Bombarding energy dependence of ^{168}Er(d, t) transitions*

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Angular distributions have been measured for transitions populating residual states in ^{167}Er through the ^{168}Er(d,t) reaction at E_d = 15, 13, 11, and 9 MeV. Many of these transitions were previously seen to have angular distribution shapes which agreed with distorted-wave Born-approximation predictions at 17 MeV and these transitions also agree well with distorted-wave Born-approximation predictions at the lower bombarding energies. A small but significant number of transitions exhibited anomalously shaped angular distributions at 17 MeV and these continue to have anomalous angular distributions at the lower bombarding energies. Spectroscopic factors do not vary significantly over the bombarding energy range $9 \le E_d \le 17$ MeV. Spectroscopic factor disagreements between recent analyses of 17 MeV angular distributions and previously reported analyses of individual spectra at 12 MeV presumably do not result from a bombarding energy dependence but rather from the large uncertainties inherent in applying the distorted-wave Born-approximation analysis without having complete angular distributions.

[NUCLEAR REACTIONS ^{168}Er(d,t), E_d=15, 13, 11, and 9 MeV measured $\sigma(\theta)$; DWBA analysis, deduced levels, l values, spectroscopic factors. Enriched targets.]

I. INTRODUCTION

Recent measurements of (d,t) transitions at 17 MeV on several rare earth targets have indicated that transitions to a small but significant number of levels show angular distributions whose shapes cannot be fitted with distorted-wave-Born-approximation (DWBA) calculations.[1-4] In addition to these transitions with anomalous angular distributions, many other transitions whose angular distributions can be well reproduced with DWBA calculations yield spectroscopic factors which disagree significantly with predictions of the Nilsson model (with Coriolis coupling included) and with spectroscopic factors extracted from previously reported[5] cross sections for 12 MeV (d,t) transitions. The previous 12 MeV studies did not measure complete angular distributions, so it is not clear whether the transitions observed to be anomalous at 17 MeV are also anomalous at 12 MeV. Neither is it clear whether the extraction of spectroscopic factors from the 12 MeV spectra should be regarded as sufficiently accurate that disagreement of these spectroscopic factors with those extracted from the ≈17 MeV angular distributions should be disturbing. On the other hand, most of our information[6] on single particle/hole states in deformed nuclei has been extracted from individual spectra for (d,p) and (d,t) reactions at 12 MeV, so it is well worth investigating the quality of this information.

Additionally, Erskine has measured excitation functions for (d,p) and (d,t) transitions on ^{238}U and ^{232}Th between 8.5 and 12 MeV and he reports a strong bombarding energy dependence of his DWBA-extracted spectroscopic factors.[7] He finds this bombarding energy dependence to be especially strong for transitions which involve large angular momentum transfers. In this paper we report

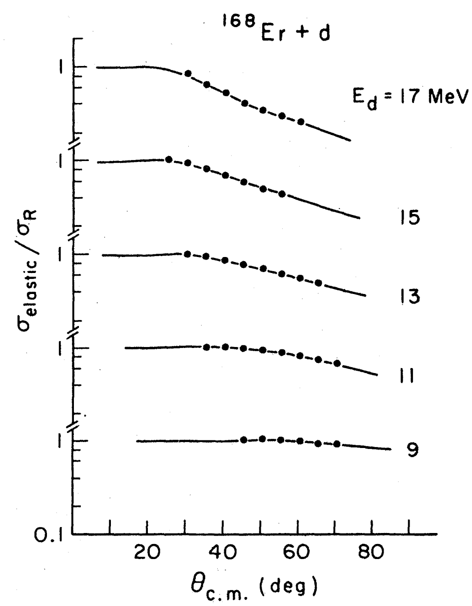

FIG. 1. Cross sections for elastic scattering of 17, 15, 13, 11, and 9 MeV deuterons from ^{168}Er.

an investigation of the bombarding energy dependence of $^{168}Er(d,t)\,^{167}Er$ transitions. Cross sections for these transitions have been measured to study both the possible energy dependence of the spectroscopic factors and the behavior at lower energies of the transitions which appear anomalous at 17 MeV.

