Decoherence-protected holonomic gates with reduced requirements for physical resources

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We study the implementation of holonomic quantum gates preserved from arbitrary single-qubit and two-qubit decoherence or errors using a dynamical decoupling approach. To integrate the holonomic feature with the dynamical decoupling technique, we require only one auxiliary qubit for single-qubit gates and no auxiliary qubit for two-qubit gates. Our scheme uses far fewer physical qubits to achieve this integration compared to existing proposals in the literature. Moreover, the scheme is based on system interactions that can be experimentally realized with superconducting qubits. These advantages suggest that our scheme is a promising step toward developing noise-resistant quantum devices.

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I. INTRODUCTION

One of the key challenges to further progress quantum computation is to realize noise-resilient gates using available experimental techniques. Over the years, various schemes have been proposed to achieve geometric gates that preserve quantum gates from imprecise control of system parameters [1–6]. Among these schemes, one promising approach is the use of holonomic gates, which can be implemented using either higher energy levels [7] or auxiliary qubits [8]. The use of higher energy levels raises concerns about decoherence in the higher excited states. This makes using auxiliary qubits more attractive, as they are more stable over time than higher energy levels.

Dynamical decoupling (DD) offers an active method to reduce the effects of noise on qubits and extend qubit coherence [9–15]. The effectiveness of DD is well established and has been extensively studied both theoretically and experimentally; however, if not properly designed, DD can unintentionally disrupt the desired gate Hamiltonian(s) while attempting to remove decoherence or errors. Therefore, ensuring compatibility between DD and the desired gate Hamiltonian(s) is crucial when implementing the DD technique in physical systems. This compatibility can be achieved through the control of system parameters or by establishing commutation between DD pulses and desired gate Hamiltonian(s), among other methods. Previous studies have demonstrated that DD can effectively combat arbitrary single-qubit and two-qubit decoherence or errors [16,17].

Single-qubit decoherence or errors arise from system-environment interaction of the form $H_{\rm SB} = \sum_{s,i} \sigma_{s,i} \otimes \vec{B}$, where σ_s $(s = x, y, z)$ represents the corresponding Pauli matrices and $\vec{B}$ describes the operator acting on the environment. The interaction causes fluctuations in $\sigma_{s,i}$ on the ith qubit. Two-qubit decoherence or errors arise from noisy couplings between two qubits, described by $H_{\rm inter} = \sum \sigma_{s,i}\sigma_{t,j}$, where $s, t = x, y, z$. These may result from spurious interqubit couplings or $H_{\rm SB}$ according to perturbation theory.

To address various types of noise, the above methods have been incorporated into established schemes [18–22].

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Traditionally, the implementation of single-logical-qubit gates requires two or three physical qubits, while two-logical-qubit gates necessitate four to six physical qubits, allowing for the simultaneous preservation of geometric and DD advantages. Consequently, the need for a sufficient supply of physical qubits is an unavoidable requirement in these known schemes, which may pose a challenge in practical experiments where such resources might not be readily available. This is the first shortcoming hindering the development of the integration of the two methods. Secondly, in the known schemes, the required gate Hamiltonians involve multiple two-qubit interactions that may not be easily achievable or may induce more errors due to the complexity of the gate Hamiltonians. The experiment-unfriendly gate Hamiltonians impose another hindrance to integration.

In this work, we present a scheme to integrate holonomic and DD approaches based on experimentally achievable Hamiltonians with single-qubit terms, aided by one two-qubit coupling. The simple Hamiltonians lead to desirable tunability in system parameters with current experimental techniques, which enables the easy application of the DD technique in the system. We show that our scheme is protected from both single-qubit and two-qubit decoherence or errors. Specifically, one auxiliary qubit is required to facilitate the realization of single-qubit holonomic gates and no auxiliary qubit is needed for executing two-qubit holonomic gates. The auxiliary qubit is used for single-qubit holonomic gates to eliminate concerns about the decoherence of higher energy levels, which would be an extra source of errors if higher energy levels were used instead of an auxiliary qubit. Although using the auxiliary qubit requires slightly more physical qubits, fortunately, one auxiliary qubit can be shared by different data qubits because the auxiliary qubit remains in its ground state during system evolution.

