ACKNOWLEDGMENTS

The technical assistance of J. Bicek and the cooperation of the cyclotron group are gratefully acknowledged. The helpful discussions with D. Gloeckner and his communication of calculations prior to publication are appreciated.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]L. R. Medsker and D. J. Horen, Nucl. Data B8 No. 1 (1972).
[2]H. Ohnuma and J. L. Yntema, Phys. Rev. 176, 1416 (1968).
[3]M. R. Cates, J. B. Ball, and E. Newman, Phys. Rev. 187, 1682 (1969).
[4]H. Ohnuma and J. L. Yntema, Phys. Rev. 179, 1211 (1969).
[5]J. R. Erskine and R. H. Vonderohe, Nucl. Instr. Methods 81, 221 (1970).
[6]J. R. Comfort, Argonne National Laboratory Physics Division Informal Report No. PHY-1970B (unpublished).
[7]We are grateful to Dr. P. D. Kunz for making this program available to us.
[8]D. Gloeckner, private communication.
[9]K. H. Bhatt and J. B. Ball, Nucl. Phys. 63, 286 (1965).
[10]J. Vervier, Nucl. Phys. 75, 17 (1965).

PHYSICAL REVIEW C VOLUME 7, NUMBER 1 JANUARY 1973

Study of ^{93}Nb Levels with the ^{96}Mo(p, α)^{93}Nb Reaction

Yong Sook Park, H. D. Jones, and D. E. Bainum
U.S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland 21005
(Received 7 September 1972)

Levels of ^{93}Nb have been studied with the ^{96}Mo(p,α) reaction at 15-MeV incident energy. Angular distributions for transitions to a dozen prominent levels below 2 MeV have been measured with 30- to 40-keV experimental resolution over $15° \leq \theta_L \leq 150°$, and they have been compared with distorted-wave Born-approximation (DWBA) calculations. Angular distributions for p-^{96}Mo and α-^{93}Nb elastic scattering have also been measured, and proton and α-optical-model potential parameter sets have been determined by a search procedure. Of the three sets of α-potential parameters, the one which best reproduced the measured (p,α) angular distributions of known low-lying levels in ^{93}Nb was chosen as the standard set for the rest of the (p,α) calculations. The conventional triton-cluster form factor was used exclusively for the (p,α) DWBA calculations which lead to J^π assignments. A semimicroscopic form factor was used in a few test cases in order to check the sensitivity of calculations to the choice of form factor. Reliable J^π assignments have been made for levels below 1.5 MeV based on the j dependence for $l=1$(p,α) transfers, as well as for higher angular momentum transfers observed in the present work; and they are in excellent agreement with those determined from recent Coulomb-excitation and ^{90}Zr(α,$p\gamma$) works. Dominant three-nucleon shell-model configurations to which the levels of interest belong have been proposed.

I. INTRODUCTION

Low-lying levels in ^{93}Nb have been studied extensively in recent years by a number of authors with ^{93}Nb(n, $n'\gamma$) inelastic scattering.[1,2] Owing to the complex scheme inherent to γ-decay modes, however, only a few consistent J^π assignments are found among these works. Recently a major discrepancy in spin assignments for low-lying positive-parity states has been resolved by Stelson *et al.*[3] via Coulomb excitation, the results of which are well described within the framework of a weak-coupling model in which ^{92}Zr is treated as a core. The more recent Coulomb-excitation work by Kreger and Seaman,[4] however, proposed tentative spin assignments for levels at 809, 950, and 979 keV which differ from those made by Stelson *et al.*[3]

The level scheme of ^{93}Nb has been studied extensively in the past by several theoretical groups[5] on the basis of a shell model in which ^{88}Sr is treated as the core. On the other hand, experimental spectroscopic studies of the levels via direct reactions have been limited to a few proton

single-particle transfer reactions. Ohnuma and Yntema[6] investigated levels of ^{93}Nb via the ^{94}Mo$(d, ^3\mathrm{He})$ reaction, but due primarily to poor experimental resolution, unique J^π assignments were limited to a few low-lying levels. The first comprehensive spectroscopic study of ^{93}Nb was recently made by Cates, Ball, and Newman[7] via the ^{92}Zr$(^3\mathrm{He}, d)$ reaction. The results from the study were, in general, well described by a simple shell-model picture. More comprehensive shell-model calculations[5] also predict high angular momentun states at energies as low as 1.5 MeV excitation in ^{93}Nb. Population of high angular momentum states is not favored in $(^3\mathrm{He}, d)$ or $(d, ^3\mathrm{He})$ primarily due to the momentum-matching effect.[8] A level with a spin as high as $\frac{17}{2}$ has been identified at this energy in the $(n, n'\gamma)$ and $(p, p'\gamma)$ work by Rogers *et al.*[1] Recently Zisman and Harvey[9] employed (α, t) and (α, d) reactions to populate high-spin states in niobium isotopes. From this work a level at 1.33 MeV was assigned to $\frac{17}{2}^+$ and a level at 1.48 MeV tentatively to $\frac{15}{2}^+$. The extent to which the (α, d) reaction can be used to study the spectroscopy of ^{93}Nb is limited of course because of the nonzero target spin.

In the present work, levels in ^{93}Nb have been studied with the ^{96}Mo(p, α) reaction. Direct (p, α) reactions have been known as one of the most reliable direct reactions with which unique J^π values of final states can be determined when the reactions are studied with 0^+ targets. This is so primarily because of the strong j dependence observed in measured (p, α) angular distributions. Also, the (p, α) j dependence has been known to be reproduced most consistently by distorted-wave Born-approximation (DWBA) calculations when the spin-orbit coupling term is included for distorted-wave channels.[10] To date, a strong j dependence has been recognized only for $l=1(p, \alpha)$ and (α, p) transfers.[11-15] Recognition of the j dependence for $l=2$ and 3 transfers has not been firmly established. There have been, to date, few reports on the j dependence associated with higher angular momentum transfers. The (p, α) reactions on medium-mass nuclei should provide one of the best opportunities for checking the j dependence for high-l transfers, because the reaction is expected to favor population of levels with high angular momentum transfers largely due to the angular-momentum-matching effect mentioned above.

It is known that the ambiguities in α-potential parameter sets have drastic effects on both the magnitudes and shapes of the predicted angular distributions for direct (d, α) reactions.[16] To date, no detailed studies of the α-parameter sensitivity have been reported for DWBA analyses of (p, α) reactions.

The choice of a realistic form factor appears to be very important for a direct (p, α) DWBA calculation, mainly because the cross section is proportional to a coherent sum of the transition amplitudes over all the three-nucleon shell-model configurations present in the nuclear wave functions.[17] In a simple situation in which all three nucleons are picked up from the same shell orbit, the effect of the coherence in the total angular momenta of the neutron pair was treated in detail by Bayman and Rost.[18] Nolen[19] successfully tested this model for some $f_{7/2}$-shell nuclei on the assumption that the two neutrons are coupled to spin 0.

In order to deal with the situation in which all three nucleons are picked up from different shell orbits, another approach for obtaining form factors was introduced by Drisko.[20] A simple product of three single-particle harmonic-oscillator wave functions is taken, and then it is properly matched to a tail characteristic of the separation energy of the triton-cluster in the target. Based on this technique, Fulmer and Ball[21] were able to reproduce the measured angular distributions for the ^{90}Zr and ^{89}Y(p, α) transitions to a few low-lying states.

On the other hand, a number of authors[13,15] have successfully demonstrated that a simple phenomenological triton-cluster form factor is adequate for extraction of l transfers or even j transfers for some cases. The form factor is computed by binding a triton with proper cluster quantum numbers at the experimental separation energy in a Woods-Saxon well; thus knowledge of the nuclear wave functions is not needed. The primary drawback of this approach is, of course, that the use of this kind of form factor does not permit quantitative analyses of data such as extraction of spectroscopic strengths.

