Clustered quark matter at finite temperature

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The properties of clustered quark matter at finite temperature are studied. It is shown that clustered quark matter can exist only at low temperatures and then it is difficult to detect the new phase by high-energy heavy-ion collisions.

Quantum chromodynamics (QCD) has been considered the best candidate for strong interaction theory. Polyakov[1] and Susskind[2] have shown that little QCD in the strong coupling limit exhibits a deconfinement transition at a finite temperature. It is expected that quarks are confined in individual hadrons at low temperatures and low densities, and form the quark matter phase at high temperatures or at high densities.[4] Recently Clark *et al.* proposed that a new phase of hadronic matter, clustered quark matter, may exist at zero temperature.[3]

Clustered quark matter, as shown in Fig. 1, consists of two parts; clusters, each of which is viewed as consisting of three interacting quarks in a color-zero, spin-$\frac{1}{2}$ and isospin-$\frac{1}{2}$ state; and quarks in the surrounding medium. Clustered quark matter cannot be stabilized only by a Coulombic quark interaction. A large bag is introduced to overcome this problem. The whole system of clustered quark matter is contained in a large bag. The bag constant gives the quantity of the energy per volume necessary to form the bag. The external pressure produced by the difference between the two vacuums inside ($B\neq0$) and outside ($B=0$) the large bag keeps the clustered quark matter stable. Clustered quark matter differs from usual nuclear matter. Clark *et al.* show that clustered quark matter gains an energetic, advantage at zero temperature.[3] Since one of the possible ways to investigate the phase transition exists in high-energy heavy-ion collisions, where matter is in a high-temperature and high-density state, it is useful to study the properties of the new hadronic phase at finite temperature, which will have a direct influence on the investigation of high-energy heavy-ion collisions. In this paper, we extend the work of Clark *et al.* on the properties of clustered quark matter from zero to finite temperature.

The mass of the cluster, as in Ref. 3, is given by

$$M_c=\frac{c}{R}\ , \tag{1}$$

where R is the radius of the cluster. The strong coupling constant varies with baryon number density ν; we use the following linear parametrization, in the lowest order approximation:

$$c=a+\left[\frac{c^0-a}{\alpha_s(2\nu_0)}\right]\alpha_s(\nu)\ , \tag{2}$$

$$\alpha_s=\frac{\alpha_s^0}{1+\ln[(\nu/2\nu_0)^{1/3}]}\ . \tag{3}$$

The value of c at $\nu=2\nu_0$ ($\nu_0=0.16\ \mathrm{fm}^{-3}$, the normal density of nuclear matter) is denoted by c^0. It is related to the mass of the nucleon in bag model calculations:

$$M_N=\tfrac{4}{3}c^{3/4}(4\pi B)^{1/4}\ . \tag{4}$$

Taking $a=4$, $\alpha_s^0=0.6$, $M_N=939$ MeV, and $B^{1/4}=171$ MeV,[3,5] we obtain $c^0=2.84$.

The three quarks making up a cluster exercise an outward pressure which is given by

$$\begin{aligned}P&=-\frac{\partial M_c}{\partial V_c}=\frac{c}{4\pi R^4}\\&=\frac{M_c^4}{4\pi c^3}\\&=\frac{M_c}{3V_c}\ ,\end{aligned} \tag{5}$$

where V_c is the volume of a cluster. This internal cluster pressure must be in equilibrium with the pressure of quarks and other clusters in the surrounding medium. We assume that clusters and free quarks are in chemical equilibrium, with as many clusters being destroyed as there are new clusters being formed. The resulting equilibrium condition is

$$3\mu_q=\mu_c\ . \tag{6}$$

The baryon number of the system is composed of two parts, which are clusters and massless free quarks. The baryon number density is

$$\nu=\frac{b}{\Omega}=\frac{\int d\Omega_a}{\Omega}\frac{1}{(2\pi)^3}\left[\int\frac{g_c d^3P}{e^{(\epsilon_c-\mu_c)/T}+1}+\tfrac{1}{3}\int\frac{g_q d^3P}{e^{(\epsilon_q-\mu_q)/T}+1}\left[1-\frac{2\alpha_s}{\pi}\right]\right]\ , \tag{7}$$

