

Nambu–Jona-Lasinio models beyond the mean-field approximation

P. P. Domitrovich, D. Bückers, and H. Mütter

Institut für Theoretische Physik, Universität Tübingen, D-7400 Tübingen, Germany

(Received 17 March 1993)

Inspired by the model of Nambu and Jona-Lasinio, various Lagrangians are considered for a system of interacting quarks. Employing standard techniques of many-body theory, the scalar part of the quark self-energy is calculated including terms up to second order in the interaction. Results obtained for the single-particle Green's function are compared with those which only account for the mean-field or Hartree-Fock term in the self-energy. Depending on the explicit form of the Lagrangian, the second-order contributions range between 4 and 90 percent of the leading Hartree-Fock term. This leads to a considerable momentum dependence of the self-energy and the effective mass of the quarks.

PACS number(2): 11.40.Fy, 12.40.Aq, 24.85.+p

I. INTRODUCTION

The so-called NJL model of Nambu and Jona-Lasinio was proposed more than 30 years ago [1]. In its original form, this model was constructed as a theory of nucleons interacting via an effective two-body interaction. Today, the fermions described by this model are usually reinterpreted as quarks, and, with this interpretation, the NJL model has become extremely popular over the past few years (see, e.g., the recent review by Klevansky [2] and the references quoted within it).

There are quite a few reasons for the popularity of the NJL model: A very important one is the fact that the Lagrange density is constructed to obey some of the symmetries of QCD. In particular, a model is constructed which permits the investigation of chiral symmetry of light quarks and the dynamical breaking of this symmetry. Dynamically broken chiral symmetry generates effective masses for the constituent quarks which are much larger than the bare current quark masses.

The attraction of the NJL model has increased as a result of successes regarding systematic attempts to determine effective theories which account for the most relevant degrees of freedom of QCD at low energies. It has been shown that the NJL model can be understood as a low-energy approximation to QCD [3,4]. A bosonization of the NJL Lagrangian leads to an effective Lagrangian in terms of meson fields, which in general shows satisfactory agreement with low-energy meson data [5]. In this sense, the NJL model can be described as an effective quark theory based on low-energy QCD, which permits the evaluation of certain observables. For example, it may provide useful predictions concerning properties such as the medium or temperature dependence of masses and coupling constants of low-energy hadrons [6,7], despite the fact that it does not predict confined quarks.

Most studies of the NJL model in its fermionic representation have been restricted to the mean-field or

Hartree-Fock (HF) approximation. In the present investigation, we advance beyond the HF approach and include in the definition of the irreducible self-energy of the quarks all terms up to second order in the interaction [see Figs. 1(a) and 1(b)]. The second-order terms of Fig. 1(b), describing the coupling of a quark to a configuration of a quark q plus a $q\bar{q}$ excitation, can be understood as the term of lowest-order describing the contribution to the quark self-energy by the exchange of a collective $q\bar{q}$ excitation, i.e., meson exchange. Note that, however, in contrast to, e.g., Cao, Shakin, and Sun [8], we not only want to consider the effects of π exchange, but also to include all relevant $q\bar{q}$ channels accounting for collective as well as noncollective modes.

Having determined the quark self-energy in this improved approximation, one could determine the corresponding single-particle Green's function and evaluate the polarization propagator, which contains the information about the $q\bar{q}$ excitation modes. This polarization propagator contains the contributions represented

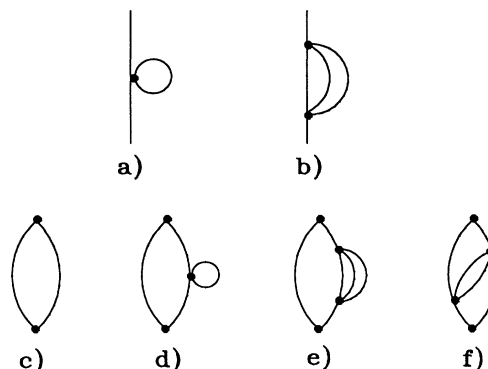


FIG. 1. Diagrams displaying the Hartree-Fock self-energy (a) and the self-energy of second-order in the interaction (b). The diagrams displayed in (c)–(f) exhibit various contributions to the polarization propagator as discussed in the text.

by the diagrams of Figs. 1(c)–1(e), among others, where the fermion lines represent the Green's functions of the bare quarks. The diagram of Fig. 1(e), in particular, is taken into account by our improved definition of the quark self-energy. This contribution might be significant, especially for the polarization propagator in the scalar-isoscalar channel describing the so-called σ meson. In this case, the coupling of the $q\bar{q}$ excitation modes to the $2q2\bar{q}$ configurations, such as, e.g., the 2π states, should be important. It is well known that the simple NJL model predicts a σ meson with a mass of about twice the constituent quark mass. This scalar meson is frequently compared to the σ meson, which is required for a one-boson-exchange description of the nucleon-nucleon interaction [9]. This meson is more representative of the exchange of two correlated pions than a well-defined $q\bar{q}$ meson.

However, evaluating the polarization propagators for the various meson modes by employing single-particle Green's functions with the inclusion of terms such as Fig. 1(b) in the self-energy for quarks, but using the bare interaction in the Bethe-Salpeter equation, would lead to an approach which obviously does not fulfill the Goldstone theorem. This means that, even for a Lagrangian which is truly invariant under a chiral transformation, the mass of the Goldstone boson associated with this symmetry would be different from zero. An approach which obeys the Goldstone theorem can only be expected if the single-particle propagator and the residual interaction used to calculate the polarization propagator are chosen in a consistent way. This implies that besides the diagram displayed in Fig. 1(e), terms represented by Fig. 1(f) must also be taken into account. Figure 1(f) can be interpreted as the leading contribution of the so-called induced interaction, which has been investigated in the many-body theory of Fermi liquids for such cases as nuclear matter [10,11] and liquid ^3He [12].

The present investigation should be considered as a first step towards such a consistent treatment of the NJL model beyond the mean-field approximation. For that purpose, we want to study the importance of the second-order terms in the self-energy [Fig. 1(b)] as compared to the Hartree-Fock contribution displayed in Fig. 1(a). Several Lagrangians will be considered, adjusting the coupling strength in such a way that the same constituent mass is obtained for quarks of zero momentum in each model. Following this Introduction, Sec. II will

describe the various Lagrangians employed, and Sec. III sketches the techniques used to evaluate the self-energy. The results of the calculations are discussed in Sec. IV, and the main conclusion will be summarized in Sec. V.

