Longitudinal spin susceptibility and spin dynamics of field-induced spin-density waves in Bechgaard salts

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(Received 1 December 1986)

Making use of a Green's function describing field-induced spin-density waves (FISDW's) in Bechgaard salts, we calculate the longitudinal spin susceptibility. We find that the longitudinal susceptibility has a pole at $\omega^2=(1-\bar{U})v_F^2q^2$ at all temperatures, which describes the longitudinal spin wave. Here $\bar{U}$ is the dimensionless Coulomb energy. Furthermore, the static spin susceptibility is independent of temperature. The spin dynamics of a FISDW are also considered. We predict magnetic solitons in FISDW's.

I. INTRODUCTION

It is now well established that the field-induced spin-density waves (FISDW's) (Ref. 1) in relaxed $(TMTSF)_2ClO_4$ and $(TMTSF)_2PF_6$ under pressure are described by a Green's function[2-4] similar to the one for a Bardeen-Cooper-Schrieffer (BCS) superconductor.

We shall consider in this paper the longitudinal spin susceptibility of a FISDW within mean-field theory. In the course of our analysis we discover a close analogy between the spin susceptibility in a FISDW and the one[5,6] in superfluid ^{3}He. For example, the longitudinal spin susceptibility for small ω and $v_Fq\,(=\zeta)$ has a simple expression

$$\chi_{33}(\omega,q)=\tfrac{1}{2}N_0\zeta^2[(1-\bar{U})\zeta^2-\omega^2]^{-1}\ , \tag{1}$$

where N_0 is the electron density of states at the Fermi surface and $\bar{U}=UN_0$ is the dimensionless Coulomb energy. Equation (1) implies the existence of a longitudinal spin wave with dispersion

$$\omega^2=(1-\bar{U})v_F^2q^2 \tag{2}$$

for all temperatures.

So far we have neglected the dipole interaction between the electron spins. Although the dipole interaction energy is much smaller than the condensation energy, this energy breaks the rotational symmetry of the spin direction in the ground state of a FISDW. As we shall see later, the spin of a FISDW is linearly polarized in the x-y plane perpendicular to the external magnetic field along the z axis. Furthermore, the dipole energy favors the configuration with the spin polarized along the y direction, perpendicular to the best conducting direction. Then this dipole energy gives rise to a mass term in the longitudinal spin wave, again similar to the one in superfluid ^{3}He; Eq. (2) is now replaced by

$$\omega^2=(1-\bar{U})v_F^2q^2+\Omega^2(T)\ , \tag{3}$$

where $\Omega(T)$ is the dipole frequency. For $T>T_c$, $\Omega(T)$ is zero, while in the vicinity of $T=T_c$, $\Omega(T)$ increases with decreasing temperature like $(T_c-T)^{1/2}$. Indeed, we estimate $\Omega(T)\simeq 10^{-3}\Delta(T)$, where $\Delta(T)$ is the temperature-dependent quasiparticle energy gap. We construct an effective Lagrangian which describes the spin dynamics of a FISDW. With the help of this Lagrangian we predict the existence of magnetic solitons in a FISDW similar to the ones[7] in superfluid ^{3}He-A.

II. DYNAMIC SPIN SUSCEPTIBILITY

In this section we shall calculate $\chi_{33}(\omega,q)$ $(=\langle[S_3,S_3]\rangle)$ the dynamic longitudinal spin susceptibility.[8] Again as in superfluid ^{3}He, the fluctuation of $S_3=\frac{1}{2}\Psi^\dagger\sigma_3\Psi$ (the spin parallel to the static external field) couples to the fluctuation of the FISDW order parameter $\delta\Delta=(1/\sqrt{2})\Psi^\dagger\rho_1\sigma_2\Psi$ in a FISDW and it is necessary to handle these fluctuations on an equal footing. Here, σ_i and ρ_i are Pauli spin matrices operating on the ordinary spin space and on the momentum space (i.e., the doublet formed by the right-going and the left-going components along the chain direction which is taken along the x axis), respectively. The corresponding spinor is given by

$$\Psi^\dagger=(\psi^\dagger_{p\uparrow},\psi^\dagger_{p\downarrow},\psi^\dagger_{p-Q\uparrow},\psi_{p-Q\downarrow})\ , \tag{4}$$

where p is the electron momentum along the x direction and Q is the x component of $\mathbf{Q}$; $Q=2p_F+q_x$.

