Confined quantum systems in one dimension and conductance oscillations in narrow channels

Karel Vacek and Ayao Okiji
Department of Applied Physics, Osaka University, Suita, Osaka 565, Japan

Norio Kawakami
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606, Japan

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An exactly solvable electron model of a confined system with inverse-square interaction is presented. The ground state is given by the Jastrow-product wave function of power-law form. We then apply the results to conductance oscillations in semiconductor nanostructures by generalizing the approach of Johnson and Payne to single-electron charging effects. Due to the internal spin degrees of freedom, there appear two independent periods of the conductance oscillations in very narrow channels even at zero temperature.

I. INTRODUCTION

One-dimensional quantum models with long-range $1/x^2$ interaction have attracted renewed interest recently. There are several classes of Hamiltonians which are known to be integrable.[1-4] For instance, a family of models with periodic boundary conditions has been studied extensively, which gives the models with $1/\sin^2 x$ interaction.[2-4] Another interesting class of Hamiltonians has been provided by the quantum systems confined with the harmonic potential.[1,2] The effect of the internal spin degrees of freedom has been studied quite well for the former case: the exact wave functions and the corresponding eigenenergies have been clarified almost completely for various kinds of multicomponent models.[5-9] However, this is not the case for the confined systems, for which the single-component model has been mainly investigated.[1,10]

Besides intrinsic theoretical interest in such confined electron systems with spin degrees of freedom, there is also a strong experimental motivation for their investigation. For example, experiments on one-dimensional electron systems have been reported for very narrow channels in Si or GaAs semiconductor nanostructures.[11-14] The conductance of a narrow channel displays periodic oscillations, which have been ascribed to the single-electron charging effect of the confined segment in the one-dimensional electron gas.[15-17] A solvable electron model with $1/x^2$ interaction[18,19] provides an analytical tool to investigate the correlation effects on such experiments.

In this paper, we would like to study an exactly solvable electron model for a *confined system* with the $1/x^2$ interaction including the *exchange effects*. In Sec. II, we introduce the model by generalizing the original one[1] to the case including the spin degrees of freedom. We then find the ground-state eigenfunction in the form of the Jastrow product, and obtain the ground-state energy exactly. The solution is obtained by generalizing techniques developed in Refs. 1–3 and 6. In Sec. III, we apply the results to experiments for the narrow channels in semiconductor nanostructures by introducing a weak coupling between the confined system and the reservoirs. We mainly discuss the exchange effect on conductance oscillations, which has not been taken into account in the previous studies.[18,19] In particular, we find two independent periods for the oscillations even at zero temperature, due to the internal spin degrees of freedom. Our conclusion is given in Sec. IV.

II. CONFINED MODEL WITH $1/x^2$ INTERACTION

A. Hamiltonian and Jastrow wave function

Let us consider a one-dimensional electron system confined by the harmonic potential $\frac{1}{2}m\omega_0^2 x^2$ where m is the mass of a particle. We introduce the following model Hamiltonian in units of $\hbar^2/m$ for the system of N electrons with the inverse-square pair interaction

$$H=-\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2}+\frac{m^2\omega_0^2}{2\hbar^2}\sum_{i=1}^{N}x_i^2+\sum_{j>i}\frac{\lambda(\lambda+P_{ij}^{\sigma})}{(x_j-x_i)^2}\ , \tag{1}$$

where $\lambda\geq 0$ is the dimensionless interaction parameter and P_{ij}^{σ} is the spin-exchange operator of two particles. The spin-exchange interaction is chosen here so as to reproduce the noninteracting limit correctly. Note that this interaction can lift a pathological degeneracy of the ground state for the multicomponent model[19] arising from peculiar short-range properties of $1/x^2$ interaction in one dimension. This point is crucial to confront the results with conductance-oscillation phenomena, which will be mentioned again later. The above Hamiltonian with the harmonic potential is a generalization of the Calogero model,[1] and may be regarded as a variant of the SU(ν) Sutherland model introduced by several authors for the periodic case.[6,8] We will see in the next section that the model exhibits quite different behaviors from those discussed in Ref. 19.