II. VARIOUS LAGRANGIANS FOR A NJL MODEL

In its original form [1], the NJL model was designed to describe a system of interacting nucleons, but has more recently been reinterpreted as a quark Lagrangian of identical form:

$$\mathcal{L}_D^{(A)} = \bar{\psi}(i\not{\nabla} - m_0)\psi + G_A [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau\psi)^2], \quad (1)$$

such that the fermion field ψ now represents quarks with SU(2) flavor (τ denoting the Pauli matrices for the flavor degrees of freedom) and SU(3) color. For vanishing current quark mass m_0 , this Lagrangian is invariant under a chiral transformation. Particular to this study, A denotes the first of the four models considered and D implies that only the direct part of the contact interaction defined in Eq. (1) is included.

Because of the presence of a contact interaction, the NJL model is nonrenormalizable and therefore requires a regularization that can be effected through the use of a cutoff scheme. Throughout this study, we will use the simplest, three-momentum noncovariant cutoff scheme. This means that all integrals over internal momenta are restricted to three-momenta $\mathbf{p}^2$ less than a cutoff parameter Λ^2 after carrying out the p_0 integration. As most of the qualitative features of the model are not very sensitive to the kind of regularization scheme employed, we expect that the qualitative results obtained in the present study should also be independent of the cutoff procedure.

Note that the quark-quark interaction in the model defined by Eq. (1) is assumed to be a pure color scalar. Therefore, we have suppressed the unit SU(3) operator acting on the color part of the spinors ψ . The color degrees of freedom become important when we want to account for the effects of the exchange terms in the two-fermion interaction. For a contact interaction such as the one defined in Eq. (1), the exchange terms can easily be determined by writing down the Fierz transformation of the direct interaction part in Eq. (1) and subtracting the exchange terms generated in this way from $\mathcal{L}_D$:

$$\mathcal{L}^{(A)} = \mathcal{L}_D^{(A)} - \mathcal{L}_E^{(A)} \quad (2)$$

$$\begin{aligned} &= \bar{\psi}(i\not{\nabla} - m_0)\psi + \frac{13}{12}G_A [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2] - \frac{1}{4}G_A(\bar{\psi}\gamma_\mu\lambda\psi)^2 \\ &\quad + \frac{1}{8}G_A [(\bar{\psi}i\gamma_5\tau\lambda\psi)^2 + (\bar{\psi}\lambda\psi)^2] - \frac{1}{8}G_A [(\bar{\psi}\tau\lambda\psi)^2 + (\bar{\psi}i\gamma_5\lambda\psi)^2] \\ &\quad - \frac{1}{12}G_A [(\bar{\psi}i\gamma_5\psi)^2 + (\bar{\psi}\tau\psi)^2] - \frac{1}{6}G_A(\bar{\psi}\gamma_\mu\psi)^2 + \dots \end{aligned} \quad (3)$$

One can see that the inclusion of the exchange terms not only leads to a renormalization of the coupling constant for those terms already existing in the direct part, but also generates additional interaction terms of both the color-scalar and color-vector type (λ refers to the Gell-Mann matrices acting on the color component of ψ). To shorten the notation, we have omitted the tensor and pseudovector components in the interaction. Also, the mesons associated

with the tensor and pseudovector interactions are not considered as relevant at nuclear length scales and are therefore neglected in this study.

An alternate choice for a Lagrangian, which in the limit of massless quarks is also symmetric under a chiral transformation and contains a color-scalar direct interaction term, is given by

$$\mathcal{L}_D^{(B)} = \bar{\psi}(i\not{\nabla} - m_0)\psi - G_B(\bar{\psi}\gamma_\mu\psi)^2. \quad (4)$$

Including the exchange terms in the same way as discussed above provides

$$\begin{aligned} \mathcal{L}^{(B)} = & \bar{\psi}(i\not{\nabla} - m_0)\psi - \frac{13}{12}G_B(\bar{\psi}\gamma_\mu\psi)^2 - \frac{1}{8}G_B[(\bar{\psi}\gamma_\mu\lambda\psi)^2 + (\bar{\psi}\gamma_\mu\tau\lambda\psi)^2] \\ & - \frac{1}{12}G_B(\bar{\psi}\gamma_\mu\tau\psi)^2 + \frac{1}{6}G_B[(\bar{\psi}\psi)^2 + (\bar{\psi}\tau\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau\psi)^2] \\ & + \frac{1}{4}G_B[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}\tau\lambda\psi)^2 + (\bar{\psi}i\gamma^5\lambda\psi)^2 + (\bar{\psi}i\gamma^5\tau\lambda\psi)^2] + \dots \end{aligned} \quad (5)$$

Starting from the path integral formulation of QCD, attempts have been made to eliminate the gluon degrees of freedom by trying to derive or motivate effective Lagrangians for quarks in the low-energy domain. Employing a current expansion of the effective quark action [13], or using the field strength approach [3,14], one obtains a quark-quark interaction defined in terms of color-vector currents. Using the nomenclature introduced before, the corresponding NJL-type Lagrangian would be written as

$$\mathcal{L}_D^{(C)} = \bar{\psi}(i\not{\nabla} - m_0)\psi - G_C(\bar{\psi}\gamma_\mu\lambda\psi)^2. \quad (6)$$

After the inclusion of the exchange terms, we obtain

$$\begin{aligned} \mathcal{L}^{(C)} = & \bar{\psi}(i\not{\nabla} - m_0)\psi - \frac{11}{12}G_C(\bar{\psi}\gamma_\mu\lambda\psi)^2 - \frac{4}{9}G_C[(\bar{\psi}\gamma_\mu\psi)^2 + (\bar{\psi}\gamma_\mu\tau\psi)^2] \\ & + \frac{1}{12}G_C(\bar{\psi}\gamma_\mu\tau\lambda\psi)^2 - \frac{1}{6}G_C[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}\tau\lambda\psi)^2 + (\bar{\psi}i\gamma^5\lambda\psi)^2 + (\bar{\psi}i\gamma^5\tau\lambda\psi)^2] \\ & + \frac{8}{9}G_C[(\bar{\psi}\psi)^2 + (\bar{\psi}\tau\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau\psi)^2] + \dots \end{aligned} \quad (7)$$

