Effects of spatial anisotropy on the order of fluctuation-driven transitions

Daniel Blankschtein and Amnon Aharony
Department of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel
(Received 16 September 1981)

The application of pressure to quasi-$(d-1)$-dimensional systems increases the effective dimensionality of their fluctuations. This is shown to turn fluctuation-driven continuous phase transitions into first-order ones via tricritical points. The resulting pressure dependence of the magnetic field-temperature phase diagrams of metamagnets is accordingly described.

The crucial role played by *critical fluctuations* in determining the *order* of a phase transition has been the subject of intensive recent research. It has been shown that a variety of phase transitions, which are predicted by Landau's theory[1] (no fluctuations) to be *continuous,* are driven *first order* by critical fluctuations.[2] Recently we have shown that critical fluctuations can also transform an otherwise *first-order* transition into a continuous one. We have emphasized that these *fluctuation-driven continuous transitions always occur in the vicinity of tricritical points.*[3]

The role played by critical fluctuations is strongly correlated to the *spatial dimensionality* of the system, d. This role becomes more important as d is lowered.[4] In particular, critical fluctuations may change the *order* of a phase transition at low d, even if they cannot do so at higher d. For example, the three-state Potts[5] model undergoes a *first-order* transition at $d=3$ (Ref. 6) and a *continuous* one at $d=2$.[7] The "destruction" of this first-order transition follows from the enhancement of critical fluctuations when the dimensionality is lowered. To substantiate this picture, the three-state Potts model was recently analyzed in a $d=3$ dimensional system with a finite thickness, L, in one direction.[8] It was shown numerically that for sufficiently large L ($L \gg \xi_0$, where ξ_0 is the finite correlation length characteristic of the first-order transition) the transition is *first order* (as in $d=3$), whereas for $L \ll \xi_0$ it becomes *continuous* (as in $d=2$). The example of the Potts model leads one to expect that lowering the effective dimensionality will always decrease the tendency of systems to undergo first-order transitions. The present note aims at a systematic analytic study of this expectation. We consider a general fluctuation-driven continuous transition, in the vicinity of the tricritical point, and show that *an increase in the effective dimensionality can indeed turn it first order.* Earlier work concentrated on reducing the effects of fluctuations by breaking the symmetry of the order parameter.[3,9] In contrast, we present here the first systematic analytic demonstration of such a change without this type of symmetry breaking. The effects of fluctuations are reduced by an effective increase of the spatial dimensionality.

We demonstrate our argument by considering a quasi-$(d-1)$-dimensional system, in which the (nonzero) coupling between spins on different $(d-1)$-dimensional hyperplanes, $J_\perp$, is much weaker than the intraplanar one, $J_\parallel$. When such systems undergo continuous transitions, they exhibit a $(d-1)$-dimensional critical behavior over a wide range of temperatures. However, they always cross over to the d-dimensional behavior, asymptotically close to the critical temperature.[10,11] A variation of the anisotropy may thus serve as a change in the effective dimensionality of the system. It is our aim here to study the effects of such a variation in the vicinity of a *tricritical point.* A tricritical point is usually reached by varying both the temperature, T, and a nonordering field, Δ.[12] We shall show below that strengthened interactions in the interplane direction (caused, e.g., by external stress) imply a shift of the tricritical point in the T-Δ plane. Within our model, this shift is towards a narrower range of continuous transitions.[13]

Some of the most studied tricritical points occur in *metamagnets,* in which the spins are ferromagnetically coupled in the layers and have a relatively weak antiferromagnetic coupling between adjacent layers. A high uniform magnetic field usually turns the (otherwise continuous) antiferromagnetic transition first order. Known examples include $FeCl_2$,[14] $FeBr_2$,[15] $Ni(NO_3)_2 \cdot 2H_2O$,[16] and many others.[17]

