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# Structure of the 8.65-8.69-MeV Doublet in $^{11}C^{\dagger}$

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The 8.65- and 8.69-MeV states in $^{11}C$ have been studied with the $^{10}B(^3He,d)^{11}C$ reaction, with a resolution of 9 keV (full width at half maximum). Comparison of absolute spectroscopic strengths for this reaction and for the $^{10}B(d,p)^{11}B$ reaction to the 9.19-MeV ($J^{\pi}=\frac{7}{2}^+$) and 9.27-MeV ($J^{\pi}=\frac{5}{2}^+$) states in $^{11}B$ shows unambiguously that the spin sequence in $^{11}C$ is the same as in $^{11}B$. An additional outcome of the present experiment is a measurement of the width of the 8.69-MeV state: $\Gamma_{TOTAL}=15\pm1$ keV. The natural width of the 8.65-MeV state is considerably less than the experimental resolution width of 9 keV.

## I. INTRODUCTION

The mirror nuclei $^{11}B$ and $^{11}C$ have been extensively studied by a number of different experimental techniques,[1] and a great deal is known[1] about their level structure and mirror correspondences below 10 MeV in excitation. However, there do persist a few unanswered questions concerning states in this excitation region. In particular, the 8.65-8.69-MeV doublet in $^{11}C$ has been identified as the mirror of the 9.19-9.27-MeV doublet in $^{11}B$ (see Fig. 1), but the spin sequence, which is known[1] to be $\frac{7}{2}^+$, $\frac{5}{2}^+$ in $^{11}B$, has not been definitively established in $^{11}C$. An earlier study of the $^{10}B$-$(^3He,d)^{11}C$ reaction[2] suggested that the ordering in $^{11}C$ was the same as in $^{11}B$, but in that study the $^{11}C$ states were not completely resolved. Further studies[3] have not clarified the issue. We have performed a high-resolution study of the $^{10}B(^3He,d)$ reaction in order to establish the spin sequence in $^{11}C$, by comparison of these data with those of the analog reaction[4] $^{10}B(d,p)$ leading to the 9.19-9.27-MeV states in $^{11}B$.

## II. EXPERIMENTAL PROCEDURE

The reaction was induced by an 18-MeV $^3He^{++}$ beam produced by the University of Pennsylvania tandem Van de Graaff accelerator. The reaction products were momentum-analyzed in a multiangle magnetic spectrograph and were recorded in nuclear emulsions. The emulsions were discriminately scanned to produce composite deuteron energy spectra at 18 laboratory angles from 3.75 to 67.5° in 3.75° intervals. The yields were converted to cross sections taking into account proper solid angle, target thickness, and incident-charge–integration-normalization factors. The target was a self-supporting isotopically enriched (97%) $^{10}B$ foil having a thickness of 15 $\mu g/cm^2$, representing an energy loss of 4 keV for 18-MeV $^3He^{++}$ ions.

A deuteron energy spectrum showing the region of interest is shown in Fig. 2. The peaks labeled $^{17}F$ and $^{12}C_{12}$ indicate the relatively small amounts of $^{16}O$ and $^{11}B$ contamination present in the $^{10}B$ target. No $^{12}C$ contamination was detectable. The good resolution [9 keV full width at half maximum (FWHM)] allowed a determination of the natural width of the 8.69-MeV state. The width was calculated assuming that the measured width is quadratically related to the sum of the natural width $\Gamma$ and the resolution width (9 keV FWHM). The average value of the width extracted at four forward angles is $\Gamma=15\pm1$ keV. The 8.69-MeV state is slightly unbound[1] (by ~1 keV) to proton decay,

