

On-demand shaped-photon emission based on a parametrically modulated qubit

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In the circuit quantum electrodynamics architectures, to realize a long-range quantum network mediated by a flying photon, it is necessary to shape the temporal profile of emitted photons to achieve a high transfer efficiency between two quantum nodes. In this work, we demonstrate an alternative single-rail and dual-rail time-bin-shaped photon generator without additional flux-tunable elements, which can act as a quantum interface of a point-to-point quantum network. In our approach, we adopt a qubit-resonator-transmission-line configuration, and the effective coupling strength between the qubit and the resonator can be varied by parametrically modulating the qubit frequency. In this way, the coupling is directly proportional to the parametric modulation amplitude and covers a broad tunable range beyond 20 MHz for the sample we used. Additionally, when emitting shaped photons, we find that the spurious frequency shift (−0.4 MHz) due to parametric modulation is small and can be readily calibrated through chirping. We develop an efficient photon field measurement setup based on the data-stream processing of GPU. Utilizing this system, we perform photon temporal profile measurement, quantum state tomography of photon field and quantum process tomography of single-rail quantum state transfer based on a heterodyne-measurement scheme. The single-rail encoding state-transfer fidelity of shaped photon emission is 90.32%, and that for unshaped photons is 97.20%, respectively. We believe that the fidelity of shaped photon emission is mainly limited by the qubit coherence time. The results demonstrate that our method is hardware efficient, simple to implement, and scalable. It could become a viable tool in a high-quality quantum network utilizing both single-rail and dual-rail time-bin encoding.

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I. INTRODUCTION

In the realm of quantum networking, it is necessary to coherently link various quantum nodes such as quantum processors, memories, and sensors together [1–3]. The core of a quantum network is the transmission of quantum states and the generation of remote quantum entanglement. In optical contexts, due to the low efficiency of single-photon generation, one typically generates

heralded entanglement probabilistically via coincidence counts of flying photons from independent sources [4–6]. In contrast, in the domain of superconducting quantum computing, circuit QED architectures [7–9] have enabled substantial coupling efficiency between superconducting qubits, microwave resonators, and one-dimensional (1D) waveguides. The microwave photon field emitted by the qubits is confined in 1D waveguides, eliminating the need for spatial mode matching, which is key to on-demand, high-efficiency quantum state transfer. Many recent experiments are based on Cirac-Zoller-Kimble-Mabuchi (CZKM) schemes [10]. Therein, single photons are pitched and caught directly by symmetrically shaping and time-reversal quantum control in emission and receiving nodes [11–14].

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Regarding the devices used in microwave photon emission and receiving, there are two primary configurations. In the first case, one directly couples quantum emitters to 1D waveguides [15–25], while in the other case one employs a buffer resonator or qubit for indirect coupling [26–35]. The former can, in principle, release and catch shaped photons more rapidly and is exposed to fewer dissipation channels. However, most implementations [15,16,18,19,21–25] require a flux-tunable loop of Josephson junctions to adjust the mutual inductance between the emitter and the waveguide, causing significant frequency shifts in the emitter that are challenging to calibrate accurately. To compare, with the aid of the extra resonator (qubit), the configurations with buffer support indirect tunable coupling like the cavity-assisted Raman process [36] and dual-rail encoding protocols that are more robust against photon-transfer loss [29,37] and phase-reference error between sender and receiver [38]. The drawback is therefore the additional loss induced by the buffer and the limited bandwidth of the emitted photon, which is constrained by both the coupling strength between the quantum emitters and the buffer and the decay rate of the buffer [39]. For all the methods mentioned above, quantum emitters experience unintended frequency shifts while photon shaping. Additionally, the relationship between effective coupling strength and control pulse amplitude is either nonlinear [15,17,18] or has a constrained linear range [26,28,29].

In this work, we propose and experimentally demonstrate another shaped photon generator employing a buffer emission resonator. We modulate the qubit parametrically by applying a time-varying magnetic flux and thus use the first-order sideband of a parametrically modulated qubit to induce an effective coupling between the transmon qubit and the emission resonator. The coupling strength is found to be proportional to the parametric modulation pulse amplitudes. The spurious frequency shift of the qubit is small, attributed to the modest amplitude of the necessary parametric modulation pulse. Consequently, the photon shaping and frequency-shift calibration processes are straightforward and easy to implement experimentally. It avoids the use of any additional flux-tunable components with Josephson junctions and serves as a quantum interface functioning as both the sender and receiver in a point-to-point quantum network. Additionally, with the resonator, it is capable of single-rail and dual-rail time-bin photon emission. Through quantum process tomography, we achieve a fidelity of single-rail quantum state transfer with a shaped photon at 90.32% and with an unshaped photon at 97.20%.

II. FLUX MODULATION OF A FREQUENCY TUNABLE QUBIT

Our scheme includes a flux-modulated frequency tunable superconducting transmon qubit capacitively

coupled to two coplanar waveguide resonators, one for dispersive readout, the other one for shaped photon emission, as schematically shown in Fig. 1(a). We use a sinusoidal parametric modulation pulse added to the dc-SQUID loop of the transmon $\Phi(t) = \Phi_{\text{dc}} + \Phi_{\text{ac}} \cos(\omega_m t + \theta_m)$, where Φ_{dc} is the static flux bias, ω_m is the frequency of parametric modulation and θ_m is the initial phase of parametric modulation. For a frequency tunable transmon, we have $\omega_q(\Phi) = \frac{1}{\hbar} (\sqrt{8E_C E_J(\Phi)} - E_C)$ where $E_J(\Phi) = E_{J\Sigma} \cos(\pi \Phi / \Phi_0)$, $E_{J\Sigma}$ is the total Josephson energy, E_C is the capacitive energy of the transmon, Φ_0 denotes the magnetic flux quantum. Due to the nonlinear relationship between the magnetic flux and transmon frequency, the parametric modulation leads to the variation of qubit frequency consists of different frequency components $\omega_q(t) = \bar{\omega}_q + \sum_{a=1}^{\infty} A_m^a \cos[a(\omega_m t + \theta_m)]$ ($a \in \mathbb{N}$) [40,41], where $\bar{\omega}_q$ is the average frequency of qubit during parametric modulation, A_m^a is the a th order Fourier components of longitudinal field modulation (LFM) amplitude (see Appendix C). When a weak parametric modulation pulse ($A_m^1 \ll \omega_m$), which has a frequency of $\omega_m \approx \omega_E - \omega_q$ is applied to the qubit, where ω_E is the frequency of the emission resonator, ω_q is the frequency of qubit, neglecting the far-detuning terms, the interaction Hamiltonian under two-level approximation is

$$\begin{aligned} \hat{H}_{\text{int}}/\hbar &= g_{qE} J_1 \left(\frac{A_m^1}{\omega_m} \right) e^{i\theta_m} \hat{a} \hat{\sigma}_+ + \text{h.c.} \\ &\approx g_{qE} \frac{A_m^1}{2\omega_m} e^{i\theta_m} \hat{a} \hat{\sigma}_+ + \text{h.c.} \\ &\approx \frac{g_{qE}}{2\omega_m} \Phi_{\text{ac}} \frac{d\omega_q}{d\Phi} \Big|_{\Phi_{\text{dc}}} e^{i\theta_m} \hat{a} \hat{\sigma}_+ + \text{h.c.}, \quad (1) \end{aligned}$$

where $J_1(x)$ is the first-order Bessel function [when $x \rightarrow 0$, $J_1(x) \approx x/2$], g_{qE} is the coupling coefficient between the qubit and emission resonator, A_m^1 is the first-order Fourier components of LFM amplitudes, θ_m , here, becomes the phase angle of the first-order sideband coupling. Equation (1) demonstrates an effective first-order side-band coupling between the qubit and emission resonator $g_{\text{eff}} = g_{qE} A_m^1 / (2\omega_m) e^{i\theta_m} \propto \Phi_{\text{ac}} e^{i\theta_m}$ that is linear to the parametric modulation pulse amplitude Φ_{ac} and the phase can be controlled by the modulation pulse θ_m . Notice that the coupling is the first-order coupling and as a result we can get a wide range of tunable coupling with a weak parametric modulation pulse. When we apply a weak parametric modulation with a time-dependent amplitude and phase $\Phi(t) = \Phi_{\text{dc}} + \Phi_{\text{ac}}(t) \cos(\omega_m t + \theta_m(t))$, the first-order effective coupling will also gain time-dependent amplitude and phase, which is linear to the parametric

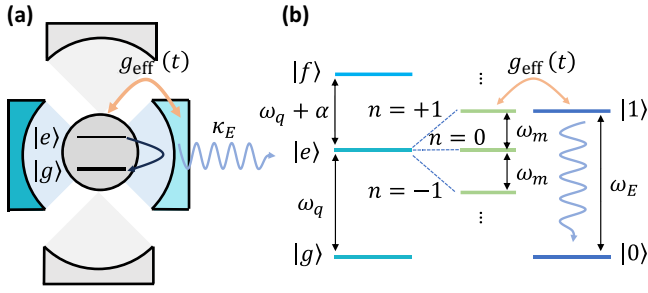


FIG. 1. Schematic diagrams of sample layouts and energy levels. (a) Our sample layout, which has two resonators coupled to the qubit, one for dispersive readout, the other for shaped photon emission. The effective coupling $g_{\text{eff}}(t)$ between the qubit and emission resonator is controlled by parametric modulation. Decay rate of the emission resonator is denoted by κ_E . (b) The energy levels of transmon and emission resonator where ω_q is the angular frequency of transmon, α is the anharmonicity of transmon and ω_E is the angular frequency of emission resonator. Modes of the readout resonator are not shown explicitly. The light blue levels represent the lowest three levels of the transmon, the green levels represent the zeroth and first-order sidebands of parametric modulation, and the dark blue levels represent the zero-photon and one-photon levels of the emission resonator. Using the first-order sideband of parametric modulation we couple the transmon qubit to emission resonator and modulate the amplitude dynamically to shape the emitted photon.

modulation pulse.

$$g_{\text{eff}}(t) = \frac{g_{qE}}{2\omega_m} \Phi_{\text{AC}}(t) \left. \frac{d\omega_q}{d\Phi} \right|_{\Phi_{\text{dc}}} e^{i\theta_m(t)} \propto \Phi_{\text{ac}}(t) e^{i\theta_m(t)}. \quad (2)$$

Thus, we can get arbitrary emitted photon shape [42] through controlling the envelope and phase of parametric modulation pulses.

