \bar{\nu}\nu$ at the NA64 $_{\mu}$ experiment is given by [39,134]

$$N_{\text{signal}}^{4\text{D,NA64}_{\mu}} = \frac{N_{\text{MOT}} n_{\text{Pb}}}{\langle dE_{\mu}/dy \rangle} \int_{E_{\mu,\text{min}}}^{E_{\mu,\text{max}}} dE_0 \times \int_{x_{\text{min}}}^{x_{\text{max}}} dx \frac{d\sigma_{2 \rightarrow 3}^{Z'}}{dx} \text{Br}(Z' \rightarrow \bar{\nu}\nu) N(m_{Z'}), \quad (4.1)$$

where $d\sigma_{2 \rightarrow 3}^{Z'}/dx$ and $\text{Br}(Z' \rightarrow \bar{\nu}\nu)$ represent the differential cross section and the branching ratio for the event, respectively (refer to Sec. III for the details). N_{MOT} is the total number of muons on target, and $n_{\text{Pb}} = 3.3 \times 10^{22} \text{ cm}^{-3}$ [127] denotes the number density of lead nuclei in the target. The quantity $\langle dE_{\mu}/dy \rangle$ represents the average energy loss per unit length for muons traversing the lead target. When looking at energies between a few GeV and about 160 GeV, muon energy loss is mostly caused by ionization processes [109]. It is safe to assume that $\langle dE_{\mu}/dy \rangle$ is independent of energy because it does not change much with the muon momentum in this range. For lead, we adopt the value $\langle dE_{\mu}/dy \rangle = 12.7 \times 10^{-3} \text{ GeV/cm}$ [127]. The integration is performed over the penetrating muon energy E_{μ} , ranging from

$$\begin{aligned} E_{\mu,\text{min}} &= E_{\mu,\text{beam}} - L_T \langle dE_{\mu}/dy \rangle, \\ E_{\mu,\text{max}} &= E_{\mu,\text{beam}}, \end{aligned} \quad (4.2)$$

where $E_{\mu,\text{beam}} \approx 160 \text{ GeV}$ is the initial beam energy and $L_T = 20 \text{ cm}$ is the total target length [134]. The integration range for the energy fraction x is taken as in Eq. (3.15), ensuring physical consistency in the production process [44].

$N(m_{Z'})$ is a normalization factor treated as a function of the Z' mass. This factor accounts for background normalization and has been calibrated using the 4D results reported by the NA64 $_{\mu}$ experimental analysis [given in Fig. 3 of [57]] for the current bound corresponding to $N_{\text{MOT}} \approx 2 \times 10^{10}$. For the projected sensitivity with $N_{\text{MOT}} = 10^{13}$ (future bound), we assume a background-free scenario, and thus take $N(m_{Z'}) = 1$. A 90% confidence level exclusion bound is imposed by requiring $N_{\text{signal}} > 2.3$ in the $(m_{Z'}, g')$ parameter space [109,135].

In the case of the 5D scenario, the Z' boson is replaced by a tower of KK excitations, each denoted $Z'^{(n)}$. We introduce the following notation for convenience:

$$N_{\text{signal}}^{5\text{D},a} = \sum_{n=1}^{n_{\text{KKmax}}} N_{\text{signal}}^{(n),a} \quad (a = \text{Experiments}). \quad (4.3)$$

The corresponding signal yield is obtained by modifying Eq. (4.1) for each KK mode as

$$\begin{aligned} N_{\text{signal}}^{(n),\text{NA64}_{\mu}} &= \frac{N_{\text{MOT}} n_{\text{Pb}}}{\langle dE_{\mu}/dy \rangle} \int_{E_{\mu,\text{min}}}^{E_{\mu,\text{max}}} dE_0 \\ &\times \int_{x_{\text{min}}^{Z'^{(n)}}}^{x_{\text{max}}^{Z'^{(n)}}} dx \frac{d\sigma_{2 \rightarrow 3}^{Z'^{(n)}}}{dx} \text{Br}(Z'^{(n)} \rightarrow \bar{\nu}\nu) N(M_n), \end{aligned} \quad (4.4)$$

where, as discussed in the first part of this section, the total number of signal events is provided as the sum of each $Z'^{(n)}$ contribution, up to the n_{KKmax} -mode. Here, the variable for integration x should be understood as $x = E_{Z'^{(n)}}/E_0$, where its domain is appropriately recognized as $x_{\text{min}}^{Z'^{(n)}} = M_n/E_0$ and $x_{\text{max}}^{Z'^{(n)}} = 1 - m_{\mu}/E_0$ [see Eq. (3.15)].

$N(M_n)$ is the normalization factor, defined as a function of the KK mode mass M_n . For the current bound, this factor is taken to be the same as that in the 4D scenario in order to account for background contributions. For the projected future bound under the assumption of a background-free analysis, we set $N(M_n) = 1$.

B. M³: Muon missing momentum experiment at Fermilab

The M³ experiment is a proposed fixed-target initiative at the Fermilab Accelerator Complex, designed to explore new muon-philic particles that decay invisibly [39,40]. The experimental concept builds upon the design principles of the LDMX experiment [136,137], utilizing a high-intensity 15 GeV muon beam to probe final states characterized by substantial missing momentum, which may signal the emission of undetected particles such as dark photons or new gauge bosons. The signal process is

$$\mu^- A(W) \rightarrow \mu^- A(W) Z', \quad Z' \rightarrow \text{invisible},$$

where the target is made of tungsten (W) [40].

The detector architecture features a precision tracking system positioned both upstream and downstream of a thick tungsten target of approximately $50X_0$, enabling accurate measurement of the incoming and outgoing muon momenta. A downstream electromagnetic calorimeter and hadronic calorimeter system serves to veto visible final-state activity and suppress SM backgrounds. The experimental program is expected to proceed in two phases [40]:

- (i) *Phase 1*: $N_{\text{MOT}} = 10^{10}$ muons on target and is primarily sensitive to light mediators with masses $m_{Z'} \lesssim 500 \text{ MeV}$.
- (ii) *Phase 2*: $N_{\text{MOT}} = 10^{13}$ muons with enhanced veto capabilities, targeting background rejection at the 10^{-13} level.

In the 4D framework, the expected number of signal events for invisible decays of the type $Z' \rightarrow \bar{\nu}\nu$ is given by [39,40]

$$N_{\text{signal}}^{4\text{D},\text{M}^3} = \frac{N_{\text{MOT}} n_{\text{W}}}{\langle dE_{\mu}/dy \rangle} \int_{E_{\mu,\text{min}}}^{E_{\mu,\text{max}}} dE_0 \times \int_{x_{\text{min}}}^{x_{\text{max}}} dx \frac{d\sigma_{2 \rightarrow 3}^{Z'}}{dx} \text{Br}(Z' \rightarrow \bar{\nu}\nu) \alpha(x, m_{Z'}), \quad (4.5)$$

which takes a similar form as in Eq. (4.1) for the NA64_μ experiment. $n_{\text{W}} = 6.3199 \times 10^{22} \text{ cm}^{-3}$ denotes the number density of tungsten nuclei in the target material [127], while $\langle dE_{\mu}/dy \rangle = 22.1 \times 10^{-3} \text{ GeV/cm}$ represents the average energy loss per unit length for muons traversing tungsten [39]. The integration over the penetrating muon energy E_{μ} extends as in Eq. (4.2), whereas the incident muon beam energy is taken to be $E_{\mu,\text{beam}} \approx 15 \text{ GeV}$ [40]. The target thickness is set to $L_T = 17.52 \text{ cm}$, corresponding to a material configuration with 100% tungsten purity and equivalent to a radiation length of $50X_0$ [127,134]. The integration limits for the energy fraction x are determined as in Eq. (3.15), which ensure the physical validity of the production process [44].

The factor $\alpha(x, m_{Z'})$ represents the detector acceptance as a function of x and $m_{Z'}$, extracted by interpolating the data provided in Fig. 5 of [40]. In this analysis, we approximate the decay probability density per unit length as unity, justified by the thick-target regime, the invisibility of the final-state particles, and the fact that all geometric and detector efficiency effects are already incorporated into the acceptance factor $\alpha(x, m_{Z'})$, as described in [40]. A 90% confidence level exclusion bound is imposed by requiring the predicted number of signal events to satisfy $N_{\text{signal}} > 2.3$ [138].

In ED extensions, the Z' boson is generalized to a tower of KK excitations, denoted by $Z'^{(n)}$. The corresponding signal yield generalizes Eq. (4.5) as

$$N_{\text{signal}}^{(n),\text{M}^3} = \frac{N_{\text{MOT}} n_{\text{W}}}{\langle dE_{\mu}/dy \rangle} \int_{E_{\mu,\text{min}}}^{E_{\mu,\text{max}}} dE_0 \times \int_{x_{\text{min}}^{Z'^{(n)}}}^{x_{\text{max}}^{Z'^{(n)}}} dx \frac{d\sigma_{2 \rightarrow 3}^{Z'^{(n)}}}{dx} \text{Br}(Z'^{(n)} \rightarrow \bar{\nu}\nu) \alpha(x, M_n), \quad (4.6)$$

where the total signal is defined as in Eq. (4.3). The factor $\alpha(x, M_n)$ denotes the detector acceptance as a function of the energy fraction x and the n th KK boson mass M_n , and is extracted using the same interpolation procedure employed in the 4D case.

C. MuSIC: Muon Synchrotron Ion Collider

The Muon Synchrotron Ion Collider (MuSIC) is a proposed high-energy muon-proton collider designed to explore physics beyond the SM, particularly in the muon sector [43,46]. It builds upon existing infrastructure at Brookhaven National Laboratory and has been envisioned as a successor to the Electron-Ion Collider following its projected completion in the 2040s. In addition to probing

new regimes of deep inelastic scattering in nuclear physics, MuSIC offers a promising opportunity to investigate rare muon processes, precision measurements, and the origin of the muon anomalous magnetic moment $(g-2)_{\mu}$.

We focus on a similar scenario that the baseline configuration of MuSIC assumes a center-of-mass energy of $E_0 = 8.9 \text{ TeV}$ and an integrated luminosity of $L_I = 2000 \text{ fb}^{-1}$.¹³ One compelling physics target is the search for light muon-philic vector bosons Z' , which may couple to $L_{\mu} - L_{\tau}$, and decay visibly to a muon pair. The Z' boson can be produced in muon-nucleus collisions via a bremsstrahlung-like process:

$$\mu^- A(\text{Au}) \rightarrow \mu^- A(\text{Au}) Z', \quad Z' \rightarrow \mu^- \mu^+,$$

where a high-energy muon interacts with a stationary gold (Au) nucleus as a target [139].