The interlayer coupling ($J_\perp$) becomes stronger by applying either hydrostatic pressure or uniaxial stress perpendicular to the layers.[18] We thus predict that application of such pressure, at a fixed temperature *above* the tricritical point, may turn the continuous transition into first order. This agrees with some experiments on $FeCl_2$ and $FeBr_2$.[19]

Some quasi-two-dimensional antiferromagnets, e.g., Rb_2CoF_4,[20] K_2CoF_4,[21] or $CoCs_3Br_5$,[22] order antiferromagnetically in the layers, and practically exhibit two-dimensional critical behavior. As far as we know, even high magnetic fields have not yet been able to turn these transitions first order. We predict that tricriticality will be reached if one also applies

pressure.[23]

The tricritical points of metamagnets were considered, using renormalization-group (RG) techniques, by Nelson and Fisher.[24] However, they were concerned mainly with the effects of the field, and did not consider in detail the effects of the spatial anisotropy, on which we concentrate here.

As a model, we start with the n-component ferromagnetic spin Hamiltonian

$$\mathcal{H} = -\sum_{\vec{R},\vec{\delta}} J(\vec{\delta})(\vec{S}_{\vec{R}} \cdot \vec{S}_{\vec{R}+\vec{\delta}}) \ , \tag{1}$$

where the exchange interactions $J(\vec{\delta})$ have different values for $\vec{\delta}$ in the $(d-1)$-dimensional layers (denoted by $\|$) or perpendicular to these layers ($\perp$). We now follow the standard methods.[25] Using the weight function for the continuous spins, $\exp(-\frac{1}{2}|\vec{S}|^2 - f_4|\vec{S}|^4 - f_6|\vec{S}|^6 - \cdots)$, the partition function becomes $Z = \int \prod_{\vec{R}} d^n S_{\vec{R}} \exp(\bar{\mathcal{H}})$, with

$$\bar{\mathcal{H}} = -\frac{\mathcal{H}}{k_B T} - \sum_{\vec{R}} (\tfrac{1}{2}|\vec{S}_{\vec{R}}|^2 + f_4|\vec{S}_{\vec{R}}|^4 + f_6|\vec{S}_{\vec{R}}|^6 + \cdots) \ . \tag{2}$$

We next make a Fourier transformation,

$$\vec{S}_{\vec{R}} = \frac{1}{N} \sum_{\vec{q}} e^{i\vec{q}\cdot\vec{R}} \vec{\sigma}_{\vec{q}} \ , \tag{3}$$

where the summation is over the N points of the Brillouin zone (BZ)

$$|q_\perp| < \pi/a_\perp \ , \quad |q_\parallel^\alpha| < \pi/a_\parallel \tag{4}$$

[α denotes the $(d-1)$ components of $\vec{q}_\parallel$]. In the thermodynamic limit, $(1/N)\sum_{\vec{q}}$ will become $a_\parallel^{d-1} a_\perp \int_{\vec{q}}$, where $\int_{\vec{q}}$ denotes $(2\pi)^{-d}\int d^d q$ with $\vec{q}$ bounded by Eq. (4).

Expanding the Fourier transform of $J(\vec{\delta})$,

$$\hat{J}(\vec{q}) = \hat{J}(0) - j_\perp (a_\perp q_\perp)^2 - j_\parallel |a_\parallel \vec{q}_\parallel|^2 + O(q^4) \ , \tag{5}$$