In addition, it should be noted that parametric modulation itself results in qubit frequency shift $\Delta = \Delta_{\text{dc}} + \Delta'_{\text{Lamb}}$, where $\Delta_{\text{dc}} = \bar{\omega}_q - \omega_q(\Phi_{\text{DC}})$ is the average frequency shift during parametric modulation and Δ'_{Lamb} is the Lamb-shift variation induced by the zero-order sideband [43] (see Appendix C). The frequency shift is approximately quadratic to parametric modulation amplitude. Therefore, we can experimentally characterize (see Appendix D) and calibrate the frequency shift with a chirp pulse $\Phi_m(t) = \Phi_{\text{ac}}(t) \cos(\omega_m t + \theta_m(t))$ where $\dot{\theta}_m(t) = -\Delta(t)$.

For our device, the peak frequency of transmon is $\omega_{q,p}/2\pi = 5.997$ GHz, and the frequency of readout and emission resonator is $\omega_{R,p}/2\pi = 4.177$ GHz and $\omega_{E,p}/2\pi = 7.142$ GHz, respectively. During the experiment, we bias our transmon qubit slightly away from the peak frequency using a static external magnetic flux, setting $\Phi_{\text{dc}}/\Phi_0 = 0.04$ where Φ_0 denotes the magnetic flux

quantum. By doing so, the qubit maintains a relatively high coherence time T_2^* , meanwhile, we can circumvent the undesired simultaneous coupling of the first and the second sidebands. At that point, the frequency of qubit and emission resonator is $\omega_q/2\pi = 5.947$ GHz and $\omega_E/2\pi = 7.139$ GHz, respectively. Emission resonator frequency shifts due to the Lamb shift induced by large dispersive coupling between the qubit and emission resonator. We then perform weak parametric modulation pulses to use the effective coupling of the first-order sideband of the qubit and emission resonator to shape the photon temporal profile and characterize the photons.

III. PHOTON SHAPING AND DYNAMICAL FREQUENCY CALIBRATION BY CHIRPING

For shaped photon generation, we excite the qubit by conventional charge drive, control the effective coupling between the qubit and emission resonator. Then, shaped photon is leaked out from the emission resonator. Due to number-phase uncertainty, the temporal envelop cannot be determined experimentally from the photon field in the Fock state such as $|1\rangle$. To observe the envelope of the photon, we need to prepare a state with uncertain photon numbers, such as $(|0\rangle + |1\rangle)/\sqrt{2}$. So, we first excite the qubit with a $\pi/2_{ge}$ pulse from $|g\rangle$ to $(|g\rangle + |e\rangle)/\sqrt{2}$ then use the first-order sideband parametric modulation pulse to eliminate the qubit $|e\rangle$ state population and emit a half photon state $(|0\rangle + |1\rangle)/\sqrt{2}$ from the emission resonator and obtain a so-called π_{e0g1} pulse. Because our qubit has a negligible thermal excitation (about 0.2% $|e\rangle$ population, see Appendix H), we perform the experiment without extra qubit initialization. Then, using the heterodyne-detection method [44] with a linear amplification chain [with a Josephson parametric amplifier (JPA) [45,46] and the quantum efficiency is $\eta = 0.26$, see Appendix E] and a linear voltage detector (a high-speed data acquisition card with the data-stream processing of GPU) (see Appendix B) we derive the time-dependent quadrature voltages and power of the photon field. We use different pulse envelopes of parametric modulation pulses [see Fig. 2(a)] such as flat-top pulse, which is square pulse convoluted with Gaussian function with $2\sigma = 4$ ns to have a smooth rising edge, $\sin^2$ pulse, which has a form of $\Phi_{\text{ac}}(t) = A_{\text{ac}} \sin(\pi t/T)$ and no need to truncate and sech pulse, which has the form of $\Phi_{\text{ac}}(t) = A_{\text{ac}} \text{sech}(\kappa t)$ truncated to the length of $4\pi/\kappa$. As a comparison, flattop pulse generates the unshaped photon without frequency calibration by chirping, while other pulses generate the shaped photons with frequency calibration by chirping. The average amplitude results are shown in Fig. 2(b), the average power results are shown in Fig. 2(c).

Following Ref. [26], we apply a symmetry factor to characterize the symmetry of both amplitude and phase of photon temporal profile by autocorrelation of the measured

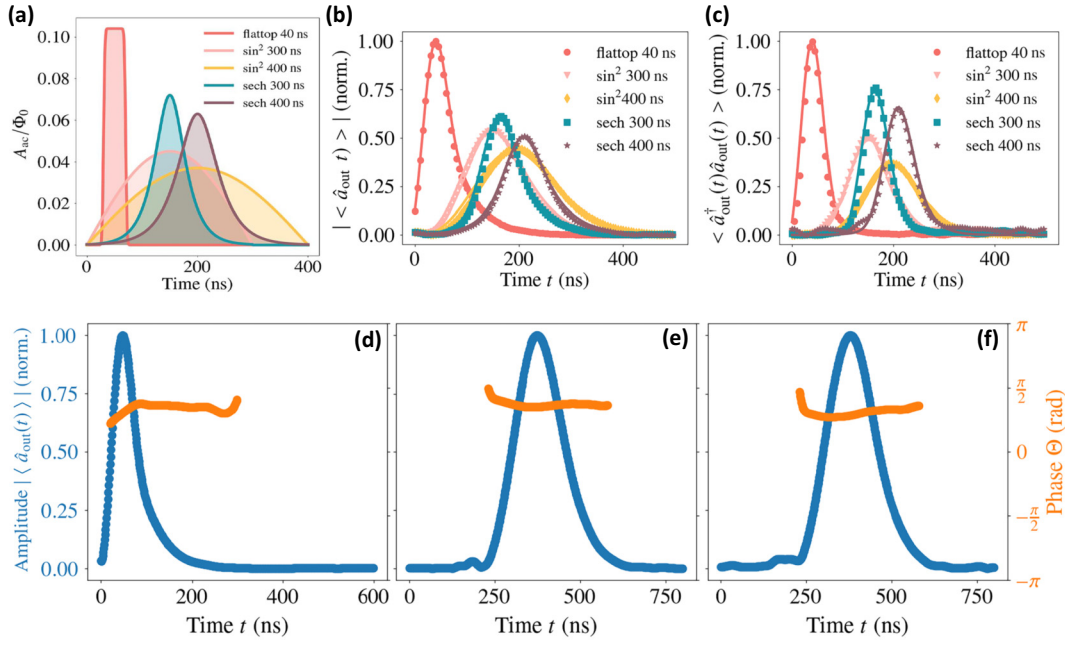


FIG. 2. Photon temporal profile of $(|0\rangle + |1\rangle)/\sqrt{2}$ state using different control pulses (flat-top, $\sin^2$, and sech envelopes). Flat-top pulse generates the unshaped photon without frequency calibration by chirping, other pulses generate the shaped photons with frequency calibration by chirping. The measured data is averaged for 3×10^6 times. (a) Different parametric modulation pulses (π_{e0g1} pulses) used to eliminate the excitation of qubit and generate photon. (b) The amplitude of the output photon field. The dot is the experimental data, the solid lines are the numerical simulations using QuTiP [47]. (c) The power of output photon field. The dot is the experimental data, the solid lines are the numerical simulations using QuTiP. (d) The amplitude and phase of unshaped photon generated by 40 ns flat-top pulse. (e) The amplitude and phase of shaped photon generated by 400 ns $\sin^2$ pulse with frequency calibration by chirping. (f) The amplitude and phase of shaped photon generated by 400 ns $\sin^2$ pulse without frequency calibration by chirping.

quadratures normalized by the quadrature amplitudes:

$$s = \max_{t_0} \frac{\int \langle \hat{a}_{\text{out}}(t_0 - t) \rangle^* \langle \hat{a}_{\text{out}}(t) \rangle dt}{\int |\langle \hat{a}_{\text{out}}(t) \rangle|^2 dt}. \quad (3)$$

In our case, the decay rate of emission resonator is about $\kappa_E/2\pi = 5.2$ MHz. Thus, the bandwidth upperband of our shaped photon is $\Gamma/2\pi \leq \kappa_E/2\pi = 5.2$ MHz. Truncated to $4\pi/\Gamma$, the minimal symmetrical photon time length is about 385 ns [39], thus the appropriate parametric modulation pulse length is 300 to 400 ns. We evaluate the symmetry of the unshaped photon generated by flat-top pulse and shaped photon generated by $\sin^2$ and sech pulses in Table I. We experimentally find that for pulses with the same length, sech-type parametric modulation pulses always generate photons that have a lower symmetry factor than that of $\sin^2$ -type pulses. We note that if we define a symmetry factor without phase by replacing the $\langle \hat{a}_{\text{out}}(t_0 - t) \rangle$, $\langle \hat{a}_{\text{out}}(t) \rangle$ with $|\langle \hat{a}_{\text{out}}(t_0 - t) \rangle|$ and $|\langle \hat{a}_{\text{out}}(t) \rangle|$ in Eq. (3), this absolute symmetry factor of photon generated by 400-ns sech-type pulse is 0.9974 while that of the 400-ns $\sin^2$ -type pulse is 0.9917 (see Table I), which means the absolute amplitude results of sech-type pulses are more symmetrical than $\sin^2$ -type pulses. However, the phase of sech-type pulses drifts more (not shown in the

figure). We believe it is attributable to the distortion of the parametric modulation pulse shape, which is caused by the unknown transfer function of the flux line. The sech-type pulses have sharper peaks and larger amplitudes for the same π_{e0g1} pulse length, thus they are more distorted and result in a spurious frequency shift that are calibrated mistakenly. As a result, the most symmetrical photon is created by the $\sin^2$ pulse of $A_{\text{ac}} = 0.037\Phi_0$ and 400-ns length whose symmetry factor is over 0.99 with a residue $|e\rangle$ -state excitation of $0.13 \pm 0.02\%$.

We demonstrate the amplitude ($|\langle \hat{a}_{\text{out}}(t) \rangle|$) and phase ($\Theta(t) = \arg(\langle \hat{a}_{\text{out}}(t) \rangle)$) of the most symmetrically shaped photon and compare the results of that with and without frequency calibration by chirping together with unshaped photon in Figs. 2(d)–2(f). We note that compared to unshaped photon [in Fig. 2(d)] the shaped photon [in Fig. 2(e)] has a significant symmetry enhancement, the symmetry factor is enhanced from 0.8933 to 0.9906. Compared to the phase of unshaped photon, the phase of shaped photon almost stays constant, too. However, the symmetry factor of shaped photon without frequency calibration by chirping [in Fig. 2(f)] drops because of the phase accumulation of photon induced by dynamical frequency shift caused by varying amplitude of the parametric modulation pulse. In spite of this, it is not much, the symmetry factor

TABLE I. The symmetry factor of different parametric modulation pulses. L represents the length of parametric modulation pulse, A_{ac} represents the amplitude of parametric modulation pulse, s represents the symmetry factor defined by Eq. (3), and s_{abs} represents the symmetry factor without phase, which replaces $\langle \hat{a}_{out}(t_0 - t) \rangle$, $\langle \hat{a}_{out}(t) \rangle$ with $|\langle \hat{a}_{out}(t_0 - t) \rangle|$ and $|\langle \hat{a}_{out}(t) \rangle|$ in Eq. (3). Flattop pulse generates the unshaped photon without frequency calibration by chirping, other pulses generate the shaped photons with frequency calibration by chirping.