In the 4D case, the number of observable events from displaced vertex signatures is given by [139]

$$N_{\text{signal}}^{4\text{D},\text{MuSIC}} = L_I \int d\gamma d\eta \frac{d\sigma_{2 \rightarrow 3}^{Z'}}{d\gamma d\eta} \text{Br}(Z' \rightarrow \mu^+ \mu^-) \times \epsilon_{\text{det}} \mathcal{P}(\ell_{\text{min}}, \ell_{\text{max}}), \quad (4.7)$$

where γ and η are the Lorentz boost factor and the pseudorapidity of Z' in the lab frame, respectively. The relations and the kinematically allowed domains have been given by Eqs. (3.18) and (3.19), respectively. Note that the differential production cross section in (γ, η) coordinates is expressed as

$$\frac{d\sigma}{d\gamma d\eta} = \frac{m_{Z'}}{E_0} \text{sech}\eta \sin\theta \frac{d\sigma}{dx d\cos\theta}, \quad (4.8)$$

with $x = \gamma m_{Z'}/E_0$ and $\theta = \sin^{-1}(\text{sech}\eta)$. Refer to Eq. (3.2) for $d\sigma_{2 \rightarrow 3}^{Z'}/(dx d\cos\theta)$.

The detection efficiency is assumed to be $\epsilon_{\text{det}} = 0.86$ for the dimuon channel [140]. Since the measurement of $Z' \rightarrow \mu^- \mu^+$ is made by a displaced detector far from the target, we should take into account the reduction of the signals due to the decay of Z' during the propagation. The decay probability density is modeled as [139]

$$P(\ell) = \frac{e^{-\ell/\ell_{Z'}}}{\ell_{Z'}}, \quad \text{with } \ell_{Z'} := \frac{\gamma}{\Gamma_{Z'}}, \quad (4.9)$$

where $\Gamma_{Z'}$ is the total decay width of a static Z' , and the boost factor γ describes the lifetime dilation for observers in the lab frame. The probability that the Z' boson decays

¹³We have verified that, for this configuration of E_0 and L_I , the result obtained by taking only the $n_{\text{KK}} = 1$ case of our five-dimensional $U(1)_{L_{\mu}-L_{\tau}}$ theory (that will be shown in Fig. 4, for $\epsilon_4 = 0$) is in close agreement with the result for the four-dimensional effective Z' description coupled with the muons calculated in Fig. 2 of [139].

within the fiducial volume of interest, denoted by $\mathcal{P}(\ell_{\min}, \ell_{\max})$, is obtained by integrating Eq. (4.9) as

$$\mathcal{P}(\ell_{\min}, \ell_{\max}) := \int_{\ell_{\min}}^{\ell_{\max}} d\ell P(\ell), \quad (4.10)$$

with the minimum distance $\ell_{\min} = 1$ mm and the maximum distance $\ell_{\max} = 30$ m, corresponding to the location of the downstream detector capable of reconstructing the decay products [139]. We impose a threshold of $N_{\text{signal}} > 5$ signal events to define the exclusion limit for the MuSIC experiment [45].

In the 5D case, the vector boson Z' originates from a tower of KK excitations $Z'^{(n)}$, each contributing to the signal. The total number of displaced dimuon events is obtained, as in Eq. (4.3), by summing over each KK mode:

$$N_{\text{signal}}^{(n), \text{MuSIC}} = L_I \int d\gamma d\eta \frac{d\sigma_{2 \rightarrow 3}^{Z'^{(n)}}}{d\gamma d\eta} \text{Br}(Z'^{(n)} \rightarrow \mu^+ \mu^-) \times \epsilon_{\text{det}} \mathcal{P}^{(n)}(\ell_{\min}, \ell_{\max}), \quad (4.11)$$

where the kinetic variables γ and η should be understood as those for the vector boson with mass M_n . Refer to Eq. (3.20) for the form of the differential cross section without ignoring the muon mass. We should also be careful about the corresponding decay probability $\mathcal{P}^{(n)}(\ell_{\min}, \ell_{\max})$. The superscript (n) indicates that both the production cross section and decay probability are mode-dependent. In particular, the decay length for each KK mode is

$$\ell_{Z'^{(n)}} := \frac{\gamma}{\Gamma_{Z'^{(n)}}}, \quad (4.12)$$

which impacts the probability $\mathcal{P}^{(n)}(\ell_{\min}, \ell_{\max})$ for decays occurring within the detector volume. Due to increasing masses and typically shorter lifetimes of higher KK modes, only a subset of modes contributes appreciably within the experimental geometry.

D. Future high-energy muon beam dump experiment

A high-energy muon beam dump setup provides a cost-effective and complementary strategy for probing weakly coupled new physics, particularly in regimes that may be inaccessible at high-luminosity colliders [45, 111]. In this configuration, a 1.5 TeV muon beam is directed onto a dense, thick target such as water [45]. This enhances the probability of producing new light mediators like the Z' boson via a bremsstrahlung-like process. These mediators can travel downstream before decaying into visible particles, which are subsequently detected via a spectrometer system placed after the shielding. The signal process is

$$\mu^- A(\text{H}_2\text{O}) \rightarrow \mu^- A(\text{H}_2\text{O}) Z', \quad Z' \rightarrow \mu^- \mu^+,$$

where the target is made of water (H_2O) [45].

For the 4D scenario, the number of signal events along the beamline is given by [45]

$$N_{\text{signal}}^{4\text{D}, \text{FMDB}} = N_\mu \int_{x_{\min}}^{x_{\max}} dx \frac{\rho \ell_{Z'}}{m_T} \frac{d\sigma_{2 \rightarrow 3}^{Z'}}{dx} \text{Br}(Z' \rightarrow \mu^+ \mu^-) \times (e^{L_{\text{tar}}/\ell_{Z'}} - 1) e^{-(L_{\text{tar}} + L_{\text{sh}})/\ell_{Z'}} (1 - e^{-L_{\text{dec}}/\ell_{Z'}}), \quad (4.13)$$

where $N_\mu = 10^{22}$ is the total number of muons incident on the target [45], the quantity $m_T = A/N_0$ represents the mass of a target atom, with A being the atomic mass number and N_0 Avogadro's number, while $\rho (= 0.997 \text{ g/cm}^3)$ is the target density of H_2O [127]. See Eq. (4.9) for the definition of $\ell_{Z'}$.

Like the MuSIC experiment, the signal part $Z' \rightarrow \mu^- \mu^+$ is measured by a displaced detector. We adopt the specific form of the decay probability for this experimental setup as discussed in [45]. The lengths L_{tar} , L_{sh} , and L_{dec} correspond to the target, shielding, and decay regions, respectively. For numerical analysis, we adopt benchmark values $L_{\text{tar}} = 10$ m, $L_{\text{sh}} = 10$ m, and $L_{\text{dec}} = 100$ m [45]. We impose a threshold of $N_{\text{signal}} > 5$ signal events to define the exclusion limit for this experiment [45]. In this regime, radiative energy losses are assumed to be negligible in this setup, allowing the energy distribution function to be treated as sharply peaked around the initial muon energy.

In the case of the 5D extension, the signal receives contributions from a tower of KK modes $Z'^{(n)}$, as in Eq. (4.3). The contribution from the n th mode is given as

$$N_{\text{signal}}^{(n), \text{FMDB}} = N_\mu \int_{x_{\min}^{Z'^{(n)}}}^{x_{\max}^{Z'^{(n)}}} dx \frac{\rho \ell_{Z'^{(n)}}}{m_T} \frac{d\sigma_{2 \rightarrow 3}^{Z'^{(n)}}}{dx} \text{Br}(Z'^{(n)} \rightarrow \mu^+ \mu^-) \times (e^{L_{\text{tar}}/\ell_{Z'^{(n)}}} - 1) e^{-(L_{\text{tar}} + L_{\text{sh}})/\ell_{Z'^{(n)}}} (1 - e^{-L_{\text{dec}}/\ell_{Z'^{(n)}}}), \quad (4.14)$$

where the variable for integration x should be understood as $x = E_{Z'^{(n)}}/E_0$ [also for $x_{\min}^{Z'^{(n)}}$ and $x_{\max}^{Z'^{(n)}}$; see Eq. (3.15)]. Refer to Eq. (4.12) for the definition of $\ell_{Z'^{(n)}}$.

V. RESULTS

We report our results, illustrating the exclusion sensitivity of NA64 μ , M^3 , MuSIC, and a forthcoming muon beam dump experiment to the $U(1)_{L_\mu - L_\tau}$ gauge boson in 5D. The analysis is performed for two benchmark scenarios: $n_{\text{KKmax}} = 1$, signifying the lowest KK excitation, and $n_{\text{KKmax}} = 5$, including the first five KK modes. We remind readers that the following four effective parameters affect results, g' , m_{KK} , ϵ_4 , and y_{SM} (in addition to n_{KKmax}). For simplicity, we introduce the notation $\tilde{y}_{\text{SM}} := y_{\text{SM}}/R$. Here, we explicitly state that the DIS effect mentioned in Sec. III A is ignored in all of the following plots. For

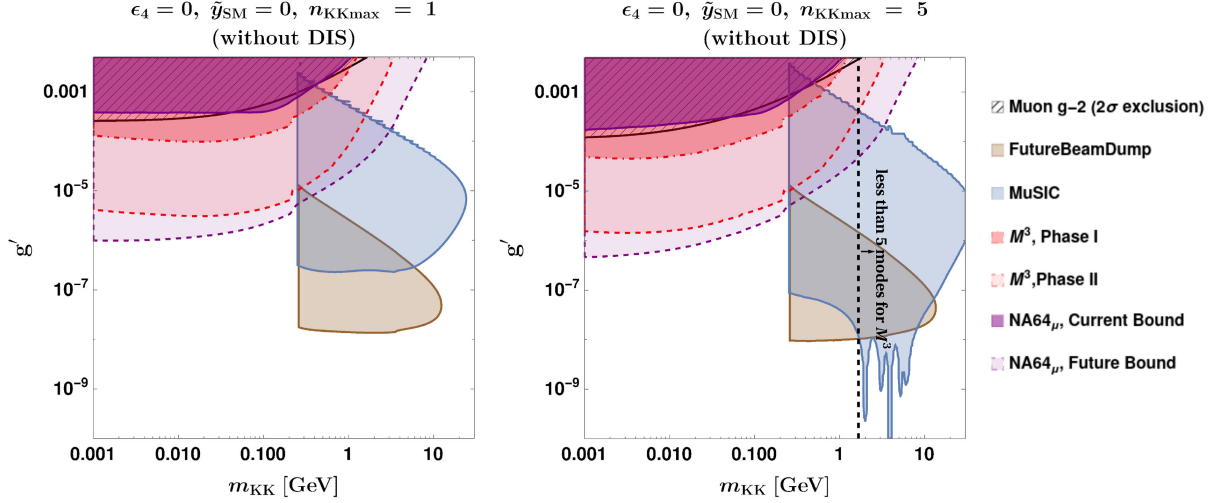


FIG. 4. Summary of current/future exclusion limits in the (m_{KK}, g') parameter space for the 5D $U(1)_{L_\mu-L_\tau}$ model under the benchmark choices $\epsilon_4 = 0$ and $\tilde{y}_{SM} = 0$. They illustrate the sensitivity of various experiments for two KK mode truncations: $n_{KKmax} = 1$ (left panel) and $n_{KKmax} = 5$ (right panel). For the meaning of the vertical black dotted line in the right panel, refer to footnote 15. For the definitions of each exclusion limit via the beam dump experiments, refer to Sec. IV.

the legitimate reason for this, refer to the final paragraph of this section.