Based on the observation of the Ha-Haldane ansatz used for systems with periodic boundary conditions,[6] we now write down the Jastrow-type trial function for systems with the confinement

$$\Psi(x_1\sigma_1,\ldots,x_N\sigma_N)=\left[\prod_{j>i}|x_j-x_i|^{\lambda}(x_j-x_i)^{\delta_{\sigma_j\sigma_i}}\exp\left[i\frac{\pi}{2}\,\mathrm{sgn}(\sigma_j-\sigma_i)\right]\right]\prod_{i=1}^{N}\exp\left[-\frac{m\omega_0}{2\hbar}x_i^2\right], \tag{2}$$

where σ_i is the particle spin. This wave function satisfies the Pauli principle, $M_{ij}\Psi=-\Psi$, where M_{ij} is a particle-exchange operator. It is also instructive to rewrite the wave function out of two parts,

$$\Psi(x_1\sigma_1,\ldots,x_N\sigma_N)=\prod_{j>i}|x_j-x_i|^{\lambda}\Psi_0(x_1\sigma_1,\ldots,x_N\sigma_N), \tag{3}$$

where Ψ_0 is the exact ground-state wave function of the noninteracting case ($\lambda=0$). For the above trial wave function, therefore, the effect of interaction is assumed to be taken into account completely by the Jastrow factor $|x_j-x_i|^{\lambda}$, as is the case for all other integrable $1/x^2$ systems. In particular, the expression (3) is quite general for the $1/x^2$ models, which is also applicable to the periodic models by choosing an appropriate wave function Ψ_0.[6]

B. Ground-state energy

We now wish to show that the wave function (2) satisfies the Schrödinger equation $H\Psi=E\Psi$ with the eigenenergy E. We limit ourselves to the proof in the sector $x_1\le x_2\le\cdots\le x_N$ which can be easily extended to the whole configuration space. We start by applying the kinetic term and the confining-potential term on the ansatz eigenfunction. Following a technique outlined in Ref. 1, this action turns out to be

$$\frac{1}{\Psi}\left[\frac{1}{2}\sum_{i=1}^{N}\left[-\frac{\partial^2}{\partial x_i^2}+\frac{m^2\omega_0^2}{\hbar^2}x_i^2\right]\right]\Psi=\frac{m\omega_0}{2\hbar}[\lambda N(N-1)+N_\uparrow^2+N_\downarrow^2]-A, \tag{4}$$

where the number of particles with the spin up (down) is $N_\uparrow$ ($N_\downarrow$), $N_\uparrow+N_\downarrow=N$. The term A can be brought to the shape

$$A=A^{(\mathrm{I})}+A^{(\mathrm{II})}+A^{(\mathrm{III})}=\sum_{k<l}\frac{\lambda(\lambda-1)}{(x_k-x_l)^2}+\sum_{k<l}\frac{2\lambda\delta_{\sigma_k\sigma_l}}{(x_k-x_l)^2}+\sum_{i\neq k\neq l}\frac{\lambda\delta_{\sigma_i\sigma_k}}{(x_i-x_k)(x_i-x_l)}. \tag{5}$$

Consider next the interaction term. In order to determine the action of the spin-exchange operator on the ansatz eigenfunction, we first calculate the action of the coordinate-exchange operator Y_{kl},

$$\frac{Y_{kl}\Psi}{\Psi}=(-1)^{\delta_{\sigma_k\sigma_l}}\prod_{i\neq kl}\left[\frac{x_i-x_l}{x_i-x_k}\right]^{\delta_{\sigma_i\sigma_k}-\delta_{\sigma_i\sigma_l}}. \tag{6}$$

According to the antisymmetric nature of electrons, we rewrite the action of the spin-exchange operator as $P_{kl}^{\sigma}\Psi=Y_{kl}M_{lk}\Psi=-Y_{kl}\Psi$ for any $k<l$. Hence the interaction term on the ansatz eigenfunction gives

$$B=\frac{1}{\Psi}\left[\sum_{j>i}\frac{\lambda(\lambda+P_{ij}^{\sigma})}{(x_j-x_i)^2}\right]\Psi=B^{(\mathrm{I})}+B^{(\mathrm{II})}+B^{(\mathrm{III})}$$
$$=\sum_{k<l}\frac{\lambda(\lambda-1)}{(x_k-x_l)^2}+\sum_{k<l}\frac{2\lambda\delta_{\sigma_k\sigma_l}}{(x_k-x_l)^2}+\sum_{k<l}\frac{\lambda}{(x_k-x_l)^2}\left[1-\prod_{i\neq kl}\left[\frac{x_i-x_l}{x_i-x_k}\right]^{\delta_{\sigma_i\sigma_k}-\delta_{\sigma_i\sigma_l}}\right](1-\delta_{\sigma_k\sigma_l}). \tag{7}$$