Pulse shape	L	A_{ac}	s	s_{abs}
flattop	40 ns	$0.104\Phi_0$	0.8933	0.9567
$\sin^2$	300 ns	$0.045\Phi_0$	0.9710	0.9904
$\sin^2$	400 ns	$0.037\Phi_0$	0.9906	0.9917
sech	300 ns	$0.072\Phi_0$	0.9112	0.9864
sech	400 ns	$0.063\Phi_0$	0.9424	0.9974

is still about 0.9862 (compared to 0.9906 of the result with frequency calibration by chirping) because of the small frequency shift (about -0.4 MHz) of the weak parametric modulation $A_{ac} = 0.037\Phi_0$ (see Appendix D).

IV. PHOTON EMISSION WITH DIFFERENT RABI ANGLES

To further verify the photon-emission process, we prepare our qubit in states with different Rabi angles θ , $\cos(\theta/2)|g\rangle + \sin(\theta/2)e^{i\phi}|e\rangle$. In Fig. 3, we produce single-rail photon and dual-rail time-bin photon with control pulse sequences shown in Figs. 3(a), 3(d), 3(g), and 3(i) respectively. For single-rail photon generation, we use the single-qubit rotation gate $R_{ge}(\theta, \phi)$ and the parametric modulation π_{e0g1} pulse, which facilitates the process $|e0\rangle \rightarrow |g1\rangle$ to eliminate the excitation of qubit and map the quantum information to the emission resonator and finally into the propagating mode. Notably, we can identify that the amplitude $\text{Re}(\langle \hat{a}_{out}(t) \rangle)$ changes sign along with Rabi angle per period of π because the off-diagonal elements of the density matrix of the qubit changes sign, which represents the process of photon emission maintaining phase coherence. Moreover, by comparing the unshaped and shaped results in Figs. 3(b) and 3(c) and Figs. 3(e) and 3(f), the shape of the photon remains symmetric and is unaffected by variations in the Rabi angles.

As for the dual-rail time-bin photon-generation protocol [as shown in Figs. 3(g) and 3(i)], we need the $|f\rangle$ state to be an ancillary state to temporarily store the quantum information from $|e\rangle$ state for the second time-bin photon emission. Therefore, we use a π_{ef} pulse to interchange the coefficients of the qubit $|e\rangle$ state and the $|f\rangle$ state, then use a π_{ge} pulse to map the $|g\rangle$ state to the $|e\rangle$ state and use the π_{e0g1} pulse to map the quantum information stored in the $|g\rangle$ state into the first time-bin photon. Next, we use another π_{ef} pulse to reload the quantum information of the $|e\rangle$ state from the $|f\rangle$ state, and finally use

the second π_{e0g1} pulse to map the quantum information stored in the $|e\rangle$ state into the second time-bin photon. In general, we generate entangled states like $\cos(\theta/2)|01\rangle + \sin(\theta/2)e^{i\phi}|10\rangle$, where $|01\rangle$ represents one photon in the early time-bin mode, $|10\rangle$ represents one photon in the latter time-bin mode. In the subspace of each mode, the quantum state does not have any coherence. Thus, we neither observe nonzero quadrature amplitudes of the time-bin photon nor show the amplitude results here. Results of power measurements demonstrate the quantum state mapping between the qubit and time-bin photon although the phase information is omitted. We notice that the second time-bin photon has less power than the first one because of the limited energy-relaxation time of the $|f\rangle$ state. Moreover, the shaped photon remains symmetric at both the first time-bin mode and the second and is still unaffected by variations in the Rabi angles.

V. QUANTUM STATE AND PROCESS TOMOGRAPHY OF PROPAGATING MODES

We then perform quantum state tomography [48,49] to characterize the state fidelity of quantum state of the most symmetrical shaped photon ($\sin^2$ 400 ns pulse) of our protocol. We use the matched filter of the same shape of $(|0\rangle + |1\rangle)/\sqrt{2}$ state quadratures mentioned above to maximize the detection efficiency. Then we generate the “ON” histogram containing both signal and noise ($\hat{S}_{ON} = \hat{a} + \hat{h}^\dagger$) from the amplification chain and the “OFF” histogram only contains noise ($\hat{S}_{OFF} = \hat{h}^\dagger$) from the amplification chain from 3×10^7 repeats of experiments. Next, we estimate the gain from the amplification chain using the relation $\langle \hat{a}^\dagger \hat{a} \rangle = |\langle \hat{a} \rangle|^2$ for $1/\sqrt{2}(|0\rangle + |1\rangle)$ state. Finally, we extract the signal moments $\langle (\hat{a}^\dagger)^n \hat{a}^m \rangle$ ($n, m \in \mathbb{N}$, truncated to $m + n \leq 4$, which is enough for single-photon characterization), which contains the information of the propagating mode signal quantum state from the histograms (see Appendix E). Then, we derive the Wigner function from signal moments and use maximum-likelihood estimation (MLE) methods [50] to determine the density matrix of the signal mode. The typical results of shaped photon states $(|0\rangle + |1\rangle)/\sqrt{2}$ and $|1\rangle$ with frequency calibration by chirping are shown in Fig. 4. We then use the definition of fidelity of mixed quantum states [51] to calculate the fidelity of $(|0\rangle + |1\rangle)/\sqrt{2}$ and $|1\rangle$ state, the fidelity is defined as

$$F(\rho, \sigma) = \left(\text{Tr} \sqrt{\sqrt{\rho} \sigma \sqrt{\rho}} \right)^2, \quad (4)$$

where σ is the measured density matrix and ρ is the ideal density matrix. As a comparison, we also perform quantum state tomography of the shaped photon without frequency calibration by chirping. The results along

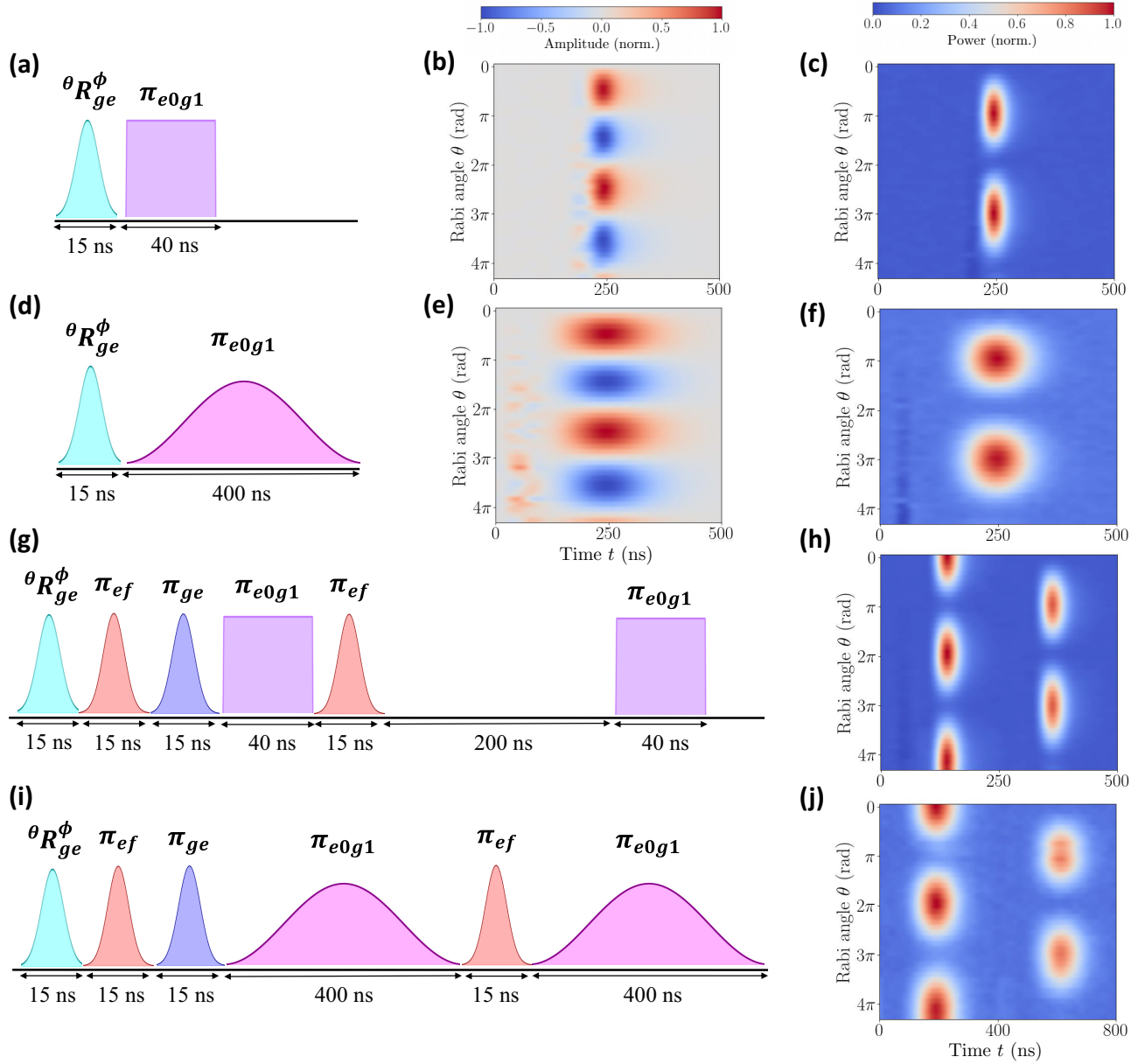


FIG. 3. Fock state photon and time-bin photon of different Rabi angles. The data is measured by heterodyne detection and averaged for 3×10^6 times. (a) Control sequence to generate unshaped Fock state photon. (b),(c) Amplitude (Re $\langle \hat{a}_{out}(t) \rangle$) and power ($\langle \hat{a}_{out}^\dagger(t) \hat{a}_{out}(t) \rangle$) of different Rabi angle (from 0 to 4π , all the following are the same) unshaped Fock state photon. (d) Control sequence to generate shaped Fock state photon. (e),(f) Amplitude (Re $\langle \hat{a}_{out}(t) \rangle$) and power ($\langle \hat{a}_{out}^\dagger(t) \hat{a}_{out}(t) \rangle$) of different Rabi angle-shaped Fock state photon. (g) Control sequence to generate unshaped time-bin photon. (h) Power ($\langle \hat{a}_{out}^\dagger(t) \hat{a}_{out}(t) \rangle$) of different Rabi angle-unshaped time-bin photon. (i) Control sequence to generate shaped time-bin photon. (j) Power ($\langle \hat{a}_{out}^\dagger(t) \hat{a}_{out}(t) \rangle$) of different Rabi angle-shaped time-bin photon.