Our scheme therefore has three main advantages over known approaches. First, it is not resource-demanding, as only a few auxiliary qubits are needed. For example, four auxiliary qubits are needed given a 2D circuit of 16 data qubits, with one auxiliary qubit shared by four data qubits. Second, the gates in our scheme are protected not only from single-qubit but also from two-qubit decoherence or errors. Lastly, our scheme is experimentally realizable due to the simple forms of the desired gate Hamiltonians. The experiment-friendly nature of our scheme makes it a timely contribution to the development of noisy intermediate-scale quantum devices. By combining geometric and DD features, our scheme enhances the designs of these devices.

II. HOLONOMIC QUANTUM GATES

Our scheme uses some single-qubit gates and control-NOT gates to effectively achieve the desired qubit-qubit interactions for implementing a universal set of DD-holonomic integrated quantum gates. The preliminarily required gates are as follows:

$$\begin{aligned}\Pi_1 &= H,\\ \Pi_2 &= \cos\frac{\pi}{4}\sigma_x + \sin\frac{\pi}{4}\sigma_y,\\ \Pi_3 &= \textsc{cnot},\end{aligned} \tag{1}$$

where H represents the Hadamard gate and CNOT represents the control-NOT gate. These do not usually possess geometric features but can be protected by DD techniques from single-qubit and/or two-qubit decoherence or errors, given tunable system parameters.

We first illustrate the implementation of two-qubit holonomic gates with a system Hamiltonian for two physical qubits with single-qubit terms only, $H_1 = (\Omega_{y,1}/2)\sigma_{y,1} + (\delta_1/2)\sigma_{z,1} + (\delta_2/2)\sigma_{z,2} - (\Omega_{x,2}/2)\sigma_{x,2}$, where $\Omega_{y,1}$ and $\Omega_{x,2}$ represent controllable external drivings on qubits 1 and 2 in the y and x directions, respectively, and $\delta_{1,2}$ are the detunings of qubit frequencies for qubits 1 and 2. When $\Omega_{y,1} = \delta_2 = J_0$ and $\Omega_{x,2} = \delta_1 = J_1$, we obtain an equivalent form of H_1 as follows:

$$H_1 = \frac{J_0}{2}\sigma_{y,1} + \frac{J_1}{2}\sigma_{z,1} + \frac{J_0}{2}\sigma_{z,2} - \frac{J_1}{2}\sigma_{x,2}. \tag{2}$$

We then entangle the two qubits with entangling operation, so that the Hamiltonian becomes $\bar{H}_1 = (\Pi_1 \otimes \Pi_1)\Pi_3' H_1 \Pi_3'^{\dagger}(\Pi_1^{\dagger} \otimes \Pi_1^{\dagger})$, where $\Pi_3' = (I_2 \otimes \Pi_2)\Pi_3(I_2 \otimes \Pi_2^{\dagger})$ and I_2 represents the 2×2 identity matrix. The Hamiltonian $\bar{H}_1$ is rewritten as

$$\bar{H}_1 = \frac{J_0}{2}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2}) + \frac{J_1}{2}(\sigma_{x,1} - \sigma_{x,1}\sigma_{z,2}). \tag{3}$$

Define $\Omega = \sqrt{J_0^2 + J_1^2}$ and $\theta = \tan^{-1}(J_0/J_1)$. At $T = \pi/\Omega$, the evolution operator resulting from $\bar{H}_1$ is written in the basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ as [23]

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & \cos(2\theta) & -\sin(2\theta)\\ 0 & 0 & -\sin(2\theta) & -\cos(2\theta)\end{pmatrix}. \tag{4}$$