As in our earlier papers[2,3] we consider a quasi-two-dimensional electron system with the quasiparticle spectrum given by

$$E(p)=-2t_a[\cos(ap_1)-\cos(ap_F)]-2t_b\cos[b(p_2-eHx)]-2t_c\cos(cp_3) \tag{5}$$

with

$$t_a/t_b/t_c\simeq 10/1/0.03\ .$$

Here the Fermi momentum is either $(\pi/4)a^{-1}$ or $(3\pi/4)a^{-1}$ and we introduce a magnetic field perpendicular to the x-y plane by $p_2\to p_2-eHx$. We assume further that the quasiparticles are interacting via the on-site Coulomb potential U (the Hubbard model). Since p_2 and p_3 are cyclic within mean-field theory the Green's function in a FISDW is given by

$$\left[i\omega+i\rho_3 v_F\frac{\partial}{\partial x}+\rho_1[\sigma_1\Delta_1(x)-\sigma_2\Delta_2(x)]\right]g(x,x')=\delta(x-x')\ , \quad (6)$$

where

$$\begin{aligned}
&\Delta_1(x)=\mathrm{Re}\Delta(x),\quad \Delta_2(x)=\mathrm{Im}\Delta(x)\ ,\\
&\Delta(x)=\Delta e^{i[\Phi(x)-\Phi(0)]}\\
&\quad=\Delta e^{-i\Phi(0)}\sum_{n=-\infty}^{\infty} I_n e^{ik(n-N)(x-x_0)}\ ,\\
&\Phi(x)=q_x(x-x_0)+\beta\cos[k(x-x_0)]-\alpha\sin[2k(x-x_0)]\ ,\\
&I_n=I_n(\alpha,\beta)=i^n\sum_{l=-\infty}^{\infty} J_l(\alpha)J_{n-2l}(\beta)\ ,\\
&\alpha=h^{-1}\cos(bq_y),\quad \beta=\eta h^{-1}\sin(\tfrac{1}{2}bq_y)\ ,\\
&h=evbH/2t_b',\quad \eta=2t_b/t_b'\ ,\\
&t_b'=-\tfrac{1}{4}t_b^2\cos(ap_F)/t_a\sin^2(ap_F)\ ,\\
&k=beH,\quad x_0=(p_2-\tfrac{1}{2}q_y)(eH)^{-1}\ ,\\
&Q=(2p_F+q_x,\pi/b+q_y,\pi/c)\ ,\\
&q_x=-Nk\ ,
\end{aligned}\quad (7)$$

and $J_n(z)$ is the Bessel function and N is an integer.

We have written down a simplified version of the 4×4 Green's function, which can be easily constructed following the procedure given in Ref. 3. For example, $\Delta(x)$ in this generalized version contains further ρ_3 dependence like $\Delta(x-\rho_3 x_1)e^{\pm i\rho_3\mathbf{Q}\cdot\mathbf{x}}$ and $x_1=(\pi/2)k^{-1}$. On the other hand, all these extra factors cancel out in the course of our analysis except the factor $e^{\pm i\rho_3\mathbf{Q}\cdot\mathbf{x}}$. Therefore, most of the relevant retarded products are obtained from $g(x,x')$ as given in Eq. (6). Furthermore, we shall neglect in Eq. (6) all but the largest component I_N for simplicity,[3] so that we have $\Delta(x)=\Delta I_N=\Delta_N$. Also, we choose Δ so that $\Delta(x)$ is a real constant. As we shall see later, this choice corresponds to having taken the spin of the FISDW oriented along the x direction, since the phase of $\Delta(x)$ describes the spin direction measured from the x direction.