with fidelity of the shaped photon with frequency calibration by chirping are all shown in Table II. The fidelity of $1/\sqrt{2}(|0\rangle + |1\rangle)$ state is $94.51 \pm 0.27\%$ and $94.24 \pm 0.06\%$, respectively. The fidelity of the $|1\rangle$ state is $86.24 \pm 0.29\%$ and $85.64 \pm 0.17\%$, respectively. The fidelity of the $|1\rangle$ state represents the qubit-photon energy exchange efficiency, in other words, the emission efficiency of our device and protocol. We observe that the fidelity of the

shaped photon remains largely unaffected in the absence of frequency calibration. This can be attributed to the slight photon phase accumulation caused by the small amplitude of the parametric modulation, aligning with the photon time-dependent phase results illustrated in Figs. 2(d) and 2(e). The primary cause of infidelity is the limited internal quality factor of the emission resonator and the limited relaxation time of the qubit, which mainly results from the

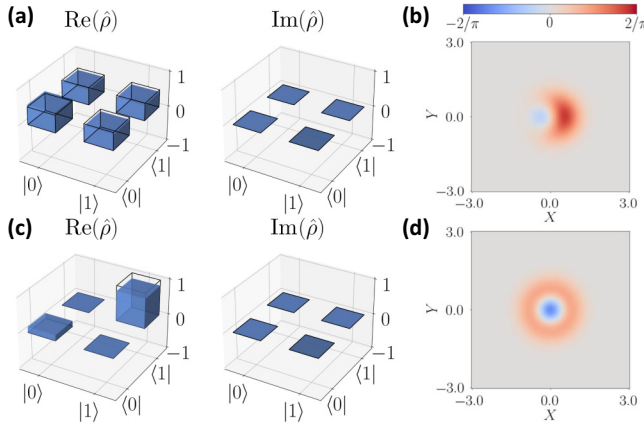


FIG. 4. Quantum state tomography of the typical shaped itinerant photon state with frequency calibration by chirping. The measured data is averaged for 3×10^7 times. (a) The density matrix of the generated $(|0\rangle + |1\rangle)/\sqrt{2}$ state. (b) Wigner function of the generated $(|0\rangle + |1\rangle)/\sqrt{2}$ state. (c) The density matrix of generated $|1\rangle$ state. (d) Wigner function of generated $|1\rangle$ state.

Purcell effect induced by the emission resonator. It can be mitigated by Purcell-filter design [52–56] and fabrication improvement [57,58].

Then by measuring the first-order $\langle \hat{a} \rangle$, second-order $\langle \hat{a}^\dagger \hat{a} \rangle$, and fourth-order $\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle$ moments of the photon field of different Rabi angles θ , as shown in Fig. 5(a) [we only show $\text{Re}(\langle \hat{a} \rangle)$ because $\text{Im}(\langle \hat{a} \rangle) \approx 0$], we confirm our shaped photon emission is in the single-photon subspace. We obtain that the second-order correlation function $g^{(2)}(0) = \langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle / (\langle \hat{a}^\dagger \hat{a} \rangle)^2$ is -0.060 ± 0.035 and -0.035 ± 0.007 for $1/\sqrt{2}(|0\rangle + |1\rangle)$ and $|1\rangle$ state, respectively, which is a significant signal of photon antibunching and sub-Poissonian photon statistics [59]. Our results of the fourth-order moment $\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle$ are all near zero but slightly negative as in Ref. [60].

Finally, we prepare six mutually unbiased states to perform quantum process tomography (QPT) [61,62] verifying the quality of quantum state transfer of our device and protocol. For each result shown in Figs. 5(b) and 5(c) we repeat for 3×10^8 times and derive the χ matrix of quantum state-transfer process, we define χ matrix as follows:

$$\mathcal{E}(\rho) = \sum_{m,n} \chi_{mn} A_m \rho A_n^\dagger, \quad (5)$$

TABLE II. Fidelity of typical states of shaped photons measured by photon quantum state tomography.

States	$(0\rangle + 1\rangle)/\sqrt{2}$	$ 1\rangle$
With freq. calibration	$94.51 \pm 0.27\%$	$86.24 \pm 0.29\%$
Without freq. calibration	$94.24 \pm 0.06\%$	$85.64 \pm 0.17\%$

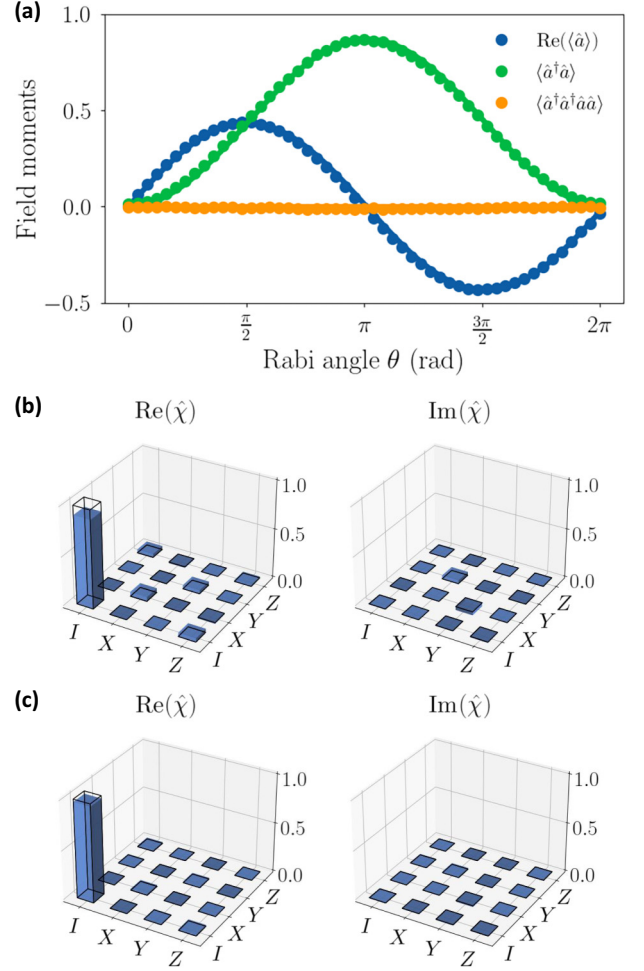


FIG. 5. Typical moments of shaped photon states and quantum process tomography of quantum state transfer. (a) The first-order, second-order, and fourth-order moments of different Rabi angles. The dot is the experimental data averaging for 3×10^8 repeats and the solid line is the theoretical prediction considering the limited emission efficiency. (b) χ matrix of quantum state transfer using shaped photon derived by quantum process tomography. (c) χ matrix of quantum state transfer using unshaped photon derived by quantum process tomography.

where $\mathcal{E}$ represents a quantum operation, A_m is the two-dimensional complete operator base set $\{I, \sigma_x, \sigma_y, \sigma_z\}$, χ_{mn} is the matrix element of the χ matrix. We recall the definition of the quantum process tomography fidelity by

$$F(\chi_{\text{ideal}}, \chi) = \left(\text{Tr} \sqrt{\sqrt{\chi_{\text{ideal}}} \chi \sqrt{\chi_{\text{ideal}}}} \right)^2, \quad (6)$$

where χ is the measured process matrix, χ_{ideal} is the ideal process matrix. The fidelity of quantum state transfer based on shaped photon is 90.32% and unshaped photon is 97.20% as a comparison. The QPT fidelity of the unshaped photon is higher than that of the shaped photon. Since unshaped photons are produced by a 40-ns flattop pulse

and shaped photons are generated by a 400-ns $\sin^2$ pulse, the qubit experiences more significant decay during the generation of shaped photons. This results further illustrate that the limited energy relaxation time of the qubit is the main limitation of the emitted photon fidelity.

VI. PROTOCOLS FOR QUANTUM STATE TRANSFER AND REMOTE ENTANGLEMENT GENERATION

To fully realize a point-to-point quantum network, we design a receiving protocol matched with the single-rail and dual-rail photon-emission protocol (as shown in Fig. 6). For single-rail encoding quantum state transfer, we prepare a state $\alpha |g\rangle + \beta |e\rangle$ then use π_{e0g1} pulses to pitch and catch [Fig. 6(a)] the shaped photon. There are two choices of single-rail remote entanglement generation protocol [Figs. 6(b) and 6(c)]: the first one generates qubit-photon entanglement with the help of the $|f\rangle$ state but does not require the $\pi/2_{e0g1}$ pulse, which brings about the process $|e0\rangle \rightarrow 1/\sqrt{2}(|e0\rangle + |g1\rangle)$ and can use the π_{e0g1} pulse calibrated in quantum state-transfer process without revising the experiment much; the second one does not require the $|f\rangle$ state and instead uses the $\pi/2_{e0g1}$ pulse to generate qubit-photon entanglement to perform “half-pitch and full-catch” process in this case we must calibrate the $\pi/2_{e0g1}$ pulse at the emission node and recalibrate the π_{e0g1} pulse at the receiving node. In contrast to Refs. [26,28,30,31], our protocol can avoid the use of the $|f\rangle$ state. This is essential for us to obtain a higher transfer efficiency because the energy relaxation time of the $|f\rangle$ state T_{1ef} is lower than that of the $|e\rangle$ state T_{1ge} .

For the dual-rail time-bin photon, we must utilize the $|f\rangle$ state as an ancillary state in both emission node and receiving node. For time-bin encoding quantum state transfer, we generate the time-bin photon at emission node and then catch them one by one [Fig. 6(d)]. For time-bin encoding remote entanglement generation, we first generate entanglement between the qubit and two time-bin modes then catch the time-bin mode one by one at the receiving node [Fig. 6(e)]. The time-bin photon is robust against photon-transfer loss [29] (with the cost of repetition rate) and phase reference error between sender and receiver [38]. Therefore, it is particularly useful for schemes using commercial circulators to ensure the communication channel unidirectionality and avoid the backaction of the reflected photon, which also causes unavoidable photon-transfer loss.