II. EXPERIMENTAL DETAILS, DATA PRESENTATION AND DISTORTED-WAVE BORN-APPROXIMATION ANALYSIS

9, 11, 13, and 15 MeV deuterons from the University of Pittsburgh three stage Van de Graaff accelerator were used to bombard ~75 $\mu g/cm^2$ Er_2O_3 targets (enriched to 97.7% in ^{168}Er). The targets had been evaporated on 30 $\mu g/cm^2$ C backings. Outgoing tritons were detected in nuclear emulsions placed in the focal plane of an Engle split-pole spectrograph. All details of the extraction of yields and normalization of successive measurements to form angular distributions are identical to those reported for this same reaction at 17 MeV bombarding energy.[4] At 9 MeV bombarding energy, the (d,t) cross sections were very small and required a long exposure time to accumulate reasonable yields; for this reason, measurements were made at only three angles (90°, 110°, and 130°) for this energy. At all other bombarding energies listed above, complete angular distributions were measured.

Absolute cross sections for all bombarding energies (including 17 MeV) were determined by

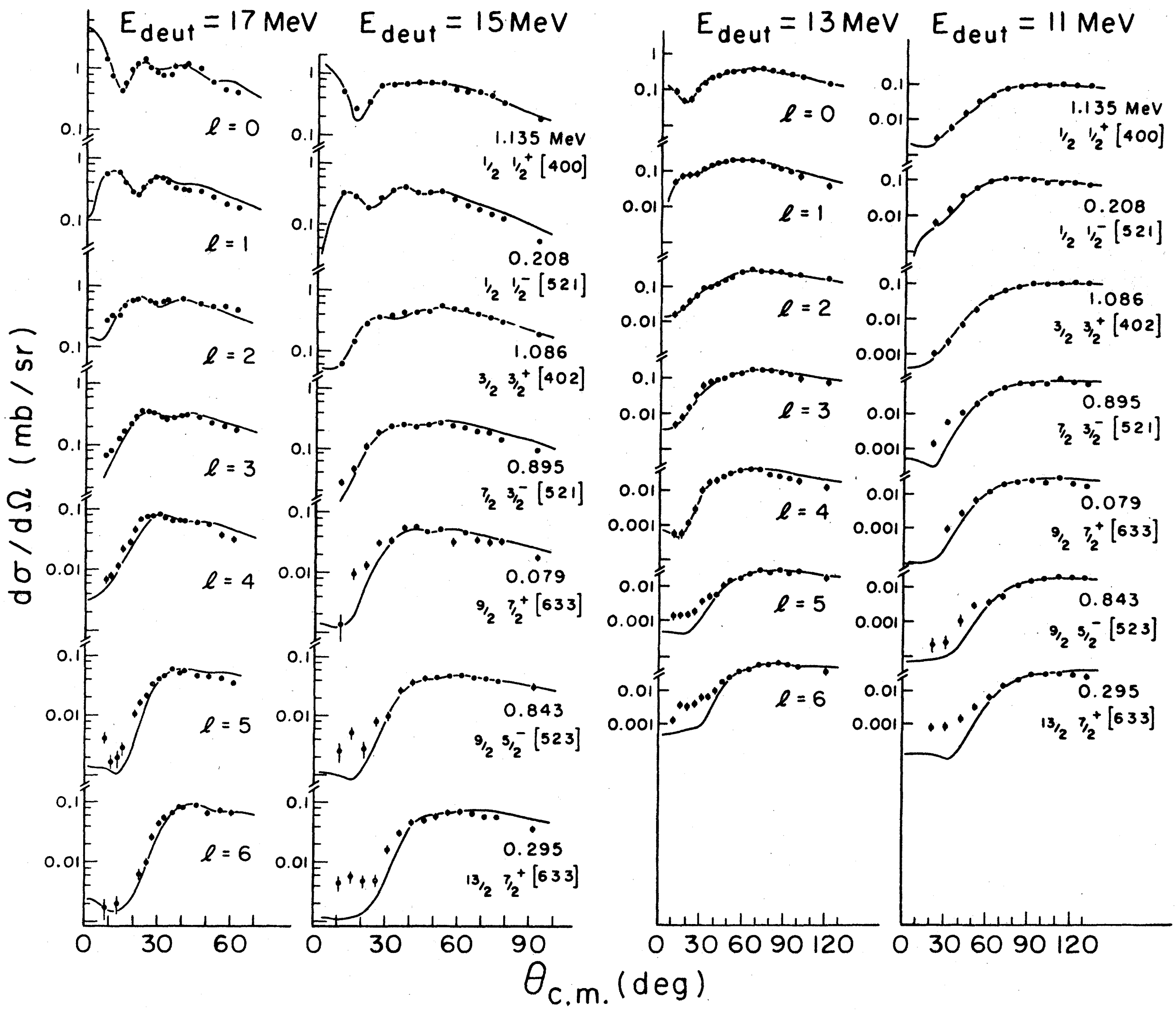

FIG. 2. Angular distributions for $^{168}Er(d,t)^{167}Er$ transitions at the indicated bombarding energies. The solid curves are DWBA calculations.

