Study of Yamaguchi-Type Potentials with Repulsion in the $^3S_1$ Eigenchannel*

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It is shown that separable potentials including repulsion and tensor components with Yamaguchi form factors are unable to give a reasonable fit simultaneously to $^3S_1$ phase shifts and the following deuteron data: the binding energy, the quadrupole moment, and the electric form factor. It is also found that a good phase-shift fit cannot be obtained from potentials of this type if the D-state probability of the deuteron desired is higher than 4%.

The appeal of analytical simplicity has led to considerable efforts being devoted in the last few years to construct separable models for the nucleon-nucleon interaction. For the $^3S_1$ eigenchannel, Yamaguchi[1] first constructed a separable potential of a particularly simple form. This potential fits the binding energy of the deuteron α, the quadrupole moment of the deuteron Q_d, and the low-energy phase shifts. However, it has no repulsion to make the phase shifts negative at high energies. Also, the corresponding deuteron wave function, which is of the Hulthen type, overestimates the electric form factor F_{el}. With the idea that addition of repulsion will improve the Yamaguchi potential, Mehrotra and Sen Gupta[2] (hereafter we refer to this work as I) constructed a separable potential (designated in this paper by V_1) which was claimed to fit the deuteron data [we use this term to collectively refer to Q_d, α, and the electric (F_{el}) and magnetic (F_{mag}) form factors of the deuteron], as well as the $^3S_1$ phase-shift data.

The study of nonlocal two-term separable potentials phase equivalent to a given local potential indicates that the former have off-shell elements that behave very differently from those of the latter.[3] On this basis it is difficult to understand the results of I. On the other hand, if these results are true, we obtain a very satisfactory deuteron wave function that is analytical in form. A careful recalculation by us has revealed considerable discrepancies in the results of I. In particular, the values of the deuteron parameters: Q_d, α, the D-state probability P_D, and the magnetic moment μ_d; the low-energy parameters: the triplet scattering length a_t and the effective range r_{0t}; and the deuteron form factors F_{el} and F_{mag} quoted in I are incorrect. The recalculated electric form factor F_{el} fits the experimental data only up to about $q^2 \approx 4$ fm^2. Using the same form as V_1 we have constructed an alternative potential V_2 which fits Q_d, α, and the $^3S_1$ phase shifts, but F_{el} is still reasonable only up to $q^2 \approx 4$ fm^2. In fact, on the basis of our investigation we believe that it is impossible to construct a potential of the form under study that gives a satisfactory agreement simultaneously with the deuteron and the phase shift data. In Table I we list the parameters of the potentials V_1 and V_2 and the values of α, Q_d, a_t, r_{0t}, P_D, and μ_d derived from them. The electric form factor $F_{el}(q^2)$ for various values of q^2 is given in Table II. For the potential V_1 we indicate results quoted in I, as well as those of our calculation. The discrepancies in the results of I are obvious. We also attempt to construct potentials of the same form with the D-state probability of the deuteron P_D higher than 4%. These potentials reproduce Q_d, α, a_t, and r_{0t}; however, it is impossible to obtain a good phase-shift fit with any of them. In fact, the best phase-shift fit is given by only those potentials which yield $P_D \approx 4\%$.

For completeness, we outline the construction of our potential. It is of the form

$$V(\vec{p}\,|\,\vec{p}\,') = -\frac{\lambda}{M}\,[g(\vec{p})g(\vec{p}\,') - h(\vec{p})h(\vec{p}\,')], \tag{1}$$

where

$$g(\vec{p}) = C(p) + T(p)\,\frac{S(\vec{p})}{\sqrt{8}}, \tag{2}$$

$$C(p) = 1/(\beta^2 + p^2), \tag{3}$$

$$T(p) = -tp^2/(\gamma^2 + p^2)^2, \tag{4}$$

$$h(\vec{p}) = np^2/(\bar{\beta}^2 + p^2)^2, \tag{5}$$

and the tensor operator

$$S(\vec{p}) = \frac{3\,(\vec{\sigma}_1\cdot\vec{p})(\vec{\sigma}_2\cdot\vec{p})}{p^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2\,. \tag{6}$$

Here β, γ, and $\bar{\beta}$ are the range parameters; and the parameters λ, t, and n characterize the strength, respectively, of the central attraction, tensor, and central repulsion components of the potential. Substituting the above in the Schrödinger equation we get the following deuteron wave func-

TABLE I. Parameters and low-energy properties of the nucleon-nucleon interaction in the $^3S_1$ eigenchannel. The notation is explained in the text. The first and the second column give the results for the potential V_1, respectively, as quoted in I and as recalculated here. The third column gives the results for the potential V_2 constructed here, while the experimental results are indicated in the fourth column. Experimental values of a_t and r_{0t} are taken from R. Wilson, Comments Nucl. Phys. 2, 141 (1968).