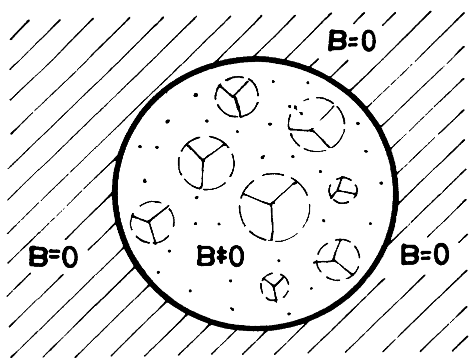

FIG. 1. Shown is clustered quark matter. Quark clusters are indicated by the encircled areas containing three connected dots. The shaded area is the normal hadronic vacuum. Dots represent quarks. The figure is adapted from Ref. 3.

where $\epsilon_c=(P^2+M_c^2)^{1/2}$, $\epsilon_q=(P^2+M_q^2)^{1/2}=P$, $g_c=4$, and $g_q=12$ are statistical degeneracy parameters and b is the total baryon number of the system. The index a in $\int d\Omega_a$ signifies that the integration over volume is to be carried out avoiding the excluded volume comprised of the proper volume of clusters. This integration leads to the factor Δ, which is just the available volume, i.e., the total volume Ω minus the volume of all clusters. The reduction in volume due to the existence of finite-size clusters is expressed by

$$\frac{\Delta}{\Omega}=1-\frac{N_c V_c}{\Omega}\ , \tag{8}$$

where V_c is the average volume of a cluster and N_c is the cluster number of the system. The second term in Eq. (7) has been corrected for the effect of the perturbative quark-quark interaction through order α_s by the factor $[1-(2\alpha_s/\pi)]$.[3,4]

The total energy of the system is composed of two parts, which are gases of clusters and quarks. The energy density of a gas of massive clusters contributes

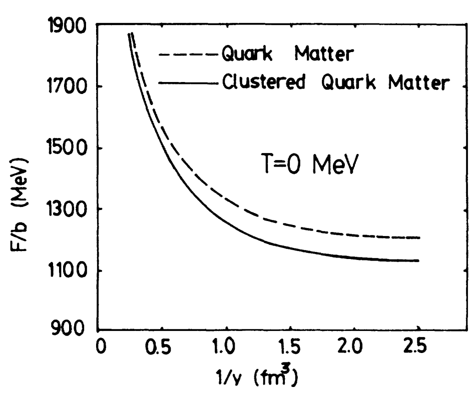

FIG. 2. At temperature $T=0$ MeV, the energy per baryon as a function of the inverse baryonic density v_0/v is shown. The figure is adapted from Ref. 3.

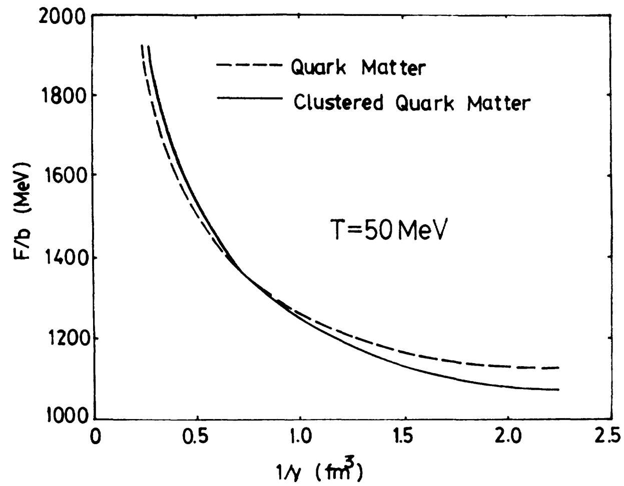

FIG. 3. At temperature $T=50$ MeV, the free energy per baryon as a function of the inverse baryonic density v_0/v is shown.