simultaneously measuring both elastic scattering angular distributions and a (d, t) spectrum with a telescope of Au-Si surface barrier detectors. The resolution for the (d, t) spectrum was not sufficient to resolve individual levels, but the cross section from an isolated group of levels could always be accurately related to the yields measured with high ($\lesssim$ 10 keV) resolution in the spectrograph at the same angle for the well resolved members of the same group. The elastic scattering angular distributions are shown in Fig. 1. Representative (d, t) angular distributions are shown in Fig. 2–4. The error in the absolute cross sections for the (d, t) measurements should be less than 10%. Statistical counting errors are shown in Figs. 1–4 wherever they are significant.

DWBA calculations were performed with the code DWUCK[8] using the same parameters that were used to analyze the 17 MeV transitions.[4] These parameters are listed in Table I and all significant details of the calculations are discussed in Ref. 4. Elastic scattering cross sections calculated with the $d+^{168}$Er parameters of Table I are shown as solid curves in Fig. 1. The solid curves shown in Figs. 2–4 are DWBA calculations performed using the parameters of Table I and assuming the values of orbital (l) and total (j) angular momentum transfer indicated in the figures.

Table II lists, for each observed transition, excitation energy, l value, Nilsson-model assignment (if available), a measured spectroscopic factor for each bombarding energy, and Nilsson-mo-

