Reply to "Comment on 'Acoustic plasma modes'"

A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907

J. Appel
Institute of Theoretical Physics II, University of Hamburg, D-2000 Hamburg 36, West Germany
(Received 21 July 1983)

There are potentially three acoustic plasma modes in a two-component Fermi liquid, provided the two effective masses are sufficiently different. In the normal state two of these modes are overdamped, but in the superconducting state all three should propagate with little damping.

During a study[1] of the effects of exchange and correlation on the frequency and damping of acoustic plasmons we discovered two additional eigenvalues among the solutions of the coupled Boltzmann equations. The theoretical method which we employed involved a search for real eigenvalues. Landau damping of the modes (caused by one-electron excitations) was then evaluated from the "golden rule" transition rate. This method is mathematically valid and (we think) physically more transparent than an alternative procedure employing complex eigenvalues at the outset, provided the damping is small, i.e., when the quality factor of the mode satisfies $Q >> 1$. If the damping is moderately large, then one must determine the complex eigenvalues self-consistently, as Oliva and Ashcroft[2] emphasize.

However, the "intrinsic" frequency of a heavily damped mode is still of interest. Consider a classical, damped harmonic oscillator described by the equation

$$\ddot{x} = -\omega_0^2 x - \gamma \dot{x} \ , \tag{1}$$

where ω_0 is the intrinsic frequency and γ is the damping coefficient. The (complex) eigenvalues for the solutions, $\sim \exp(-i\omega t)$, are

$$\omega_\pm = -\tfrac{1}{2} i\gamma \pm [\omega_0^2 - (\tfrac{1}{2}\gamma)^2]^{1/2} \ . \tag{2}$$

When the oscillator is overdamped, i.e., when

$$\gamma \geqslant 2\omega_0 \ , \tag{3}$$

it does not oscillate at all. It only undergoes exponential decay. The real part of $\omega_\pm$ is zero.

Nevertheless, the intrinsic frequency ω_0 is still of physical interest. For example, suppose an oscillatory force, of frequency ω, is applied to such a system. The spectral density of the response (the power dissipation versus ω, normalized to unity) is

$$P(\omega) = \frac{2\pi^{-1}\gamma\omega^2}{(\omega^2 - \omega_0)^2 + \gamma^2\omega^2} \ . \tag{4}$$

The spectral shape of this dissipation depends strongly on the intrinsic frequency ω_0 even though, with $\gamma > 2\omega_0$, the "oscillator" cannot oscillate. Landau damping of modes in metals leads to the same spectral density[3] as that given by Eq. (4). ω_0 remains an important parameter of the system. It describes the intrinsic "springiness" associated with a particular generalized displacement.

We disagree with Oliva and Ashcroft[2] when they suggest that we overlooked the importance of damping. Our intent was not to calculate the eigenvalues analogous to $\omega_\pm$ in Eq. (2), but rather the parameter equivalent to ω_0. On investigating the Landau damping we found (and stated) that the new "modes" were overdamped. We took it for granted that most readers of the Physical Review would be familiar with terms commonly used for elementary oscillators, including Eqs. (1)–(4), and that they know that an overdamped oscillator cannot oscillate, and that an overdamped mode (of wave vector $\vec{q}$) cannot propagate. We also remarked on our intentional neglect of the frequency shift caused by damping (since it is small for the Fröhlich mode). Whether or not one considers the new roots as "modes" is perhaps merely a matter of taste. Their *undamped* velocity was (and is) of interest to us.

Our investigation of acoustic plasmons stemmed from their possible importance in superconductors. We cite the work of Frölich,[4] Takada,[5] and Ruvalds.[6] Many superconductors have both heavy-mass *d* bands and light-mass *s-p* bands, so acoustic plasmons are likely to occur. It is well known from the pioneering experiments of Bömmel[7] that Landau damping of acoustic waves drops rapidly to zero below the superconducting transition temperature T_c. Furthermore, the superconducting energy gap 2Δ causes a negligible change ($\sim 10^{-5}$) in acoustic velocities.[8] Consequently, we conjecture that *all three* acoustic plasma modes might propagate (with negligible damping) in the superconducting state near $T=0$, provided $\hbar\omega < 2\Delta$. Brillouin[9] or Raman[10] scattering techniques could be applicable for detection. A theoretical study of these possibilities seems worthwhile. It would be exciting to find all three modes, and to observe two of them disappear as T approaches T_c.

[1]J. Appel and A. W. Overhauser, Phys. Rev. B 26, 507 (1982).
[2]J. Oliva and N. W. Ashcroft, Phys. Rev. B 28, 1067 (1983), preceding comment.
[3]M. Danino and A. W. Overhauser, Phys. Rev. B 26, 1569 (1982).
[4]H. Fröhlich, Proc. Phys. Soc. London 1, 544 (1968).
[5]Y. Takada, J. Phys. Soc. Jpn. 43, 1627 (1977).
[6]J. Ruvalds, Adv. Phys. 30, 677 (1981).
[7]H. E. Bömmel, Phys. Rev. 96, 220 (1954).
[8]G. A. Alers and D. L. Waldorf, IBM J. Res. Dev. 6, 89 (1962).
[9]J. R. Sandercock, in *Light Scattering in Solids III*, edited by M. Cardona and G. Güntherodt (Springer, New York, 1982), p. 172.
[10]M. V. Klein, in Ref. 9, p. 121.