One can see from Eqs. (4), (5), and (7) that the application of the Hamiltonian produces not only the constant term but also the multiparticle terms. If these multiparticle terms cancel each other completely ($A-B=0$), we get the eigenenergy by the first term of (4). We will show that this is indeed the case.

We would like to prove here that $A-B=0$ by employing a technique analogous to that used for the periodic case.[6] Note first that the spin-independent parts (namely, the first terms $A^{(\mathrm{I})}$ and $B^{(\mathrm{I})}$) and the parts for the same spins $\sigma_k=\sigma_l$ (namely, the second terms $A^{(\mathrm{II})}$ and $B^{(\mathrm{II})}$) of the expressions for A and B are the same. Furthermore, by using the following relation,

$$\left[\frac{x_i-x_l}{x_i-x_k}\right]^{\delta_{\sigma_i\sigma_k}-\delta_{\sigma_i\sigma_l}}=1+(x_k-x_l)\left[\frac{\delta_{\sigma_i\sigma_k}}{x_i-x_k}-\frac{\delta_{\sigma_i\sigma_l}}{x_i-x_l}\right], \tag{8}$$

where $\sigma_k\neq\sigma_l$, we can rewrite the part containing pairs with different spins,

$$B^{(\mathrm{III})}=\frac{1}{2}\sum_{k\neq l}\frac{\lambda}{(x_k-x_l)^2}\left\{1-\prod_{i\neq kl}\left[1+(x_k-x_l)\left[\frac{\delta_{\sigma_i\sigma_k}}{x_i-x_k}-\frac{\delta_{\sigma_i\sigma_l}}{x_i-x_l}\right]\right]\right\}(1-\delta_{\sigma_k\sigma_l}). \tag{9}$$

So, our objective is now to rearrange the expression for $B^{(\mathrm{III})}$ in order to cancel it with the $A^{(\mathrm{III})}$ term. To this end let

us expand the above product over $i \neq kl$ in expression (9). One finds that the constant term $B_0^{(\mathrm{III})}$ vanishes, and the linear term $B_1^{(\mathrm{III})}$ is equal to $A^{(\mathrm{III})}$. The terms of higher order, $B_q^{(\mathrm{III})}$, $2 \leq q \leq N-2$, can be simplified by introducing *clusters* of $q+2$ particles where $r+1$ particles have the spin up and $s+1$ particles have the spin down, $r+s=q$. Namely, let us write $B_q^{(\mathrm{III})}$, $q \geq 2$, as a sum over such clusters as

$$B_q^{(\mathrm{III})} = -\frac{\lambda}{2}\frac{1}{q!} \sum_{k \neq \alpha_1 \neq \cdots \neq \alpha_r \neq l \neq \beta_1 \neq \cdots \neq \beta_s} b_q(k\alpha_1 \cdots \alpha_r l \beta_1 \cdots \beta_s) , \tag{10}$$

where

$$b_q(k\alpha_1 \cdots \alpha_r l \beta_1 \cdots \beta_s) = (-1)^s (x_k - x_l)^{r+s-2} \prod_{i=1}^{r} \frac{1}{x_{\alpha_i} - x_k} \prod_{j=1}^{s} \frac{1}{x_{\beta_j} - x_l} \tag{11}$$

for $\sigma_k = \sigma_{\alpha_1} = \cdots = \sigma_{\alpha_r} \neq \sigma_l = \sigma_{\beta_1} = \cdots = \sigma_{\beta_s}$. As shown in the Appendix, the sum of b_q ($q \geq 2$) over the permutations $k \leftrightarrow \alpha_i$, $i = 1,2,\ldots,r$, and $l \leftrightarrow \beta_j$, $j = 1,2,\ldots,s$, is zero within the particles of the same spin in the cluster of $q+2$ particles:

$$\sum_{\{k\alpha_1 \cdots \alpha_r\}} \sum_{\{l\beta_1 \cdots \beta_s\}} b_q(k\alpha_1 \cdots \alpha_r l \beta_1 \cdots \beta_s) = 0 , \tag{12}$$

where $q \geq 2$ is fixed, and r and s are arbitrary (satisfying only $r+s=q$). Consequently, we have shown that contribution from the terms of higher order $B_q^{(\mathrm{III})}$, $2 \leq q \leq N-2$, is identically zero, and that $A - B = 0$ holds. Therefore, we find the wave function (2) to be the eigenfunction of the Hamiltonian (1).

The eigenenergy $E(N_\uparrow, N_\downarrow)$ is expressed, according to expression (4), by a remarkably simple formula

$$E(N_\uparrow, N_\downarrow) = \tfrac{1}{2}\hbar\omega_0[\lambda(N_\uparrow + N_\downarrow)(N_\uparrow + N_\downarrow - 1) + N_\uparrow^2 + N_\downarrow^2] . \tag{13}$$

This expression also ensures the result predicted in Ref. 20. Although it is not easy to prove rigorously that the above eigenstate indeed corresponds to the *ground state,* one can see evidence for it in several limiting cases. In the limit of $\omega_0 \to 0$, the Hamiltonian (1) becomes a positive definite operator with the lowest eigenvalue of zero, as was shown by Polychronakos.[8] One can see that the formula (13) actually produces the value of zero in such a limiting case, which proves that the energy (13) indeed corresponds to the ground-state energy. Also, in case of electrons with a single spin component, it is seen that the wave function (2) with the energy (13) reduces to the exact ground-state wave function.[1,2,8] From these observations we believe that the eigenfunction (2) with the energy (13) generally describes the exact ground state of the Hamiltonian (1).

The ground-state eigenenergy $E_0(N_\uparrow, N_\downarrow)$ is characterized by $N_\uparrow = N_\downarrow$ (for N even) or $N_\uparrow = N_\downarrow \pm 1$ (for N odd). The ground state is also the state with the minimal spin. In the limit of weak interactions ($\lambda \to 0$), the formula (13) reproduces the eigenenergy expected for noninteracting electrons with the internal spin degrees of freedom,

$$E(N_\uparrow, N_\downarrow, \lambda \to 0) = \tfrac{1}{2}\hbar\omega_0(N_\uparrow^2 + N_\downarrow^2) .$$

It is instructive to note here that the correct noninteracting energy cannot be reproduced if we omit the exchange term from the Hamiltonian, as has been done in Ref. 19. This pathological behavior comes from a peculiar property of $1/x^2$ interaction (short-range divergence property) in one dimension, which gives rise to a large number of degeneracies of the ground state. Therefore, the exchange term used in this paper is crucial to correctly describe interacting electrons in terms of $1/x^2$ interaction from the weakly to the strongly correlated regimes.[6,8]

III. CONDUCTANCE OSCILLATIONS IN NARROW CHANNELS

In this section, we will apply our model to the transport of electrons through a narrow channel of the semiconductor nanostructure. The one-dimensional electron gas is confined by impurities[12] or by constrictions[13] to a finite segment of the channel. We would like to model correlated electrons in such segment by Hamiltonian (1), where we substitute the mass m of a particle by the effective mass m^* of electrons in Si or GaAs. Experimentally, both the confining potential and the electron-electron interaction potential are difficult to estimate. Generally, these potentials are believed to be device dependent. According to this uncertainty in the shape of the confining and the interaction potentials, the parameters ω_0 and λ may be treated empirically to achieve the best fit to the experiment. We wish to mention here that applications of the $1/x^2$ models to conductance oscillations have been done in Refs. 18 and 19, and several characteristic properties have been explained. However, the models have been restricted to spinless fermions so far, in which the exchange effect is completely neglected.