and ignoring higher-order terms, we end up with

$$\begin{aligned}\bar{\mathcal{H}} = &-\tfrac{1}{2} a_\parallel^{d-1} a_\perp \int_{\vec{q}} \left(1 - \frac{\hat{J}(0)}{k_B T} + \frac{j_\perp}{k_B T}(a_\perp q_\perp)^2 + \frac{j_\parallel}{k_B T}|a_\parallel \vec{q}_\parallel|^2\right)(\vec{\sigma}_{\vec{q}} \cdot \vec{\sigma}_{-\vec{q}}) \\ &- f_4 (a_\parallel^{d-1} a_\perp)^3 \int_{\vec{q}_1}\int_{\vec{q}_2}\int_{\vec{q}_3} (\vec{\sigma}_{\vec{q}_1}\cdot\vec{\sigma}_{\vec{q}_2})(\vec{\sigma}_{\vec{q}_3}\cdot\vec{\sigma}_{-\vec{q}_1-\vec{q}_2-\vec{q}_3}) \\ &- f_6 (a_\parallel^{d-1} a_\perp)^5 \int_{\vec{q}_1}\int_{\vec{q}_2}\int_{\vec{q}_3}\int_{\vec{q}_4}\int_{\vec{q}_5} (\vec{\sigma}_{\vec{q}_1}\cdot\vec{\sigma}_{\vec{q}_2})(\vec{\sigma}_{\vec{q}_3}\cdot\vec{\sigma}_{\vec{q}_4})(\sigma_{\vec{q}_5}\cdot\vec{\sigma}_{-\vec{q}_1-\vec{q}_2-\vec{q}_3-\vec{q}_4-\vec{q}_5}) \ .\end{aligned} \tag{6}$$

The spatial anisotropy is reflected, at this stage, by the different coefficients of $|\vec{q}_\parallel|^2$ and $q_\perp^2$ and by the noncubic BZ [Eq. (4)]. We now change the scales of $\vec{q}$ and of $\vec{\sigma}_{\vec{q}}$ (which are dummy integration variables), in order to bring (6) to a more convenient form. This is done in three steps:

(A) We rescale $q_\perp$, $q_\perp \to \tilde{q}_\perp = q_\perp (j_\perp a_\perp^2 / j_\parallel a_\parallel^2)^{1/2}$, to make the coefficients of $|\vec{q}_\parallel|^2$ and $\tilde{q}_\perp^2$ equal. The cutoff of $\tilde{q}_\perp$ is now changed into $\Lambda = (\pi/a_\parallel) \times (j_\perp/j_\parallel)^{1/2}$.

(B) We now make both cutoffs dimensionless, by rescaling $\vec{q}$ into $\vec{q}/\Lambda$. The cutoff of the new $q_\perp$ is now equal to unity, while that of $\vec{q}_\parallel$ becomes $(j_\parallel/j_\perp)^{1/2} > 1$ (we assume that $j_\parallel > j_\perp$).

(C) The above two steps introduce extra factors of $(j_\parallel a_\parallel^2 / j_\perp a_\perp^2)^{1/2} \Lambda^d$ in front of each integration, and an additional factor Λ^2 in front of $\vec{q}^2(\vec{\sigma}_{\vec{q}}\cdot\vec{\sigma}_{-\vec{q}})$. This coefficient now becomes equal to unity if we replace $\vec{\sigma}_{\vec{q}}$ by $\tilde{\vec{\sigma}}_{\vec{q}}$,

$$\vec{\sigma}_{\vec{q}} \to \tilde{\vec{\sigma}}_{\vec{q}} = \vec{\sigma}_{\vec{q}} \left(\frac{j_\parallel a_\parallel^2}{k_B T}\right)^{1/2} \left(\frac{j_\parallel a_\parallel^2}{j_\perp a_\perp^2}\right)^{1/4} (\Lambda^{d+2} a_\parallel^{d-1} a_\perp)^{1/2} \ . \tag{7}$$