In Fig. 4, The benchmarks are presented assuming $\epsilon_4 = 0$ and $\tilde{y}_{SM} = 0$.¹⁴ Note that the details of our signal estimations in the four experiments have been propounded in Sec. IV, based on the signal cross section formulas in Sec. III.¹⁵ Here, we provide some additional comments

¹⁴Complementary plots for $\epsilon_4 = 0$, but $\tilde{y}_{SM} \neq 0$ is available as Fig. 7 in Appendix A, where a weak dependence on $\tilde{y}_{SM}$ is observed.

¹⁵Regarding several minor handling points, we will comment on them below. (i) Since the beam energy in the M^3 experiment is 15 GeV, it is kinematically impossible to directly produce up to the fifth KK mode when the m_{KK} is greater than ~ 2 GeV, corresponding to the area on the right separated by the vertical dotted line in the right panel of Fig. 4. In this region, we consider the contribution of all kinematically possible KK vector bosons with n_{KK} less than 5; (ii) as introduced in Sec. II, in calculating the mixing of the fields, we performed an expansion using the epsilon parameter and retained only the lowest-order term. However, this is not necessarily justified when the mass of a given unperturbed KK particle approaches the mass of the Z boson sufficiently closely. If $\epsilon_4 \neq 0$, this failure is materialized in calculations arising from the factor $(\frac{\epsilon_n m_{Z,0}^2}{M_n^2 - m_{Z,0}^2})$, which manifests in mixing [see Eqs. (2.14)–(2.16)]. To avoid the catastrophe, in the calculations with $\epsilon_4 \neq 0$ of MuSIC and the future beam dump, we will impose the following *ad hoc* cutoffs for the KK modes, $M_i \lesssim 45$ GeV ($i = 1, 2, 3, 4, 5$). Via the mode-by-mode reaches for $\epsilon_4 = 0$ (up to the 5th mode) in Fig. 5, namely $m_{KK} \lesssim 10$ GeV in MuSIC and the future muon beam dump, the introduction of the above *ad hoc* cutoffs may change the results very little. To improve this situation, it is necessary to take into account higher-order perturbations relating to mixing; however, as this improvement is only meaningful when m_{KK} is very large, we shall not engage in any further quantitative discussion of this point in this paper.

from a kinematic perspective regarding the results of experiments observing muon pairs in decays. When observing the $Z'^{(n)} \rightarrow \mu^- \mu^-$ signal experimentally, it is common practice to place the detector at a point considerably distant from the collision point in order to suppress background events. In this case, a signal can only be detected if each $Z'^{(n)}$ particle decays inside a displaced detector, where the averaged lifetime of $Z'^{(n)}$ should be within a certain range. This is the cause of the “island structure” observed in the experimental results, as seen, e.g., in Fig. 4. To clarify further, in $Z'^{(n)} \rightarrow \mu^- \mu^+$, the reason there is no restriction in the region where $m_{KK} < 2m_\mu \sim 0.2$ GeV is that the decay process of interest ceases to occur on shell in this region.

An appreciable improvement in sensitivity is noted with the addition of KK modes. For example, increasing n_{KKmax} from one to five significantly enhances the exclusion reach for both NA64 μ and M^3 . Note that we have established that truncating the KK tower at $n_{KKmax} = 5$ provides a sufficiently precise and convergent approximation for our purpose, to use the accuracy of the log-log plot to ascertain the current global status and future projections of the parameter space. You can find some experimental circumstances in NA64 μ , M^3 in Figs. 8 and 9 in Appendix A. Furthermore, please refer to Appendix B regarding the accuracy of the adopted $n_{KKmax} = 5$.

We will briefly summarize the information obtained from the latest results on the muon $(g-2)$. As a sensational announcement, the long-standing anomaly between the SM prediction and the experimental results was resolved as $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 38(63) \times 10^{-11}$, which is consistent within $\sim 0.6\sigma$ [91]. Currently, this experimental value can be used to exclude specific areas of new physics coupled with

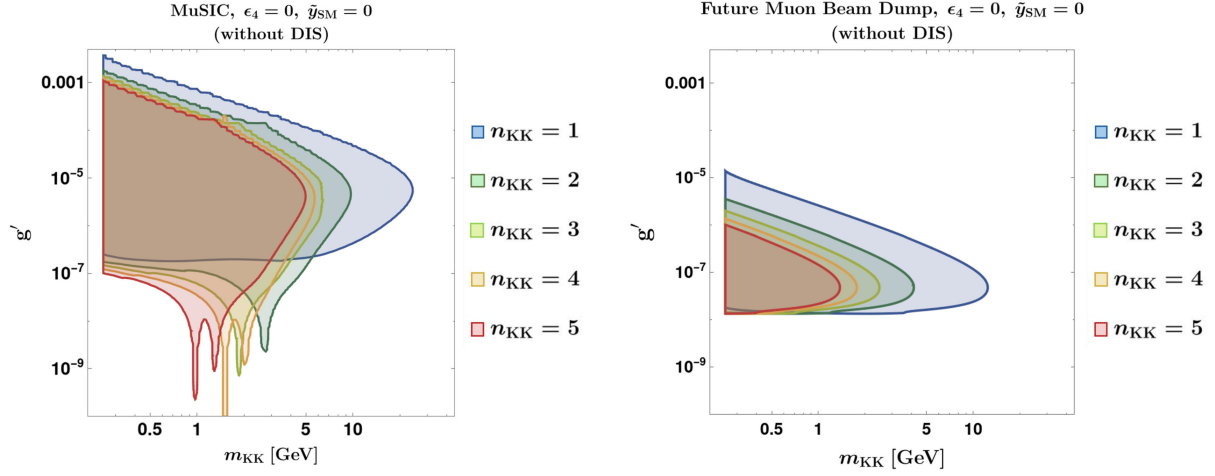


FIG. 5. Comparison of MuSIC (left) and future beam dump (right) sensitivities for individual n_{KK} modes for $\tilde{y}_{SM} = 0$ and $\epsilon_4 = 0$.

muons. As shown in Fig. 4, it is notable that the constraint on our scenario is comparable to the current experimental constraint from NA64 $_{\mu}$. Note that we used the formula for evaluating KK gauge boson contributions to the muon ($g - 2$) available in Sec. 2.4 of [92].

Here, we comment further on the constraints from neutrino-trident production [via the Chicago–Columbia–Fermilab–Rochester (CCFR) measurement [141] of the neutrino trident cross section] on our scenario. As shown in Fig. 2 of [142] (prior to its experimental exclusion last year), in a four-dimensional model containing a single corresponding Z' , a portion of the parameter space capable of explaining the muon ($g - 2$) anomaly evaded the CCFR experimental constraints; even within the excluded region, it was not particularly far removed from the CCFR constraint boundary. Furthermore, as the measurement uncertainties have been improved in the update of last year’s muon ($g - 2$) results, we reach the notion that the constraints derived from the CCFR experimental data via neutrino trident production are at least not significantly better than those derived from the latest muon ($g - 2$) data. We therefore decided not to discuss the current constraints on neutrino trident production in this paper, as they require substantial calculations and do not affect our main conclusions based on current experimental constraints.¹⁶

The most phenomenologically interesting prospect arises from the future observation of massive gauge boson decays to muon pairs at a displaced detector in upcoming experiments, MuSIC, and a forthcoming muon beam dump experiment. These effective search regions are determined

by the lifetimes of massive gauge bosons, where feebly interacting light massive particles near ~ 1 GeV are the primary targets for exploration.

With improved sensitivity, compared to the single-mode case, adding more KK modes to the MuSIC experiment creates a fascinating region in the parameter space. The enhancement can be attributed to the configuration of the mode functions f_n , which gain significance with the inclusion of additional KK modes [see Ref. [92] for further information]; also, for a given mass, a kinematic resonance arises due to ultrahigh incident energy as shown in Fig. 10 in Appendix B.¹⁷ Conversely, in the forthcoming muon beam dump experiment, the incorporation of elevated KK modes does not result in any significant enhancement in sensitivity. This difference can be explained by focusing on the future charted domains by individual KK modes. In Fig. 5, we figure out the “ $N_{\text{signal}} > 5$ ” regions for the first five KK modes separately.

¹⁶In the future, once experimental results have been updated—for example, from the DUNE experiment—it is expected that further constraints will be obtained from neutrino trident production. See [143–145] for details of prospects for the 4D scenario.

¹⁷To understand this, it is best to focus on Eq. (3.21), which is the form obtained after integrating over $\theta_{Z'^{(n)}}$ in the differential cross section expressed in terms of the kinematic parameters $\theta_{Z'^{(n)}}$ and x (although the analysis in the MuSIC experiment is performed using different kinematic parameters, η and γ ; see the second last paragraph of Sec. III A). Here, the factor $(M_n^2(1 - x) + m_\mu^2 x^2)$ appearing in the denominator becomes zero at a point where x is very slightly greater than unity, provided that $M_n \gg m_\mu$. The maximum possible value of x in terms of kinematics is given by Eq. (3.15), and this approaches unity as the beam’s incident energy increases. Note that the focused energy in the MuSIC experiment ($E_0 = 8.9$ TeV) is significantly higher than that in other experiments. Since the value of x_{max} is always less than unity, no divergence occurs; only kinematic enhancement takes place. Furthermore, when considering the final result—the number of signals—it is necessary to account for the x -integral and the effects of decay before reaching the displaced detector [as formulated in Eq. (4.11) for the MuSIC experiment].

In the experimental setup of the future muon beam dump experiment, as shown in the right panel of Fig. 5, the domain of the $(i+1)$ -mode is located just inside of that of the i -mode ($i = 1, 2, 3, 4$). Therefore, considering the general property that the cross section for generating the high KK mode is suppressed compared to that for the low KK mode, it is most reasonable that the restriction from the first mode alone provides a good approximation, as shown in Fig. 4.

In contrast, in the MuSIC experiment setup, the initial muon beam energy is extremely high ($= 8.9$ TeV), so the production cross sections for high KK modes do not decrease so rapidly. Furthermore, this high initial energy may enable clear identification of differences in the lifetimes of KK modes. These observations can be clearly discerned from the left panel of Fig. 5. Therefore, in the MuSIC experiment, a significant increase in the searchable region based on the existence of multiple KK modes is clearly observable.

Moreover, the fact that the muon pair signal allows the parent particle's mass to be reconstructed holds significant importance in our scenario. In the overlapping regions of the searchable areas for each mode in Fig. 5, the existence of multiple massive gauge bosons with different masses can be confirmed from the reconstruction of muon pair signals after accumulating the targeted amount of data. Therefore, muon pair signal surveys using displaced detectors provide a powerful tool for directly distinguishing whether the $U(1)_{L_\mu-L_\tau}$ interaction originates in five dimensions or in four dimensions.