II. EXPERIMENTAL PROCEDURES AND RESULTS

A. ^{96}Mo$(p, \alpha)^{93}$Nb Reaction

The (p, α) angular distributions were measured with 15-MeV protons from the Ballistic Research Laboratories (BRL) tandem accelerator over the angular range of $15° \leq \theta_L \leq 150°$. Detailed descriptions of the scattering facilities and the detection method used in this experiment were presented previously.[16,22] α particles were detected with an array of four 300-μm-thick surface-barrier detectors which were cooled to -30°C. Variations in the target thickness, target angles, and beam-charge collection were monitored by measuring elastic scattering at a fixed angle during the entire run. The target was 97%-enriched ^{96}Mo isotope evaporated onto a 20-μg/cm^2-thick carbon

backing, and its thickness was about 150 μg/cm^2.

An α-particle energy spectrum measured at $\theta_L = 60°$ is shown in Fig. 1. The over-all energy resolution of 30 keV for 20-MeV α particles limited the present investigation to levels below 2 MeV excitation in ^{93}Nb. Even so, nearly 50% of the observed levels appear to be multiplets unresolved in the present work when compared with those observed in Ref. 2. The levels are labeled with the excitation energies assigned in the present study based on energy calibrations obtained from the well-known ^{27}Al$(p, \alpha)^{24}$Mg reaction. The energies are accurate to within ±5 keV for levels below 1 MeV and ±10 keV for levels above. Arrows below 1.4 MeV indicate the expected locations of levels identified in Ref. 2.

Relative cross sections were obtained with monitor normalization, and the absolute cross-section scales were then fixed by the best over-all fit to the "charge-normalized" absolute cross sections in the manner described in Ref. 16. The measured angular distributions for transitions to prominent levels populated in the present (p, α) reaction are shown in Figs. 6 and 7, in which they are identified by the excitation energies and the (l, j) transfers (for the triton cluster) extracted from DWBA analysis of the data.

Most of the measured angular distributions exhibit forward peaking and a strong diffractionlike pattern; and they are well predicted by DWBA calculations. Thus it is most likely that the stronger (p, α) transitions to the levels of interest in the present study proceed predominantly via a direct-reaction mechanism.

B. ^{96}Mo(p, p) and Nb(α, α) Elastic Scattering

The ^{96}Mo(p, p) angular distribution was measured over the angular range of $20° \leq \theta_L \leq 140°$ at 15 MeV, the same incident proton energy for the ^{96}Mo(p, α)-^{93}Nb run. The α-^{93}Nb elastic scattering angular distribution was measured at 22 MeV rather than at 18 MeV, the maximum α energy for (p, α). This preference of a higher energy was based on the attempt to reduce the ambiguity in selecting unique optical-potential parameter sets.

The detection setup for the elastic scattering runs was basically the same as that for the (p, α) experiment. The ^{96}Mo target exposed in the (p, α) experiment and 2-mm surface-barrier detectors were used for the proton scattering. The Nb target was made of natural niobium evaporated onto a 20-μg/cm^2-thick carbon backing; its thickness was about 110 μg/cm^2.

The cross section for each angle was measured at least twice with different detectors to check reproducibility, and normally the overlapping points agreed within 5%. Weighted averages of the cross sections were adopted as the final values. The absolute cross-section scales were fixed on the basis of both the monitor and charge normalization in the same manner as in the (p, α)

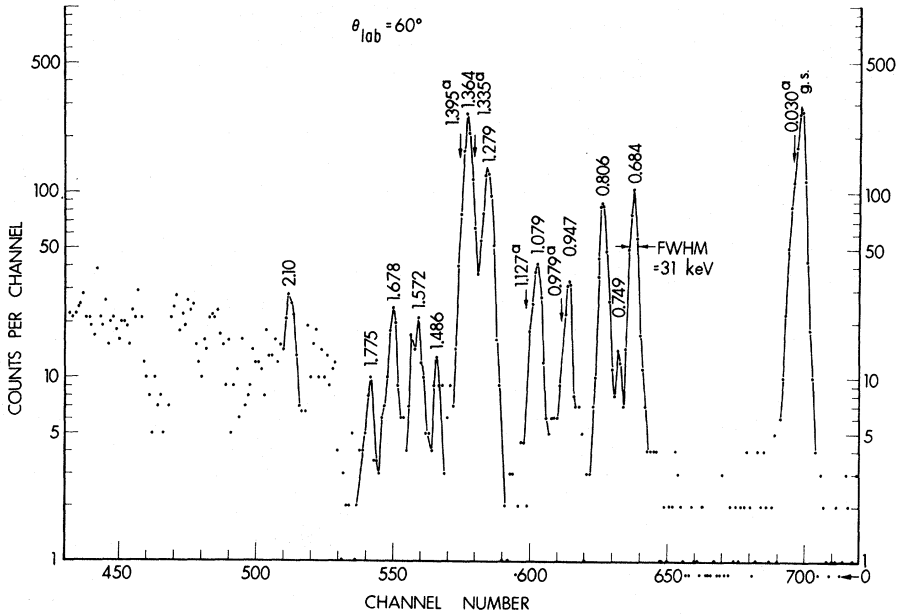

FIG. 1. An α spectrum from the ^{96}Mo$(p, \alpha)^{93}$Nb reaction at 15 MeV. Levels are labeled with their measured excitation energies. Excitation energies marked with the suffix a are those determined in Ref. 2.

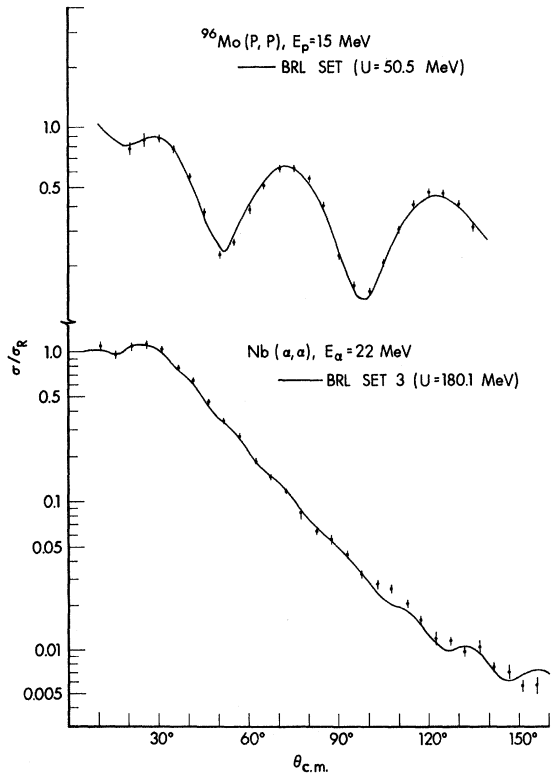

FIG. 2. The ratios of the measured p-^{96}Mo and α-Nb elastic scattering cross sections to the Rutherford cross sections. The curves are the optical-model fits to the data and are based on the parameter sets determined in this work (listed in Table I).

case. The measured p and α elastic scattering angular distributions are shown in Fig. 2, in which they are compared to optical-model predictions.

C. Cross-Section Errors

Random errors are represented by the sizes of the error bars drawn through the data points on all of the measured angular distributions. They are primarily due to counting statistics, background subtractions, and uncertainties in the solid angles of the movable detectors; for the (p, α) data the error bars also include uncertainties introduced in fitting procedures for the peak integration of imperfectly resolved adjacent levels. The dominant contributions to the absolute cross-section scale errors originate from uncertainties in the target thickness for both the (p, α) and elastic scattering measurements. The thicknesses of the ^{96}Mo and Nb targets quoted earlier were determined by measuring Coulomb scattering at small angles. They are believed to be accurate to within 10%. The over-all absolute cross-section scale errors are estimated to be 13%.