Substitution in $\bar{\mathcal{H}}$ and renaming $\vec{q}$ and $\vec{\sigma}$ finally yields

$$\begin{aligned}\bar{\mathcal{H}} = &-\tfrac{1}{2}\int_{\vec{q}} (r + q^2)(\vec{\sigma}_{\vec{q}}\cdot\vec{\sigma}_{-\vec{q}}) - u_4 \int_{\vec{q}_1}\int_{\vec{q}_2}\int_{\vec{q}_3} (\vec{\sigma}_{\vec{q}_1}\cdot\vec{\sigma}_{\vec{q}_2})(\vec{\sigma}_{\vec{q}_3}\cdot\vec{\sigma}_{-\vec{q}_1-\vec{q}_2-\vec{q}_3}) \\ &- u_6 \int_{\vec{q}_1}\int_{\vec{q}_2}\int_{\vec{q}_3}\int_{\vec{q}_4}\int_{\vec{q}_5} (\vec{\sigma}_{\vec{q}_1}\cdot\sigma_{\vec{q}_2})(\vec{\sigma}_{\vec{q}_3}\cdot\vec{\sigma}_{\vec{q}_4}) \\ &\times (\vec{\sigma}_{\vec{q}_5}\cdot\vec{\sigma}_{-\vec{q}_1-\vec{q}_2-\vec{q}_3-\vec{q}_4-\vec{q}_5}) \ ,\end{aligned} \tag{8}$$

where $\int_{\vec{q}}$ now denotes $(2\pi)^{-d}\int d^dq$ with

$$|q_\perp|<1,\quad |q_\parallel^\alpha|<(j_\parallel/j_\perp)^{1/2}\ , \tag{9}$$

and

$$u_4=f_4\pi^{d-4}\left(\frac{k_BT}{j_\parallel}\right)^2\left(\frac{j_\perp}{j_\parallel}\right)^{(d-5)/2}\ , \tag{10}$$

$$u_6=f_6\pi^{2d-6}\left(\frac{k_BT}{j_\parallel}\right)^3\left(\frac{j_\perp}{j_\parallel}\right)^{d-4}\ , \tag{11}$$

$$r=[k_BT-\hat{j}(0)]/j_\perp\pi^2\ . \tag{12}$$

Note that the geometrical anisotropy, $(a_\parallel/a_\perp)$, has completely dropped out of the calculation. The only important effects are those of the anisotropy in the coupling, $j_\perp/j_\parallel$.

The only difference between Eq. (8) and the Hamiltonian which is usually used in RG calculations (except for the initial values of r, u_4, and u_6) is now reflected by the unusual shape of the BZ, Eq. (9). Equation (8) now also contains fluctuations $\vec{\sigma}_{\vec{q}}$ with $1<|q_\parallel^\alpha|<(j_\parallel/j_\perp)^{1/2}$. Since these additional fluctuations have short wavelengths, we may integrate them out of the partition function and remain with a cubic BZ, for which all $|q^\alpha|<1$. This integration may be carried out using a perturbation expansion in u_4 and u_6. The only effects of this integration are shifts in the coefficients r, u_4, u_6, etc. These now acquire effective values, which are functions of the anisotropy $(j_\parallel/j_\perp)$. Since the resulting Hamiltonian is identical in form to that of the isotropic system, its continuous transitions will be described by the isotropic fixed point, and its critical behavior will be the same as that of the d-dimensional isotropic system.[11,24] It is for this reason that the anisotropy $(j_\perp/j_\parallel)$ may be considered a "*redundant*" variable.[26]

Unlike earlier work,[11,24] we now consider in detail the new effective coefficients. In order to obtain quantitative estimates for these coefficients, we now follow the usual routine,[25] and replace the BZ by an ellipsoid,

$$q_\perp^2+\frac{j_\perp}{j_\parallel}|\vec{q}_\parallel|^2<1\ . \tag{13}$$

(An integration over the rectangular BZ will give the same monotonic dependence on $j_\perp/j_\parallel$.) In order to turn this BZ into the (usual) sphere $|\vec{q}|<1$, we now integrate over the $\vec{\sigma}_{\vec{q}}$'s with $|\vec{q}|>1$. In spherical coordinates,

$$q_\perp=q\cos\theta,\quad q_\parallel=q\sin\theta\ ,$$

this implies

$$1<q^2<Q(\theta)^2=\left(\cos^2\theta+\frac{j_\perp}{j_\parallel}\sin^2\theta\right)^{-1}\ . \tag{14}$$