A few remarks are in order about the spin configuration of the FISDW. The spin in a FISDW has, in general, x and y components. Furthermore, the mean-field values of S_+ $(=S_1+iS_2)$ and S_- $(=S_1-iS_2)$ are related by $\langle S_+\rangle^*=\langle S_-\rangle$.

In general this implies that the spin configuration is either helical $\langle S_+\rangle\sim e^{\pm i\mathbf{Q}\cdot\mathbf{x}}$ or linearly polarized $\langle S_+\rangle\sim\cos(\mathbf{Q}\cdot\mathbf{x})$. In the former case, however, for each spin component the energy gap opens up only at one of two Fermi surfaces, while the latter case the energy gap opens up at both of the Fermi surfaces. Therefore, the condensation energy of the linear (i.e., Ising-like) SDW is almost twice that of the helical SDW. This means that the ground state of a FISDW is linearly polarized. Our choice of the off-diagonal elements in Eq. (6) corresponds to this linear SDW.

Since the spin fluctuation couples to the fluctuation of the order parameter $\delta\Delta$, we obtain within a mean-field calculation[8]

$$\langle[s_3,s_3]\rangle=\langle[s_3,s_3]\rangle_0+2U(\langle[s_3,s_3]\rangle_0\langle[s_3,s_3]\rangle+\langle[s_3,\delta\Delta]\rangle_0\langle[\delta\Delta,s_3]\rangle)\ , \quad (8a)$$

$$\langle[\delta\Delta,s_3]\rangle=\langle[\delta\Delta,s_3]\rangle_0+2U(\langle[\delta\Delta,s_3]\rangle_0\langle[s_3,s_3]\rangle+\langle[\delta\Delta,\delta\Delta]\rangle_0\langle[\delta\Delta,s_3]\rangle)\ , \quad (8b)$$

$$\langle[\delta\Delta,\delta\Delta]\rangle=\langle[\delta\Delta,\delta\Delta]\rangle_0+2U(\langle[\delta\Delta,s_3]\rangle_0\langle[s_3,\delta\Delta]\rangle+\langle[\delta\Delta,\delta\Delta]\rangle_0\langle[\delta\Delta,\delta\Delta]\rangle)\ , \quad (8c)$$

where $\langle\ \rangle_0$ means the thermal average with $U=0$ (i.e., the effects of U are only included through the single-particle Green's functions). The second terms in the right-hand side of Eqs. (8a)–(8c) arise from the Coulomb interaction

$$U\int dx\,[n(x)]^2\cong-2U\int dx\,[s_3(x)]^2\cong-2U\int dx[\delta\Delta(x)]^2 \quad (9)$$

which is easily obtained from rearranging the electron field operators. For example, a similar approach has been used to study the spin-wave spectrum[6] in superfluid ^{3}He.

The retarded products $\langle[s_3,s_3]\rangle_0$, etc. are evaluated by standard methods. We find

$$\langle[s_3,s_3]\rangle_0(\mathbf{q},\mathbf{q}')=\delta_0\chi^0_{33}(\mathbf{q})\ , \quad (10a)$$

$$\langle[s_3,\delta\Delta]\rangle_0(\mathbf{q},\mathbf{q}')=(\delta_{\mathbf{Q}}+\delta_{-\mathbf{Q}})\chi^0_{3\Delta}(\mathbf{q})\ , \quad (10b)$$

$$\langle[\delta\Delta,\delta\Delta]\rangle_0(\mathbf{q},\mathbf{q}')=\delta_0\chi^0_{\Delta\Delta}(q)+(\delta_{2\mathbf{Q}}+\delta_{-2\mathbf{Q}})\chi^{0\prime}_{\Delta\Delta}[\tfrac{1}{2}(\mathbf{q}+\mathbf{q}')]\ , \quad (10c)$$

where

$$\delta_{\mathbf{p}}=(2\pi)^3\delta(\mathbf{q}-\mathbf{q}'+\mathbf{p})\ . \quad (11)$$

