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Novel features of quantum Hall plateaus for varying interface charge

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A systematic study of quantum Hall plateaus has been made in Si metal-oxide–semiconductor field-effect transistors having driftable Na^+ ions. For low interface ion densities ($N_{ox} \lesssim 2\times10^{11}$ cm^{-2}) the positions of plateaus for filled spin and valley levels exhibit unexpected shifts with respect to those for completely filled Landau levels, and the widths of plateaus for filled Landau levels increase more rapidly with interface charge than do those for filled spin and valley levels. These results have interesting implications for features of the density of states of the system and for aspects of interface electron scattering.

Since the discovery[1] of the quantum Hall effect (QHE) in the two-dimensional electron gas formed in the inversion layer of a Si metal-oxide-semiconductor field-effect transistor (MOSFET) there has been great experimental and theoretical interest in this unique phenomenon.[2–6] In a perpendicular magnetic field the density of states of the electron gas in this system is separated into fully quantized levels which are sketched in the insert to Fig. 1. Each Landau level is split into two spin states which are further split into two valley levels arising from the degeneracy of the conduction band at the Si(100) surface. As these levels of equal degeneracy are filled by increasing the gate voltage V_g, the Hall resistivity exhibits a series of plateaus with values $\rho_{xy}=h/e^2i$, $i=1,2,3,\ldots,$ to remarkably high accuracy. The diagonal resistivity ρ_{xx} is nearly zero for the corresponding values of V_g.

The physical origin of the quantized values of Hall resistivity (for at least the so-called "integer" plateaus) is coming to be well understood.[4–6] The finite widths of the plateaus are less well understood and generally have been associated with pinning of the Fermi level by localized states in tails of the bands corresponding to the broadened levels of the inversion layer.[3,5,6] In the usual Mott picture of localization the plateau width in gate voltage then corresponds to the number of electrons in localized states above the upper mobility edge of the band which has just been filled plus those below the lower mobility edge in the band next higher in energy. However, the microscopic nature of this localization and its role in the QHE are not yet well understood.

To gain insight into the role of localized states in giving Hall plateaus we have studied MOSFET's with mobile positive charge in the oxide. This system in which the number of positive charges at the interface N_{ox} can be varied is particularly attractive for a systematic study of the effects of localization and scattering of the inversion-layer electrons in the QHE. Charge which is drifted to the Si-SiO_2 interface has been found to give reduced carrier mobilities as a result of increased carrier scattering. The Na^+ ions in the oxide near the interface can give rise directly to carrier scattering in the inversion layer of the semiconductor. Alternately, the Na^+ ions can cause impuritylike electron states to be bound in the inversion layer at energies below those for the extended states.[7] Then the Na^+ and the associated localized impurity electrons give rise to local fluctuations in the po-

