

Reconfigurable Terahertz Quarter-Wave Plate for Helicity Switching Based on Babinet Inversion of an Anisotropic Checkerboard Metasurface

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Dynamic helicity switching by utilizing metasurfaces is challenging because it requires deep modulation of polarization states. To realize such helicity switching, this paper proposes a dynamic metasurface functioning as a switchable quarter-wave plate, the fast axis of which can be dynamically rotated by 90° . The device is based on the critical transition of an anisotropic metallic checkerboard, which realizes the deep modulation and simultaneous design of the switchable states. After verifying the functionality of the ideally designed device in a simulation, we tune its structural parameters to realize practical experiments in the terahertz frequency range. By evaluating the fabricated sample with vanadium dioxide, the conductivity of which can be controlled by temperature, its dynamic helicity switching function is demonstrated.

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I. INTRODUCTION

Dynamic polarization control technology has been fundamental for flat-panel displays, polarization-sensitive spectroscopy, highly sensitive measurements with the lock-in detection technique, and data transmission. In the optical frequency domain, liquid-crystal and electro-optic-crystal modulators have been widely used to achieve dynamic polarization modulation. However, in lower frequency ranges such as the terahertz frequency range [1], which is potentially important for spectrally resolving giant and biomolecules, the wavelength is several orders of magnitude larger than that in the optical frequency range. Thus, the thickness of conventional dielectric devices at terahertz frequencies must become much larger than that at optical frequencies and it is difficult to reach ultrathin dynamic devices.

To overcome this difficulty, researchers have been developing artificially designed surfaces, called “metasurfaces” [2–6]. Most metasurfaces are partially made of metals, the highly conductive nature of which is applicable to subwavelength devices. Recently, the progress of

metasurfaces has motivated research into dynamic terahertz polarization control utilizing photoswitchable chiral metamolecules [7], gate-tunable polarization modulators with graphene [8], chiral microelectromechanical systems (MEMS) spirals [9,10], and a MEMS polarization converter with a tunable cross-conversion rate [11]. In particular, Zhao *et al.* realized a tunable wave plate with actuated microcantilever arrays to convert a linear polarization to transmitted circular ones [12], while a similar function of a MEMS metadvice for reflected waves was discussed in Ref. [13]. Phase change materials such as vanadium dioxide have also been applied to dynamic quarter-wave plates with variable working frequencies [14,15] and reconfigurable polarization modulators of which output is switchable between linear and circular polarization states [16,17]. Despite these remarkable achievements, designing switchable metasurfaces that have desired functionalities with deep modulation contrast is still challenging. The difficulty arises from the simultaneous design to realize multiple functionalities, which usually interfere with each other.

To resolve this issue, an extremely thin metallic checkerboard can be a key structure. The electromagnetic properties of metallic checkerboards critically depend on their

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connection states [18–22]. Between the connected and disconnected states, extraordinary frequency-independent characteristics have been revealed for the metallic checkerboards with resistive and random connections [23–26]. The checkerboard critical behavior is closely related to the checkerboard special property: the *local* electrical conductivity switching at the interconnection points results in *global* Babinet inversion, which is the generalization of the interchange between the metallic and vacant areas. We can interpret this situation to mean that local conductivity switching can induce a global *phase transition*, which exhibits critical changes in the electromagnetic scattering property. This transition is preferable to realizing not only the deep modulation but also simultaneous design of dynamic metasurfaces, because the original and switched states are related through the structural symmetry of Babinet inversion. This theory has been demonstrated to realize the capacitive-inductive reconfigurable filter [27] and extended to anisotropic checkerboards to realize a reconfigurable polarizer [28]. Even planar chirality switching has been theoretically proposed [29].