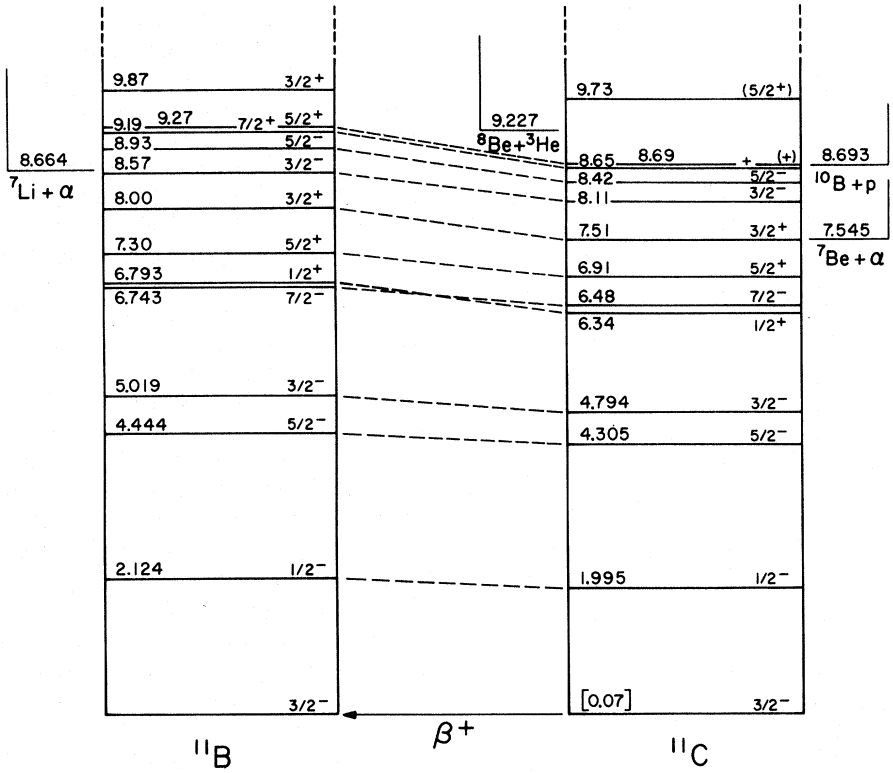


FIG. 1. Energy-level diagram of the mirror nuclei $^{11}$B-$^{11}$C up to 10 MeV in excitation. The dashed lines indicate mirror correspondences. Spin and parity assignments are summarized in Ref. 1.

but the natural width probably all lies in the $\alpha$-particle decay channel since it is unbound to $\alpha$ decay[1] by 1.15 MeV.

## III. ANALYSIS AND DISCUSSION

The distorted-wave Born-approximation (DWBA) formalism has been used to describe both the present ($^3$He, $d$) data and the previously measured ($d$, $p$) reaction data of Hinds and Middleton,[4] leading to the 8.65-8.69-MeV and 9.19-9.27-MeV doublets in $^{11}$C and $^{11}$B, respectively. In a simple picture the two reactions are analogous and should populate mirror states with equal spectroscopic strengths. The reactions are somewhat different in that on a purely kinematical basis the ($d$, $p$) reaction on $^{10}$B should favor an $l=0$ transition, while the ($^3$He, $d$) reaction should favor $l=2$. However, the difference is readily calculable within the DWBA.

The experimental angular distributions were compared to the distorted-wave calculations (obtained with the code DWUCK[5]) by the relation

$$\sigma_{\text{exp}}(\Theta_{\text{c.m.}}) = NC^2 \frac{(2J_f+1)}{(2J_i+1)} \sum_{lj} \frac{S_{lj}\sigma_{\text{DWBA}}(\Theta_{\text{c.m.}})}{2j+1},$$

TABLE I. Optical-model parameters used in DWBA analysis of $^{10}$B($^3$He, $d$)$^{11}$C and $^{10}$B($d$, $p$)$^{11}$B reactions.