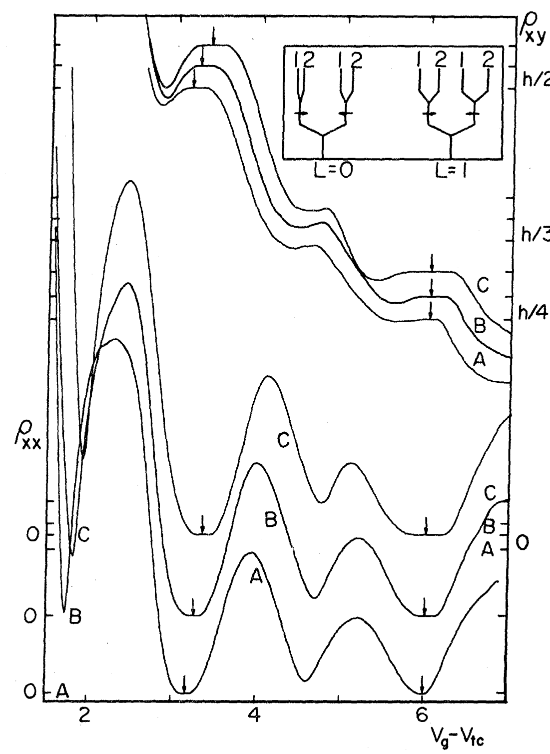

FIG. 1. Effect of added positive interface charge on Hall plateaus at 13 T and 1.35 K: for curves A, $\Delta N_{ox}=0$; B, $\Delta N_{ox}=1.4\times10^{10}$ cm^{-2}; C, $\Delta N_{ox}=4.5\times10^{10}$ cm^{-2}. The two vertical scales are the same and origins have been shifted for clarity. The insert shows a sketch of the energy levels of the system. The data shown in this figure correspond to the fillling of the lowest Landau level $L=0$.

tential and to increased scattering of the inversion-layer electrons. In the present paper we report striking systematic variations in properties of quantum Hall plateaus with variations in the concentration of Na^+ ions at the interface.

Hall bars 250 μm wide by 1300 μm long with mobile positive charge, presumably Na^+, in the gate oxide were used. The samples fall into two categories: (1) high mobility ($\mu_{max} > 20\,000$ cm^2/V s) with maximum driftable oxide charge $\Delta N_{ox} < 10^{11}$ cm^{-2}, and (2) good mobility ($\mu_{max} \approx 10\,000$ cm^2/V s) with maximum $\Delta N_{ox} > 3\times 10^{12}$ cm^{-2}. The second set shows a well-defined impurity band when $\Delta N_{ox} > 5\times 10^{11}$, indicating that the mobile positive charge at the interface was distributed quite uniformly. The amount of interface charge was reversible to within $\Delta N_{ox} < 10^{10}$ cm^{-2}. The conductivity thresholds V_{tc} were determined by a linear extrapolation to zero of the transconductance $(d\sigma/dV_g)$ at 77 K.[7] The amount of positive charge per unit area drifted to the interface was taken to be $\Delta N_{ox} = C\Delta V_{tc}/e$, where C is the capacity per unit area of the device, and ΔV_{tc} is the change in the conductivity threshold. The position of the conductivity threshold with the minimum N_{ox} indicates that in this case N_{ox} is then less than 2×10^{10} cm^{-2} and therefore values of ΔN_{ox} quoted here are quite close to the total N_{ox}. The positions of the Hall plateaus above V_{tc} and their widths in gate voltage were measured using ac techniques. Source-drain current excitation (100 nA at $\sim$1 KHz) was used to measure ρ_{xx} and ρ_{xy}, and gate modulation with 100-nA dc source-drain current was used to measure $d\rho_{xx}/dn$ and $d\rho_{xy}/dn$ as functions of V_g. Here

$$n = C(V_g - V_{tc})/e \qquad (1)$$

is the electon density per unit area in the inversion layer. The position of a given Hall plateau was taken to be the position of the ρ_{xx} minimum at a temperature (1.3–4.2 K), where ρ_{xx} shows a well-defined minimum rather than a flat region in V_g. In determining Hall plateau widths we follow the usual method[3] in which the width is taken to be the region in V_g or n for which $\rho_{xy} = h/ie^2 \pm \Delta$, where Δ is a small quantity associated with experimental resolution. The width at 13 T and 1.35 K, for example, was that for which $d\rho_{xy}/dn < 3.5\times 10^{-10}$ Ω cm^2, where n is given by Eq. (1), and the corresponding Δ is $\sim 1\,\Omega$.

Two distinct regimes of behavior are observed in the data, one for low interface charge density ($N_{ox} \lesssim 2\times 10^{11}$ cm^{-2}) and correspondingly high mobilities, and the second for higher N_{ox} and lower mobilities.[8] In the present paper we report and discuss the results for the regime of low N_{ox} which have interesting implications for features of the density of states of the system and for aspects of interface electron scattering. The interpretation of results in this regime is more straightforward because the low-density Na^+ can be pictured as essentially isolated ions.