Finally, we would also like to consider a Lagrangian of the form that has been considered in Ref. [15] (without introducing a momentum dependence of the coupling constant):

$$\mathcal{L}_D^{(D)} = \bar{\psi}(i\not{\nabla} - m_0)\psi + G_D[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}i\gamma^5\tau\lambda\psi)^2], \quad (8)$$

which leads to

$$\begin{aligned} \mathcal{L}^{(D)} = & \bar{\psi}(i\not{\nabla} - m_0)\psi + \frac{11}{12}G_D[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}i\gamma^5\tau\lambda\psi)^2] + \frac{1}{6}G_D(\bar{\psi}\gamma_\mu\lambda\psi)^2 \\ & - \frac{8}{9}G_D(\bar{\psi}\gamma_\mu\psi)^2 + \frac{4}{9}G_D[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma^5\tau\psi)^2] \\ & + \frac{1}{12}G_D[(\bar{\psi}i\gamma^5\lambda\psi)^2 + (\bar{\psi}\tau\lambda\psi)^2] - \frac{4}{9}G_D[(\bar{\psi}\tau\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2] + \dots \end{aligned} \quad (9)$$

Comparing Eqs. (3), (5), (7), and (9), one can see that these four models yield quite different ratios between the coupling constants of the corresponding individual terms. Assuming the same cutoff parameter Λ , one would obtain in the Hartree-Fock approximation applied to the models (A–D) the same constituent quark mass m_{HF}^* from the so-called gap equation

$$m_{\text{HF}}^* = m_0 + \tilde{G} \frac{24}{(2\pi)^3} \int_0^\Lambda d^3p \frac{m_{\text{HF}}^*}{\sqrt{\mathbf{p}^2 + (m_{\text{HF}}^*)^2}}, \quad (10)$$

if the coupling strengths in front of the scalar-isoscalar terms in Eqs. (3), (5), (7), and (9) are chosen to be identical to $\tilde{G}$. This means

$$\tilde{G} = \frac{13}{12}G_A = \frac{1}{6}G_B = \frac{8}{9}G_C = \frac{4}{9}G_D. \quad (11)$$

However, by choosing the coupling constants in this way, we see that the strength in front of the other interaction terms will be quite different. Therefore, depending

on which model (A–D) we choose, we obtain different strengths for the residual interaction in the various channels even if we adjust the coupling constants in such a way that the Hartree-Fock approximation leads to the very same gap equation.

III. CALCULATION OF THE SELF-ENERGY

In calculating the self-energy contributions that are second order in the interaction, we will use the identification of momenta exhibited in Fig. 2. Note that we need only consider diagrams representing the direct terms since the exchange terms are taken into account by means of a Fierz transformation, as discussed above. Adopting the Feynman rules presented on pp. 107–108 of [16], the second-order term can be written as

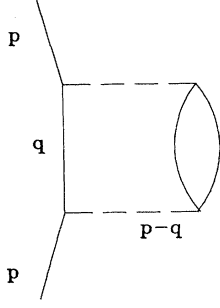


FIG. 2. Diagrams representing the second-order contribution to the self-energy for a fermion with momentum p . The labels for the momenta of the intermediate fermion (q) and the polarization and boson propagators ($p - q$) are identical to those used in Eq. (12).

$$\Sigma_{\gamma\delta}(p) = i \sum_{\epsilon\eta} \int \frac{d^4 q}{(2\pi)^4} \Gamma_{\gamma\epsilon}(p-q) \Delta(p-q) g_{\epsilon\eta}(q) \times \Gamma_{\eta\delta}(p-q) \Pi^{(0)}(p-q). \quad (12)$$

As an example, we will discuss the case in which all interaction vertices are given by the scalar part of the Lagrangian:

$$\mathcal{L} = \tilde{G}(\bar{\psi}\psi)^2. \quad (13)$$

This implies that the vertices Γ , which are independent of the momentum variables, are given by

$$\Gamma_{\alpha\beta}(q) = \sqrt{2\tilde{G}} \delta_{\alpha\beta}, \quad (14)$$

with α and β referring to the indices of the color matrices, flavor matrices, and Dirac spinors. In Eq. (12), $g_{\epsilon\eta}(q)$ refers to the propagator of a quark with momentum q and is multiplied by an identity matrix in color space and flavor space. Also, the “interacting boson propagator” $\Delta(p-q)$ can be replaced by (-1) , due to the contact interaction. The polarization propagator in its irreducible (lowest order form) $\Pi^{(0)}$ is defined for the various excitation modes λ by

$$\Pi_{\lambda}^{(0)}(q) = -i \int \frac{d^4 k}{(2\pi)^4} \text{tr}[\Gamma_{\mu\nu}^{\lambda}(q) g_{\nu\rho}(k) \Gamma_{\rho\sigma}^{\lambda}(q) \times g_{\sigma\mu}(k-q) \Delta(q)], \quad (15)$$

where the trace is over the three sets of indices mentioned above. Inserting the vertices for the scalar excitation modes [Eq. (14), $\lambda = s$] and assuming for the Green’s function g a Hartree-Fock self-energy for quarks which is characterized by a constant effective mass m^* [see Eq. (10)], the imaginary part of the polarization propagator at zero Fermi energy is given by

$$\text{Im}\Pi_s^{(0)}(q) = \frac{\tilde{G}n_f n_c}{4\pi^2} \int \frac{d^3 k}{E_k^* E_{k-q}^*} \times [E_k^* E_{k-q}^* + \mathbf{k}(\mathbf{k} - \mathbf{q}) - m^{*2}] \times \delta(|q_0| - E_k^* - E_{k-q}^*) \Theta(\Lambda - |\mathbf{k}|) \times \Theta(\Lambda - |\mathbf{k} - \mathbf{q}|), \quad (16)$$

where n_f and n_c stand for the number of flavors (2) and colors (3), respectively. Furthermore,

$$E_k^* = \sqrt{\mathbf{k}^2 + m^{*2}}. \quad (17)$$

The real part of the polarization propagator is related to the imaginary part by a dispersion relation

$$\text{Re}\Pi_{\lambda}^{(0)}(q) = \frac{1}{\pi} \text{P} \int_0^{\infty} dq'_0 \text{Im}\Pi_{\lambda}^{(0)}(q'_0, \mathbf{q}) \times \left[\frac{1}{q'_0 - q_0} + \frac{1}{q'_0 + q_0} \right]. \quad (18)$$