In this study, we propose a reconfigurable metasurface based on the anisotropic checkerboard transition to realize helicity switching, which requires the deep modulation and simultaneous design of the switchable states. As shown in Fig. 1, this metasurface converts incident linear polarization to right- or left-circular polarization, depending on the connection states. The device functions as a dynamic quarter-wave plate, the fast axis of which can be rotated by 90° . Compared to a previously studied dynamic polarizer based on an anisotropic checkerboard [28], of which only the amplitude manipulation function was investigated, the dynamic quarter-wave plate requires the simultaneous control of both the amplitude and phase for each polarization. We show that the anisotropic checkerboard can also function as a dynamic quarter-wave plate because of a general constraint between the transmission amplitude and phase, and we demonstrate its functionality in the terahertz frequency range. In this study, we assume a harmonic time dependence $\exp(j\omega t)$, where we use j for the imaginary unit.

II. PRINCIPLE

First, we design an ideal dynamic quarter-wave plate based on the anisotropic checkerboard to demonstrate our strategy for realizing fast axis rotation. Before going into detailed discussion, we explain the general restriction imposed on the complex transmission amplitudes, derived from energy conservation law. Consider a thin electric metasurface located at $z = 0$ in a vacuum. An electromagnetic wave having a complex electric field $\tilde{\mathbf{E}}_0 \exp(-jk_0 z)$ with $k_0 > 0$ enters the metasurface from $z < 0$ to $z > 0$. The transmitted and reflected components propagating along the z direction with the same polarization

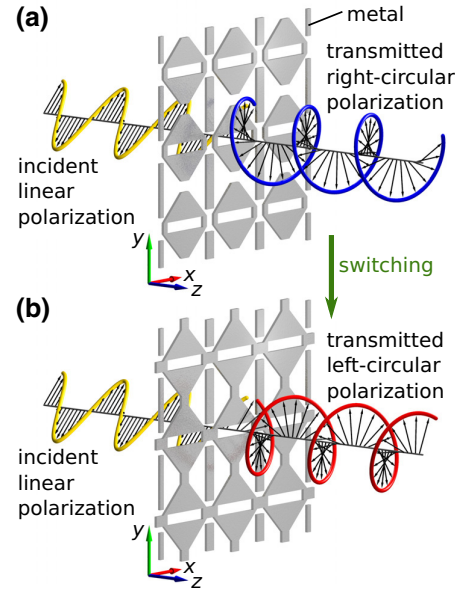


FIG. 1. Schematics for dynamic helicity switching for the transmitted wave by utilizing the transition between the (a) disconnected and (b) connected anisotropic checkerboards.

$\tilde{\mathbf{E}}_0$ are denoted by $\tilde{t}\tilde{\mathbf{E}}_0 \exp(-jk_0 z)$ and $\tilde{r}\tilde{\mathbf{E}}_0 \exp(jk_0 z)$, respectively. Here, $\tilde{t}$ ($\tilde{r}$) is a complex transmission (reflection) coefficient. If there is no energy loss caused by resistive elements, polarization conversion, or diffraction into higher-order propagating modes, the following equation must be satisfied from the electric field continuation $1 + \tilde{r} = \tilde{t}$ at $z = 0$ and energy conservation law $|\tilde{t}|^2 + |\tilde{r}|^2 = 1$:

$$|\tilde{t}|^2 + |1 - \tilde{t}|^2 = 1. \quad (1)$$

This implies that $\tilde{t}$ must be on the circumference of the dashed circle shown in Fig. 2(a). The quarter-wave plate function requires $\tilde{t}_y = \pm j\tilde{t}_x$, where $\tilde{t}_i$ is $\tilde{t}$ for a normally incident i -polarized wave. If we assume Eq. (1) for each polarization, the only possible choice for a quarter-wave plate is $\tilde{t}_x = (1 + j)/2$, $\tilde{t}_y = (1 - j)/2$ or $\tilde{t}_x = (1 - j)/2$, $\tilde{t}_y = (1 + j)/2$ [30,31]. Under these conditions, $T_x = T_y = 1/2$ is satisfied for the power transmission $T_x = |\tilde{t}_x|^2$, $T_y = |\tilde{t}_y|^2$.

