Letter

Negative superfluid density and spatial instabilities in driven superconductors

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We consider the excitation of Higgs modes via the modulation of the Bardeen-Cooper-Schrieffer coupling within the Migdal-Eliashberg-Keldysh theory of time-dependent superconductivity. Despite the presence of phonons, which break integrability, we observe Higgs amplitude oscillations reminiscent of the integrable case. The dynamics of quasiparticles follows from the effective Bogolyubov–de Gennes (BdG) equations, which represent a Floquet problem for the Bogolyubov quasiparticles. We find that when the Floquet-Bogolyubov bands overlap, the homogeneous solution formally leads to a negative superfluid density, which is no longer proportional to the amplitude of the order parameter. This result indicates an instability, which we explore using spatially resolved BdG equations. Spontaneous appearance of spatial inhomogeneities in the order parameter is observed and they first occur when the superfluid density becomes unphysical. We conclude that the homogeneous solution to time-dependent superconductivity is generally unstable and breaks up into a complicated spatial landscape via an avalanche of topological excitations.

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Introduction. Magnetic field expulsion [1] is one of the hallmarks of superconductivity. It originates from the diamagnetic supercurrent $\mathbf{j}_s$ which is related to the gauge potential $\mathbf{A}$ via London's equation $\mathbf{j}_s = -\frac{n_s}{m}\mathbf{A}$, where m is the effective electron mass and n_s is the superfluid density. The latter also characterizes the stiffness of phase fluctuations θ in superfluids with the classical free energy being given by $\delta F \propto n_s \int d\mathbf{r}(\nabla\theta)^2$ [2]. The thermodynamic stability of bulk samples requires $n_s > 0$ and the negative values of the superfluid density resulting in a paramagnetic Meissner response have been associated with the instability of the system towards the formation of spatial inhomogeneities. Well-known examples include the Fulde-Ferrell [3] and Larkin-Ovchinnikov [4] states, which occur in superconductors in the presence of a magnetic field or spin imbalance in neutral superfluids. Negative values of n_s were also theoretically predicted for the odd-frequency superconducting states, implying they are thermodynamically unstable [5]. Many such scenarios are characterized by emergent Bogolyubov Fermi surfaces that possess an enhanced density of states of quasiparticles. This results in a large paramagnetic contribution to the electromagnetic response, thereby rendering n_s negative.

In this Letter, we demonstrate that negative values of n_s can occur in conventional superconductors in the presence amplitude (Higgs) oscillations of the order parameter, which implies an instability of the homogeneous solution. Different proposals for the generation of Higgs oscillations were extensively studied in the past and include interaction quenches [6–8] and an ultrafast terahertz pumping [9–12]. The oscillatory behavior of the superfluid density in the presence of the Higgs excitations was shown to lead to the parametric amplification/generation of photons [13,14]. In this Letter, we demonstrate that, in addition to the oscillatory behavior, the superfluid density can acquire a divergent negative static contribution. We attribute this component to the emergent Floquet-Bogolyubov bands, in an ideal case where the electron-hole recombination is impossible [15], characterized by a divergent density of states (DOS). Interaction with phonons broadens the DOS, reducing this singularity. Finally, we simulate a quasi-one-dimensional disordered superconductor and observe proliferation of solitons, concurrently with n_s acquiring negative values in a uniform case.

We start by numerically evaluating the superfluid density [16] within the Bardeen-Cooper-Schrieffer (BCS)-Holstein model. For simplicity, we consider the quench protocol for inducing the Higgs oscillations, i.e., we assume the coupling inducing pairing is rapidly changed in time. The full Hamiltonian reads $H(t) = H_{\rm BCS}(t) + H_{\rm el-ph} + H_{\rm dis}$:

$$H_{\rm BCS} = \sum_{k,\sigma} \xi_{\mathbf{k}} \psi^\dagger_{\mathbf{k},\sigma}\psi_{\mathbf{k},\sigma} - \lambda(t)\nu_0^{-1}\sum_{\mathbf{q}} \varrho_{\mathbf{q}}\varrho_{-\mathbf{q}}, \qquad (1)$$

$$H_{\rm dis} = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},\mathbf{q},\sigma} U_{\mathbf{q}} \psi^\dagger_{\mathbf{k}-\mathbf{q},\sigma}\psi_{\mathbf{k},\sigma}, \qquad (2)$$