In this paper, we will focus our attention on the scalar part of the self-energy, which is obtained from Eq. (12) with the aid of the relation

$$\Sigma^{(s)} = \frac{1}{4n_f n_c} \text{tr}(\Sigma). \quad (19)$$

Taking the real part of the scalar part of the self-energy for the scalar interaction and utilizing the dispersion relation (18), we obtain

$$\delta \text{Re}\Sigma_s^{(s)}(p) = \text{Re} \left[\frac{-i\tilde{G}}{2n_f n_c} \int \frac{d^4 q}{(2\pi)^4} \text{tr}[g(q)] \Pi_s^{(0)}(p-q) \right] \quad (20)$$

$$= -\frac{\tilde{G}m^*}{4\pi^3} \int_0^{\Lambda} \frac{\mathbf{q}^2 dq}{E_q^*} \int_{-1}^{+1} dx \times \left\{ \text{P} \int_0^{\infty} dq_0 \frac{\text{Im}\Pi_s^{(0)}(q_0, |\mathbf{p} - \mathbf{q}|)}{q_0 + p_0 + E_q^*} + \text{P} \int_0^{\infty} dq_0 \frac{\text{Im}\Pi_s^{(0)}(q_0, |\mathbf{p} - \mathbf{q}|)}{q_0 - p_0 + E_q^*} \right\}. \quad (21)$$

The integration variable x stands for the cosine of the angle between $\mathbf{p}$ and $\mathbf{q}$. From Eq. (21), one can verify immediately that the scalar contribution indeed yields results which only depend on the absolute value of the zero component p_0 .

If instead of the scalar part of the interaction (13) one now considers a pseudoscalar interaction

$$\mathcal{L} = \tilde{G}(\bar{\psi}i\gamma^5\psi)^2, \quad (22)$$

the corresponding imaginary part of the polarization propagator is

$$\text{Im}\Pi_{ps}^{(0)}(q) = \frac{\tilde{G}n_f n_c}{4\pi^2} \int \frac{d^3k}{E_k^* E_{k-q}^*} \left[E_k^* E_{k-q}^* + \mathbf{k}(\mathbf{k} - \mathbf{q}) + m^{*2} \right] \delta(|q_0| - E_k^* - E_{k-q}^*) \Theta(\Lambda - |\mathbf{k}|) \Theta(\Lambda - |\mathbf{k} - \mathbf{q}|). \quad (23)$$

Equation (23) can be used to evaluate the contribution to the scalar part of the self-energy originating from Eq. (22):

$$\delta\text{Re}\Sigma_s^{(ps)}(p) = \text{Re} \left[\frac{-i\tilde{G}}{2n_f n_c} \int \frac{d^4q}{(2\pi)^4} \text{tr} [i\gamma^5 g(q) i\gamma^5] \Pi_{ps}^{(0)}(p - q) \right]. \quad (24)$$

This yields a result similar to Eq. (21), except that one must replace $\Pi_s^{(0)}$ by $\Pi_{ps}^{(0)}$ and supply an overall minus sign, since $\text{tr}[i\gamma^5 g(q) i\gamma^5] = -\text{tr}[g(q)]$.

Finally, we consider the contribution of second order originating from a vector interaction:

$$\mathcal{L} = -\tilde{G}(\bar{\psi}\gamma_\mu\psi)^2. \quad (25)$$

In this case, the contribution to the self-energy is given by

$$\delta\text{Re}\Sigma_s^{(v)}(p) = \frac{\tilde{G}m^*}{4\pi^3} \int_0^\Lambda \frac{\mathbf{q}^2 dq}{E_q^*} \int_{-1}^{+1} dx \left\{ P \int_0^\infty dq_0 \frac{\sum_\phi \text{Im}\Pi_{v,\phi}^{(0)}(q_0, |\mathbf{p} - \mathbf{q}|)}{q_0 + p_0 + E_q^*} \right. \\ \left. + P \int_0^\infty dq_0 \frac{\sum_\phi \text{Im}\Pi_s^{(0)}(q_0, |\mathbf{p} - \mathbf{q}|)}{q_0 - p_0 + E_q^*} \right\}, \quad (26)$$

with

$$\text{Im} \sum_\phi \Pi_{v,\phi}^{(0)}(q) = \text{Im} \sum_\phi \left\{ -2i\tilde{G} \int \frac{d^4k}{(2\pi)^4} \text{tr} [\gamma^\phi g(k + q/2) \gamma_\phi g(k - q/2)] \right\} \quad (27)$$

$$= + \frac{\tilde{G}n_f n_c}{2\pi^2} \int \frac{d^3k}{E_k^* E_{k-q}^*} \left[E_k^* E_{k-q}^* + \mathbf{k}(\mathbf{k} - \mathbf{q}) + 2m^{*2} \right] \\ \times \delta(|q_0| - E_k^* - E_{k-q}^*) \Theta(\Lambda - |\mathbf{k}|) \Theta(\Lambda - |\mathbf{k} - \mathbf{q}|). \quad (28)$$

Since the imaginary part of the vector polarization propagator in the preceding equation is always positive, the real part of the second-order scalar self-energy due to the vector interaction has the same sign as the analogous contribution due to the pseudoscalar interaction.

The results presented thus far can easily be extended to interaction terms which are isovector rather than the isoscalar in Eqs. (13), (22), and (25). If

$$\tilde{G}(\bar{\psi}\hat{O}\psi)^2 \Rightarrow \tilde{G}(\bar{\psi}\hat{O}\boldsymbol{\tau}\psi)^2, \quad (29)$$

then one obtains the corresponding second-order contribution to the scalar part of the self-energy with an additional factor of 3, which originates from the different flavor term in the interaction. If one takes into account interaction terms which are products of vector operators in SU(3) color space, then

$$\tilde{G}(\bar{\psi}\hat{O}\psi)^2 \Rightarrow \tilde{G}(\bar{\psi}\hat{O}\boldsymbol{\lambda}\psi)^2, \quad (30)$$

and the corresponding contributions to the self-energy should be multiplied by a factor 32/9.