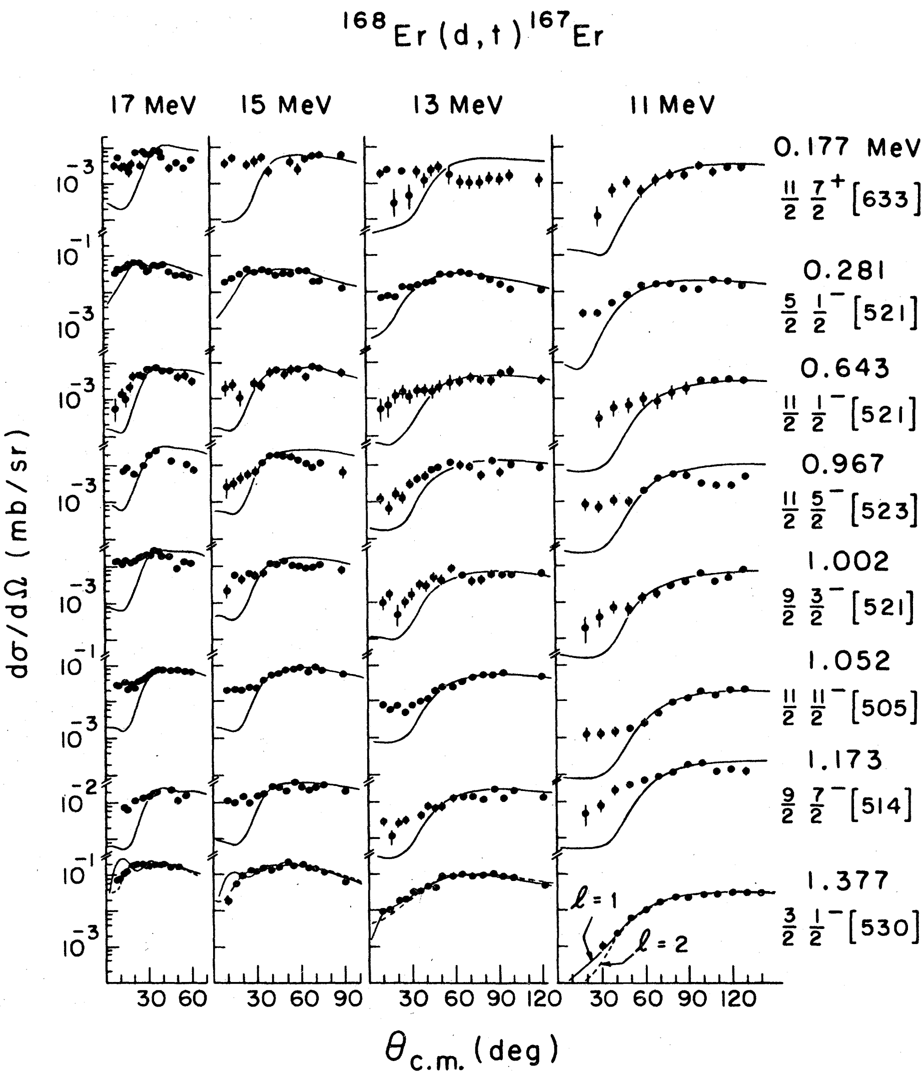

FIG. 3. Angular distributions for anomalous ^{168}Er$(d,t)^{167}$Er transitions at the indicated bombarding energies. The solid curves are DWBA calculations.

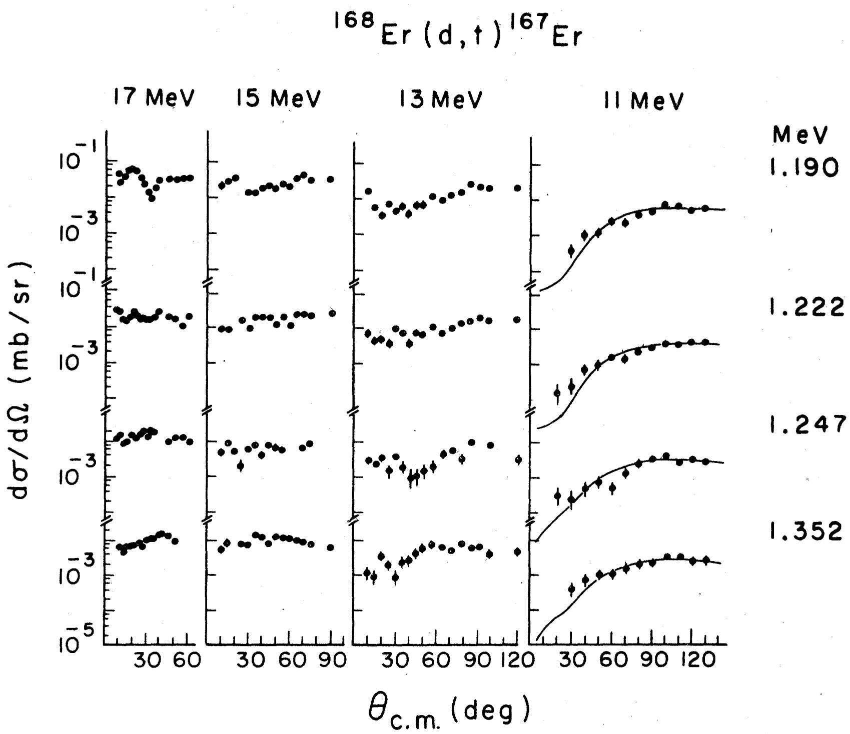

FIG. 4. Angular distributions for anomalous $^{168}Er(d,t)^{167}Er$ transitions at the indicated bombarding energies. The solid curves are DWBA calculations.

del spectroscopic factors both with and without Coriolis coupling. The Nilsson-model calculations are described in Ref. 4.