Finally, we discuss future exploration limits when the typical size of kinetic mixing is nonzero. Figure 6 provides the future expected reaches for $\tilde{y}_{\text{SM}} = 0$ under the presence of a nonzero kinetic mixing parameter, with $\epsilon_4 = 10^{-4}$ (left column), $\epsilon_4 = 10^{-5}$ (center column), and $\epsilon_4 = 10^{-6}$ (right column). As can be seen immediately from the figure, there is an expansion of the searchable region, particularly where

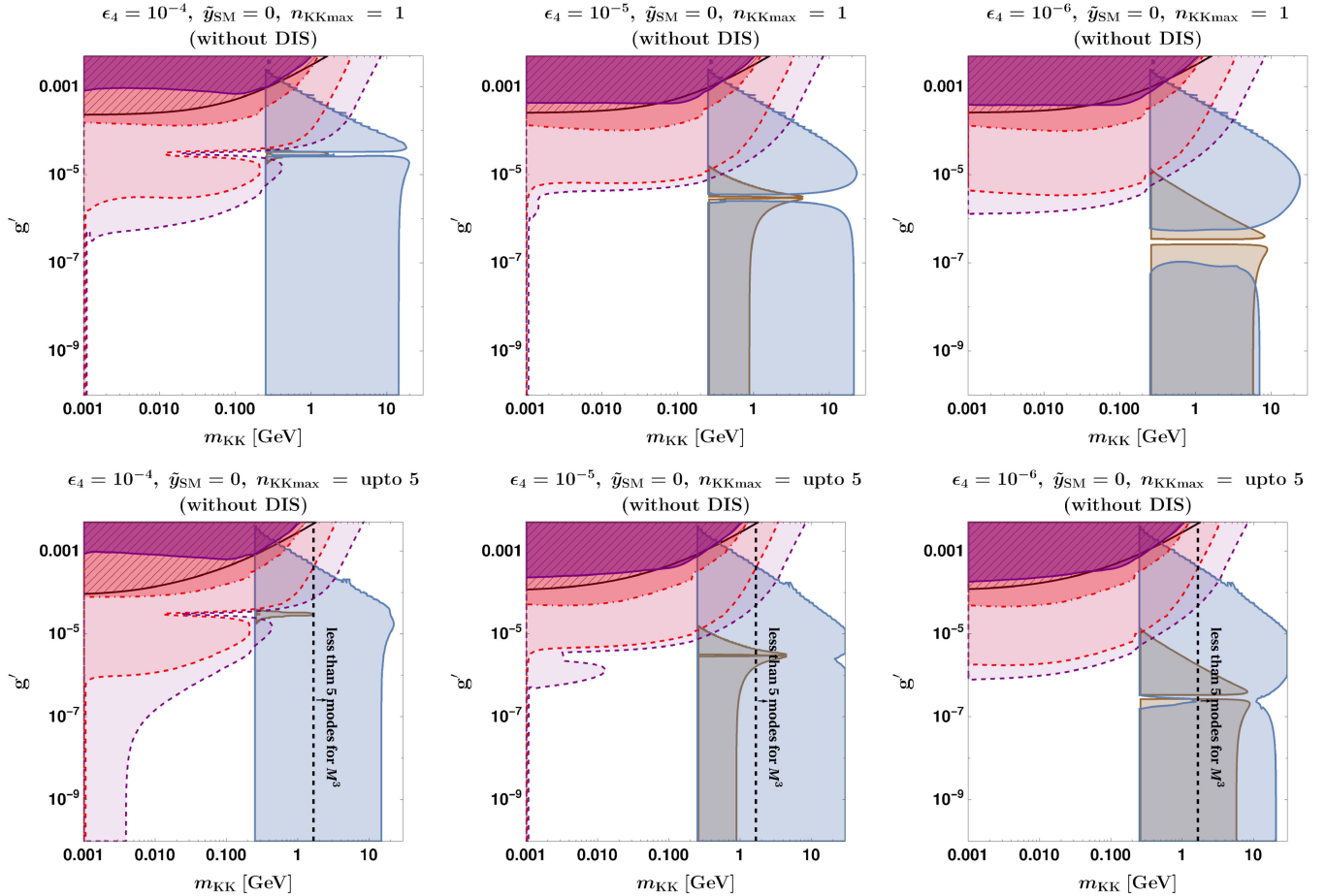


FIG. 6. Summary of current/future exclusion limits in the (m_{KK}, g') parameter space for the ED $U(1)_{L_\mu-L_\tau}$ model under the benchmark choices $\epsilon_4 = 10^{-4}, 10^{-5}$ and 10^{-6} (from left to right columns) under $\tilde{y}_{\text{SM}} = 0$. The first and second rows correspond to $n_{\text{KKmax}} = 1$ and $n_{\text{KKmax}} = \text{up to } 5$, respectively. Here, we follow the same color conventions as those for Fig. 4. The meaning of “up to 5” is as follows. We imposed the *ad hoc* cutoff $M_i \lesssim 45$ GeV ($i = 1, 2, 3, 4, 5$) to eliminate heavy modes that are phenomenologically unfavorable and exceed the current mixing approximation's validity range. Refer to footnote 15 for details.

g' is small. This can be easily understood because a new contribution is added to the effective coupling.¹⁸

It is noteworthy that, for each specific ϵ_4 coordination, a narrow region of g' that cannot be probed has been reported. This phenomenon, reported in a previous paper for $\text{E}\nu\text{ES}$ [92], is thought to be realized by the contributions from ϵ_4 and g' coincidentally canceling each other out. What has become clear here is that muon beam dump experiments also possess strong detection capabilities for even minute g' values at tiny but nonzero ϵ_4 .

Here, we will discuss the contribution of VVCS mentioned at the end of Sec. III A. As stated in paper [128], when all interactions are provided by kinetic mixing, the VVCS contributes an additional $\sim 10\%$, if the initial charged lepton energy is significantly greater than the mass of a massive gauge boson. When g' is significantly larger than ϵ_4 , we can conclude that this contribution is negligible. Conversely, when ϵ_4 is significantly larger than g' , this contribution cannot necessarily be ignored; however, when the incident lepton's energy is sufficiently high, this contribution amounts to about 10%. This parameter region, currently under investigation, $\epsilon_4 \gg g'$, is part of the focus of MuSIC and the future muon beam dump experiments. However, since the initial muon beam energies in these two experiments are extremely high, exceeding 1 TeV, the application of the aforementioned excess estimate is justified. Therefore, it can be concluded that the estimation of the searchable region in Fig. 6 is sufficiently accurate, under the outlook that a subleading extra contribution of approximately 10% in the signal cross section can be neglected.

Also, we provide a comment on the DIS-active domain. Based on the criterion in Eq. (3.13), in each experiment, the vector bosons in $m_{Z'} \gtrsim 18$ GeV (NA64 $_{\mu}$), $m_{Z'} \gtrsim 6$ GeV (M^3), $m_{Z'} \gtrsim 133$ GeV (MuSIC), $m_{Z'} \gtrsim 55$ GeV (future muon beam dump) reaches the DIS-active domain, and additional gains can enter into the picture. Up to the 5th KK mode, the above conditions can be rephrased in the sense of m_{KK} as $m_{\text{KK}} \gtrsim 2$ GeV (NA64 $_{\mu}$), $m_{\text{KK}} \gtrsim 0.7$ GeV (M^3), $m_{\text{KK}} \gtrsim 15$ GeV (MuSIC), and $m_{\text{KK}} \gtrsim 6$ GeV (Future Muon Beam Dump). In this particular domain, we believe we are underestimating the constraints and predictions in the experiments. However, as can be seen from Figs. 4 and 5 (cf. Fig. 6), in each experiment, this only applies near the heavier boundary of the corresponding exclusion limits. Therefore, within the scope discussed in this paper, it can be concluded that additional signals from deep-inelastic

scattering can appear significantly only at the very edge of the probeable region, affecting only a small fraction of the total area, and thus have little effect on the global shapes of the exclusion limits.

VI. CONCLUSIONS

In this paper, we have investigated how to survey the nature of the 5D $U(1)_{L_{\mu}-L_{\tau}}$ interactions in present and future muon dump experiments, namely, NA64 $_{\mu}$, M^3 , MuSIC, and a future muon dump experiment. These experiments fall into two categories: the first two can probe processes where feebly interacting massive particles go into invisible channels, while the latter two can probe processes where they go into muon pairs. These two types of experiments are complementary in that they allow exploration of different parameter regions of a model. In our scenario, the presence of multiple massive gauge bosons as KK particles leads to a tendency for more signals to appear compared to the corresponding four-dimensional one. In particular, the decay process into muon pairs enables mass reconstruction of the parent particle, making it possible to directly demonstrate the existence of multiple KK particles in at least some parameter regions. This provides clear evidence that the origin of the $U(1)_{L_{\mu}-L_{\tau}}$ interaction lies in five dimensions. Furthermore, the muon $(g-2)$ value, which is now consistent with the SM, can be used to exclude specific parameter regions for new particles interacting with muons. We have further discussed the nontrivial effects arising from kinetic mixing (which many other related studies have not addressed). As succinctly illustrated in Figure 6, kinetic mixing produces interesting effects, such as the extension of restrictions to regions with small g' values and the disappearance of certain restricted areas due to the cancellation of impacts between g' and ϵ_4 .

The motivation to explain the (once-observed) muon $(g-2)$ anomaly has since been lost, but as briefly mentioned in the introduction, the $U(1)_{L_{\mu}-L_{\tau}}$ interaction is still considered a meaningful interaction. For example, it is phenomenologically interesting to investigate the role of the mediator for thermal WIMP dark matter at the MeV and GeV mass scales in the current five-dimensional case. Furthermore, it is a significant issue to discuss the parameter regions that can explain the experimental results of the Hubble tension in the case of 5D $U(1)_{L_{\mu}-L_{\tau}}$, as well as those that can be ruled out due to strong inconsistencies. Also, future muon collider experiments currently in planning can also impose constraints on KK gauge bosons, which are light and interact very weakly with muons [146].

ACKNOWLEDGMENTS

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¹⁸We provide a comment on why the constraint via the future muon dump experiment becomes tiny for $\epsilon_4 = 10^{-4}$ in Fig. 6. Under the “sizable” value of ϵ_4 , each KK mode couples with SM particles quite strongly and tends to decay immediately. So, most of the KK vector bosons evanesce in the large decay volume before reaching the displaced detector. In the case of MuSIC, the size of the decay volume is relatively small, and still, the emitted KK particles are able to reach the detector region.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: SUPPLEMENTAL AND COMPLEMENTARY PLOTS

In this appendix, we provide some supplemental plots that provide relevant additional details and provide a better understanding of our analysis. Figure 7 provides a complementary analysis with those in Fig. 4, where the value of $\tilde{y}_{\text{SM}}$ is nonzero. Figures 8 and 9 show the convergence of our numerical analyses of NA64_μ and M^3 .