IV. RESULTS AND DISCUSSION

In order to display the basic features of the second-order contributions to the scalar self-energy, a few plots obtained for the scalar interaction [see Eqs. (13), (20), and (21)] are presented in Fig. 3. In evaluating these contributions, the Green's functions for the quarks have been approximated by Green's functions for fermions with a constant effective mass $m^*=313$ MeV, and the integrals have been regularized by a cutoff parameter $\Lambda = 653$ MeV (see Table II of Ref. [2]). This cutoff parameter will remain unchanged for all results presented unless explicitly stated otherwise. The choice for the coupling constant $\tilde{G}$ does not affect the qualitative features that we want to discuss in the first part of this section since all results for the second-order contributions of the self-energy can be scaled by $\tilde{G}^2$. The numerical values displayed in Fig. 3 were obtained by first employing $\tilde{G}=1.1088 \times 10^{-5} \text{ MeV}^{-2} = G_D$ and can be obtained from Eqs. (10) and (11). These self-energy values were then scaled such that the result for the vector interaction, Eq. (26), yielded a self-energy contribution of 1 MeV for

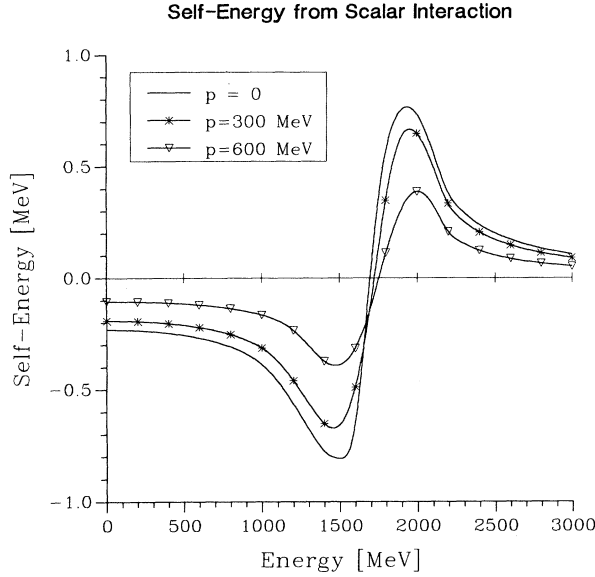


FIG. 3. Real part of the scalar term of the self-energy for quarks with momentum p , assuming a scalar interaction [Eq. (13)]. For various values of $|p|$, results are displayed as a function of p_0 . The self-energy has been calculated assuming a Green's function for quarks characterized by a constant effective mass $m^* = 313$ MeV, a cutoff parameter $\Lambda = 653$ MeV, and a coupling constant $\tilde{G} = 1.1088 \times 10^{-5}$ MeV $^{-2}$.

quarks with $|p| = p_0 = 0$ (see Fig. 5 also).

Inspecting the results in Fig. 3 for fermions with momentum $|p| = 0$ as a function of the energy p_0 , one observes, centered about $p_0 = 1500$ MeV, a resonance structure which is typical for a contribution to the self-energy of second or higher order. This resonance structure simply reflects the fact that diagrams, such as the one displayed in Fig. 1(b), describe the modification of the single-particle propagator due to the admixture of two-particle-one-hole (2p-1h) configurations. This admixture provides a nonvanishing imaginary part of the self-energy for energies p_0 which are larger than the threshold of such 2p-1h configurations. For the quark self-energy with momentum $p = 0$, this threshold is at $p_0 = 3m^*$ ($= 939$ MeV), while the largest energy leading to a nonvanishing imaginary part is determined by the cutoff parameter. The energy dependence of the real part of the self-energy is determined by these characteristics of the imaginary part because both are related to each other by a dispersion relation. It is worth recalling that the self-energy is symmetric with respect to $p_0 = 0$: $\Sigma(-p_0) = \Sigma(p_0)$.

For momenta $|p|$ larger than zero, the energy dependence is rather similar. In this case, however, one observes that the absolute value of the real part of the self-energy is slightly reduced as compared to the $|p|=0$ case. This reduction can be related to the fact that the phase space of 2p-1h configurations with momenta larger than zero is more strongly affected by the cutoff than it is for $|p|=0$.

Very similar features can be observed if a pure pseudoscalar interaction [see Eq. (22)] is considered. Results

for the real part of the scalar term of the self-energy are displayed in Fig. 4. Also, for these results, the same values of m^* , Λ , and $\tilde{G}$ were employed as those discussed above for the scalar interaction. Comparing the results of Figs. 3 and 4, one notices that the real part of the self-energy for a pseudoscalar interaction has the opposite sign as that of the self-energy for a scalar interaction. This difference can be understood by comparing Eqs. (20) and (24). Furthermore, one finds that, using the same coupling constant, the absolute value of an arbitrary self-energy point obtained from a pseudoscalar interaction is about twice as large as a corresponding point obtained from the scalar interaction. This difference is reflected by the fact that the polarization propagators $\Pi_s^{(0)}$ and $\Pi_{ps}^{(0)}$ differ [compare Eqs. (16) and (23)]. The dominance of the pseudoscalar contribution with respect to the contribution obtained from a scalar interaction is very important since chiral symmetry always relates the coupling strengths for these two interaction channels. Hence, we will always observe a partial cancellation of the second-order contributions to the self-energy obtained from the scalar and pseudoscalar interactions; however, their combined effect yields a positive contribution to the real part of the self-energy for energies p_0 below the threshold of 2p-1h configurations.

Another positive contribution to the real part of the scalar self-energy at low energies is obtained for a pure vector interaction of the form found in Eq. (25). This fact is illustrated in Fig. 5 by comparing self-energy plots obtained from the three interactions utilized in this study. These self-energies were calculated with the same coupling constant $\tilde{G}$ and scaled as discussed earlier. One finds that the contribution obtained from the vector interaction is even larger in absolute value than that obtained from the pseudoscalar interaction. Note, however, that this fact does not lead to a model-independent con-

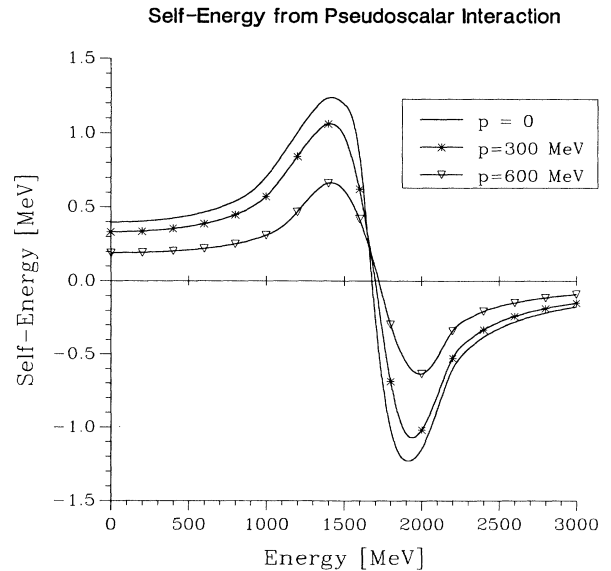


FIG. 4. Real part of the scalar self-energy for quarks with various momenta $|p|$ assuming a pseudoscalar interaction [Eq. (22)]. For further details, see the caption of Fig. 3.