III. DISCUSSION

The angular distributions shown in Fig. 2 comprise a representative sample of transitions that are well fitted with DWBA calculations. Reference 4 shows 25 such angular distributions for the 17 MeV transitions; the 7 transitions (one for each l value for 0 through 6) shown in Fig. 2 suffice to illustrate the result that the shapes of the well behaved angular distributions can be rather well reproduced by DWBA calculations throughout the present range of bombarding energies. As bombarding energy decreases, there is a slight tendency for the higher l angular distributions, particularly $l = 6$, to show larger cross sections than are predicted by the DWBA at small angles, but basically the agreement is rather good at all beam energies.

Cross sections for all transitions which were seen to have anomalous 17 MeV angular distributions are shown in Figs. 3 and 4. The transitions shown in Fig. 3 are those populating states for which Nilsson-model assignments had previously been made. Of these states, the 1.377 MeV state was probably misassigned when it was called $\frac{3}{2}\frac{1}{2}^-$

TABLE I. Optical model parameters and bound-state well parameters used in the DWBA calculation.

	V_r (MeV)	r_r (fm)	r_c (fm)	a_r (fm)	W (MeV)	W_d (MeV)	r_I (fm)	a_I (fm)	V_{so} (MeV)	r_{so} (fm)	a_{so} (fm)	λ_{so}
$^{168}Er + d$ [a]	102.2	1.15	1.15	0.81	...	17.6	1.34	0.68	...	...	...	...
$^{167}Er + t$ [b]	166.7	1.16	1.40	0.752	14.7		1.498	0.817	...	...	...	...
Bound states	[c]	1.25	1.25	0.65								25.0

[a] C. M. Perey and F. G. Perey, Phys. Rev. 132, 755 (1963).
[b] E. R. Flynn *et al.*, Phys. Rev. 182, 1113 (1969).
[c] Adjusted to give correct separation energy.

TABLE II. Spectroscopic information for ^{167}Er levels populated through the ^{168}Er (d, t) ^{167}Er reaction. Except for weakly populated states (as discussed in Ref. 4) these excitation energies are accurate to ±2 keV. Nilsson model assignments are identical to those assumed in Ref. 4.