	V_1^q	V_1^c	V_2	Expt.
β/α	7.5		7.5	
$\bar{\beta}/\alpha$	10.5		9.75	
γ/α	6.5		6.5	
λ	61.5	64.43	62.1	
n^2	29.9		26.1	
t	0.7348		0.8974	
α (fm^{-1})	0.2316	0.2316	0.2316	0.2316
Q_d (fm^2)	0.280	0.231	0.282	0.282
a_t (fm)		6.22	5.425	5.425
r_{0t} (fm)		1.79	1.779	1.749
P_D (%)	4.0	2.5	3.76	
μ_d	0.855	0.862	0.858	0.857

TABLE II. Electric form factor $[F_{el}(q^2)]$ of the deuteron. The first column lists the values of q^2 at which $F_{el}(q^2)$ is calculated for the potentials V_1 and V_2. The rest of the notation is as in Table I. We do not calculate the relativistic correction to the form factor. The experimental results are from J. E. Elias, J. I. Friedman, G. C. Hartman, H. W. Kendall, P. N. Kirk, M. R. Sogard, L. P. Van Speybroeck, and J. K. de Pagter, Phys. Rev. 177, 2075 (1969).

q^2	V_1^q	V_1^c	V_2	Expt.
0.3	0.823	0.819	0.817	0.824 ± 0.006
0.6	0.684	0.690	0.686	0.680 ± 0.004
1.0	0.554	0.566	0.560	0.552 ± 0.008
2.0	0.330	0.348	0.341	0.325 ± 0.004
3.0	0.269	0.265	0.259	0.260 ± 0.007
4.0	0.200	0.197	0.191	0.180 ± 0.005
5.0	0.132	0.151	0.145	0.134 ± 0.004
6.0	0.100	0.118	0.114	0.0933 ± 0.005
7.0	0.070	0.094	0.090	0.0745 ± 0.002
8.0	0.061	0.076	0.073	0.0594 ± 0.001
12.0	0.028	0.037	0.036	0.029 ± 0.002

tion in the coordinate space:

$$r\psi(\vec{r},\ \text{spin}) = \left[u(r) + \frac{S(\vec{r})}{\sqrt{8}}\, w(r)\right] X_1^m\,, \tag{7}$$

where

$$u(r) = \frac{\sqrt{2}\,\pi N_1}{\beta^2-\alpha^2}\,(e^{-\alpha r} - e^{-\beta r}) - \frac{\sqrt{2}\,\pi N_2 \alpha^2}{(\bar{\beta}^2-\alpha^2)^2}\left(e^{-\bar{\beta} r} - e^{-\alpha r} + \frac{\bar{\beta}^2-\alpha^2}{\alpha^2}\,\frac{\bar{\beta}}{2}\, r e^{-\bar{\beta} r}\right) \tag{8}$$

and

$$w(r) = \frac{\sqrt{2}\,\pi N_1 t}{(\gamma^2-\alpha^2)^2}\left[\left(\frac{3}{r^2} + \frac{3\alpha}{r} + \alpha^2\right)e^{-\alpha r} - \left(\frac{3}{r^2} + \frac{3\gamma}{r} + \gamma^2\right)e^{-\gamma r} + \frac{\alpha^2-\gamma^2}{2}\, e^{-\gamma r}(1+\gamma r)\right]. \tag{9}$$

Here N_1 and N_2 are normalization constants fixed by

$$\int [u^2(r) + w^2(r)]\, dr = 1 \tag{10}$$

and X_1^m is the triplet spin wave function.