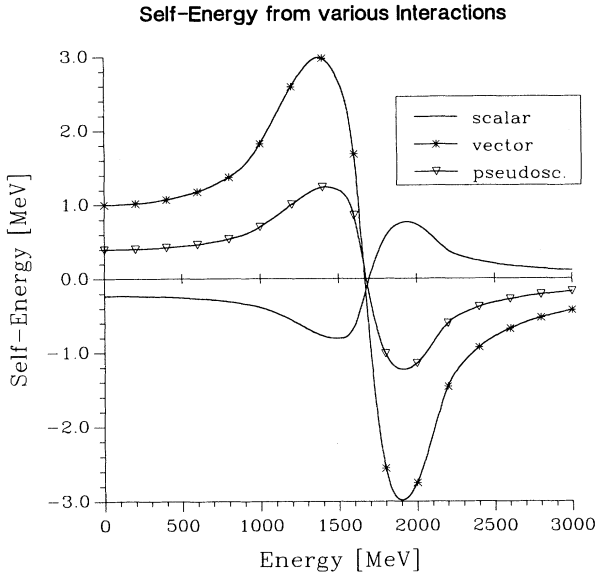


FIG. 5. Real part of the scalar self-energy for quarks with momentum $|\mathbf{p}|=0$, assuming various interaction terms. For further details, see the caption of Fig. 3.

clusion concerning the relative importance of this contribution, since the coupling constant for a vector interaction is independent of the interaction strength of the scalar and pseudoscalar interactions: Their ratio is not constrained by chiral symmetry.

Again, with the same scaling procedure for the self-energies, it is apparent that the dependence of the absolute value of the real parts of the scalar self-energies on the three-momentum $|\mathbf{p}|$ is very similar for all three interactions. This fact is displayed in Fig. 6.

In the next step of our discussion, we turn to the vari-

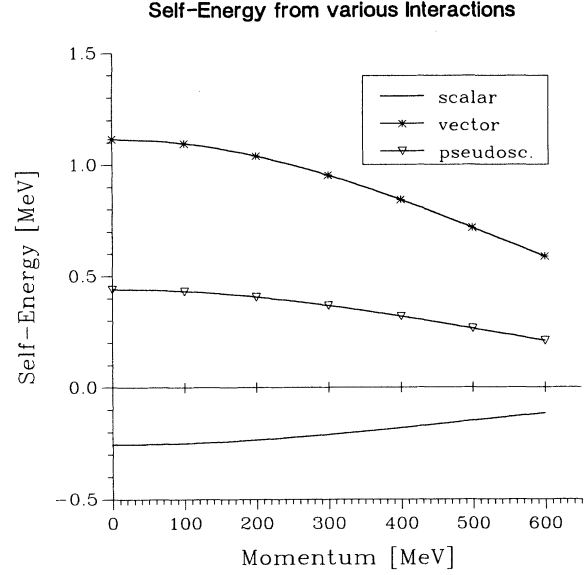


FIG. 6. Real part of the scalar self-energy for quarks with momentum p , assuming various interaction terms. Assuming $p_0 = 1$ GeV, results are presented as a function of $|\mathbf{p}|$. For further details, see the caption of Fig. 3.

ous Lagrangians discussed in Sec. II, which were denoted as models A , B , C , and D . The coupling constant for each Lagrangian is now adjusted by requesting that the sum of its Hartree-Fock and second-order contributions leads to a self-energy for $|\mathbf{p}|=0$ and $p_0 = m^* = 313$ MeV which is identical to p_0 :

$$p_0 = m_0 + \text{Re}\Sigma_s^{\text{HF}} + \Sigma_s^{(2)}(p_0, |\mathbf{p}|=0). \quad (31)$$

As an example, we consider Lagrangian A [see Eq. (3)] for which this equation takes the form

$$\begin{aligned} p_0 = m_0 &+ G_A \frac{13}{12} \frac{24}{(2\pi)^3} \int_0^\Lambda d^3p \frac{m^*}{\sqrt{\mathbf{p}^2 + m^{*2}}} \\ &+ G_A^2 \left[\delta\tilde{\Sigma}_s^{(s)}(p_0, \mathbf{0}) \left\{ \left(\frac{13}{12}\right)^2 + \left(\frac{1}{12}\right)^2 3 + \left(\frac{1}{8}\right)^2 \frac{32}{9} + \left(\frac{1}{8}\right)^2 \frac{32}{3} \right\} \right. \\ &\quad + \delta\tilde{\Sigma}_s^{(ps)}(p_0, \mathbf{0}) \left\{ \left(\frac{1}{12}\right)^2 + \left(\frac{13}{12}\right)^2 3 + \left(\frac{1}{8}\right)^2 \frac{32}{9} + \left(\frac{1}{8}\right)^2 \frac{32}{3} \right\} \\ &\quad \left. + \delta\tilde{\Sigma}_s^{(v)}(p_0, \mathbf{0}) \left\{ \left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2 \frac{32}{9} \right\} \right]. \end{aligned} \quad (32)$$

In this equation, $\delta\tilde{\Sigma}_s^{(i)}$ represents the second-order contribution to the scalar part of the self-energy originating from an interaction of type i ($i = s, ps$, or v), calculated for a coupling constant $\tilde{G} = 1$. The factors multiplying these contributions refer to the actual value of the coupling constant in this model [see Eq. (3)] multiplied by the appropriate flavor-color factors, as discussed at the

end of Sec. III. The Hartree-Fock term is calculated following the gap equation (10), and the current quark mass m_0 has been chosen to be 5 MeV. From Eq. (32), one can determine the new G_A and the coupling constants of the other models in a corresponding way. This procedure leads to the values which are quoted in the caption of Table I.