E_x (MeV)	ℓ value	^{167}Er Nilsson assignment	S_{expt} E_d = 17 MeV	15 MeV	13 MeV	11 MeV	9 MeV	S model Mixed	Unmixed
0.000	4	$\frac{7}{2}\frac{7}{2}^+$ [633]	0.009	0.009	0.013	0.012		0.0008	0.006
0.079	4	$\frac{9}{2}\frac{7}{2}^+$ [633]	0.13	0.13	0.14	0.16	0.17	0.08	0.053
0.177	(6)[a]	$\frac{11}{2}\frac{7}{2}^+$ [633]	0.24	0.27	0.28	0.32		0.013	0.009
0.208	1	$\frac{1}{2}\frac{1}{2}^-$ [521]	0.19	0.19	0.19	0.20	0.21	0.12	0.12
0.264	1	$\frac{3}{2}\frac{1}{2}^-$ [521]	0.008	0.011	0.009	0.010	0.013	0.029	0.028
0.281	(3)[a]	$\frac{5}{2}\frac{1}{2}^-$ [521]	0.10	0.092	0.094	0.11	0.12	0.089	0.084
0.295	6	$\frac{13}{2}\frac{7}{2}^+$ [633]	1.22	1.30	1.52	1.76	2.52	1.47	0.94
0.345	3	$\frac{5}{2}\frac{5}{2}^-$ [512]	0.013	0.012	0.010	0.014		0.0002	0.0009
0.414	3	$\frac{7}{2}\frac{1}{2}^-$ [521]	0.14	0.15	0.15	0.13	0.13	0.20	0.10
0.431	3	$\frac{7}{2}\frac{5}{2}^-$ [512]	0.15	0.16	0.17	0.18	0.19	0.27	0.20
0.439	5	$\frac{9}{2}\frac{1}{2}^-$ [521]	[b]	0.36	0.36	0.44		0.18	0.11
0.532	2	$\frac{3}{2}^+$ [c]	0.022	0.026	0.025	0.026			
0.573	2	$\frac{5}{2}^+$ [c]	0.014	0.013	0.016	0.015			
0.643	(5)[a]	$\frac{11}{2}\frac{1}{2}^-$ [521]	0.063	0.083	0.070	0.072		0.041	0.013
0.668	3	$\frac{5}{2}\frac{5}{2}^-$ [523]	0.12	0.14	0.14	0.11		0.14	0.13
0.711	4	$\frac{9}{2}^+$	0.027	0.033	0.030	0.039			
0.753	1	$\frac{3}{2}\frac{3}{2}^-$ [521]	0.21	0.25	0.23	0.23	0.28	0.13	0.14
0.802	1	$\frac{3}{2}\frac{1}{2}^-$ [510]	0.039	0.037	0.037	0.042		0.098	0.056
0.812	2	$\frac{5}{2}\frac{5}{2}^+$ [642]	0.054	0.075	0.088	0.086		0.004	0.005
0.843	5	$\frac{9}{2}\frac{5}{2}^-$ [523]	1.08	1.14	1.20	1.26	1.32	1.82	1.37
0.854	3	$\frac{5}{2}\frac{1}{2}^-$ [510]	[b]	0.042	0.047	0.051		0.051	0.041
0.895	3	$\frac{7}{2}\frac{3}{2}^-$ [521]	0.53	0.55	0.56	0.64	0.72	1.01	0.95
0.911	[b]	$\frac{13}{2}^+$ [c]							
0.933	4	$\frac{9}{2}\frac{5}{2}^+$ [642]	0.28	0.30	0.32	0.30		0.17	0.20
0.943	3	$\frac{7}{2}\frac{1}{2}^-$ [510]	0.047	0.050	0.057	0.055		0.21	0.020
0.967	(5)[a]	$\frac{11}{2}\frac{5}{2}^-$ [523]	0.36	0.34	0.46	0.46		0.049	0.093
1.002	(5)[a]	$\frac{9}{2}\frac{3}{2}^-$ [521]	0.58	0.50	0.60	0.63		0.27	0.65
1.052	(5)[a]	$\frac{11}{2}\frac{11}{2}^-$ [505]	1.64	2.04	1.95	1.56		1.94	1.94
1.086	2	$\frac{3}{2}\frac{3}{2}^+$ [402]	0.91	0.92	0.93	0.86	0.52	1.74	1.56
1.109	6	$\frac{13}{2}\frac{5}{2}^+$ [642]	[b]	1.89	1.75	1.95		1.14	1.67
1.135	0	$\frac{1}{2}\frac{1}{2}^+$ [400]	0.66	0.66	0.66	0.55	0.40	0.36	1.16
1.173	(5)[a]	$\frac{9}{2}\frac{7}{2}^-$ [514]	1.02	1.02	1.04	0.84		0.013	0.068
1.190	[a]								
1.205	0		0.053	0.043	0.052	0.040			
1.222	[a]								
1.247	[a]								
1.280	[b]								
1.302	3		0.054	0.052	0.042	0.050			
1.352	[a]								

TABLE II. (*Continued*)

E_x (MeV)	ℓ value	^{167}Er Nilsson assignment	S_{expt} E_d = 17 MeV	15 MeV	13 MeV	11 MeV	9 MeV	S model Mixed	Unmixed
1.377		$\frac{3}{2}\frac{1}{2}^-$ [530]	(0.13)	(0.14)	(0.13)	(0.11)		0.23	0.27
	2		0.29	0.38	0.35	0.29			
1.426	0		0.14	0.13	0.13	0.12			
1.440	0		0.015	0.016	0.018	0.020			

[a] Anomalous angular distribution shape.

[b] Populated weakly and/or obscured by stronger transitions so that no meaningful angular distribution could be extracted.

[c] States reported to have large, vibrationally mixed components by G. Lovhoiden *et al.*, Nucl. Phys. A194, 463 (1972).