With these wave functions, it is possible to express the quantities Q_d, α, a_t in terms of the range $(\beta, \bar{\beta}, \gamma)$ and the strength (λ, n, t) parameters. Using the experimental values of Q_d, α, a_t we obtain three simultaneous nonlinear equations which determine the strength parameters for a given set of range parameters. A variation of the range parameters leads to a good fit to phase-shift data. We observe that for a given set of range parameters the strength of the repulsion cannot be arbitrarily increased. Good agreement with phase-shift data can be obtained for certain sets, but the repulsion is always inadequate to yield a good fit to deuteron form factor F_{el}. We can construct potentials with varying P_D. However, the higher the P_D, the higher is the value of the parameter γ, and above $P_D \approx 4\%$ this vitiates the phase-shift fit. Here we find $\bar{\beta} \approx \gamma$, contrary to the idea that the "range" of the tensor force should be much larger than that of the repulsion. The chosen form of the repulsion $h(\vec{p})$ introduces a repulsive term of the type $e^{-\bar{\beta} r}$ in the wave function. It seems that this term does not sufficiently cut down the wave function at small nucleon-nucleon separation distances, and one needs additional terms with higher powers of $e^{-\bar{\beta} r}$. This will clearly destroy the simplicity of the separable potential. In summary, we note that, contrary to what is claimed in I, it is not possible to get a good fit simultaneously to the deuteron and $^3S_1$ phase-shift data with the Yamaguchi potentials that include repulsion. Moreover, we find that the potentials of this type which fit the phase-shift data cannot have P_D higher than 4%. These may very well be the features common with the separable potentials studied so far in the literature.

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Photoneutron Production in Thick Targets*

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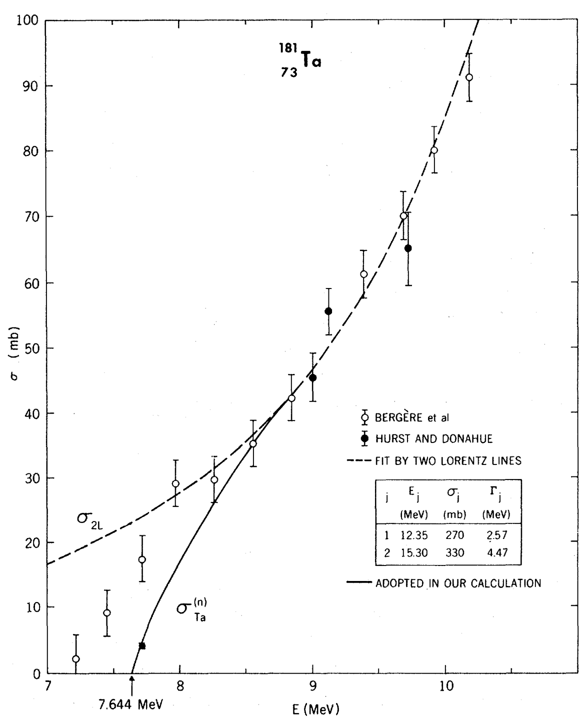
This paper reports new results on the yield of photoneutrons from thick targets bombarded with electron beams. Yields have been calculated for incident electron energies from 20 MeV down to the photonuclear cross section threshold, for tantalum and tungsten targets with thicknesses up to 12.5 g cm^{-2}. The increased yield from composite tungsten-beryllium targets has been explored.

This note is a sequel to an earlier paper[1] on bremsstrahlung and photoneutron production in thick tungsten targets irradiated by electrons with energies between 12 and 60 MeV. We have now extended our calculations – for tungsten as well as tantalum targets – to lower energies, down to the threshold for the photonuclear cross section. The energy region from 12 to 20 MeV has been covered in more detail than before. Finally, sample calculations have been made of the increase in photoneutron yield at 10 MeV obtainable with a composite tungsten-beryllium target.

As before, our calculations were made for beams of monoenergetic electrons incident perpendicularly on plane-parallel targets finite in one dimension and unbounded in the other two. A Monte Carlo method was used to simulate the resulting electron-photon cascade in the target. The thickest target treated (12.5 g cm^{-2}) was divided into 50 equal layers, and the bremsstrahlung tracklength distribution (differential in photon energy) was obtained by summing over the appropriate number of sublayers. Finally, the photoneutron yield was calculated by forming the product of the differential tracklength, the photonuclear cross section and the number of atoms per unit volume, and integrating the product over all photon energies down to threshold.

For electron bombarding energies not far above threshold, the photoneutron yield depends rather sensitively on the assumed energy dependence of the photonuclear (γ, n) cross section. The photonuclear threshold energy for $^{181}_{73}$Ta was determined by Geller, Halpern, and Muirhead[2] to be 7.640 $\pm$0.025 MeV. More recently, the atomic-mass evaluation of Wapstra and Gove[3] led to nearly the same value, 7.644$\pm$0.022 MeV. Using monoener-



j	E_j (MeV)	σ_j (mb)	Γ_j (MeV)
1	12.35	270	2.57
2	15.30	330	4.47

FIG. 1. Photonuclear cross section in tantalum near threshold.