An example of the present data is shown in Fig. 1. ρ_{xx} and ρ_{xy} are given as functions of gate voltage above V_{tc} for three values of interface charge. The minima (arrows) in ρ_{xx} and plateaus in ρ_{xy} at the higher gate voltage correspond to filling the first complete Landau level which has a degeneracy of four levels (see insert to Fig. 1). The (arrows) at lower gate voltage correspond to the filling of the first spin level. Finally, the less-pronounced minima in ρ_{xx} above and below the spin plateau are for filled valley levels. The plateau widths increase with increasing oxide charge, and the ρ_{xx} minima for filled spin and valley levels have unexpected systematic shifts with respect to those for Landau levels. The dips in ρ_{xy} at gate voltages just below the plateaus are a common sample-dependent feature of quantum Hall-effect measurements.

In looking back at the data in Ref. 1 we note a small shift of the ρ_{xx} minimum corresponding to a filled spin level with respect to that for a filled Landau level which is of the same order ($\sim 10^{11}$ cm^{-2}) as the shifts observed in our samples having no added interface charge. We also note such shifts in the case of quantum Hall steps in $Al_xGa_{1-x}As/GaAs$ heterostructures.[3] To our knowledge, however, this effect has not been addressed in any detail previously.

It is useful to discuss the shifts of the ρ_{xx} minima in terms of an apparent conductivity threshold V_t (or apparent position of the lowest electric subband) which is determined for each type of ρ_{xx} minima (filled valley, spin, or Landau level) by extrapolating the positions of the minima as a function of Landau index back to zero. The shifts of minima for filled spin levels with respect to those for filled Landau levels persist for all plateaus that can be resolved (at least five in all of the present cases), and the relative shifts for higher Landau levels decrease only slightly with increasing Landau index. In Fig. 2 the extrapolated thresholds V_t are shown as $V_t - V_{tc}$ versus the amount of drifted interface charge. Each value of V_t is averaged over at least five different magnetic fields. The systematic shifts of spin and valley thresholds with respect to the Landau-level thresholds are seen to be substantial. The measurements of the Landau thresholds are consistent with there being no shift relative to V_{tc}; there appears to be a small systematic increase in the Landau threshold positions with ΔN_{ox}, but uncertainties in the determining V_{tc} and ΔN_{ox} ($\gtrsim 2\times 10^{10}$ cm^{-2}), particularly at high ΔN_{ox},[7] are greater than this variation.

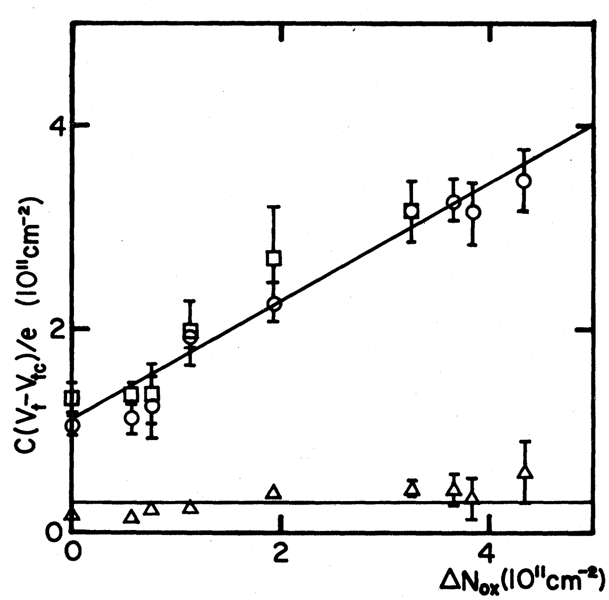

FIG. 2. The apparent conductivity thresholds V_t for filled spin levels (○), filled valley levels (□), and filled Landau levels (Δ) vs interface charge density. Error bars are uncertainties associated with the extrapolations of the data to find V_t. The line through the spin and valley data is a linear fit. The line through the Landau level data is a guide to the eye.