Using the quantum state tomography data in Sec. V, the transfer efficiency and the fidelity of the quantum state-transfer process [12] can also be estimated assuming that the receiver setup has the same specifications. Because the $|1\rangle$ state fidelity is $F_{|1\rangle} = 86.24 \pm 0.29\%$, which also represents the energy-emitting efficiency. Considering the transfer loss (including PCB, SMP adapters, circulator, low-loss waveguide, etc., 6% in total) [30,31] and the

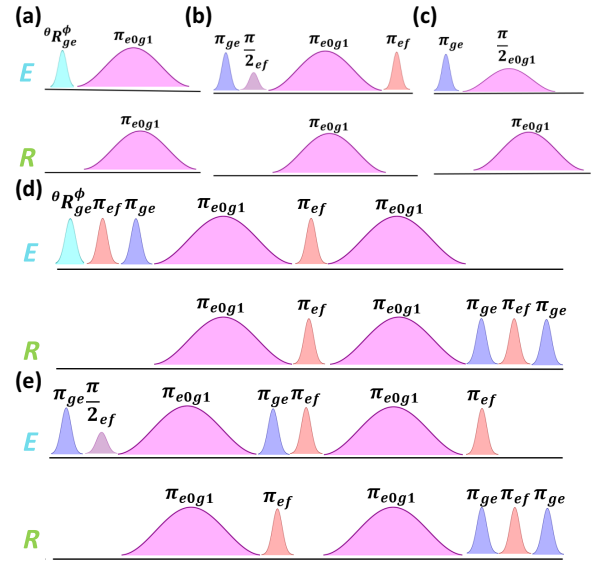


FIG. 6. Control pulse-sequence protocols for emission nodes and receiving nodes of quantum state transfer and remote entanglement generation based on Fock-state encoding and time-bin encoding. *E* represents the emission node, *R* represents the receiving node. (a) Control sequence of Fock-state-encoded quantum state transfer. (b),(c) Control sequence of Fock-state-encoded remote entanglement generation with and without $\pi/2_{e0g1}$ pulse. (d) Control sequence of time-bin-encoded quantum state transfer. (e) Control sequence of time-bin-encoded remote entanglement generation.

energy absorption inefficiency (estimated to be $F_{|1\rangle}$), we can get the transfer efficiency $\eta_t = 0.86^2 \times (1 - 0.06) \approx 0.70$ provided perfect frequency matching between the two buffer resonators. Furthermore, the fidelity of the quantum state-transfer process can be estimated to be $F_\chi = (1 + \sqrt{\eta_t})^2/4 \approx 0.84$. These results are at the same level as Refs. [28,30,31].

VII. DISCUSSION AND CONCLUSION

To summarize, our study presents an on-demand shaped photon-generation method utilizing a parametrically modulated qubit, achieving a high quantum process tomography fidelity of 90.32% for single-rail-shaped photon emissions, all without the need of additional tunable couplers. We implement slight parametric modulation pulses and use the first-order sideband coupling between the qubit and emission resonator inducing a linear relationship between parametric modulation amplitude and effective coupling covering a large tunable range up to 20 MHz. This makes the photon shaping and frequency-calibration process experimentally friendly. Although the parametric modulation also results in a spurious frequency shift of the qubit, for the optimal-shaped photon, the phase accumulation of photon caused by the shift is almost negligible and we can further calibrate the dynamical frequency shift

by chirping. Furthermore, our method is robust to crosstalk [41], enabling simultaneous execution of single qubit operations and the emission and reception of shaped photons across multiple channels on a multiqubit chip. Therefore, our approach avoids the necessity for high-power control pulses [26,28,29], enables easy amplitude and phase symmetrical photon generation and holds promise for scalable applications.

In our case, the limitation of shaped photon emission speed is the decay rate of the emission resonator. To speed up the emission process, we need to enhance the decay rate $\kappa_E/2\pi$ of our emission resonator up to 10–20 MHz. To protect the qubit from Purcell decay, Purcell filters [52–56] need to be added to the design. By enhancing the qubit energy relaxation time and speeding up the emission process, we can reach a higher emission efficiency and quantum process tomography fidelity of quantum state transfer.

For future improvements in experimental techniques, we can use absolute power calibration methods like measurement-induced dephasing [63] and ac Stark shift induced by emission resonator to get a more accurate gain of the amplification chain and absolute value of experimental data. For pulse shaping, we should further consider the transfer function of our flux line and calibrate the distortion of parametric modulation pulses and use numerical methods to optimize them.

Further using the parametric modulation sideband coupling between a higher state of qubit and emission resonator (the first-order sideband coupling between $|f0\rangle$ and $|e1\rangle$), we can generate complex 1D time-bin photon-entangled states [35]. With the help of the emission resonator we can also implement quantum nondemolition detection of itinerant photon [63,64], parity detection [65], universal quantum gate set for itinerant photons [60], and a photon-scattering gate like the Duan-Kimble gate [66] for the remote controlled-phase gate between quantum nodes [67].

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D.-N.Z., Y.-X.Z. and Z.-C.X. supervised the project. X.L. conceived the idea, designed the sample, performed the experiment, processed the data and wrote this manuscript. X.L. and S.-Y.L. built up the experiment measurement and control system with RFSoc and the data-stream processing of GPU. S.-L.Z. fabricated the sample. Z.-Y.M. and X.-H.S. fabricated the JPA. D.-N.Z. and Y.-X.Z. revised the manuscript. All authors contributed to discussions and production of the manuscript.

The authors declare no competing interests.

DATA AVAILABILITY

The data produced in this work is available from the corresponding authors upon reasonable request.

APPENDIX A: SAMPLE FABRICATIONS AND SAMPLE PARAMETERS

The device is fabricated using micronano fabrication technology, the main steps are as follows. (1) Growth and patterning of aluminum film. A 100-nm aluminum thin film is uniformly grown on a *c*-plane sapphire substrate by electron-beam evaporation (Plassys-MBE550s). Shunted capacitors, resonators, and transmission lines are patterned by laser direct writing (DWL66+) and etched by wet etching (ZX238). (2) Preparation of Josephson junctions. Josephson junctions are formed by crossing two perpendicular lines using double-angle electron-beam evaporation of aluminum through a Dolanbridge prepared by spin-coating double-layer photoresist and electron-beam exposure (Raith EBPG5200). (3) Preparation of an airbridge. The use of airbridges in devices ensures good ground-plane connectivity and reduces slotline modes [68]. 300-nm airbridges are prepared by laser direct writing (DWL66+) and wet etching (Aluminum Etchant Type D). (4) Slicing and packaging. The device is cut into $6 \times 6 \text{ mm}^2$ (DISCO-DAD323), wire bonded (Westbond-7476d) to a silver-plated printed-circuit board (PCB) and then packed in an aluminum packaging box. The sample optical microscope diagram is shown in Fig. 7.

For sample design, we engineer a large coupling (about 300 MHz) between qubit and emission resonator to have a large tunable effective coupling range where effective coupling is linear to parametric modulation amplitude. The emission resonator decay rate is designed to be large (about 5.2 MHz) to speed up the photon-emission process. The measured sample parameters are shown in Table III.

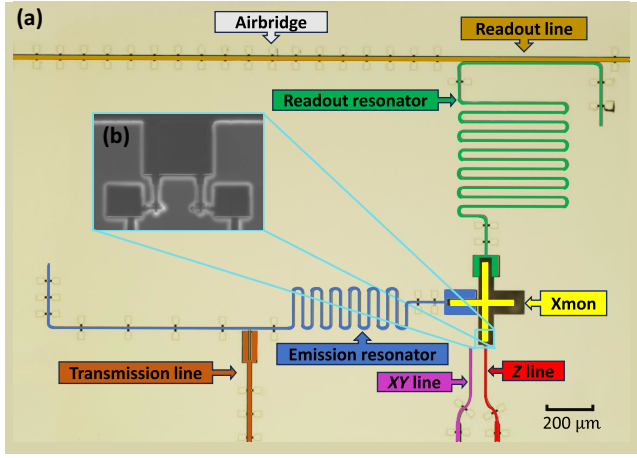


FIG. 7. Sample optical microscope diagram with false colors. (a) The main parts of the sample layout where there are one Xmon capacitively coupled to readout resonator and emission resonator. An individual XY line and Z line is to drive the qubit and change the flux of the dc-SQUID loop. We fabricated an airbridge at each CPW resonator and transmission line to prevent the propagation of parasitic slotline modes [68]. (b) The SEM image of the dc-SQUID loop, which is connected to the ground.

APPENDIX B: EXPERIMENTAL SETUP

The experimental setup in the dilution refrigerator and at room temperature is shown in Fig. 8. We mount our sample inside a magnetic shield in the mixing-chamber stage of a Bluefors dilution refrigerator. We use eccosorb infrared filters [69] in every rf microwave input coaxial line to mitigate quasiparticle excitation, 7.5-GHz low-pass filters to mitigate higher harmonics and proper attenuators to mitigate the Johnson-Nyquist noise from room temperature. The XY line and Z line of the qubit are separated and have the same attenuator and filter arrangement. The Z line is combined with a dc line, which has a RLC filter

TABLE III. Summary of sample parameters.

Parameters	
Qubit max frequency $\omega_{q,p}/2\pi$	5.997 GHz
Qubit operating frequency $\omega_q/2\pi$	5.947 GHz
Qubit anharmonicity $\alpha/2\pi$	-228 MHz
Qubit energy relaxation time for $ e\rangle$ T_{1ge}	5.539 μ s
Qubit energy relaxation time for $ f\rangle$ T_{1ef}	2.829 μ s
Qubit decoherence time for $ e\rangle$ T_{2ge}^*	2.234 μ s
Qubit decoherence time for $ f\rangle$ T_{2ef}^*	1.068 μ s
Emission resonator frequency $\omega_E/2\pi$	7.139 GHz
Emission resonator coupling decay rate $\kappa_{E,c}/2\pi$	5.2 MHz
Emission resonator internal decay rate $\kappa_{E,i}/2\pi$	0.27 MHz
Emission resonator-qubit coupling $g_{qE}/2\pi$	299.4 MHz
Readout resonator frequency $\omega_R/2\pi$	4.177 GHz
Readout resonator decay rate $\kappa_R/2\pi$	0.4 MHz
Readout resonator-qubit coupling $g_{qR}/2\pi$	48.2 MHz

and provided stable dc flux bias to qubit by a bias tee. We have two read-in-and-out loops, one is for qubit dispersive readout, the other is for the characterization of emission resonator and measurement of the photon field. Thus, we can measure the S_{21} of the readout resonator and the S_{11} of the emission resonator. A flux-driven impedance-engineered JPA [45,46], which has a flux-pump line and dc bias line combined by a bias tee is used to enhance the measurement quantum efficiency for photon quadrature measurement and tomography. An extra circulator is put between the sample and JPA to mitigate the measurement backaction of JPA. In the two read-out lines, there are both high-electron-mobility transistor (HEMT) and a room-temperature amplifier aimed at amplifying the signal to proper voltage for the analog-digital converter (ADC) to sample.