$$H_{\rm el\text{-}ph} = \sum_{\mathbf{q}} g_{\mathbf{q}}\phi_{\mathbf{q}}\varrho_{-\mathbf{q}} + \sum_{\mathbf{q}} \omega_{\mathbf{q}} a^\dagger_{\mathbf{q}} a_{\mathbf{q}}, \qquad (3)$$

where $\varrho_{\mathbf{q}} = V^{-1/2}\sum_{\mathbf{k},\sigma} \psi^\dagger_{\mathbf{k}+\mathbf{q},\sigma}\psi_{\mathbf{k},\sigma}$, $\phi_{\mathbf{q}} = a_{\mathbf{q}} + a^\dagger_{-\mathbf{q}}$, $\psi_{\mathbf{k},\sigma}$ ($\psi^\dagger_{\mathbf{k},\sigma}$), and $a_{\mathbf{q}}(a^\dagger_{\mathbf{q}})$ respectively denote the electron and phonon annihilation (creation). Their dispersions are respectively defined as $\xi_{\mathbf{k}} = k^2/2m - E_F$ and ω_q, where m is the electronic mass and E_F is the Fermi energy. g is the electron-phonon coupling strength and V is the volume of the system, ν_0 is the electron density of states at the Fermi level, λ is the strength of short-range contact interaction representing an interaction with a high-energy phonon band. $U_{\mathbf{q}}$ denotes Fourier transform of the local disorder potential. Our model includes both a local BCS pairing interaction, characterized by the constant λ, and phonons, which play the role of a bath leading to thermalization. Throughout this work, we assume the coupling to the phonon bath to be weak such that the main source of attractive interaction leading to superconductivity is represented by λ. The interaction with the electromagnetic field is included via the minimal substitution $\mathbf{k} \to \mathbf{k} - \mathbf{A}$ in the electronic dispersion.

Quench dynamics. To induce the amplitude oscillations in our model, we consider a rapid change in the BCS pairing constant at $t = 0$ from $\lambda(0^-) = \lambda_i$ to $\lambda(0^+) = \lambda_f$. We describe the time evolution within the mean-field and disorder-averaged Gor'kov equations for the nonequilibrium Green's function, defined on the Kadanoff-Baym contour [17]

$$(i\partial_t - \hat{h}_{\mathbf{k}}(t) - \hat{\Sigma}_{\mathbf{k}}\star)\hat{G}_{\mathbf{k}}(t,t') = \delta(t,t')\hat{\mathbb{1}}, \tag{4}$$

where $\hat{G}_{\mathbf{k}}$ is the contour-ordered Green's function defined as $\hat{G}_{\mathbf{k}}(t,t') = -i\langle T_C \Psi_{\mathbf{k}}(t) \otimes \Psi^\dagger_{\mathbf{k}}(t')\rangle$, with $\Psi_{\mathbf{k}} = \binom{\psi_{\mathbf{k},\uparrow}}{\psi^\dagger_{-\mathbf{k},\downarrow}}$ denoting Nambu spinors. $\hat{h}_{\mathbf{k}}(t) = \xi_{\mathbf{k}}\hat{\tau}_3 + \Delta(t)\hat{\tau}_1$ is the effective Bogolyubov–de Gennes Hamiltonian (BdG) with $\Delta(t)$ being the instantaneous part of the anomalous self-energy due to the nonretarded BCS coupling λ. $\hat{\tau}_i$ denote Pauli matrices. δ is the contour Dirac delta function, and $\star$ denotes the matrix multiplication in temporal and Nambu indices. The self-energy in Eq. (4) has contributions from the phonon and disorder scatterings $\hat{\Sigma}_{\mathbf{k}} = \hat{\Sigma}^{\mathrm{ph}}_{\mathbf{k}} + \hat{\Sigma}^{\mathrm{dis}}_{\mathbf{k}}$:

$$\hat{\Sigma}^{\mathrm{ph}}_{\mathbf{k}}(t,t') = -i\frac{1}{V}\sum_{\mathbf{k}'} g^2 D_{\mathbf{k}-\mathbf{k}'}(t,t')\hat{\tau}_3\hat{G}_{\mathbf{k}'}(t,t')\hat{\tau}_3, \tag{5}$$