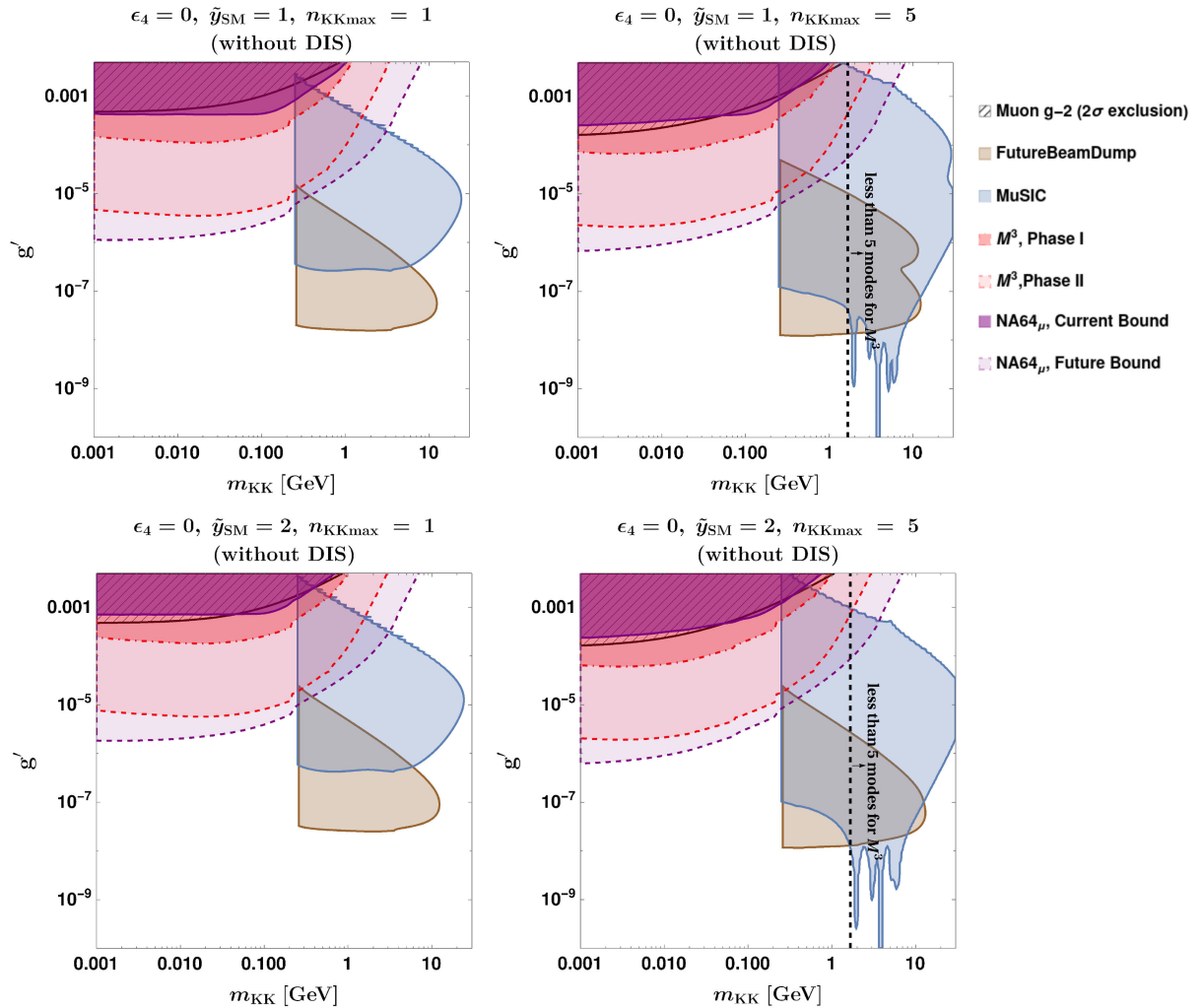
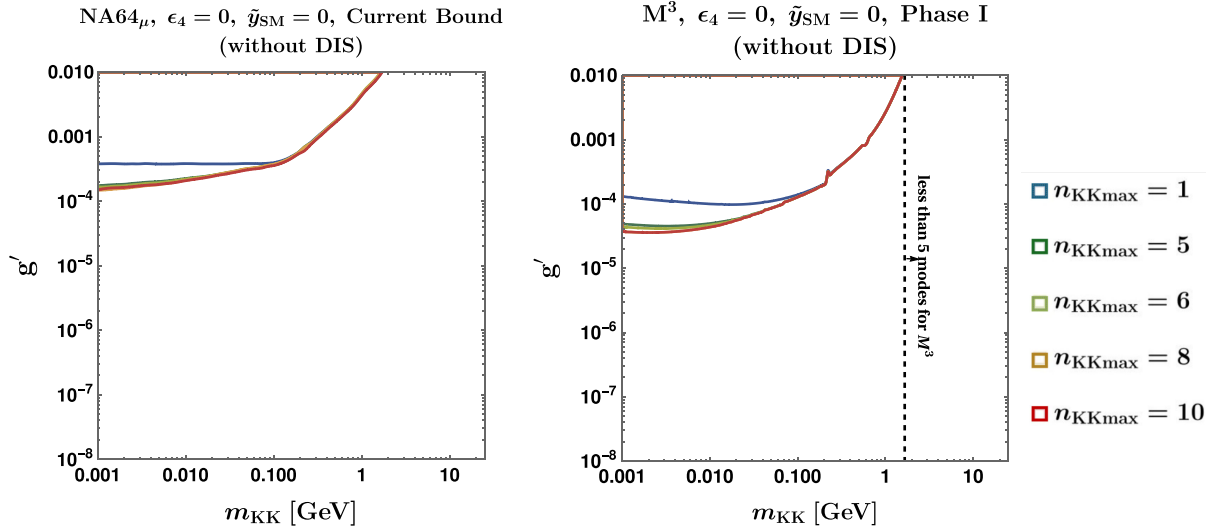
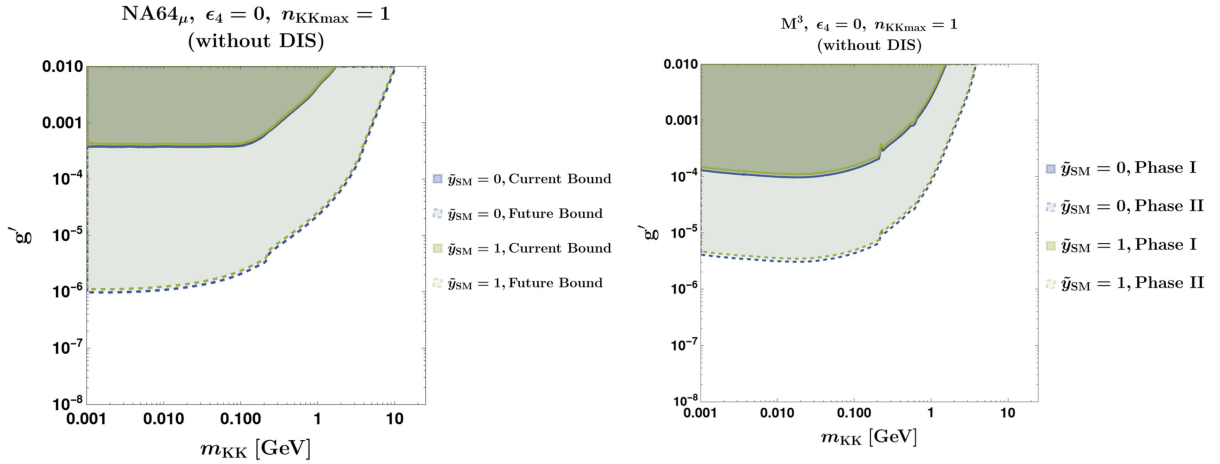


FIG. 7. Summary of current/future exclusion limits in the (m_{KK}, g') parameter space for the 5D $U(1)_{L_\mu-L_\tau}$ model under the benchmark choices $\epsilon_4 = 0$ and $\tilde{y}_{\text{SM}} = 1$ and 2. They illustrate the sensitivity of various experiments for two KK mode truncations: $n_{\text{KKmax}} = 1$ (left panels) and $n_{\text{KKmax}} = 5$ (right panels). Here, we follow the same color conventions as those for Fig. 4.


 FIG. 8. Convergence for NA64_μ and M³ experiments of n_{KKmax} modes.

 FIG. 9. Comparison of different $\tilde{y}_{\text{SM}}$ for NA64_μ (left) and M³ (right) for $n_{\text{KKmax}} = 1$.

APPENDIX B: ON OUR CHOICE OF $n_{\text{KKmax}} = 5$

For our 5D scenario, the expected number of signals in each experiment is estimated as the sum of the contributions of the n th KK mode, as shown in Eq. (4.3). As n increases, the mass of the corresponding KK particles becomes heavier, the production cross section decreases, and their contribution to the signal count diminishes. n_{KKmax} is introduced as a cutoff of the mode summation.

To investigate this issue caused by the variation in n_{KKmax} , we plot the distributions of $N_{\text{signal}}^{(n_{\text{KK}}),a}$ for the four experiments ($a = \text{NA64}_\mu, \text{M}^3, \text{MuSIC}, \text{FMBD}$), based on Eqs. (4.4), (4.6), (4.11), and (4.14), with the focused couplings and the individual target luminosities, respectively. For NA64_μ, M³ and FMBD, if m_{KK} is heavier than ~ 10 MeV, signals via higher modes immediately decouple, and the difference by ignoring more than $n_{\text{KK}} = 5$

modes will be around 10%, which seems to be therefore more than sufficient for log-log plots. Conversely, if m_{KK} is ~ 10 MeV or less, a difference of the order of $\mathcal{O}(10)\%$ or even higher may arise; however, this difference is not so large that it is barely discernible on log-log plots, and we consider $n_{\text{KKmax}} = 5$ to be sufficient for our purposes also for this region.