$$E_c=\frac{\int d\Omega_a}{\Omega}\frac{1}{(2\pi)^3}\int\frac{g_c\epsilon_c d^3P}{e^{(\epsilon_c-\mu_c)/T}+1}\ . \tag{9}$$

The energy density of a gas of nearly massless u and d quarks contributes

$$E_q=\frac{\int d\Omega_a}{\Omega}\frac{1}{(2\pi)^3}\int\frac{g_q\epsilon_q d^3P}{e^{(\epsilon_q-\mu_q)/T}+1}\left[1-\frac{2\alpha_s}{\pi}\right]\ . \tag{10}$$

The energy per baryon of the system is

$$E/b=(E_c+E_q+B)/v\ . \tag{11}$$

The relativistic equation of state of the system[3] is given by

$$P=\tfrac{1}{3}(E_c+E_q)\ . \tag{12}$$

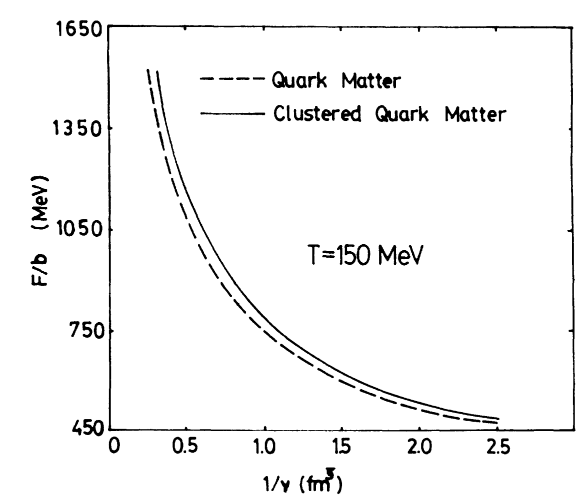

FIG. 4. At temperature $T=150$ MeV, the free energy per baryon as a function of the inverse baryonic density v_0/v is shown.

We approach this quantity as follows: M_N and $B^{1/4}$ are fixed at 939 MeV and 171 MeV, respectively, in the following calculations in comparison with Clark's result.[3] For a definite baryon number density ν, $\alpha_s(\nu)$ and $c(\nu)$ are determined by solving Eq. (3) and Eq. (2). We see that Eqs. (5)–(12) form a complete set. For a definite temperature T, we solve the self-consistent set of Eqs. (5)–(12) with the help of a computer, and then we get chemical potential $\mu_c(\nu,T)$, mass of cluster $M_c(\nu,T)$, and energy per baryon E/b. Having the thermodynamic relation $F=E-TS$, we obtain the free energy per baryon.

The free energy per baryon of clustered quark matter and usual quark matter are shown in Fig. 3 and Fig. 4, corresponding to the temperatures $T=50$ MeV and $T=150$ MeV. The result of zero temperature adapted from Ref. 3 is shown in Fig. 2 for comparison.

From the above calculations, we see that clustered quark matter gains an energetic advantage and can easily exist at low temperatures. But this energetic advantage decreases slowly with increasing temperature. The free energy difference between clustered quark matter and unclustered quark matter is not large at a temperature of $T=50$ MeV. Contrary to zero temperature, as shown in Fig. 2, it is obviously favorable for quark matter at a high temperature of $T=150$ MeV, as shown in Fig. 4. Clark *et al.*[3] point out that the influence of the great number of degrees of freedom of the quarks does not play an important role at zero temperature. Now we think that the great number of degrees of freedom of quarks increase their influence with increasing temperature.

Mclerran *et al.*,[6] Engels *et al.*,[7] and Kuti *et al.*,[8] obtained a transition temperature of $T_c \sim 200$ MeV from Monte Carlo lattice gauge theory. We also get a similar result with the use of the soliton bag model.[9] From the above discussions, we see that the system may be in the quark matter phase at higher temperatures. It is very difficult to form the clustered quark matter phase at the high temperature and high density matter created in high-energy heavy-ion collisions. That is to say, it is not easy to confirm the existence of this new hadronic matter phase in the high-energy heavy-ion collisions.

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