The holonomic property of the two-qubit gate U_1 can be seen by rewriting $\bar{H}_1$ as $\bar{H}_1 = \Omega\big(\sin\theta\,|01\rangle\langle 10| + \cos\theta\,|01\rangle\langle 11|\big) + \text{H.c.}$ Define two subspaces $\mathcal{S}_0 = \{|00\rangle, |01\rangle\}$ and $\mathcal{S}_1 = \{|10\rangle, |11\rangle\}$, with the corresponding projection operators $P_{\mathcal{S}_0} = |00\rangle\langle 00| + |01\rangle\langle 01|$ and $P_{\mathcal{S}_1} = |10\rangle\langle 10| + |11\rangle\langle 11|$. According to $\bar{H}_1$, we can observe Rabi oscillations between $|01\rangle$ and $\sin\theta\,|10\rangle + \cos\theta\,|11\rangle$, or equivalently between $\mathcal{S}_0$ and $\mathcal{S}_1$. Due to the Rabi oscillation, the system evolves out of the subspace $\mathcal{S}_j$ $(j = 1, 2)$ with time and then returns at $t = T$. In other words, the system completes a cyclic evolution in $\mathcal{S}_j$ at $t = T$. In addition to the cyclic evolution, we also find $P_{\mathcal{S}_j}(t)\bar{H}_1 P_{\mathcal{S}_j}(t) = 0$, where $P_{\mathcal{S}_j}(t) = U_1(t)P_{\mathcal{S}_j}U_1^\dagger(t)$, with $U_1(t)$ being the evolution operator obtained from $\bar{H}_1$. As a result, U_1 possesses geometric features [7,23,24].

It is apparent that U_1 also leads to a set of holonomic single-qubit gates when the first qubit is set as an auxiliary qubit. If qubit 1 is initially prepared in state $|1\rangle$, it is clearly shown that noncommuting single-qubit gates acting on qubit 2 are realizable from $U_{s1} = -\sin(2\theta)\sigma_x + \cos(2\theta)\sigma_z$. Therefore, the implementation of the single-qubit gates relies on engaging one auxiliary qubit that remains in its ground state at $t = T$.

To realize a set of universal holonomic gates, we consider another Hamiltonian

$$H_2 = \frac{J_0}{2}\sigma_{y,1} - \frac{J_1}{2}\sigma_{y,2} + \frac{J_0}{2}\sigma_{z,2} + \frac{J_1}{2}\sigma_{x,1}, \tag{5}$$

where J_0 and J_1 are similarly controllable system parameters related to external drivings and energy shifts. We then transform it to $\bar{H}_2 = (\Pi_1 \otimes \Pi_1)\Pi_3' H_2 \Pi_3'^\dagger(\Pi_1^\dagger \otimes \Pi_1^\dagger)$ to obtain

$$\bar{H}_2 = \frac{J_0}{2}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2}) + \frac{J_1}{2}(\sigma_{y,2} - \sigma_{y,2}\sigma_{z,1}). \tag{6}$$

In a similar way, we equivalently have $\bar{H}_2 = \Omega\big(\sin\theta\,|01\rangle\langle 10| + i\cos\theta\,|11\rangle\langle 10|\big) + \text{H.c.}$ The corresponding evolution operator at $T = \pi/\Omega$ is given as

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & 0 & i\sin(2\theta) \\ 0 & 0 & -1 & 0 \\ 0 & -i\sin(2\theta) & 0 & -\cos(2\theta) \end{pmatrix}. \tag{7}$$

Based on $\bar{H}_2$, Rabi oscillation now occurs between $|10\rangle$ and $\sin\theta\,|01\rangle + i\cos\theta\,|11\rangle$, and comparably between $\mathcal{S}_0' = \{|00\rangle, |10\rangle\}$ and $\mathcal{S}_1' = \{|01\rangle, |11\rangle\}$. With the two new subspaces $\mathcal{S}_0'$ and $\mathcal{S}_1'$, we introduce the corresponding projection operators $P_{\mathcal{S}_0'} = |00\rangle\langle 00| + |10\rangle\langle 10|$ and $P_{\mathcal{S}_1'} = |01\rangle\langle 01| + |11\rangle\langle 11|$. At $t = T$, the system undergoes a cyclic evolution in the subspace $\mathcal{S}_j'$ $(j = 1, 2)$, separately. Together with the condition that $P_{\mathcal{S}_j'}(t)\bar{H}_2 P_{\mathcal{S}_j'}(t) = 0$, where $P_{\mathcal{S}_j'}(t) = U_2(t)P_{\mathcal{S}_j'}U_2^\dagger(t)$ and $U_2(t)$ is the evolution operator obtained from $\bar{H}_2$, we conclude that U_2 describes holonomic two-qubit gates [7,23,24]. When qubit 2 is initially in its ground state, we obtain another type of single-qubit gate, $U_{s2} = -\sin(2\theta)\sigma_y + \cos(2\theta)\sigma_z$, acting on qubit 1.