| | $V$ (MeV) | $W$ (MeV) | $W'=4W_d$ (MeV) | $r_0$ (fm) | $a$ (fm) | $r_0'$ (fm) | $a'$ (fm) | $V_{so}$ (MeV) | $r_{0c}$ (fm) |
|---|---|---|---|---|---|---|---|---|---|
| $^{10}$B + $d$ [a] | 100 | 10 | 0 | 1.40 | 0.60 | 1.74 | 0.80 | 0 | 1.40 |
| $^{11}$B + $p$ [b] | 60.83 | 0 | 25.14 | 1.142 | 0.57 | 1.142 | 0.50 | 5.5 | 1.142 |
| $^{10}$B + $^3$He [c] | 177 | 15 | 0 | 1.138 | 0.724 | 1.602 | 0.796 | 0 | 1.14 |
| $^{11}$C + $d$ [a] | 100 | 10 | 0 | 1.40 | 0.60 | 1.74 | 0.80 | 0 | 1.40 |
| $^{10}$B + $n$ [a]<br>$^{10}$B + $p$ | Varied | ··· | ··· | 1.26 | 0.60 | ··· | ··· | $\lambda=25$ | 1.26 |

[a] Reference 7. [b] Reference 8. [c] Reference 6.

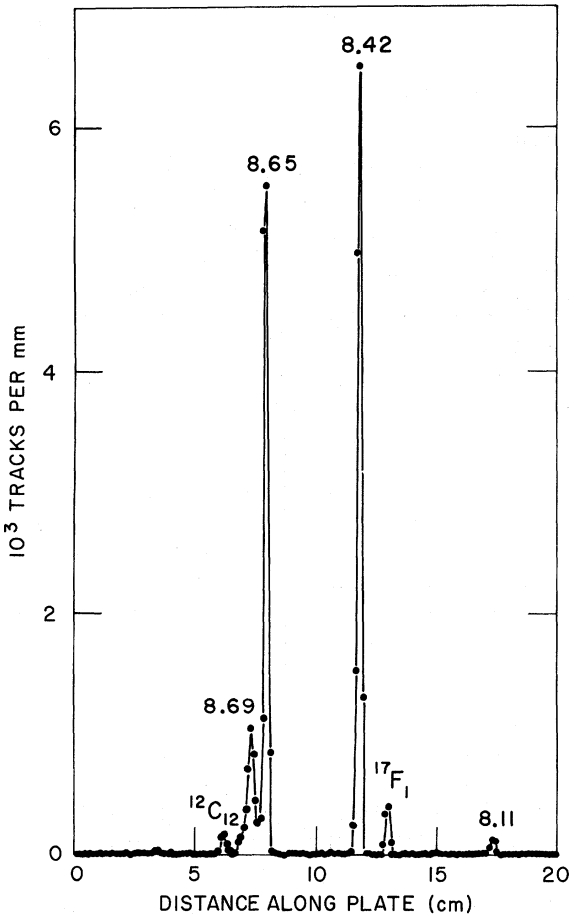


FIG. 2. Energy spectrum for the groups of interest in the $^{10}B(^3He,d)^{11}C$ reaction at a bombarding energy of 18 MeV and a laboratory angle of $11\frac{1}{4}°$. The measured resolution is 9 keV FWHM.

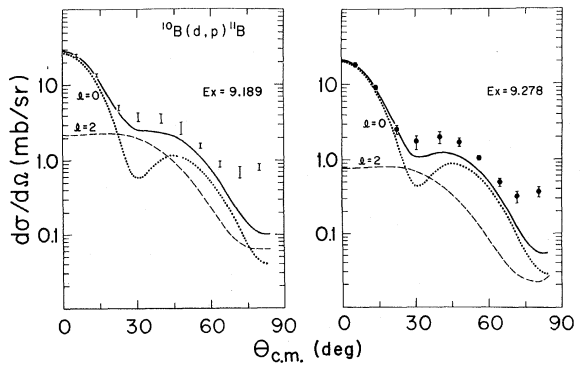


FIG. 3. Angular distributions and DWBA calculations for the $^{10}B(d,p)^{11}B$ data of Hinds and Middleton. The solid lines are admixed sums of $l=0$ and $l=2$. Resulting spectroscopic strengths are listed in Table II.

where $N$ is a reaction-dependent normalization factor, $C$ is an isospin Clebsch-Gordan coefficient, and $S$ is the spectroscopic factor. In the present case, $C^2$ is unity for both reactions, and we have used $N=4.42$ for $(^3He,d)$ and $N=1.65$ for $(d,p)$. The quantities $J_i$, $J_f$, and $j$ are the initial, final, and transferred total angular momenta, respectively. The calculations performed were local and zero range, with an integration region of 0–19 fm.