TABLE I. Various contributions to the real part of the scalar self-energy. Listed are the contributions from the Hartree-Fock term and the second-order contributions originating from the various interaction terms. Results are presented for three-momentum $|\mathbf{p}| = 0$ and energy $p_0 = m^* = 313$ MeV. Different Lagrangians have been considered for each of the models *A*, *B*, *C*, and *D* introduced in Sec. II. In each case, the coupling constant G has been adjusted in such a way that the sum of all contributions listed yields $m^* = 313$ MeV. The following coupling constants were obtained: $G_A = 4.38948 \times 10^{-6}$ MeV $^{-2}$, $G_B = 1.53437 \times 10^{-5}$ MeV $^{-2}$, $G_C = 4.80885 \times 10^{-6}$ MeV $^{-2}$, and $G_D = 7.75355 \times 10^{-6}$ MeV $^{-2}$. All entries are given in MeV.

Term	Model A	Model B	Model C	Model D
HF	297.204	159.830	267.159	215.376
$(\bar{\psi}\psi)^2$	-2.080	-0.602	-1.681	-1.092
$(\bar{\psi}\tau\psi)^2$	-0.037	-1.805	-5.043	-3.277
$(\bar{\psi}\lambda\psi)^2$	-0.098	-4.813	-0.210	-16.523
$(\bar{\psi}\tau\lambda\psi)^2$	-0.295	-14.439	-0.630	-0.410
$(\bar{\psi}i\gamma_5\psi)^2$	0.021	1.031	2.880	1.872
$(\bar{\psi}i\gamma_5\tau\psi)^2$	10.694	3.093	8.641	5.616
$(\bar{\psi}i\gamma_5\lambda\psi)^2$	0.169	8.247	0.360	0.234
$(\bar{\psi}i\gamma_5\tau\lambda\psi)^2$	0.506	24.742	1.080	89.491
$(\bar{\psi}\gamma_\mu\psi)^2$	0.213	109.945	1.818	18.901
$(\bar{\psi}\gamma_\mu\tau\psi)^2$	0.0	1.952	5.453	0.0
$(\bar{\psi}\gamma_\mu\lambda\psi)^2$	1.704	5.205	27.492	2.363
$(\bar{\psi}\gamma_\mu\tau\lambda\psi)^2$	0.0	15.614	0.682	0.0

Table I lists the various contributions to the effective mass of quarks with momentum $\mathbf{p}$ equal to zero if the coupling constant is chosen according to procedure just outlined. By construction, the sum of all contributions is equal to $m^* = 313$ MeV. The importance of individual components, however, depends to some extent on the Lagrangian chosen for the model. For example, in our model *A*, the original NJL Lagrangian of Eq. (1), the effective mass is dominated by the Hartree-Fock contribution. This can immediately be understood since the coupling constant in front of the scalar-isoscalar-color-scalar interaction term $[(\bar{\psi}\psi)^2]$ determines the importance of the Hartree-Fock term. In Eq. (3), which shows the Lagrangian with inclusion of exchange terms, this constant is large ($13/12G_A$) compared to the interaction strengths found in the other interaction channels. Because of this large coupling constant in the scalar and “pionic” channels, it is clear that, also, the second-order contributions are dominated by the contributions of these interaction channels (see first column of Table I). In this model, however, the second-order contributions are small compared to the leading HF term.

The situation is somewhat different for model *B*, where it is assumed that the direct interaction is a pure vector-isoscalar-color-scalar one [Eq. (4)]. In this case the coupling constant defining the strength in the scalar channel is rather weak ($1/6 G_B$) compared to those in the other channels. But, also, in this case more than 50% of the effective mass is due the Hartree-Fock contribution.

While for models *A* and *D* the coupling to the pseudoscalar—i.e., “pionic” excitations are dominant—the largest contributions to the second-order terms in models *B* and *C* originate from the coupling to vector excitation modes. The second-order contributions describing the coupling to the “pionic” excitations can be un-

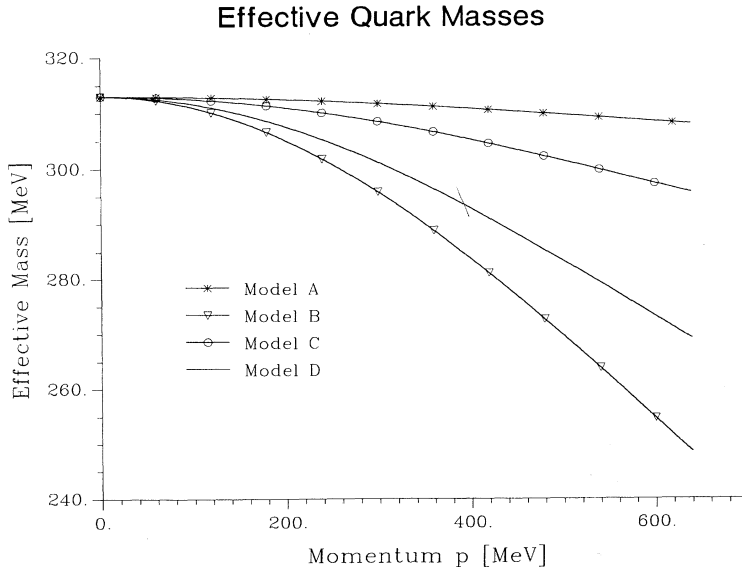


FIG. 7. Effective mass $M_{\text{eff}}(\mathbf{p})$ as defined in Eq. (34) for various momenta. Using the coupling constants provided in the caption of Table I, the effective masses approach the value 313 MeV in the limit of zero three-momentum for all four models of the Lagrangian.

derstood as a first step towards an attempt to include the Fock contributions to the quark self-energy due to pion exchange [8]. To include these pion exchange contributions in a microscopic way, one would have to replace the irreducible polarization propagator $\Pi_{ps}^{(0)}$ in Eq. (24) by the corresponding collective propagator. This would lead to an imaginary part in the quark self-energy at lower energies than the threshold discussed above and also shift the characteristic energy dependence of the real part of the self-energy displayed in Fig. 4 to lower energies. It should be kept in mind, however, that the contribution to the self-energy originating from the pionic excitations is only a part of the entire contribution.