D. p- and α-Optical-Model-Potential Parameter Search

The need for entering the proper optical-potential parameter set into DWBA calculations for reactions involving composite particles has previously been emphasized.[16] Those p and α sets which are pertinent to the present reaction had not been reported in the literature; thus, attempts were made to obtain them from optical-model-potential parameter searches on the measured elastic angular distributions described earlier.

Parameters found in recent literature for nuclei and energies which are as near as possible to those in the current study were chosen as starting sets in the search procedure. The six-parameter search scheme was adopted, in which only two parameters were allowed to vary at a time, the other four remaining fixed. Detailed accounts of the particular search procedure adopted are found in Refs. 16 and 22. The searches were made by the automatic search code JIB[23] with the SYSTEMS-86 computer at this laboratory.

The proton parameter Set A obtained by Smith from the p-Nb elastic scattering at 16.2 MeV[24] was used as the starting set for our p-Mo parameter search. A spin-orbit potential of 8 MeV was fixed in the search. The final set (BRL set) obtained does not differ much from Smith's set, and the parameters are listed in Table I in standard notation. In Fig. 2 the angular distribution based

TABLE I. Optical-model-potential parameters used in the (p, α) DWBA calculations. All of the sets (BRL sets) were determined in a six-parameter search procedure described in the text. The value of the proton spin-orbit potential was chosen arbitrarily, and the value of the spin-orbit radius was set equal to that of r_0 for the (p, α) calculations.

Channel	Set	E (MeV)	U (MeV)	r_0 (fm)	r_c (fm)	a (fm)	V_{so} (MeV)	W_s (MeV)	W_D (MeV)	r'_0 (fm)	a' (fm)	χ^2 (per point)
P-^{96}Mo		15	50.5	1.251	1.251	0.678	8.0	···	14.33	1.242	0.483	4.9
α-^{93}Nb	1	22	60.5	1.620	1.620	0.506	···	12.75	···	1.612	0.350	2.4
	2	22	149.6	1.552	1.552	0.498	···	19.37	···	1.593	0.331	4.8
	3	22	180.1	1.528	1.528	0.497	···	21.19	···	1.583	0.269	4.8

on the BRL set is fitted to the measured elastic angular distribution in the upper half of the figure.

The three α-potential parameter sets, obtained via a six-parameter search by Park, Jones, and Bainum[16] for yttrium at 20 MeV, were chosen as starting sets for the present α-Nb parameter search at 22 MeV. The values of χ^2 were improved by a factor of 5 when the final search was made. All three of the searched sets, labeled BRL sets, are listed in Table I. The quality of a typical fit is exhibited in the lower half of Fig. 2, where the result of BRL Set 3, the set with the deepest well depth, is compared with the measured elastic angular distribution. All three sets were obtained in the case of the α potentials, because of the well-known ambiguities of α-potential parameter sets. Sensitivity of (p,α) DWBA calculations to α-potential parameter sets is examined below in some detail.

III. DWBA CALCULATIONS AND PARAMETER SENSITIVITIES

A. Form Factors

Most of the DWBA calculations in the present work were made with the use of the triton-cluster form factors. The proton-potential parameter set searched in this work (listed in Table I) was used throughout for generating the distorted wave for the entrance channel. The result from a recent (d,α) investigation in the mass 90 region[16] suggested that an α-potential set with a real-well depth of about 180 MeV would be a proper set to use in the present (p,α) calculations. Thus BRL α Set 3, the deepest potential set from this work (also listed in Table I), was chosen for the α-channel distorted wave. The effects of using the remaining two sets in the calculations will be examined later. All the calculations based on the triton-cluster form factors were performed with code DWUCK[25]; they are local zero-range calculations without the use of low cutoffs in radial integrals. The spin-orbit potential of Thomas form with the usual factor of $\lambda = 25$ was used for the cluster bound-state wave function.

Calculations were made initially with a Woods-Saxon well geometry of $r_0 = 1.38$ fm and $a = 0.45$ fm, the same values used in Ref. 13. Although the calculations predicted in general the locations of the minima and maxima of the measured angular distributions correctly, they failed in reproducing the steeper slopes of the data. A series of searches were thus made for the combination of r_0 and a, with which the measured angular distributions for the transitions to the levels with known J^π could best be reproduced. The best combination was found to be one with $r_0 = 1.38$ fm and $a = 0.32$ fm; this was then used for all subsequent cluster calculations. These calculations are compared with the measured angular distributions in Fig. 6, in which the calculations have arbitrarily been normalized to the data. The calculations are labeled in the figures by the intrinsic cluster

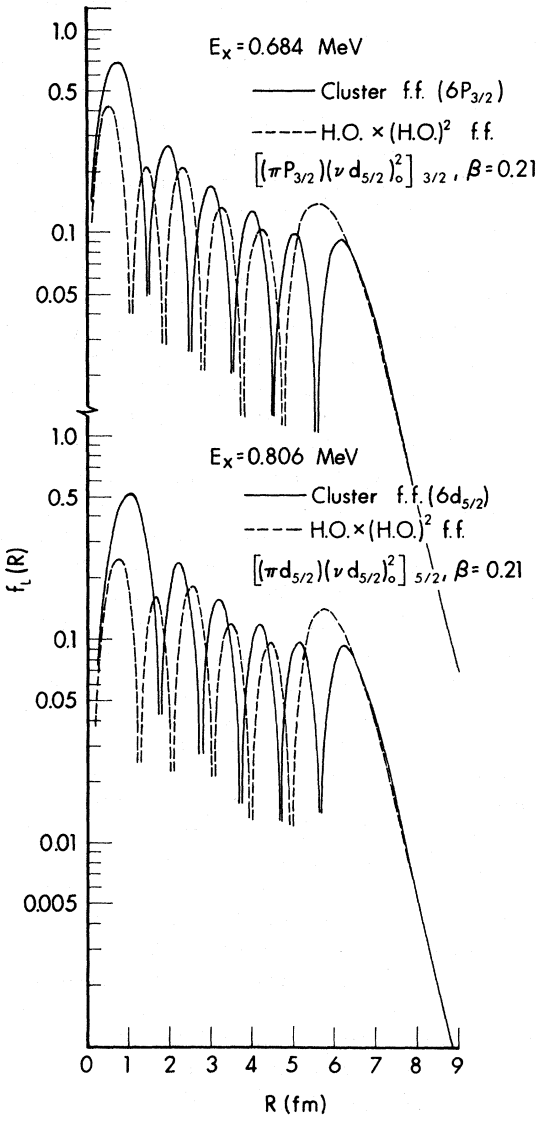

FIG. 3. Comparison between triton-cluster form factors (solid curves) and harmonic-oscillator-"product" form factors of Drisko (dashed curves). The cluster form factors are based on Woods-Saxon well geometry with $r_0 = 1.38$ fm and $a = 0.32$ fm, and the harmonic-oscillator form factors are based on the three-nucleon configurations specified in the figure (β is the size parameter of the harmonic oscillator). The latter form factors have been matched to tails characteristic of the separation energy of the triton in the target.

quantum numbers for the transfers.

A few preliminary calculations[26] were also made independently with the code JULIE which generated form factors on the basis of the semimicroscopic formalism of Drisko,[20] as mentioned previously. The sample cases chosen were the (p, α) transitions to the 0.684- and 0.806-MeV states for which $\frac{3}{2}^-$ and $\frac{5}{2}^+$ assignments, respectively, appear to be unique. Since the extent of the complete three-nucleon configurations pertinent to these transitions was not known from the literature, pure configurations were assumed for these trial cases. In Fig. 3 the two types of form factors are compared. The solid lines represent the triton-cluster form factors, the $6p_{3/2}$ transfer on top and $6d_{5/2}$ transfer on bottom. The dashed lines represent the semimicroscopic form factors, which are basically formed by taking a product of three harmonic-oscillator wave functions with the size parameter $\beta = 0.21$ fm in the usual notation, calculated for the configurations specified in the figure. These form factors have been properly matched at the nuclear surface to Hankel-function tails characteristic of the separation energies of the triton cluster in the target nucleus.