Now, we apply the checkerboard singularity to quarter-wave plate switching. Consider a dipole-nested checkerboard metasurface (D-checkerboard) at $z = 0$ in a vacuum, as shown in Fig. 2(b). The thickness of the metal is safely assumed to be zero if we consider the structure as much thinner than the typical length, such as a . The metasurface is composed of perfectly metallic parts connected with interconnection patches exhibiting variable sheet impedance Z_c . Changing Z_c from ∞ to 0, we can induce the checkerboard transition from the disconnected structure (off state) to the connected structure (on state). The metasurface is designed to be *dynamically Babinet*

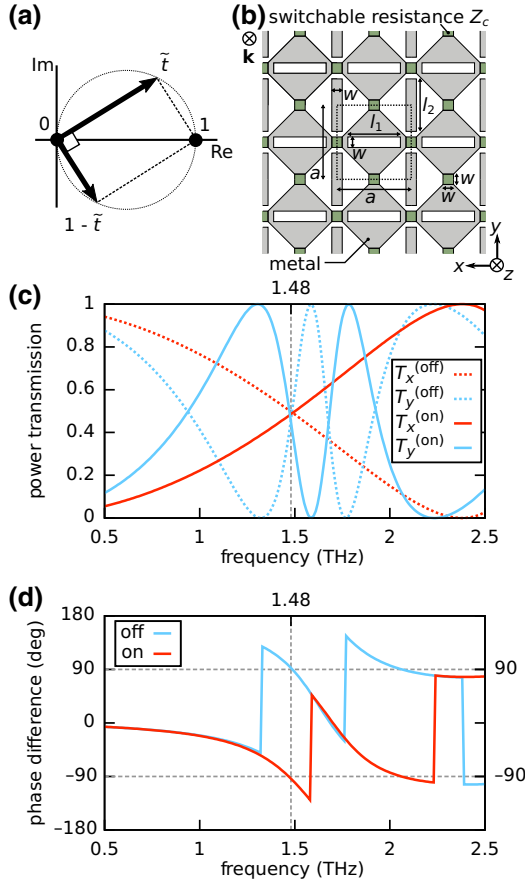


FIG. 2. (a) Constraint on a complex transmission amplitude for lossless electric metasurface without resistive elements, polarization conversion, or diffraction into higher-order propagating modes. (b) Design of a dipole-nested checkerboard metasurface. Calculated (c) power transmission ($T_x = |\tilde{t}_x|^2$, $T_y = |\tilde{t}_y|^2$) and (d) phase difference between transmitted x and y polarizations $[\arg(\tilde{t}_y) - \arg(\tilde{t}_x)]$ for the off- and on-state dipole-nested checkerboard made of perfect metal with $a = 106 \mu\text{m}$, $w = 15 \mu\text{m}$, $l_1 = l_2 = 75 \mu\text{m}$ at the normal incidence of plane waves.

invertible for $l = l_1 = l_2$; the on-state structure is obtained by a 90° rotation after Babinet inversion of the off-state structure. This special symmetry automatically leads to the transmission inversion for each polarization [28]:

$$\tilde{t}_i^{(\text{off})} + \tilde{t}_i^{(\text{on})} = 1 \quad (i = x, y), \quad (2)$$

where $\tilde{t}_i^{(\text{off})}$ and $\tilde{t}_i^{(\text{on})}$ are $\tilde{t}_i$ for the off- and on-state checkerboards, respectively. If we assume Eq. (1), the transmission inversion [$\tilde{t}_i^{(\text{off})}$ to $\tilde{t}_i^{(\text{on})} = 1 - \tilde{t}_i^{(\text{off})}$] is represented by a 90° rotation in the complex plane, as shown in Fig. 2(a). If we have $\tilde{t}_x^{(\text{off})} = (1 + j)/2$ and $\tilde{t}_y^{(\text{off})} = (1 - j)/2$, then $\tilde{t}_x^{(\text{on})} = (1 - j)/2$ and $\tilde{t}_y^{(\text{on})} = (1 + j)/2$ automatically follow, respectively. Thus, we can realize dynamic rotation of the fast axis of the quarter-wave plate by 90° . Note

that $T_x^{(\text{off})} = T_y^{(\text{off})} = T_x^{(\text{on})} = T_y^{(\text{on})} = 0.5$ holds with $T_i^{(\text{off})} = |\tilde{t}_i^{(\text{off})}|^2$ and $T_i^{(\text{on})} = |\tilde{t}_i^{(\text{on})}|^2$ ($i = x, y$).