We note that since the order parameter carries the $\mathbf{x}$ dependence $e^{\pm i\mathbf{Q}\mathbf{x}}$, the retarded function contains diagonal and off-diagonal components. The latter involve the momentum transfer from the order parameter by a multiple of $\mathbf{Q}$. However, the terms with the momentum transfer larger than those listed in Eq. (10) give only negligible contributions, since the intermediate states have large energy. Since the expressions for the retarded products are still rather complicated, we shall assume that $\mathbf{q}$ is parallel to the x axis. Then the relevant integrals are easily done (see Appendix) and we find

$$\chi^0_{33}(\mathbf{q},\omega)=a=\tfrac{1}{2}N_0(\zeta^2-\omega^2)^{-1}(\zeta^2-4\omega^2\Delta_N^2F)\ , \quad (12a)$$

$$\chi^0_{3\Delta}(\mathbf{q},\omega)=b=-i\frac{1}{\sqrt{2}}N_0\,|I_N|\,\omega\Delta_N F\ , \quad (12b)$$

$$\chi^0_{\Delta\Delta}(\mathbf{q}+\mathbf{Q},\omega)=\chi^0_{\Delta\Delta}(\mathbf{q}-\mathbf{Q},\omega)=c_1=\tfrac{1}{2}N_0\,|I_N|^2[\bar{U}_N^{-1}-(\zeta^2-\omega^2+2\Delta_N^2)F]\ , \quad (12c)$$

$$\chi^{0\prime}_{\Delta\Delta}(\mathbf{q},\omega)=c_2=N_0\,|\,I_N\,|^2\Delta_N^2 F\ , \tag{12d}$$

and $\bar{U}_N=\bar{U}\,|\,I_N\,|^2$ and

$$F=(\zeta^2-\omega^2)\int_\Delta^\infty dz(z^2-\Delta_N^2)^{-1/2}\tanh(\tfrac{1}{2}\beta z)\frac{N}{D}\ ,$$
$$N=(\zeta^2-\omega^2)^2-4z^2(\omega^2+\zeta^2)+4\Delta_N^2\zeta^2\ ,$$
$$D=N^2+64z^2\omega^2\zeta^2(\Delta_N^2-z^2)\ . \tag{13}$$

We have already encountered the function F in our analysis of the transverse spin susceptibility.[9] Putting these together we obtain

$$\chi_{33}(q,\omega)=\langle[s_3,s_3]\rangle$$
$$=D_1/D_0=\tfrac{1}{2}N_0\zeta^2[(1-\bar{U})\zeta^2-\omega^2]^{-1} \tag{14}$$

where D_0 and D_1 are determinants:

$$D_1=\begin{vmatrix}1-2Uc_1 & -b & -Uc_2\\ -2Ub & a & -Ub\\ -2Uc_2 & -b & 1-Uc_1\end{vmatrix}$$

$$=[1-2U(c_1-c_2)]\{a[1-2U(c_1+c_2)]-4Ub^2\} \tag{15}$$

and

$$D_0=\begin{vmatrix}1-2Uc_1 & 2Ub & -2Uc_2\\ -2Ub & 1-2Ua & -2Ub\\ -2Uc_2 & 2Ub & 1-2Uc_1\end{vmatrix}$$

$$=[1-2U(c_1-c_2)]\{(1-2Ua)[1-2U(c_1+c_2)]+8U^2b^2\}\ . \tag{16}$$

In spite of the complicated momentum transfer structure of individual retarded products involved, we obtain an extremely simple expression of the mean field χ_{33} as given in Eq. (14), which has essentially the same form as the one in superfluid ^{3}He-A. Equation (14) shows that in the present system the longitudinal spin wave with dispersion

$$\omega^2=(1-\bar{U})v_F^2q^2 \tag{17}$$

exists for all temperatures irrespective of the presence of a FISDW, as long as $(1-\bar{U})>0$. Furthermore, the static longitudinal susceptibility is given by

$$\chi_{33}(0,0)=\tfrac{1}{2}N_0(1-\bar{U})^{-1}\ , \tag{18}$$

independent of temperature.