The optical-model parameters used in the distorted-wave analysis for the $^3He$ and deuteron channels were taken from the literature[6,7] and constitute average real and imaginary well parameters applicable to this mass and energy range. The proton parameters were calculated from a general equation given by Watson, Singh, and Segel.[8] The bound-state form factors were computed for a Woods-Saxon potential with geometrical parameters equal to those previously used in this region of nuclei.[7] The well depth was adjusted to reproduce the experimental neutron/proton binding energy of each state. The optical-model and bound-state parameters are listed in Table I.

If the 8.65-8.69-MeV states in $^{11}C$ are indeed mirrors of the 9.19-9.27-MeV states in $^{11}B$, there are two possible cases to consider: *case (1)*: $^{11}B(9.19)$ and $^{11}C(8.65)$ are mirrors and $^{11}B(9.27)$ and $^{11}C(8.69)$ are mirrors, i.e. the level order of these two states is the same in $^{11}C$ as in $^{11}B$; *case (2)*: $^{11}B(9.19)$ and $^{11}C(8.69)$ are mirrors and $^{11}B(9.27)$ and $^{11}C(8.65)$ are mirrors, i.e. the level order of these two states in $^{11}B$ and $^{11}C$ is inverted.

In case (1), the changing of a neutron into a proton has caused the 9.27-MeV state to move downward by 40 keV relative to the 9.19-MeV state. In case (2), this transformation has caused the 9.27-MeV state to move downward by 120 keV relative to the 9.19-MeV state. In either case the 9.27-MeV state has moved downward relative to the 9.19-MeV state.

Coulomb-energy calculations show that relative to a $p$ state at a similar binding energy, a $d$ state moves upward and an $s$ state moves downward when a neutron is changed into a proton. Thus the

TABLE II. Stripping strengths $(2J_f+1)S$ in $^{11}B$ and $^{11}C$.

| $E_x$ ($^{11}B$) (MeV) | $J^\pi$ | $l=0$ | $l=2$ [a] | $E_x$ ($^{11}C$) (MeV) | $l=0$ | $l=2$ [a] |
|---|---|---|---|---|---|---|
| 9.189 | $\frac{7}{2}^+$ | 1.40 | 2.03 | 8.65 | 1.4 | 1.7 |
| 9.278 | $\frac{5}{2}^+$ | 1.02 | 0.76 | 8.69 | 1.2 | 0.6 |

[a] A $d_{5/2}$ transfer was assumed.

downward shift of the 9.27-MeV state relative to the 9.19-MeV state implies that the 9.27-MeV state has more $s$ strength than the 9.19-MeV state, or that the 9.27-MeV state has less $d$ strength than the 9.19-MeV state, or both. If the spectroscopic factors of mirror levels are equal, then in $^{11}$C we would expect that in case (1):

$S_{l=0}(8.69) > S_{l=0}(8.65)$ or

$S_{l=2}(8.69) < S_{l=2}(8.65)$, or both,

whereas in case (2):

$S_{l=0}(8.69) < S_{l=0}(8.65)$ or

$S_{l=2}(8.69) > S_{l=2}(8.65)$, or both.

The $^{10}$B$(d,p)^{11}$B data of Hinds and Middleton[4] for the 9.19- and 9.27-MeV states are shown in Fig. 3. The curves are the results of DWBA calculations. Several immediate observations can be made in regard to the data and calculations. First, the cross sections for both states are peaked at small angles, indicating the kinematic preference for $l=0$. Secondly, the cross section for the 9.19-MeV state is larger than that for the 9.27-MeV state by about 30% at the forward angles. Since $l=0$ is dominant this difference implies that the $l=0$ spectroscopic strength for the lower state is approximately 1.3 times the $l=0$ spectroscopic strength for the upper state. Thirdly, the first minimum for the lower state is much more filled in than that of the upper state, implying a much larger $l=2$ strength for the lower state than for the upper one. The spectroscopic strengths extracted from the $(d,p)$ reaction data are shown in Table II.