For electronics at room temperature, we build up a control and measurement system with radio-frequency system-on-a-chip (RFSoc) [70–72] to directly generate and sample radio-frequency signal for qubit control and readout. We use a radio-frequency digital-analog converter (rf DAC) based on RFSoc FPGA (Xilinx Zynq UltraScale+ RFSoc XCZU47DR) whose sampling rate is 8.0 GSa/s to directly generate read-in and XY control pulses without modulation using the second Nyquist zone with

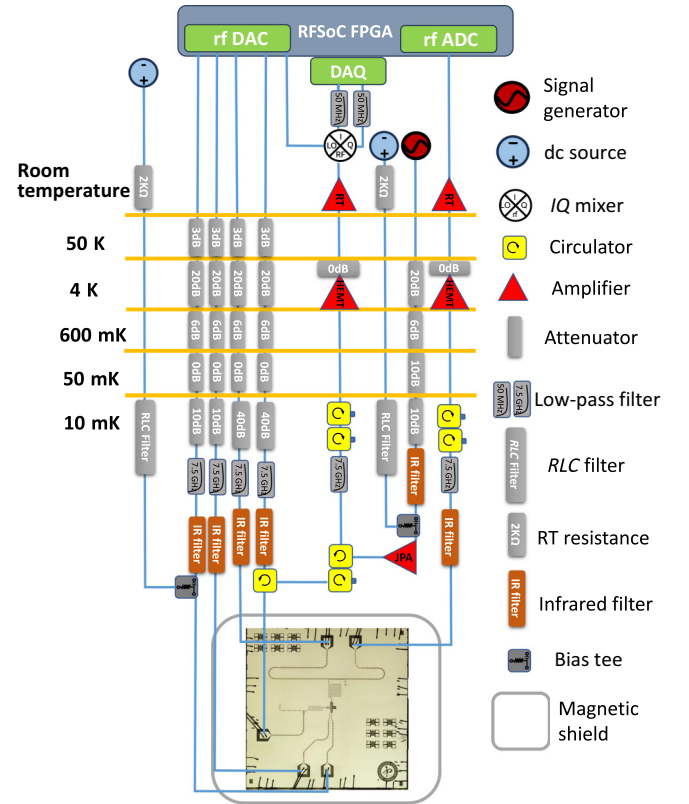


FIG. 8. Experimental setup in the dilution refrigerator and at room temperature.

the help of extra band-pass filters. The generated frequency range is 4.0–8.0 GHz and is suitable for qubit measurement and control. The mix mode [71,73] is used to have a smooth power-frequency dependence in higher Nyquist zones. The parametric modulation pulse is generated by the same rf DAC using the first Nyquist zone with the help of a 4.0-GHz low-pass filter. For qubit-readout signal sampling, we use a radio-frequency analog-digital converter (rf ADC) with 4.0-GSa/s sampling rate based on another RFSoc FPGA (Xilinx Zynq UltraScale+ RFSoc XCZU47DR) to directly sample signals at higher Nyquist zones, which are aliased within the first Nyquist zone of the rf ADC. To avoid signal aliasing from other Nyquist zones and enhance SNR we also add a band-pass filter in front of the rf ADC.

In order to meet the demands of massive data processing for low SNR experiments like microwave photon-field heterodyne measurement and tomography, we build a data-stream processing system with a NVIDIA GPU and a high-speed data acquisition (DAQ) card with 1.0-GSa/s sampling rate. We are able to process the experimental data in real time, eliminating the need for storage, which in turn reduces the time cost. In our experiment, the trigger period is set to 12 μ s, and the experiment repeated 3×10^8 times only cost 1 h in total. The data processing and transfer time between the DAQ card and the GPU is not the limitation. We can further improve the duty cycle and reduce the experimental time cost by reducing the trigger period because photon emission is just qubit reset. Furthermore, we utilize the output of a single rf-DAC channel, augmented with a room-temperature amplifier, to serve as the local oscillators (LOs) for the heterodyne measurements, guaranteeing phase locking for each measurement. All the electronic devices at room temperature are synchronized by a rubidium clock (Stanford Research Systems FS725) and triggered by the same clock and trigger distribution module.

APPENDIX C: DETAILED CALCULATIONS ABOUT PARAMETRIC MODULATION

It is convenient and reasonable to omit the readout resonator in our flux-modulating process, then consider a two-level system approximation, we will derive a Jaynes-Cummings-like Hamiltonian

$$\hat{H}/\hbar = \frac{\omega_q(\Phi(t))}{2} \hat{\sigma}_z + \omega_E \hat{a}^\dagger \hat{a} + g_{qE} (\hat{a}^\dagger \hat{\sigma}_- + \hat{a} \hat{\sigma}_+), \quad (C1)$$

where $\hat{a}(\hat{a}^\dagger)$ is the annihilation (creation) operator of the emission resonator mode, $\hat{\sigma}_-(\hat{\sigma}_+)$ is the annihilation (creation) operator of qubit $\{|g\rangle, |e\rangle\}$ subspace. Because of the nonlinearity of frequency-flux relation of transmon, when we add sinusoidal parametric modulation $\Phi(t) = \Phi_{dc} + \Phi_{ac} \cos(\omega_m t + \theta_m)$ there are high-order harmonics terms

$A_m^a \cos[a(\omega_m t + \theta_m)]$ ($a \in \mathbb{N}$) and dc offset Δ_{dc} in qubit frequency. We call the a th-order harmonics term of qubit time-dependent frequency variation the a th-order longitudinal field modulation (LFM). When the qubit is not at the sweet spot, the least order of longitudinal field modulation (LFM) is $a = 1$, and we ignore the higher-order terms, then we have

$$\begin{aligned} \omega_q(\Phi(t)) &= \bar{\omega}_q + \sum_{a=1}^{\infty} A_m^a \cos[a(\omega_m t + \theta_m)] \\ &\approx \bar{\omega}_q + A_m^1 \cos(\omega_m t + \theta_m), \end{aligned} \quad (C2)$$

where

$$\bar{\omega}_q = \omega_q(\Phi_{dc}) + \Delta_{dc}(\Phi_{dc}, \Phi_{ac}), \quad (C3)$$

$$A_m^1(\Phi_{dc}, \Phi_{ac}) = \Phi_{ac} \left. \frac{d\omega_q}{d\Phi} \right|_{\Phi_{dc}}, \quad (C4)$$

$$\Delta_{dc}(\Phi_{dc}, \Phi_{ac}) = \frac{\Phi_{ac}^2}{4} \left. \frac{d^2\omega_q}{d\Phi^2} \right|_{\Phi_{dc}}. \quad (C5)$$

Then we know that the first order of LFM is proportional to the parametric modulation amplitude $A_m^1 \propto \Phi_{ac}$ and there is a quadratic relation between Δ_{dc} and Φ_{ac} . Both LFM amplitude A_m^1 and dc offset Δ_{dc} are dependent on the working point Φ_{dc} and parametric modulation amplitude Φ_{ac} . When the Φ_{dc} is larger, A_m^1 is more sensitive to Φ_{ac} while Δ_{dc} is less sensitive to Φ_{ac} according to the frequency-flux relation of transmon with symmetric Josephson-junction SQUID [74]. In our experiment, we choose $\Phi_{dc} = 0.04$ to balance the trade-off of LFM frequency shift, tunable coupling range, and qubit coherence.

We use a time-dependent unitary transformation

$$\begin{aligned} \hat{U}(t) &= \exp \left\{ -i \left(\frac{\bar{\omega}_q}{2} \hat{\sigma}_z + \frac{A_m^1}{2\omega_m} \cos(\omega_m t + \theta_m) \hat{\sigma}_z + \omega_E \hat{a}^\dagger \hat{a} \right) \right\}, \end{aligned} \quad (C6)$$

to show sideband couplings in a rotating frame, in the rotating frame the Hamiltonian is

$$\begin{aligned} \hat{H}'/\hbar &= \hat{U} \hat{H} / \hbar \hat{U}^\dagger - i \hat{U} \partial_t \hat{U}^\dagger \\ &= g_{qE} e^{-i\Delta_{qE} t} \hat{a}^\dagger \hat{\sigma}_- \sum_{n=0}^{\infty} J_n \left(\frac{A_m^1}{\omega_m} \right) e^{i(n\omega_m t + \theta_m + \frac{\pi}{2})} + \text{h.c.}, \end{aligned} \quad (C7)$$

where $\Delta_{qE} = \bar{\omega}_q - \omega_E$ and $J_n(x)$ is the n th-order Bessel function. When A_m^1 is small (Φ_{ac} is small) and $\Delta_{qE} \approx \omega_m$,

only the zero- and first-order sidebands have a significant impact.