So far we have neglected the dipole interaction between electron spins. Since the dipole interaction energy breaks the rotation symmetry of the spin polarization of a FISDW (i.e., the dipole interaction energy depends on the phase of Δ discussed in the beginning of this section), this produces an energy gap in the spin-wave dispersion;[5,6] now the spin-wave dispersion is given by

$$\omega^2=(1-\bar{U})\psi_F^2q^2+\Omega^2(T) \tag{19}$$

and

$$\Omega^2(T)=2\pi\lambda[\mu_B\Delta(T)]^2\pi N_0\Delta(T)^{-1}\tanh[\tfrac{1}{2}\beta\Delta(T)]$$
$$\simeq 10^{-6}\Delta^2(T)\ , \tag{20}$$

where

$$\lambda=(Q_x^2-Q_y^2)/(Q_x^2+Q_y^2+Q_z^2)$$
$$\simeq 1-[2+(b^*/c^*)^2][(3b^*/a^*)^2+(b^*/c^*)^2+1]^{-1} \tag{21}$$

and $\Omega(T)=0$ for $T>T_c$. This gap should be observable by an electron spin resonance experiment. The estimated $\Omega(T)$ is around $10^3\sim10^2$ MHz and vanishes like $(T_c-T)^{1/2}$ as the temperature approaches the transition temperature T_c.

III. SPIN DYNAMICS OF FISDW

As already mentioned, the phase of the order parameter $\Delta(x)$ is related to the direction of spin polarization of a FISDW. Furthermore, the fluctuation in this polarization angle couples to that of the longitudinal spin fluctuation. When the phase is space-time dependent, we can construct an effective Lagrangian which describes the spin dynamics of a FISDW. Following a similar analysis[10] for superfluid ^{3}He we shall consider a unitary operator which induces the phase change of a FISDW order parameter $\Delta(x)\to\Delta(x)e^{i\phi}$. This change is produced by a unitary transformation of the electron field operator

$$\psi\to U\psi\ , \tag{22}$$

with

$$U=\exp\left[\frac{i}{2}\sigma_3\phi\right]\ . \tag{23}$$

Note that unlike the case of a charge-density wave,[11] there is no ρ_3 matrix in the exponent. Rather, this type of unitary operator is the same as the one in superfluid ^{3}He. When ϕ is independent of space and time (i.e., global gauge transformation), it does not change the free energy of the system except the dipole interaction energy

$$E_D=\tfrac{1}{2}\chi_0\Omega^2(T)(1-\cos^2\phi)\ , \tag{24}$$

and $\Omega(T)$ has been already defined in Eq. (20). The dipole energy favors the spin configuration with the polarization perpendicular to the chain direction. Then when ϕ depends weekly on space and time, the space and the time derivative of ϕ give rise to a coupling energy which is expressed in terms of the Lagrangian density

$$L_I=\tfrac{1}{2}\psi^\dagger\sigma_3\psi\phi_t+\tfrac{1}{2}v_F\psi^\dagger\sigma_3\rho_3\psi\phi_x\ , \tag{25}$$

where suffices x and t imply the partial derivative with respect to x and t. The first term follows from $iU^{-1}(\partial U/\partial t)$ and the second term from $U^{-1}E(p)U=U^{-1}(-iv_F\rho_3\partial_x U)$, where $E(p)$ is the quasiparticle energy already given by Eq. (5). Here we have neglected the higher-order terms in $a(p_1-p_F)$ for simplicity. Equation (25) implies that ϕ_t couples to the longitudinal spin s_3 while ϕ_x couples to the spin current. The phase Lagrangian is now given by