In the MuSIC experiment, the situation is broadly similar, but there is one significant difference. As shown in Fig. 10, for a given mass, a resonance signal arises due to ultrahigh incident energy. By ignoring modes above $n_{\text{KK}} = 5$, these resonance signals are also ignored; however, as can be seen from Fig. 4, the new contributions generated by these resonance signals tend to act to exclude regions where g' is very small. Conversely, unless one requires extremely high precision in the exclusion of regions where g' is particularly small, ignoring these

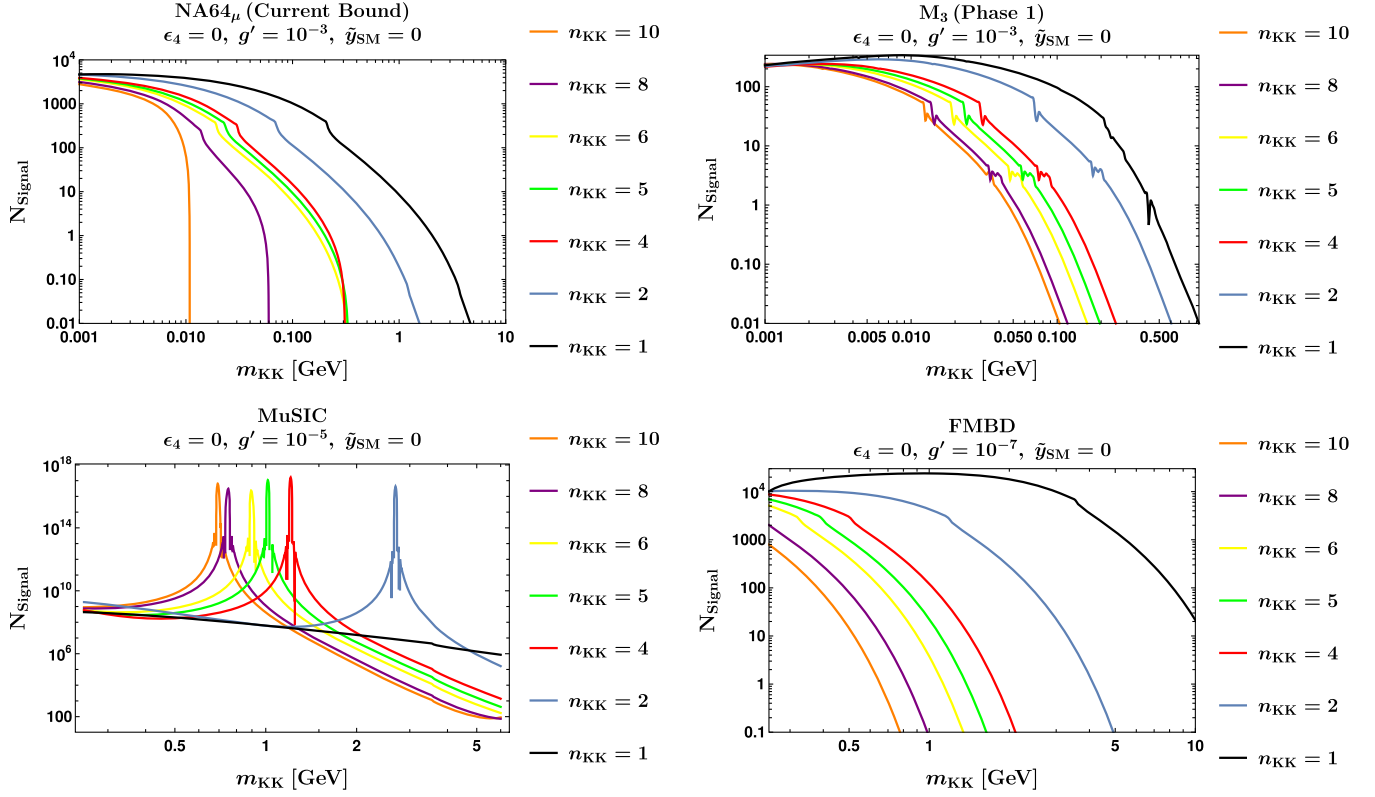


FIG. 10. Distributions of $N_{\text{signal}}^{(n_{\text{KK}}),a}$ for the four experiments ($a = \text{NA64}_\mu, \text{M}^3, \text{MuSIC}, \text{FMBD}$), based on Eqs. (4.4), (4.6), (4.11), and (4.14), with the focused couplings and the individual target luminosities, respectively.

resonance signals with high n_{KK} values does not affect the global structure of the exclusion region where g' is above a certain level.

From the above analysis, we can conclude that setting $n_{\text{KKmax}} = 5$ is necessary and sufficient for determining the regions to be excluded in each experiment (excluding the resonance region where $n_{\text{KK}} > 5$) from log-log plots.

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- [1] C. Antel *et al.*, Feebly-interacting particles: FIPs 2022 workshop report, *Eur. Phys. J. C* **83**, 1122 (2023).
 - [2] F. Bergsma *et al.* (CHARM Collaboration), A search for decays of heavy neutrinos in the mass range 0.5-GeV to 2.8-GeV, *Phys. Lett.* **166B**, 473 (1986).
 - [3] A. Konaka *et al.*, Search for neutral particles in electron beam dump experiment, *Phys. Rev. Lett.* **57**, 659 (1986).
 - [4] E. M. Riordan *et al.*, A search for short lived axions in an electron beam dump experiment, *Phys. Rev. Lett.* **59**, 755 (1987).
 - [5] J. D. Bjorken, S. Ecklund, W. R. Nelson, A. Abashian, C. Church, B. Lu, L. W. Mo, T. A. Nunamaker, and P. Rassmann, Search for neutral metastable penetrating particles produced in the SLAC beam dump, *Phys. Rev. D* **38**, 3375 (1988).
 - [6] M. Davier and H. Nguyen Ngoc, An unambiguous search for a light Higgs boson, *Phys. Lett. B* **229**, 150 (1989).
 - [7] A. Bross, M. Crisler, S. H. Pordes, J. Volk, S. Errede, and J. Wrbanek, A search for shortlived particles produced in an electron beam dump, *Phys. Rev. Lett.* **67**, 2942 (1991).
 - [8] C. Athanassopoulos *et al.* (LSND Collaboration), Evidence for $\nu_\mu \rightarrow \nu_e$ oscillations from pion decay in flight neutrinos, *Phys. Rev. C* **58**, 2489 (1998).
 - [9] P. Astier *et al.* (NOMAD Collaboration), Search for heavy neutrinos mixing with tau neutrinos, *Phys. Lett. B* **506**, 27 (2001).
 - [10] D. Banerjee *et al.* (NA64 Collaboration), Search for a hypothetical 16.7 MeV gauge boson and dark photons in the NA64 experiment at CERN, *Phys. Rev. Lett.* **120**, 231802 (2018).
 - [11] M. Anelli *et al.* (SHiP Collaboration), A facility to search for hidden particles (SHiP) at the CERN SPS, [arXiv: 1504.04956](https://arxiv.org/abs/1504.04956).

- [12] S. Alekhin *et al.*, A facility to search for hidden particles at the CERN SPS: The SHiP physics case, *Rep. Prog. Phys.* **79**, 124201 (2016).
- [13] A. Ariga *et al.* (FASER Collaboration), Technical proposal for FASER: Forward search experiment at the LHC, [arXiv:1812.09139](https://arxiv.org/abs/1812.09139).
- [14] A. Ariga *et al.* (FASER Collaboration), FASER's physics reach for long-lived particles, *Phys. Rev. D* **99**, 095011 (2019).
- [15] J. L. Feng *et al.*, The forward physics facility at the high-luminosity LHC, *J. Phys. G* **50**, 030501 (2023).
- [16] G. Aad *et al.* (ATLAS Collaboration), Search for long-lived neutral particles decaying into lepton jets in proton-proton collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, *J. High Energy Phys.* **11** (2014) 088.
- [17] G. Aad *et al.* (ATLAS Collaboration), A search for prompt lepton-jets in pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector, *J. High Energy Phys.* **02** (2016) 062.
- [18] ATLAS Collaboration, Search for light long-lived neutral particles that decay to collimated pairs of leptons or light hadrons in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, <https://cds.cern.ch/record/2799602>.
- [19] G. Aad *et al.* (ATLAS Collaboration), Search for light long-lived neutral particles that decay to collimated pairs of leptons or light hadrons in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, *J. High Energy Phys.* **06** (2023) 153.
- [20] G. Aad *et al.* (ATLAS Collaboration), Search for light long-lived neutral particles from Higgs boson decays via vector-boson-fusion production from pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, *Eur. Phys. J. C* **84**, 719 (2024).
- [21] G. Aad *et al.* (ATLAS Collaboration), Search for light neutral particles decaying promptly into collimated pairs of electrons or muons in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector, *Eur. Phys. J. C* **85**, 335 (2025).
- [22] R. Aaij *et al.* (LHCb Collaboration), Search for $A' \rightarrow \mu^+ \mu^-$ decays, *Phys. Rev. Lett.* **124**, 041801 (2020).
- [23] R. Aaij *et al.* (LHCb Collaboration), Searches for low-mass dimuon resonances, *J. High Energy Phys.* **10** (2020) 156.
- [24] S. Kanemura, T. Moroi, and T. Tanabe, Beam dump experiment at future electron-positron colliders, *Phys. Lett. B* **751**, 25 (2015).
- [25] L. Marsicano, M. Battaglieri, M. Bondi', C. D. R. Carvajal, A. Celentano, M. De Napoli, R. De Vita, E. Nardi, M. Raggi, and P. Valente, Dark photon production through positron annihilation in beam-dump experiments, *Phys. Rev. D* **98**, 015031 (2018).
- [26] E. Nardi, C. D. R. Carvajal, A. Ghoshal, D. Meloni, and M. Raggi, Resonant production of dark photons in positron beam dump experiments, *Phys. Rev. D* **97**, 095004 (2018).
- [27] J. M. Berryman, A. de Gouvea, P. J. Fox, B. J. Kayser, K. J. Kelly, and J. L. Raaf, Searches for decays of new particles in the DUNE multi-purpose near detector, *J. High Energy Phys.* **02** (2020) 174.
- [28] A. Berlin, P. deNiverville, A. Ritz, P. Schuster, and N. Toro, Sub-GeV dark matter production at fixed-target experiments, *Phys. Rev. D* **102**, 095011 (2020).
- [29] A. Celentano, L. Darmé, L. Marsicano, and E. Nardi, New production channels for light dark matter in hadronic showers, *Phys. Rev. D* **102**, 075026 (2020).
- [30] K. Asai, T. Moroi, and A. Niki, Leptophilic gauge bosons at ILC beam dump experiment, *Phys. Lett. B* **818**, 136374 (2021).
- [31] K. Asai, S. Iwamoto, Y. Sakaki, and D. Ueda, New physics searches at the ILC positron and electron beam dumps, *J. High Energy Phys.* **09** (2021) 183.
- [32] T. Moroi and A. Niki, Leptophilic gauge bosons at lepton beam dump experiments, *J. High Energy Phys.* **05** (2023) 016.
- [33] K. Asai, A. Das, J. Li, T. Nomura, and O. Seto, Chiral Z' in FASER, FASER2, DUNE, and ILC beam dump experiments, *Phys. Rev. D* **106**, 095033 (2022).
- [34] M. Battaglieri *et al.*, Dark matter search with the BDX-MINI experiment, *Phys. Rev. D* **106**, 072011 (2022).
- [35] K. Asai, S. Iwamoto, M. Perelstein, Y. Sakaki, and D. Ueda, Sub-GeV dark matter search at ILC beam dumps, *J. High Energy Phys.* **02** (2024) 129.
- [36] N. Blinov, P. J. Fox, K. J. Kelly, P. A. N. Machado, and R. Plestid, Dark fluxes from electromagnetic cascades, *J. High Energy Phys.* **07** (2024) 022.
- [37] D. Curtin, Y. Kahn, and R. Nguyen, Dark photons from charged pion bremsstrahlung at proton beam experiments, *Phys. Rev. D* **108**, 095039 (2023).
- [38] I. Schulthess and F. Meloni, New physics search with the optical dump concept at future colliders, [arXiv:2503.20996](https://arxiv.org/abs/2503.20996).
- [39] C.-Y. Chen, M. Pospelov, and Y.-M. Zhong, Muon beam experiments to probe the dark sector, *Phys. Rev. D* **95**, 115005 (2017).
- [40] Y. Kahn, G. Krnjaic, N. Tran, and A. Whitbeck, M^3 : A new muon missing momentum experiment to probe $(g-2)_\mu$ and dark matter at Fermilab, *J. High Energy Phys.* **09** (2018) 153.
- [41] S. N. Gninenko, D. V. Kirpichnikov, and N. V. Krasnikov, Probing millicharged particles with NA64 experiment at CERN, *Phys. Rev. D* **100**, 035003 (2019).
- [42] S. N. Gninenko, N. V. Krasnikov, and V. A. Matveev, Search for dark sector physics with NA64, *Phys. Part. Nucl.* **51**, 829 (2020).
- [43] D. Acosta and W. Li, A muon-ion collider at BNL: The future QCD frontier and path to a new energy frontier of $\mu^+ \mu^-$ colliders, *Nucl. Instrum. Methods Phys. Res., Sect. A* **1027**, 166334 (2022).
- [44] D. V. Kirpichnikov, H. Sieber, L. M. Bueno, P. Crivelli, and M. M. Kirsanov, Probing hidden sectors with a muon beam: Total and differential cross sections for vector boson production in muon bremsstrahlung, *Phys. Rev. D* **104**, 076012 (2021).
- [45] C. Cesarotti, S. Homiller, R. K. Mishra, and M. Reece, Probing new gauge forces with a high-energy muon beam dump, *Phys. Rev. Lett.* **130**, 071803 (2023).
- [46] D. Acosta, E. Barberis, N. Hurley, W. Li, O. Miguel Colin, Y. Wang, D. Wood, and X. Zuo, The potential of a TeV-scale muon-ion collider, *J. Instrum.* **18**, P09025 (2023).
- [47] A. S. Zhevlakov, D. V. Kirpichnikov, and V. E. Lyubovitskij, Implication of the dark axion portal for the EDM of fermions and dark matter probing with