In Fig. 4 the DWBA curves based on the form factors shown in Fig. 3 are compared with the measured angular distributions. The calculations are normalized arbitrarily to the data. The same distorting parameters, BRL p set and α Set 3, were used for both types of theoretical curves.

The microscopic form-factor calculations do not reproduce the detailed structure of the measured angular distributions as well as the cluster form-factor calculations. Most of the (p, α) transfers of interest proceed in a severe momentum-mismatch condition, and thus the contributions from the nuclear interior to the transition amplitudes may no longer be ignored.[8] The marked difference in shape inside the nucleus between the two types of the form factors shown in Fig. 3 may then be responsible in part for the major shape discrepancy between the corresponding calculations in Fig. 4. The zero-range DWBA treatment is known to be inadequate for reactions in which the momentum mismatch is severe.[8] The conventional method of applying the finite-range correction in the local-energy approximation (LEA)[27] to the zero-range DWBA formalism may not be justified in a multinucleon transfer reaction.[16] The reason that the cluster calculations without such corrections were so successful in correctly predicting the shapes of the experimental angular distributions is probably that effects due to the finite-range and nonlocality corrections may have been overshadowed by a rather arbitrary choice of Woods-Saxon well geometry. This observation is also supported by the fact that a change in the LEA finite-range correction produces an effect similar to that produced when the diffuseness of the Woods-Saxon well is changed. Recently some efforts have been made by Hird and Li[28] to derive a more sophisticated form of the finite-range correction in analogy with LEA treatments for two-nucleon transfer reactions. A minor improvement over the zero-range case has been reported by these authors for the $^{19}F(p, \alpha)^{16}O$ reaction. This method has not been tested in this work.

Another significant effect which has not been taken into consideration is the coherence effect. The (p, α) differential cross section is proportional to the coherent sums of the transition amplitudes over the total angular momenta of the neutron pair as well as over all three-nucleon configurations, as mentioned earlier. Thus the present calculations which are based on pure configurations are not likely to reproduce the detailed structure of the data. The coherence due to the mixed configurations also affects the cross-section magnitudes when the microscopic form factors are used in the calculations.[17] Since the extent of the configuration mixture pertinent to the present (p, α) reaction has not been reported to date, no attempt to extract the spectroscopic amplitudes from the measured data can be made. The use of the triton-cluster form factors in the

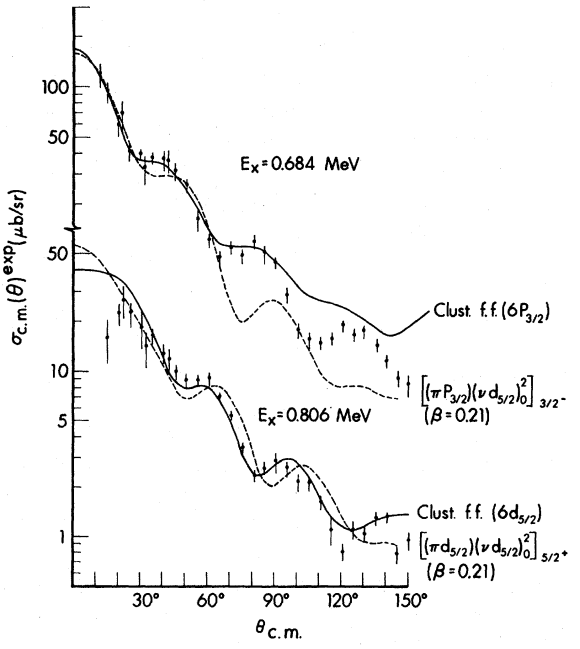

FIG. 4. Theoretical angular distributions, calculated with the use of the form factors shown in Fig. 3, are compared with data.

calculations, of course, does not permit quantitative analysis.

B. α Potential

It is known that the ambiguity of potentials for strongly absorbed particles, such as α particles, consistently contributes to the uncertainty in DWBA predictions for the reactions in which the α channel constitutes one of the distorted-wave channels. The discrete parameter sets belonging to the same family have drastically different effects on the reaction cross section. This point has been carefully and comprehensively checked for (^{3}He, α) reactions by Stock *et al.*,[29] and for (d, α) reactions by Daehnick and Park,[30] and for both reactions more recently by us.[16] In contrast, little has been reported on the α-parameter sensitivity of (p, α) reactions. A real-well depth as low as 40–60 MeV was commonly used in the past. Yet the calculations for the ^{90}Zr$(p, \alpha_0)^{87}$Y and ^{89}Y$(p, \alpha_0)^{86}$Sr reactions at 22.5 MeV in Ref. 21, for example, managed to reproduce the details of the measured angular distributions surprisingly well. On the other hand, Dittmer and Daehnick[13] had to use a well depth of 150 MeV in order to predict the j dependence for $l=1$ transfers in the ^{107}Ag, ^{109}Ag, and ^{108}Pd(p, α) reactions at 12 and 15 MeV.

In Fig. 5 zero-range cluster calculations based on three BRL α sets obtained in this work (listed in Table I) are compared with three experimental angular distributions. The one at the top is the unresolved angular distribution for the transitions