$$L=\tfrac{1}{2}\chi_0\phi_t^2-\tfrac{1}{4}v_F^2N_0\phi_x^2-\tfrac{1}{2}\chi_0\Omega^2(T)\sin^2\phi\ , \tag{26}$$

with

$$\chi_0=\tfrac{1}{2}(1-\bar{U})^{-1}N_0\ . \tag{27}$$

When ϕ oscillates around the ground-state configurations ($\phi=0$ or π), the oscillation has a dispersion given by Eq. (19), as is easily verified. Since the ground state is doubly degenerate, a FISDW has domain walls (or solitons) given by

$$\phi(x,t)=2\tan^{-1}(e^{\pm(x-vt)/\xi(v)})\ , \tag{28}$$

where

$$\xi(v)=[\Omega(T)/c]^{-1}(1-v^2/c^2)^{1/2}\ , \tag{29}$$

and $c=(1-\bar{U})^{1/2}v_F$ is the spin-wave velocity. These solitons are very similar to d solitons[7] in superfluid ^{3}He-A. The soliton carries spin and spin current given by

$$s_3(x,t)=\pm v\xi^{-1}(v)\chi_0\,\mathrm{sech}[(x-vt)/\xi(v)] \tag{30}$$

and

$$\bar{j}_{\rm spin}(x,t)=\pm\tfrac{1}{2}v_F^2\xi^{-1}(v)N_0\,\mathrm{sech}[(x-vt)/\xi(v)]\ . \tag{31}$$

The present soliton also resembles the π-soliton in a quasi-one-dimensional antiferromagnetic chain like $(CH_3)_4NMnCl_3$(TMMC).[12] Especially, if we take the chain direction and the magnetic field direction as the x and the z axis, s_3 of the present soliton is interpreted as s_1 of the π-soliton in TMMC. However, the thermal excitation of the present soliton is rather unlikely since the present soliton is semimacroscopic unlike the π-soliton in TMMC.

The direct detection of solitons may be difficult in the absence of pinning, since the localized spin-wave mode associated with a solition has zero energy and does not contribute to electron spin resonance. Perhaps these solitons will couple to the electric current, and therefore, they may be seen in the microwave absorption. However, a further study on this question is clearly desirable.

IV. CONCLUDING REMARKS

We have studied the longitudinal spin susceptibility of FISDW's within mean-field theory. We find that the spin susceptibility is independent of T. Furthermore, we find the longitudinal spin wave, which is gapless above the transition temperature ($T>T_c$) but which acquires a small energy gap ($\sim10^3$ MHz) below the FISDW transition. Therefore, the longitudinal spin wave should be accessible by electron spin resonance technique below $T=T_c$. We predict magnetic solitons in a FISDW. They are invisible by electron spin resonance except as a deficit in the total spin resonance intensity in the absence of pinning centers. On the other hand, pinning centers will produce low-frequency structure associated with magnetic solitons in the electron spin resonance.

ACKNOWLEDGMENTS

One of us (K.M.) gratefully acknowledges generous support from the Alexander von Humboldt Foundation. Both of us wish to thank Peter Fulde for his hospitality at the Max-Planck-Institut. This work is in part supported by the National Science Foundation under Grant Nos. DMR-82-14525 and DMR-86-11829.

APPENDIX

Evaluation of quantities in Eqs. (12a)–(12d). First the corresponding thermal products are calculated for $\mathbf{q}=(q,0,0)$ as follows:

$$\chi_{33}^0(\mathbf{q},i\omega_\nu)=\tfrac{1}{2}N_0T\sum_\omega\int d\xi\, d_1^{-1}d_2^{-1}[\omega\omega'-\xi(\xi+\zeta)+\Delta_N^2]$$
$$=\tfrac{1}{2}N_0\left[1-\pi T\sum_\omega\left[1-\frac{\omega\omega'+\Delta_N^2}{AB}\right]\times\frac{(A+B)}{(A+B)^2+\zeta^2}\right]\ , \tag{A1}$$