Here, we generalize the above discussion and summarize the new focus of this study. We consider a lossless switchable metasurface with periodicity in a vacuum, satisfying the following conditions: (a) the metasurface is dynamically Babinet invertible and the on-state is obtained by a 90° rotation after the Babinet inversion of the off state; (b) both off and on states are symmetric under a mirror operation with respect to a plane $x = 0$ or $y = 0$ (this leads to no cross conversion between x and y polarizations); and (c) the operating frequency is under the *diffraction frequency*, above which waves can be diffracted into higher-order propagating modes. Then, we have established the following theorem for the metasurface.

Theorem: If we have power transmission $T_x^{(\text{off})} = T_y^{(\text{off})} = 0.5$ with the different transmission phases ($\arg \tilde{t}_x \neq \arg \tilde{t}_y$) only for the off state, we automatically obtain the switching function of a quarter-wave plate, without additional tuning for the on state.

Furthermore, the trajectory of $\tilde{t}_i(\omega)$ on the circle rotates only *clockwise* with increasing the angular frequency ω , from Foster's reactance theorem [32]. Therefore, we can state the following corollary as a design principle.

Corollary: For the quarter-wave plate switching, it is enough to realize the crossing condition of $T_x^{(\text{off})} = T_y^{(\text{off})} = 0.5$ with $[dT_x^{(\text{off})}/d\omega][dT_y^{(\text{off})}/d\omega] < 0$ at $\omega = \omega_0$.

Note that these results are based not only on transmission-reflection switching, which was already introduced in our previous paper [28], but also the general restriction on the transmission amplitude and phase. The combination of these principles was not discussed anywhere and it is critical to helicity switching.

To confirm the above theory, we demonstrate an ideal reconfigurable quarter-wave plate without a substrate, in a simulation. To calculate the transmission spectra of the metasurface in a vacuum, a conventional finite-element solver (COMSOL MULTIPHYSICS) was used. All the metal sections are assumed to be perfect electric conductors. Plane waves with x and y polarizations normally enter into a metasurface from $z < 0$ to $z > 0$. Using mirror symmetry, the simulation domain is set to a quarter of the unit cell with appropriate side boundary conditions depending on the incident polarization. The top and bottom boundaries are set as ports with only fundamental modes because all calculations are performed under the diffraction frequency $f_d = c_0/a = 2.83 \text{ THz}$ (c_0 is the speed of light in a vacuum). Fixing $a = 106 \mu\text{m}$ and $w = 15 \mu\text{m}$, we search the crossing condition to realize $T_x^{(\text{off})} = T_y^{(\text{off})} = 0.5$ with $dT_x^{(\text{off})}/d\omega < 0$ and $dT_y^{(\text{off})}/d\omega > 0$ at a target frequency, by varying $l (= l_1 = l_2)$. Performing parametric sweeps, we found that the above condition is satisfied at 1.48 THz for the off-state structure with $l = 75 \mu\text{m}$. Figure 2(c)

presents the power transmission spectra for both the off- and on-state structures with $l = 75 \mu\text{m}$. We can see that the four curves cross at 1.48 THz with a power transmission of 0.5, as we predicted before. Figure 2(d) presents the phase difference $\arg(\tilde{t}_y) - \arg(\tilde{t}_x)$. The phase difference is shifted from $+90^\circ$ to -90° at 1.48 THz for the conductivity switching. These characteristics are guaranteed by transmission inversion because of the structural symmetry of the anisotropic checkerboard. Thus, we could realize a subwavelength terahertz quarter-wave plate, of which the fast axis can be dynamically rotated by 90° .