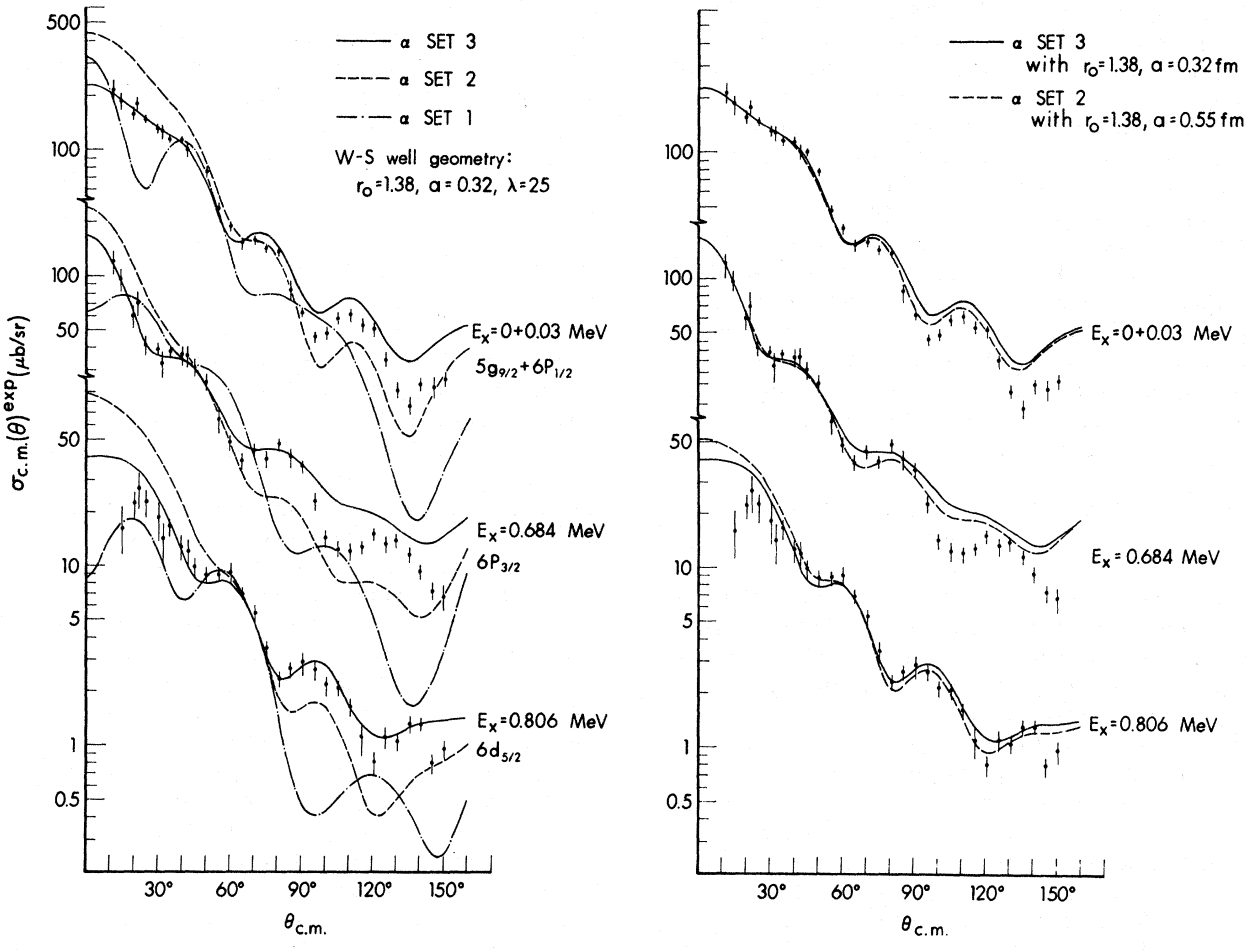

FIG. 5. Effect of variation of the α-potential set on zero-range noncutoff calculations based on triton-cluster form factors. On the left, calculations corresponding to the three BRL α sets obtained in this work (Table I) are compared with the data for three transitions. All calculations are based on the same BRL proton-potential set (Table I) and Woods-Saxon well geometry. On the right, the calculations based on the combination of α set 3 and the W-S well geometry of $r_0=1.38$ fm and $a=0.32$ fm, represented by the same solid curves used on the left, are compared with calculations based on another combination of α set 2 and the well geometry of $r_0=1.38$ fm and $a=0.55$ fm.

to the ground and first excited states. The rest are the same experimental angular distributions shown in Fig. 4. The distorting parameters for the p channel are those of the BRL set, also obtained at this laboratory (Table I).

All the calculations shown on the left-hand side used the triton-cluster form factors with a well geometry of $r_0 = 1.38$ fm, $a = 0.32$ fm, and $\lambda = 25$ as before. The calculations based on α Sets 3, 2, and 1 are represented by the solid, dashed, and broken curves, respectively, and they are arbitrarily normalized to the data. It is immediately clear that the calculations with α Set 1 (with the real-well depth of 60 MeV) are totally unacceptable. The superiority of the Set 3 calculations was expected, however, because the form-factor parameters used here are self-consistent with this set. The curves representing Set 2 with 149 MeV have slopes which are too steep; yet the stripping patterns of the data are correctly reproduced by the curves. This observation suggests the possible existence of another set of bound-state parameters which would make another optimum combination with α Set 2. On the right hand side of Fig. 5, the calculations based on the combination of α Set 3 and the well geometry of $r_0 = 1.38$ fm and $a = 0.32$ fm (the same solid curves shown on the left figure) are compared with new calculations based on the combination of α Set 2 and bound-state parameters of $r_0 = 1.38$ fm and $a = 0.55$ fm (the dashed curves). It is no longer clear which calculation is superior. Attempts to find a set of bound-state parameters which might be compatible with α Set 1 were, however, unsuccessful.

IV. DISCUSSION OF INDIVIDUAL LEVELS– J^π ASSIGNMENTS

The main tool with which the spins of final states are determined in the present work is the well-known j dependence in direct (p, α) angular distributions. Measured angular distributions for strong transitions to levels below 1.5 MeV excitation in ^{93}Nb are compared with DWBA predictions in Fig. 6. The calculations are based on the triton-cluster form factors with the Woods-Saxon well geometry of $r_0 = 1.38$ fm and $a = 0.32$ fm and on optical-model parameters (BRL p set and α Set 3) determined in this work as described earlier. A value of 8 MeV for the spin-orbit potential was used for the proton channel in all the calculations. The extracted triton-cluster angular momentum transfers and J^π values of the levels under study are summarized in Table II along with the excitation energies assigned in this work.

Ground State and 0.030-MeV Level. The J^π value of the ground state is known to be $\frac{9}{2}^+$. Shell-model calculations predict $\frac{1}{2}^-$ for the first excited state. This is confirmed by the $(p, p'\gamma)$ and $(n, n'\gamma)$ work of Rogers *et al.*[1] and supported by ^{92}Zr(^{3}He, d) and ^{94}Mo(d, ^{3}He) reaction studies.[6,7] In the present (p, α) experiment the 0.03-MeV level is not separated enough from the ground state to permit us to obtain a separate angular distribution. The unresolved angular distribution is very well reproduced by the calculated curve in Fig. 6, which represents a direct sum of the $g_{9/2}$ and $p_{1/2}$ transfers.

Taking ^{88}Sr as the core, as assumed in most shell-model calculations for the mass-90 region,[5] the ground-state wave function of ^{96}Mo may consist of

$$\{a[\pi(p_{1/2}^2)_0(g_{9/2}^2)_0] + b[\pi(g_{9/2}^4)_0]\}[\nu(d_{5/2}^4)] .$$

The (p, α) transitions to the ground and first excited states of ^{93}Nb are dominated by picking up the three nucleons from $[\pi(g_{9/2}), \nu(d_{5/2}^2)_0]_{9/2}$ and $[\pi(p_{1/2}), \nu(d_{5/2}^2)_0]_{1/2}$ configurations, respectively. The wave functions for the ground and first excited states of ^{93}Nb may then consist of

$$\{c[\pi(p_{1/2}^2)_0(g_{9/2})]_{9/2^+} + d[\pi(g_{9/2}^3)_{v=1}]_{9/2^+}\}[\nu(d_{5/2}^2)_0]$$

and

$$\{e[\pi(p_{1/2})(g_{9/2}^2)_0]_{1/2^-}\}[\nu(d_{5/2}^2)_0] ,$$

respectively.

0.684-MeV Level. A simple shell-model calculation based on a limited configuration space predicts a $\frac{3}{2}^-$ level at 0.66 MeV.[7] The state was populated both in ^{92}Zr(^{3}He, d) and ^{94}Mo(d, ^{3}He) reactions with an $l = 1$ proton transfer.[6,7] Based on the spectroscopic strength, the $\frac{3}{2}^-$ assignment was preferred over $\frac{1}{2}^-$ for this level. This preference is supported by the recent ^{90}Zr$(\alpha, p\gamma)$ experiment[31] conducted at this laboratory.

The state is populated with a moderate strength and well separated from the 0.749-MeV level in (p, α), as can be seen in Fig. 1. In Fig. 6 the measured angular distribution is compared with both $p_{3/2}$ (solid line) and $p_{1/2}$ (dashed line) triton-cluster transfers. It is immediately clear that the $\frac{3}{2}^-$ assignment is unique. It also demonstrates that the well-known j dependence associated with an $l = 1$ transfer in (p, α) can be profitably used as a strong spectroscopic tool in the present ^{96}Mo(p, α) reaction. The dominant three-nucleon configuration to which this state belongs appears to be $[\pi(p_{3/2}), \nu(d_{5/2}^2)_0]_{3/2^-}$, but the extent of a possible admixture of the type $[\pi(p_{1/2}), \nu(d_{5/2}^2)_2]_{3/2^-}$ cannot be measured at the moment.

0.749-MeV level. The J^π of this level has been firmly established as $\frac{7}{2}^+$ from recent Coulomb excitation works.[1,3] It is not seen in the (^{3}He, d) reaction by Cates, Ball, and Newman.[7] The popu-

lation of the level is not certain in the reported $(d,{}^3\mathrm{He})$ reaction[6] because of the poor experimental resolution. The level is either populated very weakly or not seen in the present (p,α) reaction. If the state belongs predominantly to a configuration of the type $[\pi(g_{7/2}),\nu(j^2)_0]$, then its absence in the (p,α) reaction at this energy is expected. No assignment can be made from the present work.