Using a linear perturbation expansion in u_4 and u_6, and assuming that near the transition r is negligible, u_6 will now remain unchanged (ignoring higher-order terms), while u_4 will transform into

$$\begin{aligned}u_4^{\rm eff}&=u_4+C\tilde{K}_du_6\int_0^\pi(\sin\theta)^{d-2}d\theta\int_1^{Q(\theta)}\frac{q^{d-1}dq}{q^2}\\&=u_4+Cu_6K_d\left[\left(\frac{j_\parallel}{j_\perp}\right)^{(d-3)/2}I_d\left(\frac{j_\perp}{j_\parallel}\right)-1\right]\Big/(d-2)\ ,\end{aligned} \tag{15}$$

where $C=3(n+4)$ is a combinatorial factor,

$$K_d^{-1}=2^{d-1}\pi^{d/2}\Gamma(d/2)\ ,$$

$$\tilde{K}_d=K_d\Big/\int_0^\pi(\sin\theta)^{d-2}d\theta\ ,$$

and

$$I_d(x)=x^{-1/2}\int_0^\pi d\theta\left(\frac{\sin^2\theta}{\sin^2\theta+\cos^2\theta/x}\right)^{(d-2)/2}\Big/\int_0^\pi(\sin\theta)^{d-2}d\theta\ . \tag{16}$$

In particular,

$$\begin{aligned}I_3(x)&=\tfrac{1}{2}(1-x)^{-1/2}\ln\frac{1+\sqrt{1-x}}{1-\sqrt{1-x}}\ ,\\I_4(x)&=2/(1+\sqrt{x})\ .\end{aligned} \tag{17}$$

A similar expression may be written for $r^{\rm eff}$. Replacing r and u_4 by $r^{\rm eff}$ and $u_4^{\rm eff}$ we can thus use Eq. (8) with all momenta $|\vec{q}|<1$, which is the usual starting point of RG calculations.[25] The effects of the original spatial anisotropy are now simply hidden in the renormalized initial conditions $r^{\rm eff}$ and $u_4^{\rm eff}$.

We now turn to the RG recursion relations. As is well known,[12] the tricritical point is represented for $d\geq 3$ by the Gaussian fixed point, near which these recursion relations have the form[27]

$$\begin{aligned}\frac{du_4}{dl}&=(4-d)u_4+CK_du_6+\cdots\ ,\\\frac{du_6}{dl}&=(6-2d)u_6+\cdots\ ,\end{aligned} \tag{18}$$

where the dots stand for higher-order terms. It is easy to check that the equation for u_4 can be rewritten as

$$\frac{d\tilde{u}_4}{dl}=(4-d)\tilde{u}_4\ , \tag{19}$$

with the scaling field[3,27]

$$\tilde{u}_4 = u_4 + Cu_6K_d/(d-2) \quad . \tag{20}$$

The tricritical point is located at $\tilde{u}_4 = 0$. Since Landau's theory[1] predicts a first-order transition for all $u_4 < 0$, we called the continuous transitions which occur for $-Cu_6K_d/(d-2) < u_4$ "fluctuation driven."[3] Note that the effects of u_6 were not included in Ref. 24.

In the present calculation we must replace the initial value of u_4 by u_4^{eff}, Eq. (15). Using also Eqs. (10) and (11) we thus find the new condition for tricriticality (TC),

$$f_4^{\text{TC}} = -f_6 CK_d \pi^{d-2} (k_B T/j_{\parallel}) I_d(j_{\perp}/j_{\parallel})/(d-2) \quad . \tag{21}$$