$$\hat{\Sigma}^{\mathrm{dis}}_{\mathbf{k}}(t,t') = -i\frac{1}{2\pi\nu_0\tau_{\mathrm{el}}}\sum_{\mathbf{k}'} \hat{\tau}_3\hat{G}_{\mathbf{k}'}(t,t')\hat{\tau}_3, \tag{6}$$

where the contour-ordered phonon propagator is defined as $D_{\mathbf{q}}(t,t') = -i\langle T_C \phi_{\mathbf{q}}(t)\phi_{-\mathbf{q}}(t')\rangle$ and $\tau_{\mathrm{el}}^{-1} = 2\pi\nu_0\overline{U_{\mathbf{q}}U_{-\mathbf{q}}}$ is the elastic scattering rate. In the following we ignore the back-action effects on the phonon propagator and assume it is in an equilibrium state at the initial temperature. We then proceed with the standard quasiclassical approximation and average the propagator over the Fermi surface $g^2_{\mathbf{q}}D_{\mathbf{q}}(t,t') \to \langle g^2_{\mathbf{q}}D_{\mathbf{q}}(t,t')\rangle_{\mathrm{FS}}$. Under these assumptions, the effective phonon propagator is fully determined by its spectral density (Eliashberg function) which we denote as $\alpha^2F(\omega) \equiv 2\nu_0 \mathrm{Im}\langle g^2_{\mathbf{q}}D^R_{\mathbf{q}}(\omega)\rangle_{\mathrm{FS}}$, where the $D^R_{\mathbf{q}}(\omega)$ is the retarded part phonon propagator. Throughout this work we consider an effective Debye model $\alpha^2F \propto \theta(\omega_{\mathrm{D}} - \omega)\omega^2$, where ω_{D} is the cutoff frequency of the phononic band. The effective electron-phonon pairing strength and the mean frequency are conventionally defined as $\lambda_{\text{el-ph}} = \int \frac{d\omega}{\omega}\alpha^2F(\omega)$, $\bar{\omega} = \lambda^{-1}_{\text{el-ph}}\int d\omega\alpha^2F(\omega)$. The time-dependent order parameter $\Delta(t)$ satisfies the self-consistency equation by replacing the phonon propagator in Eq. (5) with $\lambda\nu_0^{-1}\delta(t,t')$.We also note that we assume the Coulomb pseudopotential μ^* is included in λ. To simplify the consideration and avoid the complications due to the energy-dependent electronic density of states, we assume the metal is two-dimensional with the high-energy cutoff such that $\xi_{\mathbf{k}} \in [-E_F, E_F]$ [18]. We discuss a caveat related to such simplified dispersion in the Supplemental Material [19].

Electromagnetic response. We now compute the electromagnetic response of the system. Provided that the electron-electron interaction is short range (contact), it is sufficient to take into account only the mean-field-level contributions. The gauge-invariant current density to first order in the external gauge potential $\mathbf{A}$ comprises paramagnetic and diamagnetic contributions $\mathbf{j}_s(t) = \mathbf{j}_{\mathrm{p}}(t) + \mathbf{j}_{\mathrm{d}}(t)$, where