III. EXPERIMENT WITH OPTIMIZED STRUCTURE

Although the above theoretical consideration works quite well, a substrate is required for a practical device to hold the structures. There is one way to use double-sided substrates sandwiching the structure to assure Babinet inversion [24]. However, we use another approach to slightly modify the structure with a single-sided substrate for simple implementation. In this case, Eq. (2) barely holds, but we can empirically realize quarter-wave plate switching as follows. In the experiments, structures are fabricated on a *c*-plane sapphire substrate. A vanadium dioxide (VO_2) film grown on the substrate is used as the variable resistance sheets. It exhibits an insulator-to-metal transition around $T_c \approx 340 \text{ K}$, where the electrical conductivity varies by several orders of magnitude through the transition. To compensate for the substrate effect, we break the symmetry condition $l_1 = l_2$ and tune l_1 and l_2 individually in a simulation to realize quarter-wave plate switching (the other parameters are identical to the previous simulation). The plane wave normally enters into the metasurface at $z = 0$ from a vacuum ($z < 0$) to the sapphire ($z \geq 0$) with refractive index $n_x = n_y = n_\perp \approx 3.1$ and $n_z = n_\parallel \approx 3.4$ [33]. Ports backed with perfectly matched layers are used to calculate spectra above the diffraction frequency $f_d = c_0/(n_\parallel a) = 0.83 \text{ THz}$. Metallic parts are assumed to

be aluminum with a thickness of 400 nm and conductivity $\sigma = 22 \text{ S}/\mu\text{m}$ [34]. The variable resistive connection patch made of VO_2 with a thickness of 200 nm is represented by the sheet impedance of 500 k Ω (off state) and 10 Ω (on state), which are based on dc measurements. The transition boundary condition with the background vacuum permittivity and permeability is imposed onto both the Al and VO_2 regions with their conductivity and thickness. We evaluate the normalized transmission coefficient of the metasurface as $\hat{t}_i = \tilde{t}_i/t_0$, where $t_0 = 2/(1 + n_\perp)$ is the Fresnel coefficient for transmission from a vacuum to the sapphire [35]. To achieve quarter-wave plate switching, we tune l_1 and l_2 as much as possible to realize $\hat{T}_x^{(\text{off})} \approx \hat{T}_y^{(\text{off})} \approx \hat{T}_x^{(\text{on})} \approx \hat{T}_y^{(\text{on})}$ at some frequency, where $\hat{T}_i = |\hat{t}_i|^2$ and the superscript represents the connection states. This transmission condition approximately leads to the switching function. Through the optimization process from $l_1 = l_2 = 75 \mu\text{m}$, we reach $l_1 = 72 \mu\text{m}$ and $l_2 = 76 \mu\text{m}$. The detailed simulation results are presented in Fig. 3.

Using these target parameters, we fabricate a D-checkerboard on a *c*-plane sapphire substrate (thickness of 1.0 mm). A VO_2 thickness of 200 nm is sputtered and patterned by photolithography using wet etching. The D-checkerboard made of aluminum (thickness of 400 nm) is formed by photolithography, electron beam evaporation at room temperature, and the lift-off technique. To ensure electrical connection, the VO_2 squares are partially overlapped by the Al structures. Figure 3(a) presents a top-view microphotograph of the fabricated sample. To evaluate the D-checkerboard, we perform polarization measurements using conventional terahertz time-domain spectroscopy. The sample holder is maintained at 300 and 370 K for the off and on states, respectively. To extend the sample substrate to 3 mm for delaying multiple reflection signals, two pieces of 1 mm-thick *c*-plane sapphire substrates are placed under the sample. As a reference sample, three stacked pieces of the substrates are used. Multiple reflection signals are truncated using a rectangular time-domain window function. Collimated terahertz waves normally enter into