An important difference between the HF contribution to the self-energy and those originating in second order is the fact that the latter contributions vary with the energy and momentum of the quark under consideration. In order to visualize these effects, we define an effective mass $M_{\text{eff}}(\mathbf{p})$ which characterizes the pole term in the Green's function for a quark with momentum $\mathbf{p}$ by:

$$p_0^2 = \mathbf{p}^2 + \left[m_0 + \text{Re}\Sigma_s^{\text{HF}} + \delta\text{Re}\Sigma_s^{(2)}(p_0, \mathbf{p}) \right]^2 \quad (33)$$

$$= \mathbf{p}^2 + M_{\text{eff}}^2(\mathbf{p}). \quad (34)$$

Results for this effective mass are displayed in Fig. 7. Using the coupling constants defined in Table I, all four models must clearly provide an effective mass of 313 MeV in the limit of vanishing three-momentum. Also, in each model under consideration, the effective mass decreases as a function of increasing three-momentum. This demonstrates that, for those combinations of p_0 and $\mathbf{p}$ which solve Eq. (34), the decrease of the second-order

TABLE II. Various contributions to the real part of the scalar self-energy originating from Hartree-Fock and second-order contributions classified according to the Dirac structure only of the implemented interaction. Results are compared for quarks with momenta $\mathbf{p} = 0$ choosing two different cutoff parameters Λ . For further remarks, see the caption of Table I.

Term	Model A	Model B	Model C	Model D
$\Lambda = 653 \text{ MeV}$				
HF	297.204	159.830	267.159	215.376
scalar	-2.511	-21.658	-7.564	-21.303
ps.	11.390	37.113	12.961	92.663
vector	1.917	132.715	35.444	21.264
$\Lambda = 800 \text{ MeV}$				
HF	297.764	163.752	269.608	218.292
scalar	-2.845	-25.658	-8.694	-24.697
ps.	11.264	38.381	13.005	93.783
vector	1.816	131.524	34.080	20.623

terms of the self-energy with momentum, as displayed in Fig. 6, dominates the increase of these contributions with increasing p_0 , as displayed in Figs. 3–5. A substantial enhancement is obtained only for values of p_0 which are larger than those that solve Eq. (34). The momentum dependence is of course larger for model B than for model A since the contributions of second order are more important in the former model.

Finally, we want to investigate the influence of the value chosen for the cutoff parameter Λ . For that purpose, in Table II we provide a comparison of the various contributions to the self-energy obtained from a calculation for $\Lambda = 800 \text{ MeV}$ with the results discussed so far using $\Lambda = 653 \text{ MeV}$. The coupling constants for the various models with $\Lambda = 800 \text{ MeV}$ are also adjusted to obtain an effective mass M_{eff} of 313 MeV in the limit of vanishing three-momentum. An increase of the cutoff parameter yields a slight decrease in the total second-order contribution as compared to the leading HF term. All other features discussed remain essentially the same.

V. CONCLUSION

The effects of second-order contributions to the real part of the scalar self-energy of quarks are discussed in various models inspired by the Nambu-Jona-Lasinio (NJL) model. Depending on the explicit form chosen for the Lagrangian, these second-order terms yield contributions which range between 4% and 90% of the leading Hartree-Fock contribution. The second-order contributions depend on the energy and three-momentum of the quarks. This leads to a momentum-dependent effective quark mass which decreases with increasing momentum. The reduction can be as large as 20% for momenta near the cutoff.

In calculating the second-order contributions, a partial cancellation is observed between terms arising from the coupling to scalar excitations on one hand and vector and pseudoscalar excitations on the other. The relative importance of scalar and pseudoscalar contributions as compared to the coupling to vector excitation modes is dictated by the Lagrangian of the model considered. A restriction to the pionic modes, i.e., to restrict the self-energy contributions to those arising from pion exchange [8], is justified for certain choices of the Lagrangian only.

The investigations presented here must be considered as a first step towards a more detailed investigation of NJL inspired models beyond the Hartree-Fock approximation. Presently, we have approximated the calculation of the self-energy by assuming a Green's function for quarks with a constant effective mass. In a consistent calculation, the Green's function should be determined from the self-energy and account for the momentum dependence of the effective mass [10,17]. Since, however, this momentum dependence is apparently moderate, we think that the main features observed here should remain intact in such a self-consistent calculation.

Furthermore, one should study the properties of mesons which are obtained from a calculation of the various polarization propagators which goes beyond the

Hartree-Fock-random-phase approximation scheme that has been used until now. As has already been discussed in the Introduction (see Fig. 1), for such an investigation a consistent improvement of the quark propagator and the residual interaction used to evaluate the polarization propagators is required. In a next-step calculation, such “collective” polarization propagators could be used to improve the calculation of the self-energy. Also, one may study other terms besides the scalar contribution

to the self-energy, investigate the effects of various cutoff procedures, and explore the modifications obtained for nonvanishing baryon density and temperature.

This work has partly been supported by the Deutsche Forschungsgemeinschaft (DFG Fa 67/14-1) and the Graduiertenkolleg “Struktur und Wechselwirkung von Hadronen und Kernen” (DFG Mu 705/3-1).

-
- [1] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); **124**, 246 (1961).
 - [2] S.P. Klevansky, *Rev. Mod. Phys.* **64**, 649 (1992).
 - [3] M. Schaden, H. Reinhardt, P.A. Amundsen, and M.J. Lavelle, *Nucl. Phys.* **B339**, 595 (1990).
 - [4] R. Alkofer and H. Reinhardt, *Z. Phys. A* **343**, 79 (1992).
 - [5] D. Ebert and H. Reinhardt, *Nucl. Phys.* **B271**, 188 (1986).
 - [6] V. Bernard, U.-G. Meissner, and I. Zahed, *Phys. Rev. D* **36**, 819 (1987).
 - [7] T. Hatsuda and T. Kunihiro, *Phys. Lett. B* **185**, 304 (1987).
 - [8] Nan-Wei Cao, C.M. Shakin, and Wei-Dong Sun, *Phys. Rev. C* **46**, 2535 (1992).
 - [9] R. Machleidt, *Adv. Nucl. Phys.* **19**, 189 (1989).
 - [10] W.H. Dickhoff and H. Mütter, *Rep. Prog. Phys.* **11**, 1947 (1992).
 - [11] W.H. Dickhoff, A. Faessler, J. Meyer-ter-Vehn, and H. Mütter, *Nucl. Phys.* **A368**, 445 (1981).
 - [12] S. Babu and G.E. Brown, *Ann. Phys. (N.Y.)* **78**, 1 (1973).
 - [13] R.T. Cahill and C.D. Roberts, *Phys. Rev. D* **36**, 2419 (1985).
 - [14] H. Reinhardt, *Phys. Lett. B* **248**, 365 (1990).
 - [15] D. Bückers and H. Mütter, *Nucl. Phys.* **A523**, 629 (1991).
 - [16] B.D. Serot and J.D. Walecka, *Adv. Nucl. Phys.* **16**, 1 (1986).
 - [17] H. Mütter and L.D. Skouras, *Nucl. Phys.* (to be published).