[530] in the previous work.[5] The higher energy angular distributions are clearly better fit with l = 2 pickup patterns (dashed curves) than with l = 1 patterns (solid curves). None of the other transitions exhibits angular distribution shapes that can be fit with any l value at the higher bombarding energies. In all these latter cases the angular distribution shapes are anomalous through the entire range of bombarding energies; in several cases the anomalous behavior is less pronounced at 11 MeV but even at this energy there are significant deviations from the expected, Coulomb-dominated, l-independent angular distribution shape predicted by the DWBA.

The angular distributions shown in Fig. 4 are all for relatively strong transitions to states of excitation energy higher than 1 MeV. None of these states has been identified with any Nilsson-model configuration, and none of them shows an angular distribution that can be fitted with a DWBA calculation for any l value at 17, 15, or 13 MeV. For 11 MeV transitions to these states, the angular distributions do not show pronounced deviations from the Coulomb-dominated, l-independent DWBA predictions.

The spectroscopic factors listed in Table II remain reasonably stable as the beam energy decreases from 17 to 9 MeV. Spectroscopic factors for two l = 6 transitions (populating the 0.177 and 0.295 MeV states) increase monotonically as the bombarding energy decreases, but the spectroscopic factors extracted from 11 MeV cross sections are only ~30% larger than those extracted from the 17 MeV data. Variations of the spectroscopic factors for the other transitions do not vary monotonically with bombarding energy and represent smaller fractional deviations from the average values.

Thus no systematic spectroscopic factor variations of the sort reported by Erskine[7] are observed with the present analysis. An effect reminiscent of Erskine's can be made to appear if single angle excitation functions are used to extract bombarding-energy-dependent spectroscopic factors (as was the case in Erskine's investigation). This is illustrated in Fig. 5; on the left side of this figure excitation functions are shown for several transitions at an angle of 30°. On the right side of the figure and for the same transitions, cross sections are plotted for the angle predicted by the DWBA to have the maximum cross section. The solid curves shown in Fig. 5 represent the DWBA-predicted cross sections assuming a constant spectroscopic factor (which is always close to the average of the values listed in Table II). The peak cross section excitation functions shown on the right side of Fig. 5 are clearly in rather good agreement with the DWBA curves—thus indicating that there is no evidence for a significant beam energy dependence to the reaction analysis. The fixed-angle excitation functions on the left deviate significantly from the DWBA curves—and in just the way Erskine reported, with larger spectroscopic factors at low energies and with an increased effect for higher l values; but these deviations clearly result from departures of the measured angular distribution shapes from DWBA expectations at angles reasonably far removed from the angle of maximum cross section and not from a bombarding energy dependence that would affect the validity of spectroscopic information extracted in a more conventional DWBA analysis of complete angular distributions.

IV. SUMMARY AND CONCLUSIONS

The ^{168}Er$(d,t)^{167}$Er transitions which exhibit anomalous angular distribution shapes[4] at E_d = 17