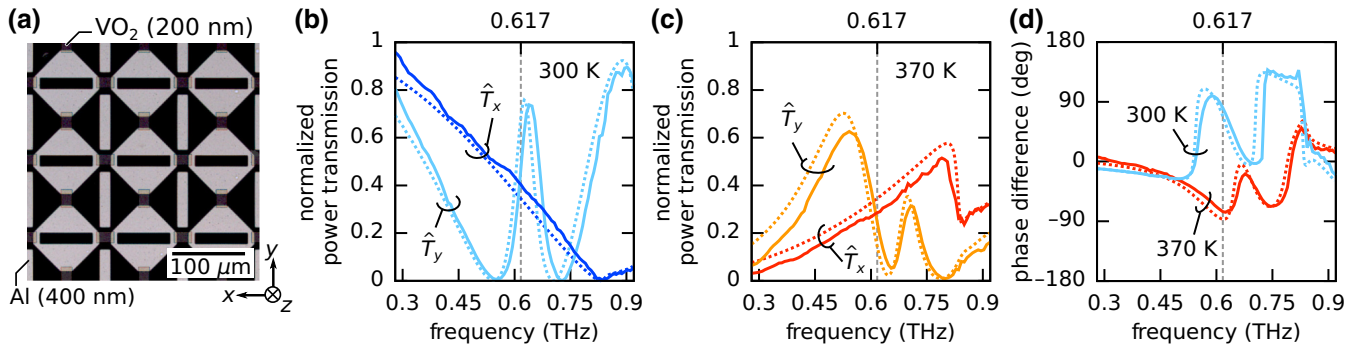


FIG. 3. (a) Microphotograph of the fabricated sample with target lengths $a = 106 \mu\text{m}$, $w = 15 \mu\text{m}$, $l_1 = 72 \mu\text{m}$, and $l_2 = 76 \mu\text{m}$. Normalized power transmission spectra of the sample at (b) 300 and (c) 370 K. (d) Phase difference between transmitted *x* and *y* polarizations for 300 and 370 K. The simulated data (dotted curves) are also depicted with the experimental data (solid).

the metasurface. Here, we define $\mathbf{e}_D = (\mathbf{e}_x + \mathbf{e}_y)/\sqrt{2}$ and $\mathbf{e}_X = (-\mathbf{e}_x + \mathbf{e}_y)/\sqrt{2}$, where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the x and y axes, respectively. Using wire grid polarizers, we measure $\hat{t}_{pq}$ ($p, q = D, X$), which is a normalized complex transmission amplitude from a normally incident wave with q polarization into a normally transmitted wave with p polarization. The normalization is performed through $\hat{E}_p(\omega)/\hat{E}_q^{(\text{ref})}(\omega)$, where $\hat{E}_p(\omega)$ represents the Fourier-transformed transmitted electric field with polarization p for the sample and $\hat{E}_q^{(\text{ref})}(\omega)$ is that with polarization q for the reference. After the measurement, we convert $\hat{t}_{pq}$ ($p, q = D, X$) into $\hat{t}_{ij}$ ($i, j = x, y$) using matrix transformation.

Figures 3(b) and 3(c) present the experimental (solid curves) and simulated (dotted) data of normalized power transmission spectra $\hat{T}_x = |\hat{t}_x|^2$ and $\hat{T}_y = |\hat{t}_y|^2$ for the off and on states. The experimental data agree quite well with the simulated data. The small discrepancy between the experimental and simulated data is due to the inaccuracy of the fabrication or measurement. At 0.617 THz, $\hat{T}_x^{(\text{off})} \approx \hat{T}_y^{(\text{off})} \approx \hat{T}_x^{(\text{on})} \approx \hat{T}_y^{(\text{on})}$ is satisfied. In Fig. 3(d), the phase difference $\arg(\hat{t}_y) - \arg(\hat{t}_x)$ is depicted. Phase difference switching from about $+90^\circ$ to -90° is observed at 0.617 THz. This result implies that we have realized fast axis rotation of the quarter-wave plate by 90° . The maximum device thickness without the substrate is 600 nm, which is deeply subwavelength at the working frequency.

To apply this functionality to the helicity switching of a circular polarization, the polarization of the transmitted wave is analyzed for a normally incident wave with D polarization. Here, we evaluate S_3/S_0 with the Stokes parameters S_3 and S_0 [36], where $S_3/S_0 = 2\text{Im}(\hat{t}_{xD}\hat{t}_{yD})/(|\hat{t}_{xD}|^2 + |\hat{t}_{yD}|^2)$ represents the z coordinate in the Poincaré sphere. The right and left circularly polarized waves correspond to $S_3/S_0 = +1, -1$, respectively.