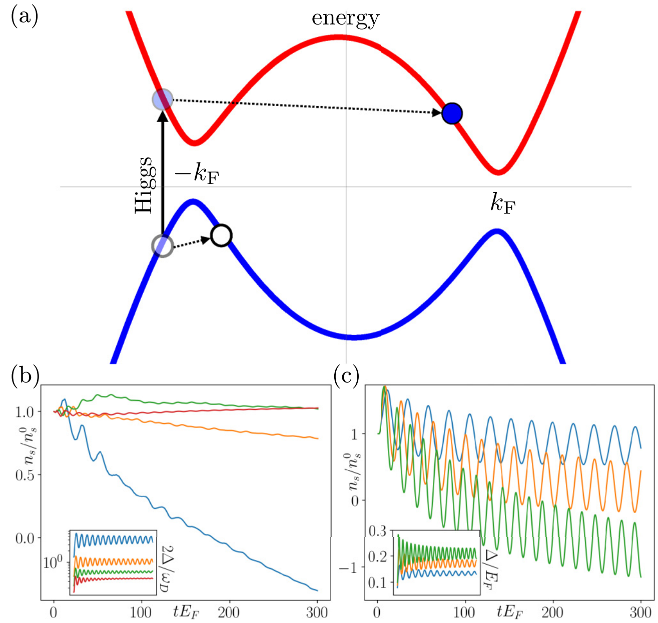

FIG. 1. Superfluid density in a quenched superconductor. (a) Schematic representation of the quasiparticle dispersion in a superconductor in the presence of external magnetic field. Arrows represent the scattering processes leading to the generation of incoherent quasiparticle population. (b) Clean superconductor with electron-phonon interaction with initial and final BCS interaction strengths $\lambda_i = 0.25$ and $\lambda_f = 0.45$ and different values of the Debye energy ω_{D}: $0.04E_F$ (blue), 0.16 (orange), 0.35 (green), 0.6 (red). (c) Disordered case, quench parameters $\lambda_i = 0.3$ and λ_f being equal to 0.4 (blue), 0.45 (orange), and 0.5 (green).

$$\mathbf{j}_{\mathrm{p}}(t) = i\frac{\mathbf{A}}{V}\sum_{\mathbf{k}}\frac{\mathbf{k}^2}{dm^2}\int_C ds\mathrm{Tr}\{\hat{G}_{\mathbf{k}}(t,s)\hat{G}_{\mathbf{k}}(s,t)\}, \tag{7}$$

$$\mathbf{j}_{\mathrm{d}}(t) = i\frac{\mathbf{A}}{V}\sum_{\mathbf{k}}\frac{1}{m}\mathrm{Tr}\{\hat{\tau}_3\hat{G}_{\mathbf{k}}(t,t+0^+)\}. \tag{8}$$

Here the Green's functions are taken in the limit $\mathbf{A} \to 0$ and d is the dimensionality of the problem. We note that the gauge potential is assumed to be static in Eqs. (7) and (8) for simplicity. The time-dependent superfluid density can be found from Eqs. (7) and (8) $\mathbf{j}(t) \equiv -\frac{n_s(t)}{m}\mathbf{A}$. Before discussing the results of a complete numerical calculation, we note that in the absence of disorder or phonon scatterings, the superfluid density does not depend on time to lowest order in Δ/E_F. This follows from the analysis of the BdG Hamiltonian in the presence of a static gauge potential $\hat{h}_{\mathbf{k}}(\mathbf{A}) \approx (\xi_{\mathbf{k}}\hat{\tau}_3 - \frac{\mathbf{kA}}{m}\hat{\tau}_0 + \frac{\mathbf{A}^2}{2m}\hat{\tau}_3) + \Delta(t)\hat{\tau}_1$. The paramagnetic term commutes with the rest of the Hamiltonian and its contribution to the response given by Eq. (7) is time independent. This implies that any dynamics of the superfluid density requires a momentum exchange mechanism as schematically shown in Fig. 1(a). In our equations of motion Eq. (4), both self-energies Eqs. (5) and (6) account for such a mechanism leading to the time-dependent n_s as shown in Figs. 1(b) and 1(c).

We first discuss the phonon-induced scattering case. By varying the Debye cutoff frequency for a fixed electron-phonon coupling strength $\lambda_{\text{el-ph}} \approx 0.075$, we observe a qualitatively different behavior of n_s. In particular, when ω_D is sufficiently large, both $n_s(t)$ and $\Delta(t)$ are approaching asymptotic values at large times, consistent with thermalization due to coupling to the phonon bath. However, when the nonequilibrium quasiparticle gap 2Δ is greater than ω_D we see a different behavior with n_s becoming negative. This configuration is also characterized by the forbidden quasiparticle recombination which hinders thermalization [15]. Similar phenomena are also observed for the disordered case for sufficiently large initial and final values of the BCS pairing strength. We also note that the oscillations of the superfluid density, which reflect the behavior of $\Delta(t)$, are more apparent in the case of a disordered superconductor. In the remainder of this Letter, we provide a qualitative explanation to these features.

Floquet-Usadel description. Let us now consider a simplified scenario where the superconducting gap is periodically modulated as $\Delta(t) = \Delta + \theta \cos \omega_0 t$, with θ and ω_0 denoting the amplitude and the frequency of the gap oscillations, respectively. In this case, we can apply the Floquet theory and find the quasienergy spectrum of the time-dependent BdG Hamiltonian $\hat{h}_{\mathbf{k}}(t)$, shown schematically in Fig. 2(a) for different values of ω_0. At $\omega_0 \approx 2\Delta$ we observe a crossing of different Floquet bands implying the possibility of a nearly resonant excitation of quasiparticles with $\xi_{\mathbf{k}} \approx 0$. Note that the resonant condition is never satisfied exactly since for $\xi_{\mathbf{k}} = 0$ the BdG Hamiltonian commutes with its oscillatory contribution.