We now introduce a weak coupling via the confining constrictions or impurities between the segment and two reservoirs[21] of the adjustable chemical potential $\mu_1 \approx \mu_2$, $\mu = (\mu_1 + \mu_2)/2$. Let the segment initially contain N electrons. The $(N+1)$th electron can enter the segment via a resonant tunneling. The resonance occurs whenever the average chemical potential μ of reservoirs aligns with the lowest unoccupied level of the segment. The resonance appears as a peak in the conductance. The position of a peak for $N \to N+1$ corresponds to a chemical potential $\mu(N) = E_0(N+1) - E_0(N)$. The zero-temperature spacing δ of two successive peaks for $N \to N+1$ and $N+1 \to N+2$ in the conductance oscillations is given by $\delta(N) = \mu(N+1) - \mu(N)$. It should be noted that in contrast to the previous model of Johnson and Payne,[19] δ can take several different values because of the spin-dependent exchange effects.

Let us now suppose that the system is *initially* in the ground state with the even number of electrons ($N_\uparrow = N_\downarrow = \frac{1}{2}N$). The increase of the chemical potential μ leads to the first conductance peak at the position $\mu(N_\uparrow+1, N_\downarrow)$. The further increase of the chemical potential μ causes the second peak at $\mu(N_\uparrow+1, N_\downarrow+1)$. The spacing between the first and the second peaks is then computed as $\mu(N_\uparrow+1, N_\downarrow+1)-\mu(N_\uparrow+1, N_\downarrow) = \hbar\omega_0\lambda$. Similarly, the spacing between the second and the third peaks is $\mu(N_\uparrow+2, N_\downarrow+1)-\mu(N_\uparrow+1, N_\downarrow+1) = \hbar\omega_0(\lambda+1)$, the spacing between the third and the fourth peaks is $\mu(N_\uparrow+2, N_\downarrow+2)-\mu(N_\uparrow+2, N_\downarrow+1) = \hbar\omega_0\lambda$, etc. It is seen that *two independent periods* of the conductance oscillations appear,

$$\delta_1 = \hbar\omega_0\lambda, \quad \delta_2 = \hbar\omega_0(\lambda+1), \tag{14}$$

reflecting *the internal spin degrees of freedom.* These two periods of conductance oscillations appear alternately; two peaks with spacing δ_1 are surrounded on each side by a peak at spacing δ_2 and vice versa. This is a fundamental extension beyond the $1/x^2$ models discussed by Tewari[18] and also beyond the Johnson-Payne model[19] which is essentially the same as the spinless fermion model vith a single period of the conductance oscillations $\delta = \hbar\omega_0(\lambda+1)$.

For the empirical fit of parameters used by Johnson and Payne $\delta \approx 8.5\hbar\omega_0$,[19] we obtain two periods $\delta_1 = 7.5\hbar\omega_0$ and $\delta_2 = 8.5\hbar\omega_0$, implying the correction due to the exchange effect is rather small for these parameters. As the interaction becomes weaker (smaller λ), however, the above exchange effect becomes conspicuous, making two periods more distinct. When the interaction strength takes the vanishing value ($\lambda \to 0$), the present model reproduces the results of free electrons including the spin degrees of freedom in the harmonic potential ($\delta_1 \to 0$, $\delta_2 \to \hbar\omega_0$).

Finally, we would like to mention briefly the relationship of our results to experiments on periodic conductance oscillations in narrow channels.[11-14] Experimentally, there are reports that the period of conductance resonances is not only a single one.[11,14] In addition to the dominant periodicity in the Fourier spectrum, several peaks have often been seen at positions corresponding to frequencies which are not harmonics of the dominant one. The origin of multiple periods, which have sometimes been observed experimentally, has been explained by the possible existence of several segments with comparable pinning energies.[11] It can be interesting to check experimentally whether one may observe two periods in the conductance oscillations due to the exchange effect at very low temperatures. If so, it may provide further knowledge of the exchange-correlation effects in one-dimensional systems.

IV. CONCLUSION

We have studied a solvable electron model for a confined one-dimensional system interacting by inverse-square interaction. We have obtained the Jastrow wave function as the eigenfunction for the Hamiltonian (1) and calculated the corresponding eigenenergy exactly. The results have been applied to the analysis of the periodic conductance oscillations in narrow channels. Though the present model includes the interaction and the exchange effect in a specific way, it clearly demonstrates that two kinds of oscillations can be expected in the single-electron charging phenomena due to the internal spin degrees of freedom.