The widths of the Hall plateaus for the highest mobility sample are shown in Fig. 3, and they are seen to increase with ΔN_{ox}. It is also seen, however, that the widths for filled Landau levels increase more rapidly with ΔN_{ox} than do those for spin levels. Each of the points in Fig. 3 is an average of six measurements at different substrate biases, and error bars give the standard deviations of the corresponding data.[9]

The experimental results obtained here can be understood on the basis of a picture in which quantum Hall plateaus arise from the pinning of the Fermi energy by localized states in the band tails. Consider first the fact that the step widths, in general, increase with ΔN_{ox} in this low-N_{ox} regime. As indicated above, the inversion-layer electrons are expected to experience greater potential fluctuations for increased oxide charge. In the standard Mott picture this leads to more localized states in the tails of each band and to movement of the mobility edges inward toward the centers of the bands. These features therefore are consistent with the observed increasing widths with increasing oxide charge for all of the plateaus.

The shifts of positions of the different ρ_{xx} minima and the different magnitudes of the changes in their associated plateau widths, however, are unexpected. The reason is that $V_t - V_{tc}$ is proportional to the number of electrons in the inversion-layer bands as given in Eq. (1). The several minima correspond to the filling of levels of constant degeneracy, and therefore their positions should remain unchanged. The finite temperatures at which the ρ_{xx} are determined cause the weakly localized states at the band edges to become delocalized and give clearly defined positions for the ρ_{xx} minima.

We argue that the relative shifts in the positions of the ρ_{xx} minima arise from particular changes in the overlaps of the corresponding bands with changing oxide charge. From the reasoning in the preceding paragraph it is seen that in order for the position of a ρ_{xx} minimum to change in gate voltage (or equivalently in density) with changing N_{ox} it is necessary that the overlap in energy of the adjacent bands change and that the band overlaps change asymmetrically. In this way the number of electrons in states below the ρ_{xx} minimum changes. The splittings between the Landau levels are $\hbar\omega_c \cong 7$ meV, spin splittings $g\mu_B B$ are smaller on the order of 2.5 meV, and valley splittings are very small on the order of 0.5 meV (spin and valley splittings depend on occupancy), and bandwidths have been estimated to be on the order of 1–1.5 meV.[10] Therefore it is natural to expect significant overlap between the states composing a given Landau level and less overlap between the adjacent states of different Landau levels. The absence of any appreciable shifts of the ρ_{xx} minima coresponding to filled Landau levels in Fig. 3 then is consistent with there being negligible overlap between the corresponding bands.

We argue furthermore that the changing overlap between the bands composing a given Landau level are expected to be asymmetrical in such a way as to cause the corresponding ρ_{xx} minima to move to higher gate voltages (or densities) as observed for several reasons: (i) The shape of a band is expected to be asymmetrical as a result of scattering by the fixed positive charges at the interface. Results of numerical studies[11] and of analytical calculations[12] for distributions of localized attractive scattering centers with densities similar to those in our experiment give Landau bands which are decidedly skew with a longer tail to the low-energy side. In the present experiments the added Na^+ ions increase the scattering and hence increase the bandwidths. For reasonable band shapes the changes in the overlap between the bands increase the number of electrons below the ρ_{xx} minima and thus move them to higher V_g. (ii) The effects of screening increase with band filling. Thus larger numbers of localized states and more strongly localized states appear at the low-energy edge of each band, and this tends to shift the ρ_{xx} minimum to higher V_g. (iii) A particular scattering mechanism arises from those Na^+ ions which form shallow impuritylike states in the inversion layer.[7,13] The lowest-energy impurity state is predominantly occupied and has either spin or valley function orthogonal to the first three states but not to the fourth of each complete Landau level. The repulsive effect from this orthogonality gives additional scattering and an upward shift in the energy of these three states. These effects increase the asymmetrical overlap among the first three bands as above and decrease the overlap corresponding to the completely filled Landau level.