The holonomic Hadamard gate can be obtained from U_{s1} with an appropriate value of θ. Geometric rotations about the x axis are realizable via two steps from U_{s2} as $e^{i\alpha\sigma_x} = U_{s2}(\theta_2)U_{s2}(\theta_1)$, where $\alpha = 2\theta_1 - 2\theta_2$. Given the holonomic Hadamard gate and rotations about the x axis, we can achieve holonomic rotations about z axis, $e^{i\alpha\sigma_z} = He^{i\alpha\sigma_x}H$. Together with the nontrivial holonomic two-qubit gates U_1 and U_2, a universal set of holonomic quantum gates can be implemented.

It should be noted that U_{s2} can be obtained from H_1 with an additional local transformation described by $\Pi_4 = e^{-i\frac{\pi}{4}\sigma_z}$, without the need to revert to H_2. This connection is easy to grasp when one realizes that H_1 and H_2 both contain only single-qubit terms and can this be related by local transformations. The newly transformed H_1 is written as $\bar{H}_1' = (\Pi_4 \otimes \Pi_4)(\Pi_1 \otimes \Pi_1)\Pi_3' H_1 \Pi_3'^\dagger(\Pi_1^\dagger \otimes \Pi_1^\dagger)(\Pi_4^\dagger \otimes \Pi_4^\dagger)$, and we obtain

$$\bar{H}_1' = \frac{J_0}{2}(\sigma_{x,1}\sigma_{x,2} + \sigma_{y,1}\sigma_{y,2}) + \frac{J_1}{2}(\sigma_{y,1} - \sigma_{y,1}\sigma_{z,2}). \tag{8}$$

In a similar way, we equivalently have $\bar{H}_1' = \Omega\big(\sin\theta\,|10\rangle\langle 01| + i\cos\theta\,|11\rangle\langle 01|\big) + \text{H.c.}$ The corresponding evolution operator at $T = \pi/\Omega$ is given as

$$U_1' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & \cos(2\theta) & i\sin(2\theta) \\ 0 & 0 & -i\sin(2\theta) & -\cos(2\theta) \end{pmatrix}. \tag{9}$$

Based on $\bar{H}_1'$, Rabi oscillations now occur between $|01\rangle$ and $\sin\theta\,|10\rangle + i\cos\theta\,|11\rangle$, and similarly between $\mathcal{S}_0$ and $\mathcal{S}_1$. At $t = T$, the system undergoes a cyclic evolution in the subspace $\mathcal{S}_j$ $(j = 1, 2)$. We can also ensure the condition $P_{\mathcal{S}_j}(t)\bar{H}_1' P_{\mathcal{S}_j}(t) = 0$, where $P_{\mathcal{S}_j}(t) = U_1'(t)P_{\mathcal{S}_j}U_1'^\dagger(t)$, with $U_1'(t)$ being the evolution operator obtained from $\bar{H}_1'$. Therefore, U_1' represents holonomic two-qubit gates [7,23,24]. If we consider qubit 1 as an auxiliary qubit, we obtain $U_{s2} = -\sin(2\theta)\sigma_y + \cos(2\theta)\sigma_z$ acting on qubit 2 when qubit 1 is in its ground state.