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APPENDIX

Here we wish to prove the identity (12). We first express the products in (11) by the Vandermonde determinants,

$$\prod_{i=1}^{r} \frac{1}{x_{\alpha_i} - x_k} = (-1)^r \frac{V^{(r)}(x_{\alpha_1} \cdots x_{\alpha_r})}{V^{(r+1)}(x_{\alpha_1} \cdots x_{\alpha_r} x_k)}, \tag{A1}$$

$$\prod_{j=1}^{s} \frac{1}{x_{\beta_j} - x_l} = (-1)^s \frac{V^{(s)}(x_{\beta_1} \cdots x_{\beta_s})}{V^{(s+1)}(x_{\beta_1} \cdots x_{\beta_s} x_l)}. \tag{A2}$$

Next, by using the binomial theorem we expand $(x_k - x_l)^{r+s-2}$. After exchanging the order of finite sums, we can rewrite the left-hand side of Eq. (12) as follows:

$$\sum_{\{k\alpha_1 \cdots \alpha_r\}} \sum_{\{l\beta_1 \cdots \beta_s\}} b_q(k\alpha_1 \cdots \alpha_r l\beta_1 \cdots \beta_s)$$
$$= \sum_{t=0}^{r+s-2} \binom{r+s-2}{t} (-1)^{t+r} \sum_{\{k\alpha_1 \cdots \alpha_r\}} \frac{x_k^{r+s-t-2} V^{(r)}(x_{\alpha_1} \cdots x_{\alpha_r})}{V^{(r+1)}(x_{\alpha_1} \cdots x_{\alpha_r} x_k)} \sum_{\{l\beta_1 \cdots \beta_s\}} \frac{x_l^t V^{(s)}(x_{\beta_1} \cdots x_{\beta_s})}{V^{(s+1)}(x_{\beta_1} \cdots x_{\beta_s} x_l)}. \tag{A3}$$

It is useful to introduce the following determinant:

$$W^{(r,u)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)=\det\begin{vmatrix} 1 & 1 & \cdots & 1 & 1 \\ x_{\alpha_1} & x_{\alpha_2} & \cdots & x_{\alpha_r} & x_k \\ \vdots & \vdots & & \vdots & \vdots \\ x_{\alpha_1}^{r-1} & x_{\alpha_2}^{r-1} & \cdots & x_{\alpha_r}^{r-1} & x_k^{r-1} \\ x_{\alpha_1}^{u} & x_{\alpha_2}^{u} & \cdots & x_{\alpha_r}^{u} & x_k^{u} \end{vmatrix}, \tag{A4}$$

where $u=r+s-t-2$. The expansion of the determinant $W^{(r,u)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)$ with respect to the last row, and the permutations of columns in the determinant $V^{(r+1)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)$, give the desired sum over permutations $k\leftrightarrow\alpha_i$, $i=1,2,\ldots,r$,

$$\sum_{\{k\alpha_1\cdots\alpha_r\}}\frac{x_k^{r+s-t-2}V^{(r)}(x_{\alpha_1}\cdots x_{\alpha_r})}{V^{(r+1)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)}=\frac{W^{(r,r+s-t-2)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)}{V^{(r+1)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)}. \tag{A5}$$

Similarly, the sum over permutations $l\leftrightarrow\beta_j$, $j=1,2,\ldots,s$, is

$$\sum_{\{l\beta_1\cdots\beta_s\}}\frac{x_l^tV^{(s)}(x_{\beta_1}\cdots x_{\beta_s})}{V^{(s+1)}(x_{\beta_1}\cdots x_{\beta_s}x_l)}=\frac{W^{(s,t)}(x_{\beta_1}\cdots x_{\beta_s}x_l)}{V^{(s+1)}(x_{\beta_1}\cdots x_{\beta_s}x_l)}. \tag{A6}$$

If $s-1\le t\le r+s-2$, then two rows of the determinant $W^{(r,r+s-t-2)}(x_{\alpha_1}\cdots x_{\alpha_r}x_k)$ are the same. If $0\le t\le s-1$, then two rows of the determinant $W^{(s,t)}(x_{\beta_1}\cdots x_{\beta_s}x_l)$ are the same. Therefore, for any t with $0\le t\le r+s-2$ at least one of the sums over permutations in Eq. (A3) is zero. Consequently, the formula (12) has been proven.

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