To find the electromagnetic response of the Higgs-driven system we apply the quasiclassical approximation to Eq. (4) in the diffusive limit, i.e., assuming that τ_{el} is the shortest timescale of the problem (except for the inverse Fermi energy). Furthermore, we focus on a steady-state regime of the driving. We consider the Keldysh contour and define $\hat{g}_{t,t'}(\mathbf{n}_k) = \frac{i}{\pi} \int d\xi_k \hat{\tau}_3 \hat{G}_{\mathbf{k}}(t,t')$, where $\mathbf{n}_k = \mathbf{k}/k$ [21]. Following the conventional approach [22], we represent the quasiclassical Green's function as $\check{g}_{t,t'} = \begin{pmatrix} \hat{g}^{\text{R}}_{t,t'} & \hat{g}^{\text{K}}_{t,t'} \\ 0 & \hat{g}^{\text{A}}_{t,t'} \end{pmatrix}$, where $\hat{g}^{\text{R,A,K}}_{t,t'}$ are the retarded, advanced, and Keldysh components. In the spatially uniform case the Green's function is independent of $\mathbf{n}_k$ and obeys

$$\partial_t \hat{\tau}_3 \check{g}_{t,t'} + \partial_{t'} \check{g}_{t,t'} \hat{\tau}_3 = [\Delta(t)\hat{\tau}_1 \overset{\star}{,} \check{g}_{t,t'}]_{t,t'} + i[\check{\Sigma}^{\text{ph}} \overset{\star}{,} \check{g}]_{t,t'}, \tag{9}$$

where $\check{\Sigma}^{\text{ph}}$ is the self-energy due to the phonon scattering equivalent to Eq. (5). The quasiclassical Green's function obeys the conventional normalization condition $\check{g}_{t,s} \star \check{g}_{s,t'} = \check{I}\delta(t - t')$, where $\check{I}$ is a 4×4 identity matrix. We note that in the uniform case the self-energy due to disorder scattering commutes with the Green's function, and therefore, does not contribute to Eq. (9). To simplify our description, we approximate $\check{\Sigma}^{\text{ph}}$ in the temporal Fourier space as follows [23–26]:

$$\check{\Sigma}^{\text{ph}}_{\omega,\omega'} = \gamma_{\text{inel}} \begin{pmatrix} \hat{\tau}_3 & 2\hat{\tau}_3 \tanh \frac{\beta\omega}{2} \\ 0 & -\hat{\tau}_3 \end{pmatrix} \delta(\omega - \omega'), \tag{10}$$

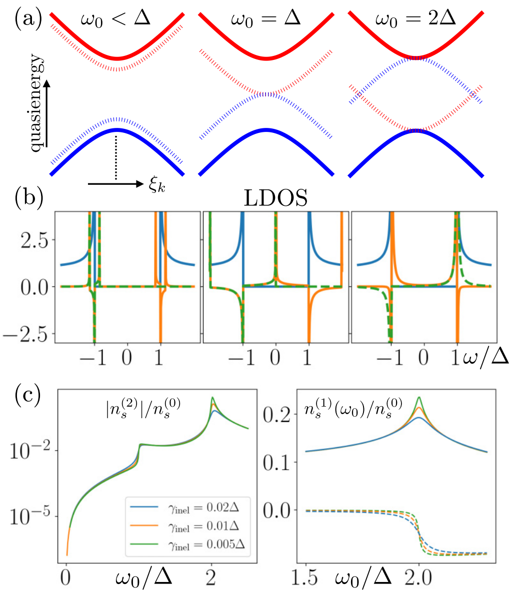

FIG. 2. Dynamic superfluid density within the Usadel-Keldysh approach. (a) Schematic representation of Floquet bands, induced by the periodic gap oscillations at different frequencies. Solid and dashed curves correspond to the Floquet index $m = 0$ and $m = 1$, respectively. (b) Floquet local density of states (LDOS) of physical fermions $\text{Re}\hat{g}^R_{0,0}(\omega)$: blue and orange curves represent the unperturbed and the second order in θ contributions (shown not to scale). Dashed green curve represents the LDOS of the occupied Fermionic states defined as $\text{Re}\hat{g}^<_{0,0}(\omega)/2$ [20] to second order in θ (shown not to scale). The panels from left to right correspond to different gap oscillation frequencies $\omega_0 = 0.15\Delta$, $\omega_0 = \Delta$, and $\omega_0 = 2\Delta$. (c) Superfluid density in diffusive limit for $\theta = 0.2\Delta$. Left and right panels depict static and dynamic components, respectively.