- NA64e, NA64 μ , LDMX, M^3 , and BABAR, *Phys. Rev. D* **106**, 035018 (2022).
- [48] I. V. Voronchikhin and D. V. Kirpichnikov, Probing hidden spin-2 mediator of dark matter with NA64e, LDMX, NA64 μ , and M^3 , *Phys. Rev. D* **106**, 115041 (2022).
- [49] D. Forbes, C. Herwig, Y. Kahn, G. Krnjaic, C. Mantilla Suarez, N. Tran, and A. Whitbeck, New searches for muonphilic particles at proton beam dump spectrometers, *Phys. Rev. D* **107**, 116026 (2023).
- [50] B. Radics, L. Molina-Bueno, L. Fields, H. Sieber, and P. Crivelli, Sensitivity potential to a light flavor-changing scalar boson with DUNE and NA64 μ , *Eur. Phys. J. C* **83**, 775 (2023).
- [51] A. S. Zhevlakov, D. V. Kirpichnikov, and V. E. Lyubovitskij, Lepton flavor violating dark photon, *Phys. Rev. D* **109**, 015015 (2024).
- [52] C. Cesarotti and R. Gambhir, The new physics case for beam-dump experiments with accelerated muon beams, *J. High Energy Phys.* **05** (2024) 283.
- [53] B. Batell, H. Davoudiasl, R. Marcarelli, E. T. Neil, and S. Trojanowski, Lepton-flavor-violating ALP signals with TeV-scale muon beams, *Phys. Rev. D* **110**, 075039 (2024).
- [54] I. V. Voronchikhin and D. V. Kirpichnikov, The bremsstrahlung-like production of the massive spin-2 dark matter mediator, *Eur. Phys. J. C* **85**, 1110 (2025).
- [55] H. Li, Z. Liu, and N. Song, Probing axion and flavored new physics with the NA64 μ experiment, *Phys. Rev. D* **112**, 115013 (2025).
- [56] H. Sieber, D. Banerjee, P. Crivelli, E. Depero, S. N. Gninenko, D. V. Kirpichnikov, M. M. Kirsanov, V. Poliakov, and L. Molina Bueno, Prospects in the search for a new light Z' boson with the NA64 μ experiment at the CERN SPS, *Phys. Rev. D* **105**, 052006 (2022).
- [57] Y. M. Andreev *et al.* (NA64 Collaboration), First results in the search for dark sectors at NA64 with the CERN SPS high energy muon beam, *Phys. Rev. Lett.* **132**, 211803 (2024).
- [58] R. Foot, New physics from electric charge quantization?, *Mod. Phys. Lett. A* **06**, 527 (1991).
- [59] X. G. He, G. C. Joshi, H. Lew, and R. R. Volkas, NEW Z-prime phenomenology, *Phys. Rev. D* **43**, 22 (1991).
- [60] X.-G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Simplest Z-prime model, *Phys. Rev. D* **44**, 2118 (1991).
- [61] R. Foot, X. G. He, H. Lew, and R. R. Volkas, Model for a light Z-prime boson, *Phys. Rev. D* **50**, 4571 (1994).
- [62] S. Baek and P. Ko, Phenomenology of $U(1)_{L_\mu-L_\tau}$ charged dark matter at PAMELA and colliders, *J. Cosmol. Astropart. Phys.* **10** (2009) 011.
- [63] S. Baek, Dark matter and muon $(g-2)$ in local $U(1)_{L_\mu-L_\tau}$ -extended Ma model, *Phys. Lett. B* **756**, 1 (2016).
- [64] S. Patra, S. Rao, N. Sahoo, and N. Sahu, Gauged $U(1)_{L_\mu-L_\tau}$ model in light of muon $g-2$ anomaly, neutrino mass and dark matter phenomenology, *Nucl. Phys.* **B917**, 317 (2017).
- [65] A. Biswas, S. Choubey, and S. Khan, Neutrino mass, dark matter and anomalous magnetic moment of muon in a $U(1)_{L_\mu-L_\tau}$ model, *J. High Energy Phys.* **09** (2016) 147.
- [66] A. Biswas, S. Choubey, and S. Khan, FIMP and muon $(g-2)$ in a $U(1)_{L_\mu-L_\tau}$ model, *J. High Energy Phys.* **02** (2017) 123.
- [67] K. Asai, K. Hamaguchi, and N. Nagata, Predictions for the neutrino parameters in the minimal gauged $U(1)_{L_\mu-L_\tau}$ model, *Eur. Phys. J. C* **77**, 763 (2017).
- [68] G. Arcadi, T. Hugle, and F. S. Queiroz, The dark $L_\mu-L_\tau$ rises via kinetic mixing, *Phys. Lett. B* **784**, 151 (2018).
- [69] A. Kamada, K. Kaneta, K. Yanagi, and H.-B. Yu, Self-interacting dark matter and muon $g-2$ in a gauged $U(1)_{L_\mu-L_\tau}$ model, *J. High Energy Phys.* **06** (2018) 117.
- [70] P. Foldenauer, Light dark matter in a gauged $U(1)_{L_\mu-L_\tau}$ model, *Phys. Rev. D* **99**, 035007 (2019).
- [71] K. Asai, Predictions for the neutrino parameters in the minimal model extended by linear combination of $U(1)_{L_e-L_\mu}$, $U(1)_{L_\mu-L_\tau}$ and $U(1)_{B-L}$ gauge symmetries, *Eur. Phys. J. C* **80**, 76 (2020).
- [72] N. Okada and O. Seto, Inelastic extra $U(1)$ charged scalar dark matter, *Phys. Rev. D* **101**, 023522 (2020).
- [73] K. Asai, S. Okawa, and K. Tsumura, Search for $U(1)_{L_\mu-L_\tau}$ charged dark matter with neutrino telescope, *J. High Energy Phys.* **03** (2021) 047.
- [74] I. Holst, D. Hooper, and G. Krnjaic, Simplest and most predictive model of muon $g-2$ and thermal dark matter, *Phys. Rev. Lett.* **128**, 141802 (2022).
- [75] A. Tapadar, S. Ganguly, and S. Roy, Non-adiabatic evolution of dark sector in the presence of $U(1)_{L_\mu-L_\tau}$ gauge symmetry, *J. Cosmol. Astropart. Phys.* **05** (2022) 019.
- [76] J. Heeck and A. Thapa, Explaining lepton-flavor non-universality and self-interacting dark matter with $L_\mu-L_\tau$, *Eur. Phys. J. C* **82**, 480 (2022).
- [77] K. I. Nagao, T. Nomura, H. Okada, and T. Shimomura, Neutrinophilic dark-matter annihilation in a model with $U(1)_{L_\mu-L_\tau} \times U(1)_H$ gauge symmetry, *Phys. Rev. D* **108**, 055032 (2023).
- [78] S. K. A., A. Das, G. Lambiase, T. Nomura, and Y. Orikasa, Probing chiral and flavored Z' from cosmic bursts through neutrino interactions, *Eur. Phys. J. C* **84**, 1224 (2024).
- [79] P. Figueroa, G. Herrera, and F. Ochoa, Direct detection of light dark matter charged under a $L_\mu-L_\tau$ symmetry, *Phys. Rev. D* **110**, 095018 (2024).
- [80] K. Asai, K. Hamaguchi, N. Nagata, and S.-Y. Tseng, Leptogenesis in the minimal gauged $U(1)_{L_\mu-L_\tau}$ model and the sign of the cosmological baryon asymmetry, *J. Cosmol. Astropart. Phys.* **11** (2020) 013.
- [81] D. Borah, A. Dasgupta, and D. Mahanta, TeV scale resonant leptogenesis with $L_\mu-L_\tau$ gauge symmetry in light of the muon $g-2$, *Phys. Rev. D* **104**, 075006 (2021).
- [82] S. Eijima, M. Ibe, and K. Murai, Muon $g-2$ and non-thermal leptogenesis in $U(1)_{L_\mu-L_\tau}$ model, *J. High Energy Phys.* **05** (2023) 010.
- [83] A. Granelli, K. Hamaguchi, N. Nagata, M. E. Ramirez-Quezada, and J. Wada, Thermal leptogenesis in the minimal gauged $U(1)_{L_\mu-L_\tau}$ model, *J. High Energy Phys.* **09** (2023) 079.
- [84] J. Wada, Majoron-driven leptogenesis in gauged $U(1)_{L_\mu-L_\tau}$ model, *Phys. Rev. D* **110**, 103510 (2024).