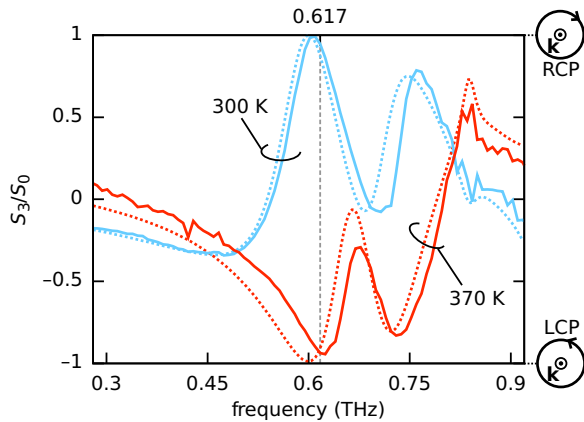


FIG. 4. S_3/S_0 of the transmitted wave for a normally incident plane wave with D polarization under 300 and 370 K. The simulated data (dotted curves) are also depicted with the experimental data (solid).

Figure 4 presents the S_3/S_0 of the transmitted wave. The helicity of the transmitted wave is dynamically switched at 0.617 THz from $S_3/S_0 = 0.95$ to -0.94 in the experiment, while the simulation shows modulation from $S_3/S_0 = 1.0$ to -0.99 at 0.60 THz. The deviation is caused by the fabrication imperfection or misalignment of the measurement system and can be improved in principle. Compared to a recently investigated MEMS quarter-wave plate [12], which realizes S_3/S_0 modulation ranging from -100% to 60% [see Fig. 5(b) in Ref. [12]], we achieve the deeper modulation no less than $\pm 94\%$ by leveraging a critical checkerboard structure. At 0.617 THz, experimental results show $|\hat{t}^{(\text{ref})}_{RD}|^2 \approx -6.6$ dB (22%) for the off state, and $|\hat{t}^{(\text{ref})}_{LD}|^2 \approx -8.8$ dB (13%) for the on state, where R and L represent right and left circular polarizations with $\mathbf{e}_R = (\mathbf{e}_x + j\mathbf{e}_y)/\sqrt{2}$ and $\mathbf{e}_L = (\mathbf{e}_x - j\mathbf{e}_y)/\sqrt{2}$, respectively, and we define an amplitude transmission of the first pulse through the sapphire plate $\hat{t}^{(\text{ref})} = t_0 t'_0 = 4n_\perp/(1 + n_\perp)^2 \approx 74\%$, with $t'_0 = 2n_\perp/(1 + n_\perp)$, except for the absorption loss of the substrate.

IV. CONCLUSIONS

We propose a terahertz metasurface functioning as a reconfigurable quarter-wave plate, of which the fast axis can be dynamically rotated by 90° . The device is based on the checkerboard critical transition, which realizes the deep modulation and simultaneous design of the off and on states of the device. Combining the constraint on complex transmission amplitudes with transmission inversion, we theoretically show that 90° fast axis rotation can be realized. Performing simulations for the ideal structure without a substrate, we confirm that the theory works well. To compensate the substrate effect for realizing practical experiments, we adjust the structural parameters by slightly breaking the structural symmetry on the interchange of the metallic and vacant area. The optimized D-checkerboard is fabricated and evaluated in the terahertz range. The experimental data agree very well with the simulated data and the fast axis rotation of the quarter-wave plate is realized at 0.617 THz. By analyzing the polarization of the transmitted wave for a normally incident D -polarization wave, we demonstrate the helicity switching function of the device. This device is applicable to chirality-sensitive spectroscopy for materials with circular dichroism and optical activity, highly sensitive measurements, and data transmission with circular polarization. The demonstrated device is practically designed with deeply subwavelength structures on thick substrates to truncate multiple reflection signals. For more efficient devices, we should further consider the elimination of multiple reflections by using a thinner substrate or an antireflective coating at the backside of the substrate. The proposed design strategy to realize the reconfigurable quarter-wave plate is not limited to the terahertz frequency range; it is applicable to other frequency

ranges, where variable resistance materials can be used. By utilizing photocarrier injection into semiconductors, ultrafast polarization control can be achieved. Broadband switching should also be studied in the future, although subwavelength devices commonly require resonant behavior, which leads to narrowband operation.

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