In the following, we discuss the execution of our protocol in superconducting systems as an example. For $\Pi_{1,2}$, we use the following Hamiltonian $H_{i,0} = (\omega_q/2)\sigma_z + \Omega_k\cos(\omega_k t + \phi_k)\sigma_x$ to realize them, where ω_q is the frequency of the qubit, and Ω_k and ϕ_k $(k = x, y)$ are the Rabi frequency and the phase of the corresponding external driving. Applying a unitary transformation $U_0 = e^{-i(\omega_{q,0}/2)\sigma_z t}$, we can obtain a frame-rotated Hamiltonian when $\omega_k = \omega_{q,0}$ and $\omega_k \gg \Omega_k$, as $H_0 = (\delta/2)\sigma_z + \Omega_k/2(e^{-i\phi_k}\sigma_+ + e^{i\phi_k}\sigma_-)$, where $\delta = \omega_q - \omega_{q,0}$ is the detuning of the qubit frequency, and $\sigma_\pm = \frac{1}{2}(\sigma_x \pm i\sigma_y)$. Moreover, Ω_k is controllable as it comes from external driving. For δ, it is also controllable in superconducting qubits as the qubit frequency can be tuned [25, 26]. Depending on the values of δ, Ω_k, and ϕ_k, we can achieve $H_{1,0} = (\Omega_x/2)\sigma_x + (\Omega_y/2)\sigma_y$ or $H_{2,0} = (\delta/2)\sigma_z + (\Omega_x/2)\sigma_x$. By appropriately letting the system evolve over time, $\Pi_{1,2}$ can be obtained as a result. For the CNOT gate, some research groups have demonstrated tunable $\sigma_z \otimes \sigma_z$

coupling and control-Z (CZ) gates with a fidelity above 99.8% experimentally in superconducting qubits [27,28]. The CNOT gate is locally unitarily related to the CZ gate, and thus both the CNOT gate and tunable $\sigma_z \otimes \sigma_z$ coupling are achievable in superconducting qubits. Similarly, H_1 and H_2 are realizable in a manner similar to that for $H_{1,0}$ or $H_{2,0}$, in the presence of two qubits.

III. IMPLEMENTATION WITH DD INTEGRATED

To implement the above-mentioned holonomic gates, two physical qubits are required for each gate, whether for single-qubit or two-qubit gates. As a result, single-qubit or two-qubit decoherence or errors may arise in experiments involving our scheme. Nonetheless, the geometric gates can be preserved from such decoherence or errors by using the DD technique. To combat arbitrary single-qubit and two-qubit decoherence or errors, represented by the error operators in the set $\mathcal{E} = \{\sigma_{s,i}, \sigma_{x,i}\sigma_{x,j}, \sigma_{x,i}\sigma_{y,j}, \sigma_{y,i}\sigma_{x,j}, \sigma_{x,i}\sigma_{z,j}, \sigma_{z,i}\sigma_{x,j}, \sigma_{y,i}\sigma_{y,j}, \sigma_{y,i}\sigma_{z,j}, \sigma_{z,i}\sigma_{y,j}, \sigma_{z,i}\sigma_{z,j}\}$, where $s = x, y, z$, we use the 16-step DD sequence given by $\left(\sigma_{z,j}[\cdot\cdot]\sigma_{z,j}\right)\left(\sigma_{z,i}[\cdot\cdot]\sigma_{z,i}\right)\left(\sigma_{z,i}\sigma_{z,j}[\cdot\cdot]\sigma_{z,i}\sigma_{z,j}\right)[\cdot\cdot]$, where $[\cdot\cdot] = \left(\sigma_{x,j}[\cdot]\sigma_{x,j}\right)\left(\sigma_{x,i}[\cdot]\sigma_{x,i}\right)\left(\sigma_{x,i}\sigma_{x,j}[\cdot]\sigma_{x,i}\sigma_{x,j}\right)[\cdot]$, and $[\cdot]$ represents the evolution operator over a period of $\tau = T/(16n)$, with T being the total time to execute the gate and n being the number of repetitions of the 16-step DD pulses [17].

The success of the DD approach depends on the tunability of system parameters to ensure that we do not inadvertently eliminate desired system Hamiltonians. Given controllable system parameters, both the necessary preliminary gates $\Pi_{1,2,3}$ and the gates $U_{1,2}$, $U_{s1,s2}$ can be made approximately robust to single-qubit and two-qubit decoherence or errors when DD pulses are applied. Fortunately, superconducting qubits or trapped ions represent promising platforms for quantum computation, as they offer adjustable system parameters [25–40]. Therefore, our scheme is potentially realizable in experiments using current techniques.