$$\chi_{3\Delta}^0(\mathbf{q},i\omega_\nu)=-\frac{i}{\sqrt{2}}\frac{N_0}{2}\,|I_N|\,\Delta_N i\omega_\nu T\sum_\omega\int d\xi\, d_1^{-1}d_2^{-1}$$
$$=-\frac{i}{\sqrt{2}}\frac{N_0}{2}\,|I_N|\,\Delta_N i\omega_\nu\pi T\times\sum_\omega\frac{1}{AB}\frac{(A+B)}{(A+B)^2+\zeta^2}\ , \tag{A2}$$

$$\chi_{\Delta\Delta}^0(\mathbf{q}\pm\mathbf{Q},i\omega_\nu)$$
$$=\tfrac{1}{2}\,|I_N|^2N_0T\sum_\omega\int d\xi\, d_1^{-1}d_2^{-1}[\omega\omega'+\xi(\xi+\zeta)]$$
$$=\tfrac{1}{2}\,|I_N|^2N_0\pi T\sum_\omega\left[1+\frac{\omega\omega'}{AB}\right]\frac{(A+B)}{(A+B)^2+\zeta^2}\ , \tag{A3}$$

$$\chi_{\Delta\Delta}^{0\prime}(\mathbf{q},i\omega_\nu)=\tfrac{1}{2}\,|I_N|^2N_0T\sum_\omega\int d\xi\, d_1^{-1}d_2^{-1}\Delta_N^2$$
$$=\tfrac{1}{2}\,|I_N|^2N_0\pi T\sum_\omega\frac{\Delta_N^2}{AB}\frac{(A+B)}{(A+B)^2+\zeta^2}\ , \tag{A4}$$

where

$$\omega=\omega_n,\quad \omega'=\omega_{n-\nu},\quad d_1=\omega^2+\xi^2+\Delta_N^2\ ,$$
$$d_2=\omega'^2+(\xi+\zeta)^2+\Delta_N^2,\quad A=(\omega^2+\Delta_N^2)^{1/2},$$
$$B=(\omega'^2+\Delta_N^2)^{1/2}\ . \tag{A5}$$

The ω sum is carried out with standard methods. Then analytical continuation of $i\omega_\nu\to\omega$, the external frequency, yields Eqs. (12a)–(12d).

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[1]Proceedings of the ICSM Conference, Kyoto, Japan 1986 [Synth. Met. **19**, 1 (1987)].

[2]K. Maki, Phys. Rev. B **33**, 4826 (1986).

[3]A. Virosztek, L. Chen, and K. Maki, Phys. Rev. B **34**, 3371 (1986).

[4]D. Poilblanc, M. Héritier, G. Montambaux, and P. Lederer, J. Phys. C **19**, L321 (1986).

[5]A. J. Leggett, Ann. of Phys. (N.Y.) **47**, 331 (1975).

[6]K. Maki and H. Ebisawa, Prog. Theor. Phys. (Kyoto) **50**, 1452 (1973); J. Low. Temp. Phys. **15**, 213 (1974).

[7]K. Maki and P. Kumar, Phys. Rev. B **14**, 118 (1976).

[8]A summary of the present results will appear in K. Maki and A. Virosztek, Synth. Met. (to be published).

[9]A. Virosztek and K. Maki, Phys. Rev. B **35**, 1954 (1987).

[10]K. Maki, Phys. Rev. B **11**, 4264 (1975).

[11]S. A. Brazovskii and I. E. Dzyaloshinskii, Zh. Eksp. Teor. Fiz. **71**, 2338 (1976) [Sov. Phys.—JETP **44**, 1233 (1976)].

[12]J. P. Boucher, L. P. Regnault, J. Rossat-Mignod, J. P. Renard, J. Bouillot, and W. G. Stirling, Solid State Commun. **33**, 171 (1980).