where γ_{inel} is an effective inelastic scattering rate and β is the inverse temperature. We note that this form of $\check{\Sigma}^{\text{ph}}$ effectively describes coupling to a nonsuperconducting fermionic reservoir. As a result, $\check{\Sigma}^{\text{ph}}$ explicitly breaks the fermion number conservation. However, within the achievable numerical precision, the density of states and the average number of fermions are exactly conserved in our model due to the particle-hole symmetry. In the following, we consider the limit when the dissipation is weak $\gamma_{\text{inel}} \to 0$.

The external gauge field can be added to Eq. (9) perturbatively, yielding the current density $\mathbf{j}_t = \pi i \frac{\sigma_{\text{N}}}{4} \mathbf{A} \text{Tr}\{\hat{\tau}_3 \check{g} \star [\hat{\tau}_3, \check{g}]\}^{\text{K}}_{t,t}$ [27], where $\sigma_{\mathbf{N}} = 2D\nu_0$ is the normal state conductivity, $D = v_F^2 \tau_{\text{el}}/3$ is the diffusion constant, ν_0 is the fermionic density of states, v_F is the Fermi velocity, and the superscript K denotes taking the Keldysh component. In equilibrium we straightforwardly find $n_s = \pi m \sigma_{\text{N}} \Delta \tanh \frac{\beta\Delta}{2}$. Our goal is now to find the corrections to the superfluid density due to the time dependence of the gap. This is readily achieved by performing the Fourier transformation of both sides in Eq. (9) with respect to t and t'. The resulting equation can be expanded in powers of θ and solved iteratively up to the second order, e.g., in mathematica. In Fig. 3(b) we provide the time-averaged Fermionic local densities of states. As expected, they reflect the Floquet spectrum of the time-dependent BdG Hamiltonian, schematically shown in

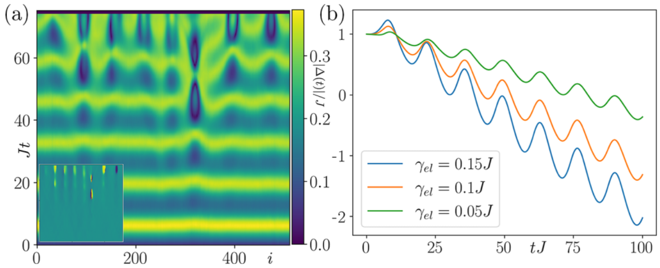

FIG. 3. Quench dynamics a one-dimensional disordered superconductor. (a) Time dependence of the spatially nonuniform gap in a one-dimensional Hubbard model after the quench with $\lambda_i = 1.2$, $\lambda_f = 2$. Inset shows the evolution of the local phase profile. The effective elastic scattering rate is $\gamma_{\rm el} = 0.1J$. (b) Numerical calculation of the superfluid density in a spatially unresolved (uniform case) tight-binding model using mean-field equations Eqs. (4) to (6).

Fig. 3(a). The first order in θ solution encodes the oscillating component of the superfluid density $n_s^{(1)}(\omega)$, shown in Fig. 3(c). For $\omega_0 \approx 2\Delta$ we observe a logarithmic singularity $n_s^{(1)}/n_s^{(0)} \propto \theta/\Delta \ln(\gamma_{\rm inel}/\Delta)$. The lowest-order static correction to n_s requires expansion up to the second order in θ (see also the Supplemental Material [19] for a detailed analysis) which can be performed numerically. For $\omega_0 \approx 2\Delta$ we find the correction having the following scaling $n_s^{(2)}/n_s^{(0)} \propto -\theta^2/(\Delta\gamma_{\rm inel})$ in the limit $\gamma_{\rm inel} \to 0$ for $\omega_0 \approx 2\Delta$.