Usually, tricriticality is achieved by causing a variation of u_4 (or f_4) as a function of some external (nonordering) field Δ (e.g., the uniform field in the metamagnet[24]). Equation (21) now represents an explicit dependence of the tricritical value f_4^{TC} on the anisotropy $(j_{\perp}/j_{\parallel})$. Ignoring the shift in $(k_B T/j_{\parallel})$ due to the anisotropy,[28] we have

$$\frac{f_4^{\text{TC}}(j_{\perp}/j_{\parallel})}{f_4^{\text{TC}}(1)} = I_d\left(\frac{j_{\perp}}{j_{\parallel}}\right) \quad . \tag{22}$$

As one sees from Eq. (17), the function $I_d(x)$ decreases monotonically (from ∞ to 1, at $d=3$) as x increases (from 0 to 1). Thus, f_4^{TC} moves to less negative values as the anisotropy becomes weaker. If we start with $f_4^{\text{TC}}(j_{\perp}/j_{\parallel}) < f_4 < f_4^{\text{TC}}(1)$ and decrease the anisotropy (e.g., by applying pressure) then f_4^{TC} may pass through f_4 and the transition will turn from continuous to first order.

Up to this point, we discussed the ferromagnetic case. For a metamagnet, one must consider a double unit call, i.e., one-half of the BZ, $q_{\perp} < \pi/2a_{\perp}$ instead of Eq. (4).[24] Defining both a ferromagnetic (S_+) and an antiferromagnetic (S_-) order parameter for each unit cell, applying a uniform field, H, and following all the steps of Nelson and Fisher[24] (including the integration of S_+ out of the partition function), we end up with an effective Hamiltonian which is exactly of the form (6) [or (8)], except that $a_{\perp} \to 2a_{\perp}$ (dropping out after the length rescaling), $\hat{J}(\vec{q})$ is the Fourier transform of the antiferromagnetic exchange (with $q_{\perp}$ measured relative to $\pi/2a_{\perp}$), and f_4 is now replaced (for small H) by[29]

$$f_4^{\text{eff}} = f_4 - AH^2 \quad . \tag{23}$$

Equation (21) may now be interpreted as yielding the dependence of the tricritical field H_{TC} on $(j_{\perp}/j_{\parallel})$, for fixed T. Assuming that the tricritical temperature T_{TC} does not change, H_{TC} becomes smaller as $j_{\perp}/j_{\parallel}$ increases (i.e., for higher pressures). This confirms our basic expectations, stated in the beginning of the paper.

In practice, T_{TC} also depends on pressure. The main source of this variation is due to the increase of $\hat{J}(0)$ [Eq. (2)], which increases linearly with $J_{\perp}$. The effects of fluctuations, which are of order f_4 and f_6, are probably much smaller.[28] Including the effects of the field,[29] we may write

$$r^{\text{eff}} = \frac{k_B T - \hat{J}(0)}{j_{\perp}\pi^2} - \frac{\delta\hat{J}(0)}{j_{\perp}\pi^2} + BH^2 \quad . \tag{24}$$

At fixed T, to leading order, the critical line will thus be shifted with pressure towards higher fields, $\delta(H_c^2) = \delta\hat{J}(0)/(j_{\perp}\pi^2 B)$. In order to check the prediction (22) we must subtract this trivial effect. The appropriate interpretation of Eq. (23) is thus that $[H_{\text{TC}}^2 - \delta\hat{J}(0)/j_{\perp}\pi^2 B]$ decreases with increasing pressure (i.e., increasing $j_{\perp}/j_{\parallel}$). In other words, H_{TC} does not increase with pressure as fast as the zero-temperature critical field H_c,

$$\frac{dH_{\text{TC}}}{dp} < \frac{dH_c}{dp} \quad . \tag{25}$$

Before discussing the experiments, we should remember the quantitative limitations of the above calculation: (a) We ignored many "irrelevant" variables, such as the coefficients of higher powers of the spin, $|\vec{S}|^8$, $|\vec{S}|^{10}$, etc., and of the momentum, $|\vec{q}|^4(\vec{\sigma}_{\vec{q}} \cdot \vec{\sigma}_{-\vec{q}})$, etc. Although these variables do not affect the critical properties, they will contribute to the effective coefficients of Eq. (8) both via the initial integration, reflected in Eq. (15) and via the scaling fields [e.g., Eq. (20)]. The effects of these variables were also not included in the calculation of the field dependence of f_4^{eff} and r^{eff}.[24] Hopefully, the qualitative trends found here will persist when these variables are included.