The $^{10}$B($^3$He, $d$)$^{11}$C angular distributions for the 8.65- and 8.69-MeV states are displayed in Fig. 4. Here, the angular momentum matching conditions are such that $l=2$ is kinematically favored. Furthermore, in the present case an $l=2$ angular distribution is peaked at 0°. Thus the observation of a larger forward-angle cross section for the 8.65-MeV state than for the 8.69-MeV state is an indication that the $l=2$ spectroscopic strength is larger for the lower state than for the upper state.

Comparison of this observation with the discussion of the $(d,p)$ data suggests that the lower state in $^{11}$C is the mirror of the lower state in $^{11}$B. This result is indeed borne out in detail by the calculations. The curves of Fig. 4 are the results of DWBA calculations, normalized to have the spectroscopic strengths in ($^3$He, $d$) equal to those in $(d,p)$. Possibility 1 is shown at the top and possibility 2 at the bottom of Fig. 4. Clearly possibility 1 is correct, and the level ordering in $^{11}$C is the same as in $^{11}$B.

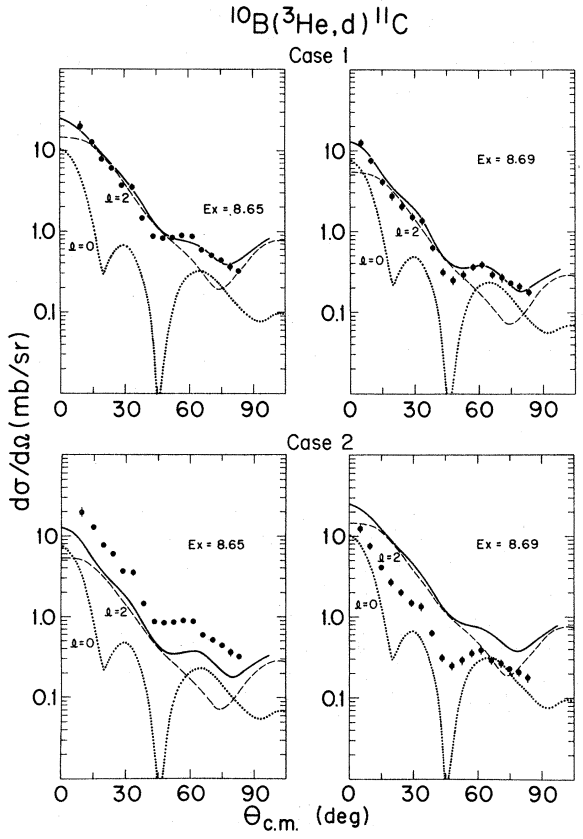


FIG. 4. Angular distributions of the $^{10}$B($^3$He,$d$)$^{11}$C reaction together with DWBA calculations normalized according to the spectroscopic factors obtained from the $(d,p)$ analysis, assuming that the level ordering in $^{11}$C is the same as in $^{11}$B (top) and is opposite (bottom). Solid lines are the admixed sums of $l=0$ and $l=2$.

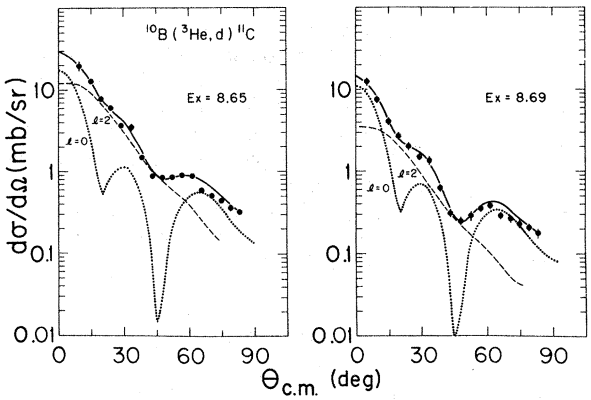


FIG. 5. Angular distributions and best-fit DWBA calculations for the $^{10}$B($^3$He,$d$)$^{11}$C reaction. Solid lines are the admixed sums of $l=0$ and $l=2$. Resulting spectroscopic strengths are listed in Table II.