The protection of $\Pi_{1,2,3}$ can be understood from the discussion regarding the performance of a universal set of decoherence-protected gates provided in Ref. [17]. For single-qubit gates $\Pi_{1,2}$, it is efficient to remove the decoherence or errors using periodic DD (PDD). This involves a base succession of pulses $\sigma_x(\cdot)\sigma_z(\cdot)\sigma_x(\cdot)\sigma_z(\cdot)$, where $(\cdot)$ represents the evolution operator over a time interval $\tau_p = T_0/(4n_p)$, with T_0 being the desired evolution time and n_p being the number of repetitions of the PDD base pulses, with appropriate adjustment of the system parameters to ensure that the desired gate Hamiltonians remain unchanged at each step. As for Π_3, it is achievable through a CZ gate, which is effectively discussed in Ref. [17], along with Hadamard gates that correspond to Π_1. To implement a CZ gate, a 16-step DD sequence is needed to counter both single- and two-qubit decoherence or errors.

TABLE I. Required system parameters for the evolution governed by H_1 at each step to ensure compatibility with the 16-step DD scheme.

Step	$\frac{J_0}{2}\sigma_{y,1}$	$\frac{J_1}{2}\sigma_{z,1}$	$\frac{J_0}{2}\sigma_{z,2}$	$-\frac{J_1}{2}\sigma_{x,2}$
1st $[\cdot]$	$J_0 \to J_0$	$J_1 \to J_1$	$J_0 \to J_0$	$J_1 \to J_1$
2nd $[\cdot]$	$J_0 \to -J_0$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_1 \to J_1$
3rd $[\cdot]$	$J_0 \to -J_0$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_1 \to J_1$
4th $[\cdot]$	$J_0 \to J_0$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_1 \to J_1$
5th $[\cdot]$	$J_0 \to -J_0$	$J_1 \to J_1$	$J_0 \to J_0$	$J_1 \to -J_1$
6th $[\cdot]$	$J_0 \to J_0$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_1 \to -J_1$
7th $[\cdot]$	$J_0 \to J_0$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_1 \to -J_1$
8th $[\cdot]$	$J_0 \to -J_0$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_1 \to -J_1$
9th $[\cdot]$	$J_0 \to -J_0$	$J_1 \to J_1$	$J_0 \to J_0$	$J_1 \to J_1$
10th $[\cdot]$	$J_0 \to J_0$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_1 \to J_1$
11th $[\cdot]$	$J_0 \to J_0$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_1 \to J_1$
12th $[\cdot]$	$J_0 \to -J_0$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_1 \to J_1$
13th $[\cdot]$	$J_0 \to J_0$	$J_1 \to J_1$	$J_0 \to J_0$	$J_1 \to -J_1$
14th $[\cdot]$	$J_0 \to -J_0$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_1 \to -J_1$
15th $[\cdot]$	$J_0 \to -J_0$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_1 \to -J_1$
16th $[\cdot]$	$J_0 \to J_0$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_1 \to -J_1$

Next, we explore the performance of the DD sequence in the protection of $U_{1,2}$, which is realized through three subevolutions. The first and third subevolutions involve combinations of $\Pi_{1,2,3}$, and the second is related to $H_{1,2}$. Since we already discussed the preservation of $\Pi_{1,2,3}$ in the previous paragraph, we now focus on the system evolution governed by $H_{1,2}$. This requires consideration of two qubits simultaneously, and 16-step DD pulses are engaged to preserve the system evolution from any error in the set $\mathcal{E}$. Some of the DD pulses may change the desired Pauli operators to negative ones, so it is necessary to tune the system parameters prevent such undesired changes. The required system parameters for the evolution controlled by H_1 and H_2 are listed in Tables I and II, respectively, for all 16 steps of the system evolution.

The protection of $U_{1,2}$ using the 16-step DD scheme can be demonstrated via numerical explorations. In this context, we assume that the preliminary gates $\Pi_{1,2,3}$ are available and ideal. In the following, we investigate the preservation of the evolution controlled by $H_{1,2}$. When considering errors or decoherence, the total Hamiltonian is written as $H_T = H_{1,2} + H_e$, where H_e is the stochastic error term [17] and is given by

$$H_e = \sum_{j,s} \lambda_j^s \sigma_{s,j} + \sum_{s,t} \lambda_{12}^{st} \sigma_{s,1}\sigma_{t,2}, \tag{10}$$

in which $j = 1, 2$ and $s, t = x, y, z$. In the expression for H_e, $\lambda_{1,2}^{x,y,z}$ and λ_{12}^{st} describe the strengths of stochastic errors that are dependent on time and act in various directions.