The zero sideband coupling with the emission resonator is a dispersive coupling, which induces Lamb shift to both qubit frequency $\tilde{\omega}_q = \omega_q + \Delta_{\text{Lamb}}$ and emission resonator $\tilde{\omega}_E = \omega_E - \Delta_{\text{Lamb}}$,

$$\begin{aligned}\Delta_{\text{Lamb}} &= \frac{\left(g_{qE} J_0\left(\frac{A_m^1}{\omega_m}\right)\right)^2}{\Delta_{qE}} \\ &= \frac{\left(g_{qE} \left(1 - \frac{(A_m^1)^2}{4\omega_m^2}\right)\right)^2}{\Delta_{qE}} + O\left(\left(\frac{A_m^1}{\omega_m^2}\right)^4\right) \\ &= \frac{g_{qE}^2}{\Delta_{qE}} \left(1 - \frac{(A_m^1)^2}{2\omega_m^2}\right) + O\left(\left(\frac{A_m^1}{\omega_m^2}\right)^4\right),\end{aligned}\quad (\text{C8})$$

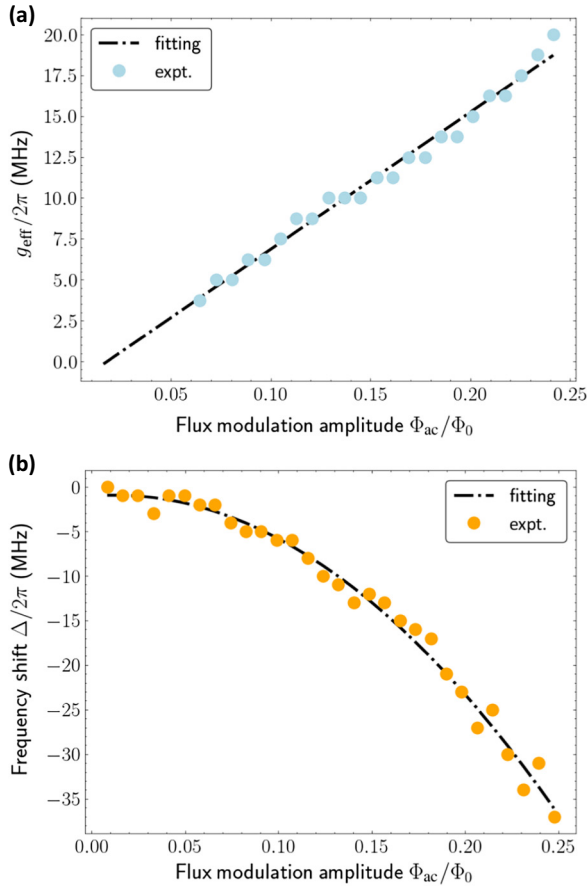


FIG. 9. Calibrations of the first-order sideband parametric modulation. (a) The relation between the first-order sideband effective coupling and parametric modulation amplitude. The dots are experimental results and the dotted line is the linear fitting results. (b) The relation between the first-order sideband parametric modulation frequency shift and parametric modulation amplitude. The dots are experimental results and the dotted line is the quadratic fitting results.

where $J_0(x)$ represents the zeroth-order Bessel function [when $x \rightarrow 0$, $J_0(x) \approx 1 - x^2/4$]. Then, the zero-order sideband-induced Lamb shift variation is

$$\begin{aligned}\Delta'_{\text{Lamb}} &= 2 \left(\Delta_{\text{Lamb}} - \frac{g_{qE}^2}{\Delta_{qE}} \right) \\ &= -\frac{g_{qE}^2}{\Delta_{qE}} \frac{(A_m^1)^2}{\omega_m^2},\end{aligned}\quad (\text{C9})$$

which represents the zero-order sideband-induced dressed frequency variation between the qubit and emission resonator and has a quadratic relation of Φ_{ac} . In our scenario, we estimate the Lamb shift to be 3.0 MHz at $\Phi_{\text{ac}} = 0.25\Phi_0$, which is an order of magnitude smaller than Δ_{DC} (about -40 MHz, see Appendix D), though, still produces an observable effect. However, it becomes nearly negligible at $\Phi_{\text{ac}} = 0.037$. It is worth noting that when the frequency of emission resonator is higher than the qubit, the sign of dc offset Δ_{dc} and Lamb shift Δ'_{Lamb} are the opposite and can compensate with each other. Generally, the dc offset induced by nonlinearity of frequency-flux relation of transmon and Lamb shift induced by the zero-order sideband coupling are both quadratic to the parametric modulation amplitude Φ_{ac} and can be calibrated together.

Considering the Lamb shift caused by the zero-order sideband, then when the first-order sideband of qubit and emission resonator are on resonance $\tilde{\omega}_E = \tilde{\omega}_q + \Delta_{\text{dc}} + \omega_m$ the Hamiltonian can be simplified to

$$\begin{aligned}\hat{H}'/\hbar &= g_{qE} J_1\left(\frac{A_m^1}{\omega_m}\right) e^{i\theta_m} \hat{a} \hat{\sigma}_+ + \text{h.c.} \\ &\approx g_{qE} \frac{A_m^1}{2\omega_m} e^{i\theta_m} \hat{a} \hat{\sigma}_+ + \text{h.c.}\end{aligned}\quad (\text{C10})$$

Finally, we get the effective coupling, which is linear to the parametric modulation amplitude and has a phase dependence to the parametric modulation phase

$$\begin{aligned}g_{\text{eff}} &= g_{qE} \frac{A_m^1}{2\omega_m} e^{i\theta_m} \\ &= g_{qE} \frac{\Phi_{\text{ac}} \frac{d\omega_q}{d\Phi} \Big|_{\Phi_{\text{dc}}}}{2\omega_m} e^{i\theta_m} \\ &\propto \Phi_{\text{ac}} e^{i\theta_m},\end{aligned}\quad (\text{C11})$$

using Eq. (C4). Then we can use a time-dependent $\Phi_{\text{ac}}(t)$ to shape the photon temporal profile and time-dependent $\theta_m(t)$ to eliminate the frequency shift $\Delta(t) = \Delta_{\text{dc}}(t) + \Delta'_{\text{Lamb}}(t)$, where $\dot{\theta}_m(t) = -\Delta(t)$.

APPENDIX D: CALIBRATIONS OF THE FIRST-ORDER SIDEBAND PARAMETRIC MODULATION

As Fig. 9(a) shows, we calibrate the relation between parametric modulation amplitude Φ_{ac} and effective first-order sideband coupling $g_{eff}/2\pi$ by the flattop pulse (square pulse convoluted with Gaussian function with $2\sigma = 4$ ns to have a smooth rising edge) shape with varying amplitudes and lengths. The effective coupling $g_{eff}/2\pi$ is greatly linear to parametric modulation amplitude Φ_{ac} . The max effective coupling $g_{eff,max}/2\pi$ is about 20 MHz at $0.25\Phi_0$ parametric modulation amplitude where Φ_0 is the magnetic flux quantum. The result shows that the first-order sideband coupling of parametric modulation in our design has a wide tunable range and can support fast photon emission.

As Fig. 9(b) shows, we calibrate the parametric modulation frequency shift caused by the average frequency shift and Lamb shift by the flattop pulse with varying amplitudes and frequencies. The frequency shift $\Delta = \Delta_{dc} + \Delta'_{Lamb}$ is quadratic to the parametric modulation amplitude Φ_{ac} and the max frequency shift is about -37 MHz at $0.25\Phi_0$ parametric modulation amplitude. For the $\sin^2$ -type photon-shaping pulse used in our experiments, the amplitude is small ($A_{ac} = 0.037\Phi_0$) and the corresponding frequency shift is only about -0.4 MHz, which causes almost negligible photon-phase accumulation during the emission process.

APPENDIX E: PHOTON-STATE TOMOGRAPHY

We use the heterodyne-detection method [44] to measure the two quadratures of the amplified single-photon signal by a high-speed data-acquisition card. We use a template function $f(t)(\int |f(t)|^2 dt = 1)$ that is the same shape as $(|0\rangle + |1\rangle)/\sqrt{2}$ state quadratures as a matched filter to maximize the detection efficiency. Then we have a time-independent signal mode $\hat{a} = \int f(t) \langle \hat{a}_{out}(t) \rangle dt$. Considering the noise generated by phase-insensitive amplifier chain [75], the final quadrature field we measured is

$$\hat{S}_0 = \sqrt{G}\hat{a} + \sqrt{G-1}\hat{h}^\dagger, \quad (E1)$$

where $\hat{a}$ is an annihilation operator of the signal mode, the $\hat{h}^\dagger$ is the creation operator of noise mode. When $G \gg 1$ then we have $\sqrt{G} \approx \sqrt{G-1}$, then we redefine the measured mode

$$\hat{S} = \frac{\hat{S}_0}{\sqrt{G}} \approx \hat{a} + \hat{h}^\dagger. \quad (E2)$$

We can generate a two-dimensional (2D) histogram $D^{[\rho_a]}(S, S^*)$ to describe the quasiprobability distribution of measured quadratures. If we assume the $\hat{a}$ and $\hat{h}$ mode are

uncorrelated, then the moments of $\hat{S}$, $\hat{a}$, and $\hat{h}^\dagger$ are related by

$$\langle (\hat{S}^\dagger)^n S^m \rangle_{\rho_a} = \sum_{i,j=0}^{n,m} \binom{m}{j} \binom{n}{i} \langle (\hat{a}^\dagger)^i \hat{a}^j \rangle \langle \hat{h}^{n-i} (\hat{h}^\dagger)^{m-j} \rangle. \quad (E3)$$

We can get the $\hat{S}$ -mode moments from our 2D histogram $D^{[\rho_a]}(S, S^*)$

$$\langle (\hat{S}^\dagger)^n S^m \rangle_{\rho_a} = \int_S (S^*)^n S^m D^{[\rho_a]}(S, S^*) G^{-\frac{n+m}{2}}. \quad (E4)$$

To obtain the moments of the noise added by the amplification chain $\langle \hat{h}^n (\hat{h}^\dagger)^m \rangle$, we generate a 2D histogram of the background without photon emission $D^{[0]\langle 0|}(S, S^*)$ from which we extract the noise moments

$$\begin{aligned} \langle \hat{h}^n (\hat{h}^\dagger)^m \rangle &= \langle (\hat{S}^\dagger)^n S^m \rangle_{|0\rangle\langle 0|} \\ &= \int_S (S^*)^n S^m D^{[0]\langle 0|}(S, S^*) G^{-\frac{n+m}{2}}. \end{aligned} \quad (E5)$$

Then by inserting Eqs. (E5) to (E3), we get a linear equation of signal moments $\langle (\hat{a}^\dagger)^n \hat{a}^m \rangle$. We obtain the signal moments from the linear equation then use the formula [49,76]

$$W(\alpha) = \sum_{n,m} \int d^2\lambda \frac{\langle (\hat{a}^\dagger)^n \hat{a}^m \rangle (-\lambda^*)^m \lambda^n}{\pi^2 n! m!} e^{-\frac{1}{2}|\lambda|^2 + \alpha \lambda^* - \alpha^* \lambda} \quad (E6)$$

to derive the Wigner function directly. The gain of amplification chain G used above is estimated by the $\langle \hat{a}^\dagger \hat{a} \rangle = |\langle \hat{a} \rangle|$ relation of the $(|0\rangle + |1\rangle)/\sqrt{2}$ state. Then we use maximum-likelihood estimation (MLE) methods to get a physical density matrix ρ_a truncated in single-photon subspace. We maximize the log-likelihood function [49]

$$\mathcal{L}_{\log} = - \sum_{n,m} \frac{1}{\delta_{n,m}^2} |\langle (\hat{a}^\dagger)^n \hat{a}^m \rangle - \text{Tr}[\rho_a (\hat{a}^\dagger)^n \hat{a}^m]|^2, \quad (E7)$$

using the convex optimization method [77] (by CVXPY [78]) to guarantee the semidefinite, Hermitian, and trace-one properties of the density matrix, where $\delta_{n,m}$ is the standard deviation of signal moments $\langle (\hat{a}^\dagger)^n \hat{a}^m \rangle$.