(b) We have approximated the BZ by an ellipsoid, Eq. (13). Again, it is quite reasonable to expect that the qualitative results should not be affected.

(c) More seriously, we ignored direct pressure dependences of the initial parameters f_4, f_6, etc. Coupling to elastic degrees of freedom may yield (after these are eliminated from the partition function) an explicit dependence of f_4 on pressure.[30] Such a dependence may change the trends found above.

With these reservations in mind, the experimental results are quite encouraging: For $FeBr_2$ one has[19] $dH_{\text{TC}}/dp = 1.65$ kOe/kbar and $dH_c/dp = 1.70$ kOe/kbar, while for the high-pressure phase of $FeCl_2$ one has[19] $dH_{\text{TC}}/dp = 0.77$ kOe/kbar and $dH_c/dp = 0.86$ kOe/kbar. Both agree with our prediction, Eq. (25). At low pressures, $FeCl_2$ has a different crystal structure and one has[21] $dH_{\text{TC}}/dp = 1.08$ kOe/kbar vs $dH_c/dp = 0.73$ kOe/kbar. Any of the above arguments may be appealed to for explaining this disagreement. One may also have to take into

account the effects of fluctuations on r^{eff}.[28]

Finally, we comment that similar arguments may be applied to cubic systems, with the quartic terms $v \sum_{\vec{R}} \sum_{\alpha} (S_{\vec{R}}^{\alpha})^4$. Again, the presence of sixth-order terms shifts the tricritical boundaries, generating fluctuation-driven continuous transitions.[3] Larger spatial anisotropy will again move these boundaries to widen the range of these continuous transitions.

In conclusion, we have demonstrated explicitly that in the vicinity of tricritical points an external pressure may turn fluctuation-driven continuous transitions into first order. We expect the same effect to result from any effective reduction of the strength of the fluctuations (e.g., by longer-range interactions, lower-spin symmetry, etc.). We hope that this calculation will stimulate more experiments, as well as alternative calculations (e.g., series, Monte Carlo).

ACKNOWLEDGMENTS

This work was supported in part by a grant from the U.S.-Israel Binational Science Foundation (BSF), Jerusalem. One of us (D.B.) is also grateful to the IBM Zürich Laboratory, and especially to K. A. Müller and T. Schneider, for their hospitality during the final stage of this work.

[1]L. D. Landau and E. M. Lifshitz, *Statistical Physics,* 2nd ed. (Pergamon, New York, 1968), Chap. XIV.

[2]See, for example, P. Bak, S. Krinsky, and D. Mukamel, Phys. Rev. Lett. 36, 52 (1976); D. Mukamel and S. Krinsky, Phys. Rev. B 13, 5065, 5078 (1976); J. Rudnick, *ibid.* 18, 1406 (1978).

[3]D. Blankschtein and A. Aharony, Phys. Rev. Lett. 47, 439 (1981).

[4]See, for example, L. J. de Jongh and A. R. Miedema, Adv. Phys. 23, 1 (1974).

[5]R. B. Potts, Proc. Cambridge Philos. Soc. 48, 106 (1952).

[6]See, for example, A. Aharony, K. A. Müller, and W. Berlinger, Phys. Rev. Lett. 38, 33 (1977), and references.

[7]R. J. Baxter, J. Phys. C 6, L445 (1973).

[8]Y. Shnidman and E. Domany, J. Phys. C 14, L773 (1981).

[9]See, for example, P. Bak, S. Krinsky, and D. Mukamel, Phys. Rev. Lett. 36, 829 (1975); E. Domany, D. Mukamel, and M. E. Fisher, Phys. Rev. B 15, 5432 (1977); M. Kerszberg and D. Mukamel, *ibid.* 23, 3943, 3953 (1981), and references.