Experiments based on superconducting qubits have substantiated our model [41,42]. The energy relaxation rates

TABLE II. Required system parameters for the evolution governed by H_2 at each step to ensure compatibility with the 16-step DD scheme.

Step	$\frac{J_1}{2}\sigma_{x,1}$	$-\frac{J_1}{2}\sigma_{y,2}$	$\frac{J_0}{2}\sigma_{z,2}$	$\frac{J_0}{2}\sigma_{y,1}$
1st [·]	$J_1 \to J_1$	$J_1 \to J_1$	$J_0 \to J_0$	$J_0 \to J_0$
2nd [·]	$J_1 \to J_1$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_0 \to -J_0$
3rd [·]	$J_1 \to J_1$	$J_1 \to J_1$	$J_0 \to J_0$	$J_0 \to -J_0$
4th [·]	$J_1 \to J_1$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_0 \to J_0$
5th [·]	$J_1 \to -J_1$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_0 \to -J_0$
6th [·]	$J_1 \to -J_1$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_0 \to J_0$
7th [·]	$J_1 \to -J_1$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_0 \to J_0$
8th [·]	$J_1 \to -J_1$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_0 \to -J_0$
9th [·]	$J_1 \to -J_1$	$J_1 \to J_1$	$J_0 \to J_0$	$J_0 \to -J_0$
10th [·]	$J_1 \to -J_1$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_0 \to J_0$
11th [·]	$J_1 \to -J_1$	$J_1 \to J_1$	$J_0 \to J_0$	$J_0 \to J_0$
12th [·]	$J_1 \to -J_1$	$J_1 \to -J_1$	$J_0 \to -J_0$	$J_0 \to -J_0$
13th [·]	$J_1 \to J_1$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_0 \to J_0$
14th [·]	$J_1 \to J_1$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_0 \to -J_0$
15th [·]	$J_1 \to J_1$	$J_1 \to -J_1$	$J_0 \to J_0$	$J_0 \to -J_0$
16th [·]	$J_1 \to J_1$	$J_1 \to J_1$	$J_0 \to -J_0$	$J_0 \to J_0$

of qubits are limited by fluctuations in the transition energies, which arise from various sources of noise. As discussed in Refs. [41] and [42], these energy relaxation rates are associated with energy shifts induced by various sources of noise, such as charge noise, low-frequency critical-current noise, and flux noise. This indicates that decoherence can be captured by modeling small fluctuations in the terms of the Hamiltonian. In the two-qubit setting, crosstalk can emerge from inadequate isolation of control lines, unintended coupling between qubits, or spurious modes, prompting us to include fluctuations in the qubit-qubit coupling terms. In practice, some of these error terms may be absent depending on the qubit design. DD techniques are effective against correlated noise [43,44], as they require noise correlation times that are longer than the DD pulse lengths. In our calculations, the stochastic strengths of the noise are modeled by samples drawn from a Gaussian distribution. As we are using a finite number of samples, these samples inevitably exhibit interdependence, thereby approximating correlated noise.

We numerically explore our scheme using superconducting transmon qubits. With transmon qubits, single-qubit decoherence may be due to the quasiparticle tunneling across Josephson junctions [45,46] and couplings to near-resonant two-level systems [46–50]. For transmon qubits, two-qubit decoherence is commonly related to microwave crosstalk [51,52] and residual interqubit coupling [53]. We consider transmon qubits with lifetimes of the order of tens to hundreds of microseconds (decay rates of 0.1 to 0.01 MHz) [48–50]. As noted in Ref. [41], energy shifts induced by errors are typically of the same order as the qubit decay rate ($\sim$ MHz). Hence, we set the magnitudes of the fluctuations in our calculations

TABLE III. Average fidelities of U_1 and $U_{s1,s2}$ in the absence (presence) of errors and DD pulses with ideal preliminarily required gates. When DD pulses are applied, we set $n = 3$, meaning the 16-step DD sequence is repeated three times.