0.806-MeV level. Rogers *et al.*[1] identified a doublet, separated by a few keV at this energy, and assigned $\frac{3}{2}^-$ and $\frac{5}{2}^+$ for the doublet. The existence of the doublet has recently been confirmed by the ^{90}Zr$(\alpha,p\gamma)$ work[31] which, however, proposes $\frac{5}{2}^+$ and $\frac{5}{2}^-$ $(\frac{3}{2}^-)$ for the doublet. The ^{92}Zr$(^3\mathrm{He},d)$ reaction[7] and the Coulomb excitation work by Stelson *et al.*[3] assign $\frac{5}{2}^+$ for a level at this energy. Thus it appears almost certain that at least the positive-parity member of the doublet is a $\frac{5}{2}^+$ state.

The doublet is strongly populated in (p,α). The unresolved angular distribution is compared with a set of calculations in Fig. 6. Our $d_{5/2}$ calculation (solid line) reproduces very well the detailed structure of the measured angular distribution except at small angles ($\theta_L < 30°$). On the other hand, the $f_{5/2}$ calculation (dashed line) does show a tendency to fall off at these small angles but

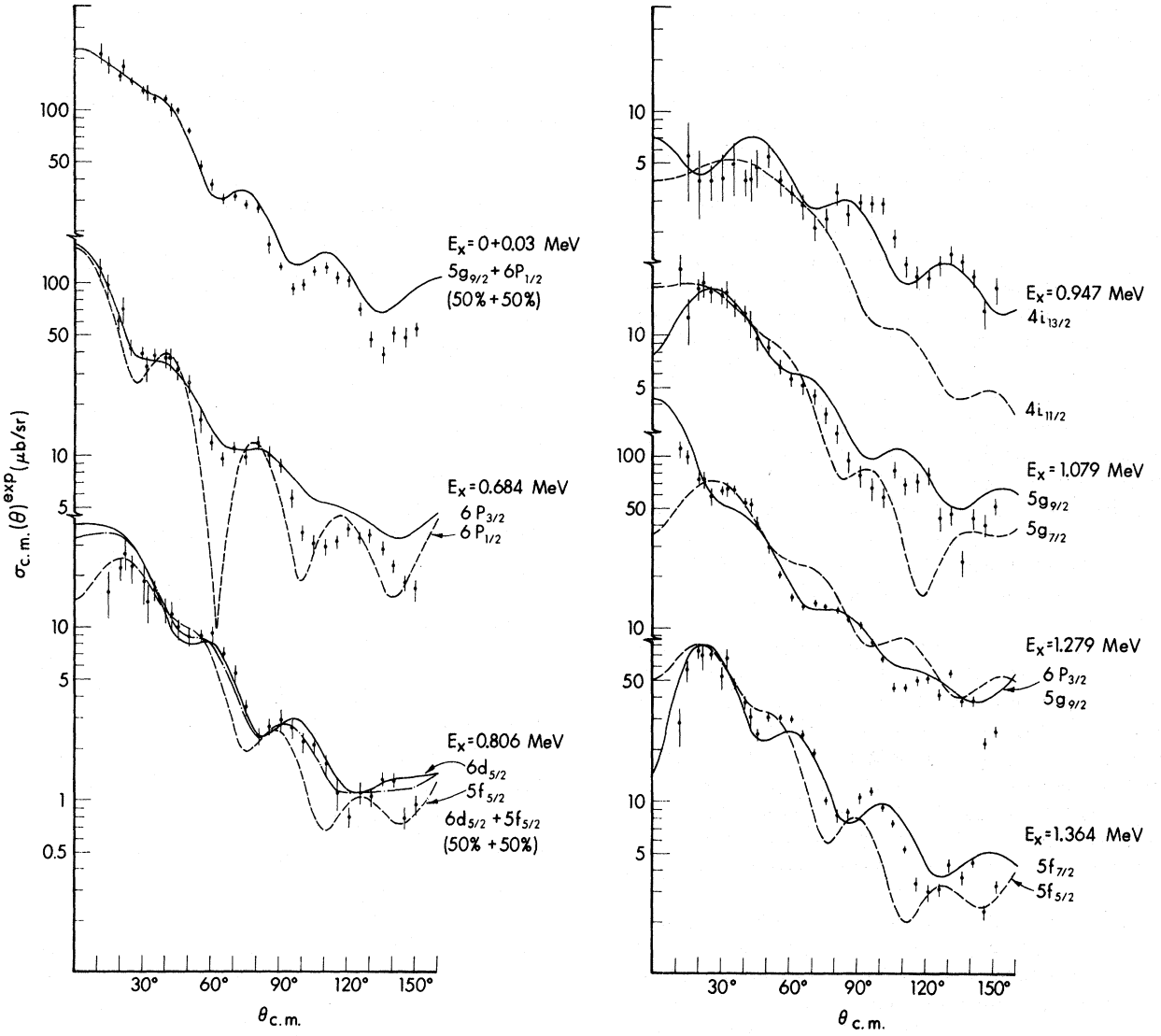

FIG. 6. Measured angular distributions for (p,α) transitions to levels in ^{93}Nb are compared with local zero-range noncutoff DWBA calculations based on BRL potential sets (α set 3) and a Woods-Saxon gemoetry of $r_0 = 1.38$ fm and $a = 0.32$ fm. The dashed curves for transitions to levels at 0.684, 0.947, 1.079, and 1.364 MeV, representing $j = l - \frac{1}{2}$ transfers, are included here purely for the purpose of showing the predicted j dependence for $l = 1$, 6, 4, and 3 transfers.

fails to predict correctly the structure at larger angles. The third curve (broken line) represents an admixture of the $d_{5/2}$ with the $f_{5/2}$ transfer, resulting in a closer agreement with the data. A sum of the $d_{5/2}$ and $p_{3/2}$ was also tried, but no improvement was found. Thus we favor tentatively the $\frac{5}{2}^-$ assignment over the $\frac{3}{2}^-$ for the negative-parity member of the doublet based purely on the quality of the shape fit.

0.947-MeV level. This level is moderately populated in (p, α) but is not resolved from the nearby 0.979-MeV level, whose existence has recently been established in Coulomb excitation,[3] $(n, n'\gamma)$,[1] and $(\alpha, p\gamma)$[31] works. The $\frac{13}{2}^+$ assignment made by these authors is confirmed by our (p, α) result, as can be seen in Fig. 6. The dashed curve, representing the $i_{11/2}$ calculation, is also compared with the data in order to demonstrate that a remarkably strong j dependence is also recognized for an angular momentum transfer as large as $l=6$. It is unfortunate that the angular distribution for the 0.979-MeV transition could not be obtained in this work, as this could complement the $l=6$ j dependence, the state known to be an $\frac{11}{2}^+$ state.[3] Cates, Ball, and Newman[7] proposed $\frac{1}{2}^-$ or $\frac{3}{2}^-$ for a level identified at 0.97 MeV based on the $l=1$ assignment. However, it seems that the level is too weakly populated in that work to permit the extraction of a reliable l value and that the $(^3\mathrm{He}, d)$ assignment is thus in doubt.

1.079-MeV level. The $\frac{9}{2}^+$ assignment made by Refs. 3, 7, and 31 is confirmed by our (p, α) angular distribution for the transition to this level, as shown in Fig. 6. It is apparent that the j dependence is also observed for the $l=4$ transfer. Rogers *et al.*[1] assigned $\frac{7}{2}^+$ for a level at 1.080 MeV, but this assignment is not supported by our (p, α) analysis.