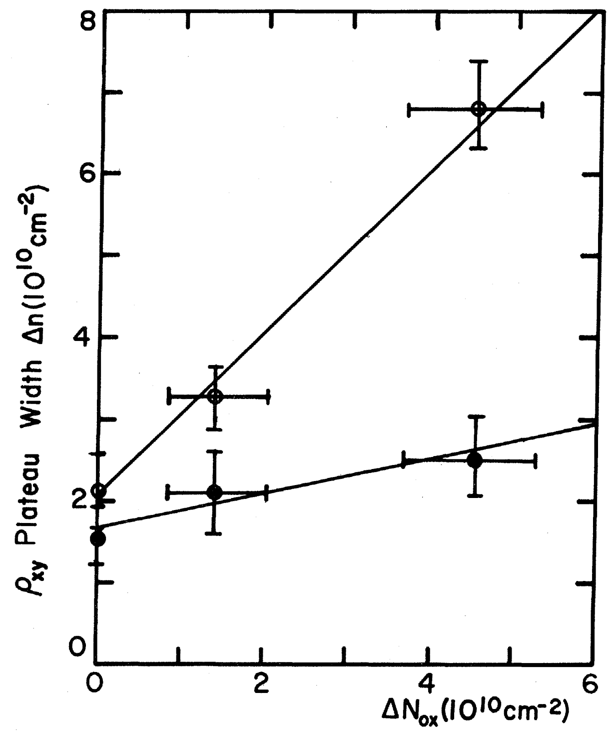

FIG. 3. Hall-plateau widths associated with filled Landau levels (upper points) and with filled spin levels (lower points) vs added interface charge.

Increased scattering for bands which have significant overlap can cause the extreme band-tail states of one band to move into a region of extended states beyond the mobility edge of the other. For adjacent bands which have negligible overlap this does not occur. This effect is consistent with the data in Fig. 3 in which the widths of quantum Hall plateaus corresponding to completely filled Landau levels, which have negligible overlap, increase more rapidly with increasing ΔN_{ox} than do those for the other plateaus which have significant band overlap.

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[1]K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).

[2]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. 54, 437 (1982); in *Proceedings of the IV International Conference on the Electronic Properties of Two-Dimensional Systems,* edited by F. Stern [Surf. Sci. 113, 1 (1983)], and references within.

[3]H. L. Störmer, D. C. Tsui, and A. C. Gossard, Surf. Sci. 113, 32 (1982); M. A. Paalanen, D. C. Tsui, and A. C. Gossard, Phys. Rev. B 25, 5566 (1982).

[4]T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. 39, 279 (1975).

[5]R. E. Prange, Phys. Rev. B 23, 4802 (1981).

[6]R. B. Laughlin, Phys. Rev. B 23, 5632 (1981); B. I. Halperin, *ibid.* 25, 2185 (1982); G. F. Giuliani, J. J. Quinn, and S. C. Ying, *ibid.* (to be published).

[7]A. Hartstein and A. B. Fowler, Phys. Rev. Lett. 34, 1435 (1975).

[8]For high ΔN_{ox} (low mobility) the plateau widths for filled Landau levels reach a maximum and then decrease for further increases in ΔN_{ox}. This behavior is consistent with that seen by Störmer *et al.* in Ref. 3. We find, in addition, however, that widths corresponding to filled spin levels continue to grow with increasing ΔN_{ox}. J. E. Furneaux and T. L. Reinecke (unpublished).

[9]The effect of substrate bias on the plateau positions was found to be smaller than our experimental uncertainty. The effect of substrate bias on plateau widths also was found to be small. Thus data for different substrate biases were averaged to give the data points.

[10]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. 36, 959 (1974).

[11]H. Aoki, J. Phys. C 10, 2583 (1977); 11, 3823 (1978).

[12]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. 36, 959 (1974); T. Ando, *ibid.* 36, 1521 (1974).

[13]B. G. Martin and R. F. Wallis, Phys. Rev. B 18, 5644 (1978); B. Vinter, *ibid.* 26, 6808 (1982).