The best-fit admixtures of $l=0$ and 2 for these two states are shown in Fig. 5, along with the data. The fits are slightly better than those of case (1) in Fig. 4, but the difference is not significant. The spectroscopic strengths extracted from the fits of Fig. 5 are listed in Table II, and are seen to be very similar to those extracted from the $(d, p)$ data. It is clear that reversing the ordering of the mirror correspondence would produce inferior agreement between measured and calculated angular distributions. For both nuclei the $S_{l=0}:S_{l=2}$ ratio for the lower state is less than 1, whereas for the upper state it is greater than 1. Furthermore, the ratio of lower : upper state strength for $l=2$ transfer is approximately the same for both nuclei.

Finally, the spectroscopic factors presented in Table II for both the $^{10}$B$(d, p)^{11}$B and $^{10}$B$(^3$He$, d)^{11}$C reactions are consistent with

$$S_{l=0}(\text{upper state}) \approx S_{l=0}(\text{lower state}),$$

and

$$S_{l=2}(\text{upper state}) < S_{l=2}(\text{lower state}),$$

again in agreement with the expectations of possibility 1.

We thus conclude that the levels have not crossed in going from $^{11}$B to $^{11}$C, and hence that $^{11}$B(9.19) and $^{11}$C(8.65) are mirrors, as are $^{11}$B(9.27) and $^{11}$C(8.69). Given the $J^\pi$ assignments in $^{11}$B, we may then assign $J^\pi = \frac{7}{2}^+$ to the 8.65-MeV state in $^{11}$C and $J^\pi = \frac{5}{2}^+$ to the 8.69-MeV state.

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# $^{69,71}$Ga$(d, p)$ and $^{71}$Ga$(d, t)$ Reactions*

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Angular distributions from the $(d, p)$ reaction on $^{69}$Ga and $^{71}$Ga have been obtained with the 12-MeV deuteron beam of the Argonne tandem Van de Graaff. Excitation energies and $l$-transfer values have been obtained for a number of levels below 2 MeV excitation. The ground state of $^{72}$Ga is not excited with observable cross section in the $^{71}$Ga$(d, p)^{72}$Ga reaction. The first level excited with an appreciable cross section is the $1^+$ level at 161 keV in $^{72}$Ga. The $(d,t)$ reaction on $^{71}$Ga proceeds to the ground state of $^{70}$Ga and, over the energy range covered, excites most of the levels observed to have a strong $l=1$ component in the $^{69}$Ga$(d, p)^{70}$Ga reaction. The excitation of the ground state of $^{70}$Ga via neutron pickup from $^{71}$Ga suggests that the neutron ground-state wave function of $^{72}$Ga has a large $(1g_{9/2})_{5/2+}{}^3$ component.

## INTRODUCTION

The energy levels of $^{70}$Ga and $^{72}$Ga have been studied by thermal-neutron capture $\gamma$ rays from the $^{69}$Ga$(n, \gamma)^{70}$Ga and $^{71}$Ga$(n, \gamma)^{72}$Ga reactions.[1,2] The present study of the $^{69}$Ga$(d, p)^{70}$Ga and $^{71}$Ga-$(d, p)^{72}$Ga was started in conjunction with the $(n, \gamma)$ studies,[1] in which primary transitions with dipole character are by far the most probable.[3] Therefore, the levels most likely to be directly populated from the $1^-$ and $2^-$ states in $^{70,72}$Ga are expected to have spins from 0 to 3. On the other hand, levels populated in the $(d, p)$ reaction at 12 MeV would be predominantly levels with an appreciable