[10]See, for example, L. L. Liu and H. E. Stanley, Phys. Rev. Lett. 29, 927 (1972), and references.

[11]A. D. Bruce, J. Phys. C 7, 2089 (1974).

[12]See, for example, E. K. Riedel and F. J. Wegner, Phys. Rev. Lett. 29, 349 (1972).

[13]Very recently, Y. Gefen, Y. Imry, and D. Mukamel [Phys. Rev. B 23, 6099 (1981)] studied the dependence of the tricritical point of the quasi-one-dimensional spin-one Ising model on the spatial anisotropy in $d=2$. Using completely different techniques, they also find a narrower first-order regime for a larger anisotropy.

[14]I. S. Jacobs and P. E. Lawrence, Phys. Rev. 164, 866 (1967); R. J. Birgeneau, G. Shirane, M. Blume, and W. C. Koehler, Phys. Rev. Lett. 33, 1098 (1974); J. A. Griffin and S. E. Schnatterly, *ibid.* 33, 1576 (1974).

[15]I. S. Jacobs and P. E. Lawrence, J. Appl. Phys. 35, 996 (1964); A. R. Fert, P. Carrara, M. S. Lanusse, M. G. Mischler, and J. P. Redoules, J. Phys. Chem. Solids 34, 223 (1974).

[16]V. A. Schmidt and S. A. Friedberg, Phys. Rev. B 1, 2250 (1970).

[17]For a general review on metamagnetic compounds see, e.g., E. Stryjewski and N. Giordano, Adv. Phys. 26, 487 (1977).

[18]A hydrostatic pressure will usually change $J_{\perp}$ faster than $J_{\parallel}$, and thus increase $(J_{\perp}/J_{\parallel})$.

[19]C. Vettier, H. L. Alberts, and D. Bloch, Phys. Rev. Lett. 1414 (1973).

[20]D. J. Breed, K. Gujamse, and A. R. Miedema, Physica (Utrecht) 45, 205 (1969); H. Ikeda and M. T. Hutchings, J. Phys. C 11, L529 (1978).

[21]H. Ikeda and K. Hirakawa, Solid State Commun. 14, 529 (1974).

[22]K. W. Mess, E. Lagendijk, D. A. Curtis, and W. J. Huiskamp, Physica (Utrecht) 34, 126 (1967).

[23]In $d=2$, with nearest-neighbor interactions only, the transition remains second order for *all* fields [e.g., K. Binder and D. P. Landau, Phys. Rev. B 21, 1941 (1980) and references]. The transition becomes first order, for low temperatures, due to next-nearest interactions (which effectively reduce fluctuations). We predict a similar effect due to a strengthened interlayer coupling. We thank C. Domb and R. Swendsen for discussions on this point.

[24]D. R. Nelson and M. E. Fisher, Phys. Rev. B 11, 1030 (1975).

[25]See, for example, A. Aharony, in *Phase Transitions and Critical Phenomena,* edited by C. Domb and M. S. Green (Academic, New York, 1976), Vol. 6, p. 357, and references.

[26]See, for example, F. J. Wegner, in Ref. 25, p. 7.

[27]See also J. Rudnick and D. R. Nelson, Phys. Rev. B 13, 2208 (1976).

[28]The shift in $(k_B T/j_{\parallel})$ due to the integration of modes with $|\vec{q}| > 1$ is of order u_4, representing higher-order corrections to Eq. (21).

[29]There is also a similar shift in the temperature variable, $r^{\text{eff}} = r + BH^2$, leading to a shift in the critical temperature of the form $T_c = T_c(H=0) - \tilde{B}H^2$.

[30]See, for example, M. de Moura, T. C. Lubensky, Y. Imry, and A. Aharony, Phys. Rev. B 13, 2176 (1976), and references.