The shell-model calculation by Auerbach and Talmi[5] predicts a group of five states near 1 MeV belonging mainly to the $[\pi(g_{9/2}), \nu(d_{5/2}{}^2)_2]_J$ configuration. The $\frac{5}{2}^+$, $\frac{7}{2}^+$, $\frac{9}{2}^+$, $\frac{11}{2}^+$, and $\frac{13}{2}^+$ levels experimentally observed at 0.806, 0.744, 1.079, 0.979, and 0.947 MeV excitation are believed to be the members of the quintuplet belonging to this configuration. It remains unexplained why the $\frac{7}{2}^+$ and $\frac{11}{2}^+$ members are not populated in the present (p, α) reaction.

1.279-MeV level. To date, a unique J^π value has not been assigned to this level. The main difficulty is that there are at least four states clustered in the span of merely 100 keV.[2] Cates, Ball, and Newman[7] were unable to resolve this level from others, yet an $l=1$ assignment was made for this transition. The Coulomb excitation work of Stelson *et al.*[3] proposes $\frac{9}{2}^+$, while various $(n, n'\gamma)$ works[1,2] fail to give a unique assignment.

The state is strongly populated and sufficiently resolved from the 1.334-MeV state at most angles in our (p, α) reaction (see Fig. 1). The $g_{9/2}$ calculation, represented by the dashed curve, is out of phase with the measured angular distribution, while the calculation for the $6p_{3/2}$ transfer reproduces remarkably well the detailed structure of the measured one. A $\frac{3}{2}^-$ state can be strongly populated in (p, α) if it belongs to $[\pi(p_{1/2}), \nu(d_{5/2}{}^2)_2]$ or $[\pi(p_{3/2}), \nu(d_{5/2}{}^2)_0]$ configuration.

1.364-MeV level. Because of the increased level density around this energy, it is not always feasible to make a one-to-one correlation between levels observed in different reactions. One of the strongest levels appears consistently at this energy in (p, α). Extreme care has been exercised to obtain the angular distribution for this transition in order to minimize possible contributions from the nearby 1.334- and 1.395-MeV levels (the existence of the 1.395-MeV level has recently been

TABLE II. Listing of level energies, angular momentum transfers of triton cluster, and J^π values assigned from the present $^{96}\mathrm{Mo}(p, \alpha)^{93}\mathrm{Nb}$ reaction.

E_x (MeV)		$(l, j)_t$	J^π
0	Not resolved	$g_{9/2}+p_{1/2}$	$\frac{9}{2}^+$
0.030 [a]			$\frac{1}{2}^-$
0.684		$p_{3/2}$	$\frac{3}{2}^-$
0.749	Weak	...	...
0.806	Doublet	$d_{5/2}$ or $d_{5/2}+f_{5/2}$	$\frac{5}{2}^+$
			$\frac{5}{2}^-(\frac{3}{2}^-)$
0.947		$i_{13/2}$	$\frac{13}{2}^+$
0.979 [a]	Weak and not resolved	...	...
1.079		$g_{9/2}$	$\frac{9}{2}^+$
1.127 [a]	Weak and not resolved	...	...
1.279		$p_{3/2}$	$\frac{3}{2}^-$
(1.334)	Not resolved	...	...
1.364		$f_{7/2}$	$\frac{7}{2}^-$
(1.395)	Weak and not resolved	...	...
1.486	Multiplet	No fit	$(\frac{15}{2}^+, \frac{17}{2}^+)$?
1.572		No fit	...
1.678		No fit	...
1.775		No fit	...
2.10		No fit	...

[a] Reference 2.

reported by Göbel, Feicht, and Vonach[2]). The measured angular distribution is well reproduced by the $5f_{7/2}$ calculation (solid line) in Fig. 6. It was also compared with the $f_{5/2}$ calculation (dashed line) in order to illustrate the j dependence associated with an $l=3$ (p, α) transfer.

The level, unresolved from the 1.279-MeV level, was strongly populated also in the ^{94}Mo$(d, ^3$He) proton-pickup reaction studied by Ohnuna and Yntema.[6] The unresolved angular distribution was reproduced by a mixture of $l=1$ and $l=3$ transfers. Our assignments of the $p_{3/2}$ and $f_{7/2}$ transfers to the 1.279- and 1.364-MeV levels, respectively, are thus in accord with the $(d, ^3$He) results. The dominant three-particle configurations to which these states belong appear to be $[\pi(p_{3/2}), \nu(d_{5/2}{}^2)_0]_{3/2^-}$ and $[\pi(f_{7/2}), \nu(d_{5/2}{}^2)_0]_{7/2^-}$, respectively.

1.486-MeV level. Shell-model calculations[5] predict $\frac{15}{2}^+$ and $\frac{17}{2}^+$ for two levels at 1.41 and 1.38 MeV, constructed mainly from $[\pi(g_{9/2}), \nu(d_{5/2}{}^2)_4]_J$ configuration. These high angular momentum states have recently been observed in (α, d) by Zisman and Harvey.[9]

Population of a $\frac{15}{2}^+$ or $\frac{17}{2}^+$ state in (p, α) would proceed by an $l=8$ transfer. The angular distribution for a level at 1.486 MeV (shown in Fig. 7) has washed-out structure, which is characteristic of high angular momentum transfers. It is then possible that this level may be either the $\frac{15}{2}^+$ or the $\frac{17}{2}^+$ state predicted by the shell-model calculation. Angular distributions have been obtained also for transitions to the states at 1.572, 1.678, 1.775, and 2.10 MeV, and they are shown in Fig. 7. Although they exhibit moderate structure, no reliable DWBA analyses have been possible, mainly due to poor statistics on data points and increased level density.

V. SUMMARY AND CONCLUSIONS

The ^{96}Mo(p, α) reaction was studied at 15 MeV in order to determine J^π values for levels below 1.5 MeV in ^{93}Nb. The extent to which the reaction can be used reliably for this purpose was carefully investigated. The main emphasis was placed on a detailed check of the sensitivity of DWBA calculations to parameter variations.

Two different methods of generating form factors for (p, α) transfers were examined in a few test cases. The first one took a simple product of three single-particle harmonic-oscillator wave functions matched to a realistic tail characteristic of the separation energy of the triton.[20, 21] The second type of form factor tried was the conventional triton-cluster form factor.

The inability of the zero-range semimicroscopic form-factor calculations to reproduce detailed shapes of measured angular distributions is viewed as due to large momentum mismatch associated with (p, α) transitions; zero-range noncutoff DWBA calculations are no longer expected to be accurate. A recent preliminary calculation[32] which includes the finite-range correction indicates that the effect in general on angular distributions is slight at forward angles but is appreciable at backward angles. Inclusion of this correction into our calculations may then be needed in order to bring the measured and calculated angular distributions to closer agreement, especially at backward angles. The effect of configuration dependence on shapes of angular distributions has not been studied here, as the shapes are expected to bear L signature.

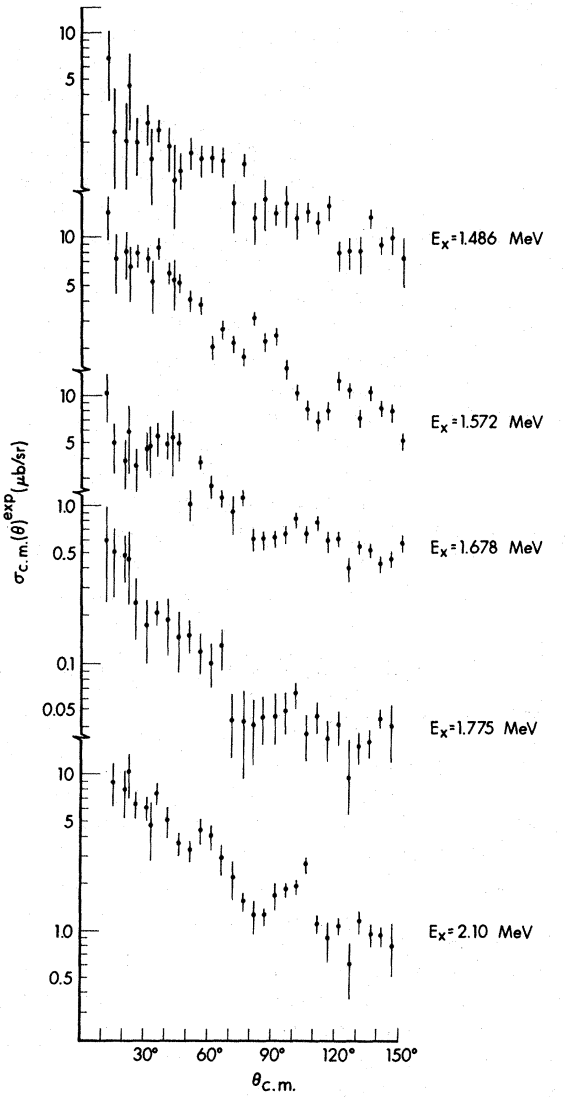

FIG. 7. Remaining measured (p, α) angular distributions. Reliable DWBA analysis was not possible primarily due to large errors assigned to data points.