By extracting the moments of noise mode $\langle \hat{h}^n (\hat{h}^\dagger)^m \rangle$, we get the average photon number of the thermal field of the amplification chain $n_{\text{noise}} = \langle \hat{h}^\dagger \hat{h} \rangle = 2.78$ and we then get the overall quantum efficiency, which includes noise added by the amplifiers, loss, and spurious reflections in

the whole amplification chain and detection inefficiency of our sampling and data-processing process [79]

$$\eta = \frac{1}{1 + n_{\text{noise}}} \approx 0.26. \quad (\text{E8})$$

We only perform single-mode photon-field tomography in this experiment. Multimode tomography for time-bin mode and photon-qubit joint tomography characterizing the entanglement between them is straightforward by expanding the methods above [49].

APPENDIX F: INITIAL STATE QUALITY FOR QUANTUM STATE TRANSFER

We use 15-ns Gaussian pulse with DRAG [80] to be the π and $\pi/2$ pulse for both $|g\rangle$ - $|e\rangle$ and $|e\rangle$ - $|f\rangle$ state to optimize the single-qubit gate fidelity and initial state-preparation fidelity. We characterize the single-qubit gate and initial state-preparation fidelity by interleaved randomized benchmarking (IRB) [81] and quantum state tomography (QST) [82]. The results are shown in Table IV.

APPENDIX G: PHOTON PHASE DEPENDENCE OF THE FIRST-ORDER SIDEBAND PARAMETRIC MODULATION

We test the photon phase dependence of the first-order sideband parametric modulation by prepare the $(|g\rangle + |e\rangle)/\sqrt{2}$ state qubit and then emit the photon with different initial phase of the parametric modulation pulse ($\theta_m = 0, \pi/2, \pi$), then measure the time-dependent quadratures and the Wigner function of the emitted photon, as shown

TABLE IV. Single-qubit gate fidelity by interleaved randomized benchmarking and initial state-preparation fidelity by quantum state tomography. I represents $|g\rangle$ state, X represents $|e\rangle$ state, $Y/2$ represents $(|g\rangle + |e\rangle)/\sqrt{2}$ state, $X/2$ represents $(|g\rangle + i|e\rangle)/\sqrt{2}$ state, $-Y/2$ represents $(|g\rangle - |e\rangle)/\sqrt{2}$ state and $-X/2$ represents $(|g\rangle - i|e\rangle)/\sqrt{2}$ state.

Single-qubit gates (states)	IRB	QST
I	99.81%	99.92%
X	99.75%	99.36%
$Y/2$	99.77%	99.89%
$X/2$	99.77%	99.90%
$-Y/2$	99.71%	99.53%
$-X/2$	99.74%	99.54%

in Fig. 10. The absolute value of quadrature $|\langle \hat{a}_{\text{out}}(t) \rangle|$ remains unchanged, while the real and imaginary value $\text{Re}(\langle \hat{a}_{\text{out}}(t) \rangle)$ and $\text{Im}(\langle \hat{a}_{\text{out}}(t) \rangle)$ exchanges absolute values and changes signs in Figs. 10(a) and 10(b), change signs in Figs. 10(a) and 10(c) representing a photon phase rotation with the phase of parametric modulation. In single-photon subspace, the Wigner function is sensitive to the relative phase of $|0\rangle$ and $|1\rangle$ state, which induces a counterclockwise rotation of Wigner function. The results in Figs. 10(b), 10(d), and 10(f) further show that the photon phase changes right along with the phase of parametric modulation. The photon phase dependence of the first-order sideband parametric modulation verifies that the effective coupling is a complex coupling $g_{\text{eff}}(t) = g_q E A_m^1(t)/(2\omega_m) e^{i\theta_m(t)}$, and we can use the phase

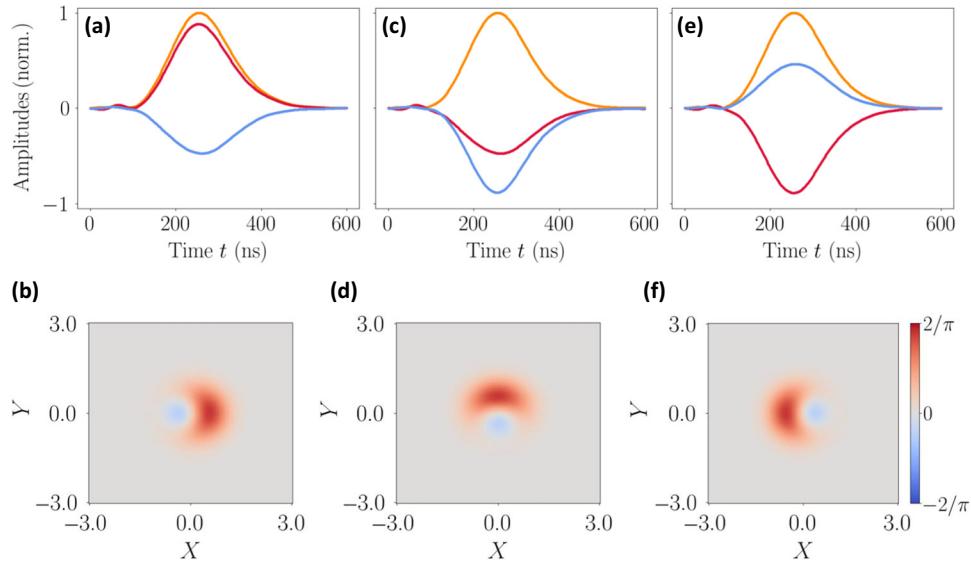


FIG. 10. The photon phase dependence of the first-order sideband parametric modulation pulse. (a),(c),(e) are the time-dependent quadratures results [orange line is $|\langle \hat{a}_{\text{out}}(t) \rangle|$, red line is $\text{Re}(\langle \hat{a}_{\text{out}}(t) \rangle)$, blue line is $\text{Im}(\langle \hat{a}_{\text{out}}(t) \rangle)$], and (b),(d),(f) are the Wigner-function results. (a),(b) The time-dependent quadratures and Wigner function of parametric modulation phase $\theta_m = 0$. (c),(d) The time-dependent quadratures and Wigner function of parametric modulation phase $\theta_m = \pi/2$. (e),(f) The time-dependent quadratures and Wigner function of parametric modulation phase $\theta_m = \pi$.

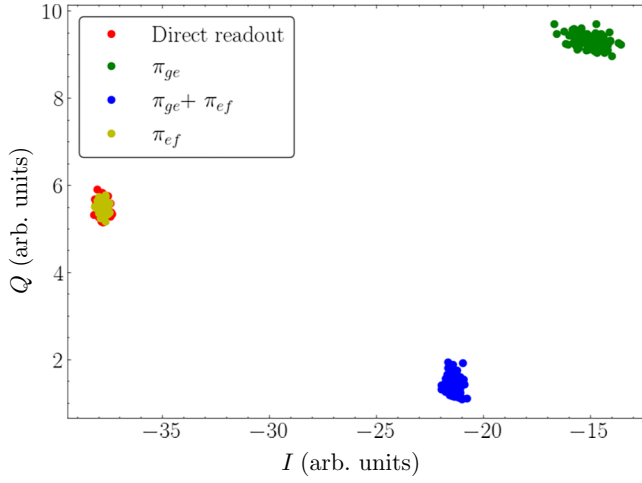


FIG. 11. The experimental test of thermal fluctuations using a high level $|f\rangle$ of qubit and four different control sequences. Each dot we average for 10^4 times, there are 100 dots for each control sequence.

of parametric modulation $\theta_m(t)$ to compensate the phase accumulation of emitted photon.

APPENDIX H: THERMAL EXCITATION TESTS

The thermal excitation results are shown in Fig. 11. We use the method from Ref. [83], which assumes the initial populations of states higher than $|e\rangle$ (e.g., $|f\rangle$) are zero. Assume that the eigenstate readout voltages are v_g, v_e, v_f for $|g\rangle, |e\rangle, |f\rangle$ states in the absence of thermal excitation, P_g, P_e is the initial population of $|g\rangle$ and $|e\rangle$ state. We use four different control pulse sequence to excite the qubit and then readout. The first sequence does not use any control pulse, the second sequence uses the π pulse of $|g\rangle - |e\rangle$ state, the third sequence uses the π pulse of $|g\rangle - |f\rangle$ state and $|e\rangle - |f\rangle$ state, the fourth sequence uses the π pulse of $|e\rangle - |f\rangle$ state. The expected values of readout voltages of the four cases are

$$v_1 = P_g v_g + P_e v_e, \quad (\text{H1})$$

$$v_2 = P_e v_g + P_g v_e, \quad (\text{H2})$$

$$v_3 = P_e v_g + P_g v_f, \quad (\text{H3})$$

$$v_4 = P_g v_g + P_f v_f, \quad (\text{H4})$$

where v_1, v_2 are the linear combinations of v_g, v_e , v_3, v_4 are the linear combinations of v_g, v_f . Then v_g can be expressed as the linear combinations of both v_1, v_2 and v_3, v_4 , we assume the linear coefficient as η and λ , then we get

$$v_g = \eta v_2 + (1 - \eta) v_1 = \lambda v_3 + (1 - \lambda) v_4. \quad (\text{H5})$$

By solving this equation, we can get

$$\eta = \frac{\text{Im}((v_3 - v_4)^* v_1 + v_4^* v_3)}{\text{Im}((v_2 - v_1)^* (v_3 - v_4))}, \quad (\text{H6})$$

$$\lambda = -\frac{\text{Im}((v_2 - v_1)^* v_4 + v_1^* v_2)}{\text{Im}((v_2 - v_1)^* (v_3 - v_4))}. \quad (\text{H7})$$

And then the thermal population P_e is

$$P_e = \frac{\eta}{2\eta - 1} = \frac{\lambda}{2\lambda - 1}. \quad (\text{H8})$$

We perform 10^6 times readout for each control sequence, from the experimental data we get the thermal population P_e is 0.23% and 0.06% by η and λ , respectively. Thus, we confirm that the upper bond of thermal population of our qubit is around 0.2% corresponding to an effective temperature of 50 mK. The results demonstrate that our device has negligible thermal excitation so that it is not necessary to initialize our qubit before the photon-emission process.

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