- [85] K. Asai, K. Hamaguchi, N. Nagata, S.-Y. Tseng, and K. Tsumura, Minimal gauged $U(1)_{L_\mu-L_\tau}$ models driven into a corner, *Phys. Rev. D* **99**, 055029 (2019).
- [86] M. Ibe, S. Shirai, and K. Watanabe, Global neutrino constraints on the minimal $U(1)_{L_\mu-L_\tau}$ model, *Phys. Rev. D* **111**, 095034 (2025).
- [87] M. Escudero, D. Hooper, G. Krnjaic, and M. Pierre, Cosmology with a very light $L_\mu-L_\tau$ gauge boson, *J. High Energy Phys.* **03** (2019) 071.
- [88] T. Araki, K. Asai, K. Honda, R. Kasuya, J. Sato, T. Shimomura, and M. J. S. Yang, Resolving the Hubble tension in a $U(1)_{L_\mu-L_\tau}$ model with the Majoron, *Prog. Theor. Exp. Phys.* **2021**, 103B05 (2021).
- [89] J. A. Carpio, K. Murase, I. M. Shoemaker, and Z. Tabrizi, High-energy cosmic neutrinos as a probe of the vector mediator scenario in light of the muon $g-2$ anomaly and Hubble tension, *Phys. Rev. D* **107**, 103057 (2023).
- [90] K. Asai, T. Asano, J. Sato, and M. J. S. Yang, Contribution of Majoron to Hubble tension in gauged $U(1)_{L_\mu-L_\tau}$ model, *Prog. Theor. Exp. Phys.* **2024**, 073E01 (2024).
- [91] R. Aliberti *et al.*, The anomalous magnetic moment of the muon in the standard model: An update, *Phys. Rep.* **1143**, 1 (2025).
- [92] D. Chakraborty, A. Chatterjee, A. Kaushik, and K. Nishiwaki, Prospects of five-dimensional $L_\mu-L_\tau$ gauge interactions in the light of elastic neutrino-electron scatterings: The scope of the DUNE near detector, *Phys. Rev. D* **110**, 095030 (2024).
- [93] E. Ponton, TASI 2011: Four Lectures on TeV Scale Extra Dimensions, in *Theoretical Advanced Study Institute in Elementary Particle Physics: The Dark Secrets of the Terascale* (2013), pp. 283–374, [arXiv:1207.3827](#).
- [94] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, The hierarchy problem and new dimensions at a millimeter, *Phys. Lett. B* **429**, 263 (1998).
- [95] L. Randall and R. Sundrum, A large mass hierarchy from a small extra dimension, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [96] L. A. Anchordoqui, I. Antoniadis, and D. Lüst, Landscape, swampland, and extra dimensions, *Proc. Sci. CORFU2023* (2024) 215 [[arXiv:2405.04427](#)].
- [97] I. Antoniadis, A possible new dimension at a few TeV, *Phys. Lett. B* **246**, 377 (1990).
- [98] T. Appelquist, H.-C. Cheng, and B. A. Dobrescu, Bounds on universal extra dimensions, *Phys. Rev. D* **64**, 035002 (2001).
- [99] T. Kakuda, K. Nishiwaki, K.-y. Oda, and R. Watanabe, Universal extra dimensions after Higgs discovery, *Phys. Rev. D* **88**, 035007 (2013).
- [100] N. Deutschmann, T. Flacke, and J. S. Kim, Current LHC constraints on minimal universal extra dimensions, *Phys. Lett. B* **771**, 515 (2017).
- [101] M. M. Flores, J. S. Kim, K. Rolbiecki, and R. R. d. A. Bazan, Updated LHC bounds on MUED after run 2, *Int. J. Mod. Phys. A* **38**, 2350002 (2023).
- [102] R. N. Saldanha, Precision measurement of the ^7Be solar neutrino interaction rate in borexino, Ph.D. thesis, Princeton University, 2012.
- [103] H. T.-K. Wong, Taiwan experiment on neutrino—history and prospects, *Universe* **3**, 22 (2015).
- [104] P. Vilain *et al.* (CHARM-II Collaboration), Measurement of differential cross-sections for muon-neutrino electron scattering, *Phys. Lett. B* **302**, 351 (1993).
- [105] P. Vilain *et al.* (CHARM-II Collaboration), Precision measurement of electroweak parameters from the scattering of muon-neutrinos on electrons, *Phys. Lett. B* **335**, 246 (1994).
- [106] R. Acciarri *et al.* (DUNE Collaboration), Long-baseline neutrino facility (LBNF) and deep underground neutrino experiment (DUNE): Conceptual design report, volume 1: The LBNF and DUNE projects, [arXiv:1601.05471](#).
- [107] B. Abi *et al.* (DUNE Collaboration), Prospects for beyond the standard model physics searches at the deep underground neutrino experiment, *Eur. Phys. J. C* **81**, 322 (2021).
- [108] B. Holdom, Two $U(1)$'s and epsilon charge shifts, *Phys. Lett.* **166B**, 196 (1986).
- [109] Y. M. Andreev *et al.* (NA64 Collaboration), Shedding light on dark sectors with high-energy muons at the NA64 experiment at the CERN SPS, *Phys. Rev. D* **110**, 112015 (2024).
- [110] I. Antoniadis and F. Rondeau, Minimal embedding of the Standard Model into intersecting D-brane configurations with a bulk leptonic $U(1)$, *Eur. Phys. J. C* **82**, 701 (2022).
- [111] J. D. Bjorken, R. Essig, P. Schuster, and N. Toro, New fixed-target experiments to search for dark gauge forces, *Phys. Rev. D* **80**, 075018 (2009).
- [112] Y.-S. Tsai, Axion bremsstrahlung by an electron beam, *Phys. Rev. D* **34**, 1326 (1986).
- [113] C. F. von Weizsäcker, Radiation emitted in collisions of very fast electrons, *Z. Phys.* **88**, 612 (1934).
- [114] E. J. Williams, Correlation of certain collision problems with radiation theory, *Kong. Dan. Vid. Sel. Mat. Fys. Med.* **13N4**, 1 (1935).
- [115] K. J. Kim and Y.-S. Tsai, Improved Weizsäcker-Williams method and its application to lepton and W boson pair production, *Phys. Rev. D* **8**, 3109 (1973).
- [116] Y.-S. Tsai, Pair production and bremsstrahlung of charged leptons, *Rev. Mod. Phys.* **46**, 815 (1974); *Rev. Mod. Phys.* **49**, 421(E) (1977).
- [117] Y.-S. Liu, D. McKeen, and G. A. Miller, Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of a new scalar boson, *Phys. Rev. D* **95**, 036010 (2017).
- [118] Y.-S. Liu and G. A. Miller, Validity of the Weizsäcker-Williams approximation and the analysis of beam dump experiments: Production of an axion, a dark photon, or a new axial-vector boson, *Phys. Rev. D* **96**, 016004 (2017).
- [119] I. V. Voronchikhin and D. V. Kirpichnikov, Implication of the Weizsäcker-Williams approximation for the dark matter mediator production, *Phys. Rev. D* **111**, 035034 (2025).
- [120] D. Gorbunov and E. Kriukova, Dark photon production via elastic proton bremsstrahlung with non-zero momentum transfer, *J. High Energy Phys.* **01** (2024) 058.
- [121] J. Blümlein and J. Brunner, New exclusion limits on dark gauge forces from proton bremsstrahlung in beam-dump data, *Phys. Lett. B* **731**, 320 (2014).
- [122] S. Foroughi-Abari and A. Ritz, Dark sector production via proton bremsstrahlung, *Phys. Rev. D* **105**, 095045 (2022).

- [123] S. Foroughi-Abari, P. Reimitz, and A. Ritz, Closer look at dark vector splitting functions in proton bremsstrahlung, *Phys. Rev. D* **112**, 015030 (2025).
- [124] R. V. Harlander, S. Y. Klein, and M. Lipp, FeynGame, *Comput. Phys. Commun.* **256**, 107465 (2020).
- [125] R. Harlander, S. Y. Klein, and M. C. Schaaf, FeynGame-2.1—Feynman diagrams made easy, *Proc. Sci. EPS-HEP2023 (2024)* 657 [arXiv:2401.12778].
- [126] L. Bündgen, R. V. Harlander, S. Y. Klein, and M. C. Schaaf, FeynGame 3.0, *Comput. Phys. Commun.* **314**, 109662 (2025).
- [127] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [128] T. Beranek and M. Vanderhaeghen, Study of the discovery potential for hidden photon emission at future electron scattering fixed target experiments, *Phys. Rev. D* **89**, 055006 (2014).
- [129] R. Mertig, M. Bohm, and A. Denner, FEYN CALC: Computer algebraic calculation of Feynman amplitudes, *Comput. Phys. Commun.* **64**, 345 (1991).
- [130] V. Shtabovenko, R. Mertig, and F. Orellana, New developments in FeynCalc 9.0, *Comput. Phys. Commun.* **207**, 432 (2016).
- [131] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 9.3: New features and improvements, *Comput. Phys. Commun.* **256**, 107478 (2020).
- [132] V. Shtabovenko, R. Mertig, and F. Orellana, FeynCalc 10: Do multiloop integrals dream of computer codes?, *Comput. Phys. Commun.* **306**, 109357 (2025).
- [133] N. Doble, L. Gagnon, G. von Holtey, and F. Novoskoltsev, The upgraded muon beam at the SPS, *Nucl. Instrum. Methods Phys. Res., Sect. A* **343**, 351 (1994).
- [134] C.-Y. Chen, J. Kozaczuk, and Y.-M. Zhong, Exploring leptophilic dark matter with NA64- μ , *J. High Energy Phys.* **10** (2018) 154.
- [135] L. Lista, *Statistical Methods for Data Analysis in Particle Physics* (Springer, New York, 2016), Vol. 909.
- [136] T. Åkesson *et al.* (LDMX Collaboration), Light dark matter experiment (LDMX), arXiv:1808.05219.
- [137] J. Mans (LDMX Collaboration), The LDMX experiment, *EPJ Web Conf.* **142**, 01020 (2017).
- [138] J. W. Rohlf, *Modern Physics from A to Z* (John Wiley and Sons, New York, 1994).
- [139] H. Davoudiasl, H. Liu, R. Marcarelli, Y. Soreq, and S. Trifinopoulos, New physics at the Muon (Synchrotron) Ion Collider: MuSIC for several scales, *J. High Energy Phys.* **03** (2025) 046.
- [140] T. Bandyopadhyay, S. Chakraborty, and S. Trifinopoulos, Displaced searches for light vector bosons at Belle II, *J. High Energy Phys.* **05** (2022) 141.
- [141] S. R. Mishra *et al.* (CCFR Collaboration), Neutrino tridents and WZ interference, *Phys. Rev. Lett.* **66**, 3117 (1991).
- [142] W. Altmannshofer, S. Gori, M. Pospelov, and I. Yavin, Neutrino trident production: A powerful probe of new physics with neutrino beams, *Phys. Rev. Lett.* **113**, 091801 (2014).
- [143] W. Altmannshofer, S. Gori, J. Martín-Albo, A. Sousa, and M. Wallbank, Neutrino tridents at DUNE, *Phys. Rev. D* **100**, 115029 (2019).
- [144] P. Ballett, M. Hostert, S. Pascoli, Y. F. Perez-Gonzalez, Z. Tabrizi, and R. Zukanovich Funchal, Z's in neutrino scattering at DUNE, *Phys. Rev. D* **100**, 055012 (2019).
- [145] T. Shimomura and Y. Uesaka, Kinematical distributions of coherent neutrino trident production in gauged $L_\mu - L_\tau$ model, *Phys. Rev. D* **103**, 035022 (2021).
- [146] D. Chakraborty, A. Chatterjee, A. Datta, A. Kaushik, and K. Nishiwaki, Aspects of a five-dimensional $U(1)_{L-L}$ model at future muon-based colliders, arXiv:2604.05041.