It is remarkable that the cluster calculations based on one fixed set of geometrical parameters ($r_0 = 1.38$ fm and $a = 0.32$ fm) are capable of reproducing so well the shapes of measured angular distributions corresponding to different angular momentum transfers. It thus appears that zero-range cluster calculations are sufficient for the purpose of shape fitting; there is no need for making the finite-range and nonlocality corrections to the zero-range calculations as long as proper well geometry and α-potential sets are used.

The problem of selecting a proper α-optical-model-potential parameter set for (p, α) DWBA calculations has not been fully resolved in the present analysis. The ambiguity in the α potential is found to be tied to that of Woods-Saxon well geometries.

The previously established j dependence observed in angular distributions for $l = 1$ (p, α) transfers has been confirmed in the present (p, α) reaction. Less conclusive evidences of the j dependence for $l = 3$, 4, and 6 have been detected in the present (p, α) reaction. The j dependence for these higher l-transfers has not been completely isolated, because for every l transfer there is only one measured angular distribution which is reproduced by the DWBA calculation for the $j = l + \frac{1}{2}$ transfer.

The J^π assignments made on the basis of j dependence are in good agreement with those recently proposed by the Coulomb-excitation work of Stelson *et al.*[3] and the ^{90}Zr$(\alpha, p\gamma)$ reaction.[31] It is clear that a simple shell-model calculation based purely on a limited three-nucleon configuration space such as $[\pi(g_{9/2}), \nu(d_{5/2}{}^2)_0]$ or $[\pi(p_{1/2}), \nu(d_{5/2}{}^2)_0]$ is not adequate for predicting high-spin states in ^{93}Nb, strongly populated via the direct ^{96}Mo(p, α) reaction. On the other hand, it remains unexplained why two members ($\frac{7}{2}^+$ and $\frac{11}{2}^+$) of the quintuplet, predicted by a shell-model calculation to belong to the $[\pi(g_{9/2}), \nu(d_{5/2}{}^2)_2]_J$ configuration, have not been populated in the present (p, α) reaction.

In conclusion, we suggest that a more refined method of obtaining microscopic form factors based on all three-nucleon configurations present in the nuclear wave functions is needed in order to fully utilize the (p, α) reaction for quantitative spectroscopic studies. A preliminary calculation based on a semimicroscopic form factor generated by the method recently proposed by Suck and Coker[33] was not successful for a few test cases in the present (p, α) reaction. A computer code based on a microscopic formalism for three-nucleon form factors is being prepared by F. Rybicki.[34]

ACKNOWLEDGMENTS

The authors are grateful to Dr. D. Eccleshall for suggesting this study. We wish to thank Dr. P. G. Roos and Dr. S. H. Suck for informative discussions and preliminary calculations. We are obliged to Dr. R. M. Tapphorn and Dr. R. Shnidman for supplying us with their results prior to publication. The services of the accelerator-operation group, under the direction of J. A. Morrissey, and of the supporting group are gratefully acknowledged. We wish to thank Mrs. Dorothy Bodt for typing the manuscript.

[1]V. C. Rogers, L. E. Beghian, F. M. Clikeman, F. Hofman, and S. Wilensky, Nucl. Phys. A142, 100 (1970), and references therein.

[2]H. Gobel, E. J. Feicht, and H. Vonach, Z. Physik 240, 430 (1970).

[3]P. H. Stelson, R. L. Robinson, W. T. Milner, F. K. McGowan, and M. A. Ludington, Bull. Am. Phys. Soc. 16, 619 (1971).

[4]M. Kregar and G. G. Seaman, Nucl. Phys. A179, 153 (1972).

[5]K. H. Bhatt and J. B. Ball, Nucl. Phys. 63, 286 (1965); N. Auerbach and I. Talmi, Nucl. Phys. 64, 458 (1965); J. Vervier, *ibid.* 75, 17 (1965).

[6]H. Ohnuma and J. L. Yntema, Phys. Rev. 176, 1416 (1968).

[7]M. R. Cates, J. B. Ball, and E. Newman, Phys. Rev. 187, 1682 (1969).

[8]See, for example, N. Austern, *Direct Nuclear Reaction Theories* (Wiley-Interscience, New York, 1970).

[9]M. S. Zisman and B. G. Harvey, Phys. Rev. C 5, 1031 (1972).

[10]See, for example, J. P. Schiffer, J. Phys. Soc. Japan, Suppl. 24, 319 (1968).

[11]L. L. Lee, Jr., A. Marinov, C. Mayer-Boricke, J. P. Schiffer, R. H. Bassel, R. M. Drisco, and G. R. Satchler, Phys. Rev. Letters 14, 261 (1965).

[12]J. A. Nolen, Jr., and C. M. Glashausser, Phys. Letters 21, 705 (1966).

[13]D. L. Dittmer and W. W. Daehnick, Phys. Rev. C 2, 238 (1970).

[14]R. O. Ginaven and A. M. Bernstein, Nucl. Phys. A154, 417 (1970).

[15]D. Bucurescu, M. Ivascu, G. Semenescu, and M. Titirici, Nucl. Phys. A189, 577 (1972).

[16]Yong Sook Park, H. D. Jones, and D. E. Bainum, Phys. Rev. C 4, 778 (1971).

[17]B. F. Bayman, Argonne National Laboratory Report No. ANL-6878, 335, 1964 (unpublished).

[18]B. F. Bayman and E. Rost, unpublished (see Ref. 19).

[19]J. A. Nolen, Jr., Ph.D. thesis, Princeton University, 1965 (unpublished).

[20]R. M. Drisko, unpublished (see Ref. 14).

[21]C. B. Fulmer and J. B. Ball, Phys. Rev. 140, B330 (1965).
[22]Yong Sook Park, H. D. Jones, D. E. Bainum, and A. W. Barrows, Jr., U.S. Army Ballistic Research Laboratories Report No. 1564, 1971 (unpublished).
[23]We are indebted to Dr. Perey for making the code JIB available to us.
[24]W. R. Smith, Argonne National Laboratory Report No. ANL-6848, 30, 1964 (unpublished).
[25]We are indebted to Dr. Kunz for making the code DWUCK available to us.
[26]The JULIE calculations were made by P. G. Roos, University of Maryland.
[27]P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. (London) 83, 701 (1964).
[28]B. Hird and T. Y. Li, Can. J. Phys. 46, 1273 (1968).
[29]R. Stock, R. Bock, P. David, H. H. Duhm, and T. Tamura, Nucl. Phys. A104, 137 (1967).
[30]W. W. Daehnick and Y. S. Park, Phys. Rev. 180, 1062 (1969); Y. S. Park, Ph.D. thesis, University of Pittsburgh, 1968 (unpublished).
[31]R. M. Tapphorn and R. Shnidman, to be published.
[32]S. H. Suck, University of Texas, private communication.
[33]S. H. Suck and W. R. Coker, Nucl. Phys. A176, 89 (1971).
[34]F. Rybicki, Temple University, private communication.