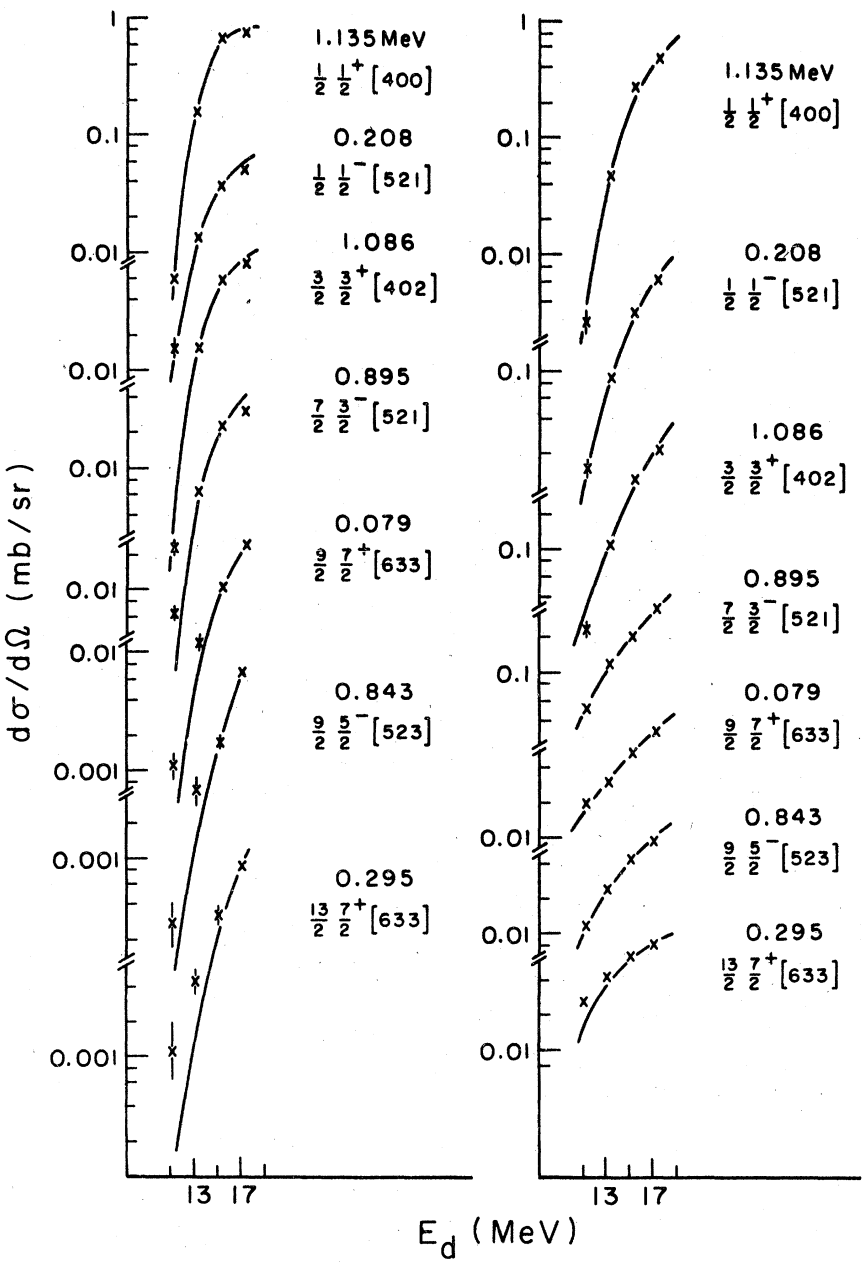

FIG. 5. Excitation functions for ^{168}Er$(d,t)^{167}$Er transitions. On the left side are shown excitation functions at a laboratory angle of 30°. On the right side the cross sections shown are taken from the angle for which DWBA calculations predict the maximum cross section. The solid curves are DWBA calculations.

MeV continue to exhibit such anomalous shapes as bombarding energy is decreased through 15 and 13 MeV. At 11 MeV the angular distributions are Coulomb dominated but deviate from DWBA predictions at least in having too large cross sections at small angles. Transitions whose angular distribution shapes follow the DWBA predictions at 17 MeV continue to show angular distributions which follow the DWBA predictions at 15, 13, and 11 MeV (except for a slight tendency for the high l transitions to have too large 11 MeV cross sections at small angles). Erskine's effect[7] (that low energy cross sections, expecially for high l transitions, yield anomalously large spectroscopic factors in a standard DWBA analysis of his excitation functions) can be understood as arising from deviations of measured cross sections from the DWBA predictions at angles which are relatively far from the angle of maximum cross section. When complete angular distributions are analyzed with the DWBA, no pronounced energy dependence of the extracted spectroscopic factors appears. Thus disagreements between (d,t) spectroscopic factors determined recently from complete angular distributions at 17 MeV over a range of rare earth targets and those reported earlier on the basis of analyses of individual spectra presumably arise from uncertainties in the analyses with individual spectra, rather than from any intrinsic energy dependence implied in the DWBA analysis. Even the presumably inaccurate spectroscopic factors (included in Table II) which arise from analyzing the anomalously shaped angular distributions show no evidence of a significant variation with bombarding energy. This latter effect is currently being investigated as part of a coupled-channels-Born-approximation analysis of the systematics of all the anomalous transitions reported in Refs. 1–4.

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