Let us now also discuss the heating of the superconductor due to the steady-state driving of Higgs oscillations. According to a simple estimate, provided in the Supplemental Material [19], the melting of the order parameter can be estimated as the second-order correction to the static gap Δ, that scales as $\Delta^{(2)} \propto \theta^2/\sqrt{\Delta\gamma_{\rm inel}}$ with $\Delta^{(2)} \ll \Delta$. We note that the same scaling can be expected for the number of excited quasiparticles in the steady state. Defining the corresponding energy scale as $E_{\rm ex} = \theta^2/\sqrt{\Delta\gamma_{\rm inel}}$, we find $n_s^{(2)}/n_s^{(0)} \approx -E_{\rm ex}/\sqrt{\Delta\gamma_{\rm inel}} \to -\infty$ and $n_s^{(1)}(2\Delta) \to 0$ in the limit $\gamma_{\rm inel} \to 0$. This implies the negative superfluid density is not induced by the heating but is rather due to the divergent DOS of Bogolyubov quasiparticles. We also note that for finite but small detuning $\delta = 2\Delta - \omega_0 > 0$ we can take the limit $\gamma_{\rm inel} \to 0$ explicitly. In this case, the scalings of $n_s^{(2)}$ and $\Delta^{(2)}$ are found to be the same with the replacement $\gamma_{\rm inel} \to \delta$.

Quasi-one-dimensional case. We now discuss the physical implications of the negative superfluid density. To this end, we consider a superconductor with a spatially inhomogeneous order parameter. To make the problem tractable, we consider a disordered one-dimensional tight-binding model with random on-site disorder. The corresponding BdG Hamiltonian can be represented in the following form:

$$\hat{H}_{\rm BdG}(t) = J\sum_i (|i\rangle\langle i+1| + |i+1\rangle\langle i|) \otimes \hat{\tau}_3 + \sum_i u_i|i\rangle\langle i| \otimes \hat{\tau}_3 + \hat{\Sigma}(t),$$

where u_i is the local disorder potential, J is the nearest-neighbor tunneling rate, and $|i\rangle$ is the state locating an electron in the cite i. The self-energy can be parametrized as $\hat{\Sigma}(t) = \sum_i(\Delta_i\hat{\tau}_+ + \Delta_i^*\hat{\tau}_- + \delta u_i\hat{\tau}_3)|i\rangle\langle i|$, Δ_i is the local gap, and δu_i is the renormalized disorder. Self-consistency equation reads (note that it also has normal components which slightly renormalize the disorder)

$$\hat{\Sigma}_i(t) = i\frac{\lambda}{2}\hat{\tau}_3\hat{G}^{\rm K}_{i,i}(t,t)\hat{\tau}_3, \tag{11}$$

where $\hat{G}^K$ is Keldysh Green's function obeying

$$\partial_t\hat{G}^{\rm K}(t,t) = -i[\hat{H}_{\rm BdG}(t), \hat{G}^{\rm K}(t,t)]. \tag{12}$$

The initial condition for the Green's function at $t = 0$ can be expressed in terms of eigenvalues γ_l and eigenvectors $|l\rangle$ of BdG Hamiltonian at $t = 0$ as follows:

$$\hat{G}^K(t=0, t=0) = i\sum_l |l\rangle\langle l| \tanh\frac{\beta\gamma_l}{2}. \tag{13}$$

We numerically self-consistently solve Eqs. (11), (12), and (13) for $N = 512$ sites. The result of the simulation is shown in Fig. 3(a). We observe the formation of topological defects in the form of phase slips. In Fig. 3(b) we provide the result of the numerical simulation of the superfluid density in a uniform one-dimensional tight-binding model using Eqs. (4) and (6). We find a good agreement between the times when the superfluid density becomes negative and when the defects start to proliferate.

Conclusion and outlook. In this work we demonstrate that the superfluid density in an amplitude-driven superconductor can become negative, resulting in proliferation of spatial inhomogeneities of the order parameter. Our analysis is applicable to both superconductors and ultracold Fermi gases. In the latter case, the Higgs oscillations can be induced by time-dependent magnetic field with the frequency matching 2Δ. We note that in static magnetic field as a consequence of the fluctuations in n_s, the superconductor is expected to emit photons at the frequency 2Δ [28]. The analysis in this Letter is limited to the s-wave symmetry of the order parameter and the analysis of more exotic configurations is left for future investigations.

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Data availability. The data that support the findings of this article are openly available [29].

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