Gate	Without errors	With errors	
	F	F without DD	F with DD
U_1	1	0.2671	0.9966
U_{s1}	1	0.1795	0.9963
U_{s2}	1	0.5772	0.9954

to be around $2\pi \times 5$ MHz. Even though current decay rates are below 1 MHz, we intentionally choose fluctuation magnitudes in the MHz range to demonstrate that our protocol remains effective for qubits with shorter coherence times, provided enough DD pulses are applied. The time dependance of the fluctuations is simulated by generating random numbers from a Gaussian distribution with a mean of $2\pi \times 5$ MHz and a standard deviation of $2\pi \times 1$ MHz when solving the differential equation numerically.

We take U_1, U_{s1}, and U_{s2} as examples to examine the protection offered by the DD technique, assuming that the preliminarily required gates are ideal. The system parameters are selected as $\Omega = 2\pi \times 40$ MHz [54] and $\theta = -\pi/8$, leading to $J_0 = 2\pi \times 40 \times \sin(-\pi/8)$ MHz and $J_1 = 2\pi \times 40 \times \cos(-\pi/8)$ MHz. From 50 randomly selected initial states, we obtain the average fidelities of the gates in the absence or presence of errors, with and without DD pulses applied. For U_1, the initial state is of the form $|\psi_i\rangle = 1/(\sqrt{\sum_{j,k=0,1} |a_{jk}|^2}) \left(\sum_{j,k=0,1} a_{jk} |jk\rangle\right)$. The gates $U_{s1,s2}$ can be realized by preparing the auxiliary qubit in its ground state initially and varying the superposition coefficients in data qubit. The numerical results are presented in Table III. As expected, the quantum gates perform perfectly in the absence of errors, as there is no approximation involved in $H_{1,2}$. In contrast, given the errors from the set $\mathcal{E}$, U_1 and $U_{s1,s2}$ are adversely influenced by the errors when no DD pulses are applied. The 16-step DD scheme effectively preserves the gates, as seen in our calculations, where we repeat the DD pulses three times. In practical cases, the preliminarily required gates $\Pi_{1,2,3}$ cannot be ideal, but they can still be approximately protected by the DD technique. The small errors in the preliminarily required gates will only slightly reduce the performance of U_1 and $U_{s1,s2}$.

IV. CONCLUSION

To summarize, we have explored a scheme for implementing decoherence-protected holonomic gates, which relies on the system Hamiltonians (2) and (5), containing single-qubit terms, with the help of three preliminarily available nongeometric gates. We have shown that the gates achieved possess geometric properties, making them robust against control imperfections in the Hamiltonians (2) and (5). To be specific, we use one auxiliary qubit to execute single-qubit holonomic gates on data qubits, avoiding the concerns related to decoherence from higher energy levels. For two-qubit holonomic gates, no auxiliary qubit is needed. Therefore, our scheme requires fewer resources than existing protocols in the literature, which typically require two or three physical qubits to form a logical qubit. We would like to highlight that the geometric protection applies only to the parameters in $H_{1,2}$ and does not extend to the parameters used in implementing the preliminarily required gates. This limitation also exists in other schemes for geometric-DD integrated quantum gates when transformations are needed to achieve the required gate Hamiltonians. On the positive side, our scheme only requires three preliminary nongeometric gates, from which a universal set of holonomic gates can be realized.

The simple form of the Hamiltonians (2) and (5) makes our scheme realizable with current experimental techniques and allows for the tunability of the parameters. As a result, the DD techniques can be incorporated into the scheme. The evolution governed by these Hamiltonians can be protected from both single-qubit and two-qubit decoherence or errors with the DD approach discussed in Ref. [17]. Moreover, the preliminary set of nongeometric gates can also be preserved from errors using the DD approach. There are two single-qubit gates and one two-qubit gate in the set. The single-qubit gates can be protected using PDD, and the two-qubit gate preserved by the 16-step DD approach. Finally, the gates in our scheme can be safeguarded from not only single-qubit but also two-qubit decoherence or errors at every step of system evolution. Our scheme offers several advantages that are particularly attractive for experimental implementation, providing a promising path toward developing quantum devices with both geometric and DD protections.

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DATA AVAILABILITY

The data from which we calculated the average fidelities of U_1 and $U_{s1,s2}$ are openly available [55].

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