

Geometric Energy Transfer in a Stückelberg Interferometer of Two Parametrically Coupled Mechanical Modes

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Geometric phase, which is acquired after a system undergoes cyclic evolution in Hilbert space, is believed to be noise resilient because it depends only on the global properties of the evolution path. We report geometric control of energy transfer between two parametrically coupled mechanical modes in an optomechanical system. The parametric pump is controlled along a closed loop to implement geometric Stückelberg interferometry of mechanical motion, in which the dynamical phase is eliminated using the Hahn-echo technique. We demonstrate that the interference based solely on the geometric phase is robust against certain noises. More remarkably, we show that the all-geometric approach can achieve a high-energy-transfer rate that is comparable to those using conventional dynamical protocols.

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I. INTRODUCTION

Strongly coupled resonators have been intensively investigated in various distinct physical systems [1–7], in which coherent control of the transport phenomena has been realized using the rich toolbox developed for the quantum systems. Specifically, for example, a two-mode mechanical system has been demonstrated as a classical analogy of a quantum two-level system, where Rabi physics was applied for full control of mechanical motion in the Bloch sphere [7–10]. More recently, studies of two-mode mechanical systems have shown that nonadiabatically traversing the anticrossing point follows the Landau-Zener (LZ) dynamics, in which the coherent splitting [11,12], Stückelberg interference [12,13], and the intriguing effect of dynamical localization [14] for mechanical motion have been experimentally achieved. In addition to the high potential of quantum information processing with respect to recent great advances in cavity optomechanics [15–19], these dynamical protocols have been widely adopted in coherent control of energy transfer

between mechanical resonators for applications such as coherent switches [14], low-power logic units [20,21], nonreciprocal transducers [22], and coherent-force sensors [23]. A continuous challenge in developing coherent mechanical devices and further extending their applications is improving the coherence of operations. Benefiting from the great advances in nanofabrications, mechanical resonators with extremely low dissipation have become available [24–26], which can significantly reduce energy dissipation during operations. Nevertheless, the coherent control of energy transfer in either optomechanical or electromechanical systems will be unavoidably subject to noises from various sources, which can cause observable phase decoherence [27,28].

So far, significant efforts have been devoted to developing noise-resilient protocols for coherent control of energy transfer in mechanical systems. For example, topological energy transfer between two modes of a mechanical resonator has recently been achieved by adiabatically encircling the exceptional point [29], where the effective Hamiltonian of the system is non-Hermitian. Alternatively, when a system undergoes cyclic evolution in Hilbert space by adiabatically changing the control parameters, it acquires a geometric phase in addition to the

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dynamical one [30,31]. Because it depends only on the global properties of the evolution path, the geometric phase is believed to be robust against certain noises and has practical applications in fault-tolerant quantum-information processing [32–35]. In order to overcome the operation speed limited by the adiabaticity, which typically requires the evolution in the parametric space at a rate significantly smaller than the typical coupling strength, numerous nonadiabatic geometric protocols have also been proposed [36–39]. Quantum logic gates based on the nonadiabatic geometric phases have been extensively implemented in the systems of magnetic resonances [40], superconductor qubits [41,42], and trapped ions [43,44].

In this paper, we demonstrate energy transfer through geometric Stückelberg interferometry of two parametrically coupled mechanical modes. We show that the geometric phase can be acquired after the cyclic evolution of the motion state between the two consecutive LZ transitions. In order to realize an all-geometric operation, the dynamical phase accumulated during the cyclic evolution is eliminated using the Hahn-echo technique. We demonstrate that noise-resilient energy transfer can be achieved using the geometric Stückelberg interferometry at an operation speed comparable with that in conventional dynamical protocols.

II. EXPERIMENT

The system under investigation is a tunable two-mode optomechanical system consisting of two cantilevers, which are elastically coupled by connecting to the same overhang [45]. By trapping one of the cantilevers (cantilever 1) inside a fiber-based cavity using a 1064-nm laser, the frequency of cantilever 1 becomes trapping power P dependent, $\omega_{1,\text{eff}} = \sqrt{\omega_1^2 + gP}$, with g representing the strength of the optical trap, while the frequency of cantilever 2, ω_2 , remains unaffected [46,47]. We use an additional weak 1310-nm laser, which is linearly coupled to the mechanical motion, to monitor the oscillation of cantilever 1. Owing to the elastic coupling, the fundamental flexural modes of the cantilevers are hybridized into two normal modes with the two cantilevers oscillating in-phase (x_-) and out-of-phase (x_+). The strength of the elastic coupling measured as the frequency difference between two normal modes at a degenerate point (where $\omega_{1,\text{eff}} = \omega_2$) is $\Delta/2\pi = 459$ Hz. Our experiments are carried out at the trapping power of $P_0 = 131 \mu\text{W}$, at which the frequencies of the in-phase and out-of-phase modes are $\omega_-/2\pi = 6234$ Hz and $\omega_+/2\pi = 6701$ Hz, respectively (see Supplemental Material [45]). Here, the system is intentionally offset from the degenerate point so that frequency fluctuation of the normal modes can be created by introducing extra noises on the trapping power P in order to study the effect of noises on the operation, as we will show later. As a result of the gentle cold-damping effect of the

1310-nm probing laser, the dissipation rates of the two normal modes are slightly different with $\gamma_-/2\pi = 0.37$ Hz and $\gamma_+/2\pi = 0.25$ Hz.

III. RESULTS AND DISCUSSIONS

In order to couple the two normal modes, a parametric pump is applied by modulating the trapping power $P(t) = P_0 + P_d \cos(\omega_d t + \theta)$. When the parametric pump is activated, the dynamics of the periodically driven two-mode mechanical system can be described by

$$\begin{pmatrix} \frac{d^2}{dt^2} + \gamma_+ \frac{d}{dt} + \omega_+^2 & 0 \\ 0 & \frac{d^2}{dt^2} + \gamma_- \frac{d}{dt} + \omega_-^2 \end{pmatrix} \begin{pmatrix} x_+ \\ x_- \end{pmatrix} - \varepsilon(t) \begin{pmatrix} 1 + \cos\alpha & \sin\alpha \\ \sin\alpha & 1 - \cos\alpha \end{pmatrix} \begin{pmatrix} x_+ \\ x_- \end{pmatrix} = 0, \quad (1)$$

where $\varepsilon(t) = (1/2)gP_d \cos(\omega_d t + \theta)$ denotes the parametric pump and α satisfies $\tan\alpha \approx [\Delta/(\omega_{1,\text{eff}}(P_0) - \omega_2)]$. When the frequency difference between the normal modes, $\delta\omega \equiv \omega_+ - \omega_-$, is compensated by the parametric pump, mixing of the two normal modes leads to a normal-mode splitting. The anticrossing phenomenon can be observed in Fig. 1(a). The strength of the parametric coupling between the two normal modes, Ω , which is proportional to the pump power, P_d , can be measured as the normal-mode splitting at the on-resonance condition $\omega_d = \delta\omega = 2\pi \times 467$ Hz [Fig. 1(b)]. In contrast to the previous Stückelberg interferometer based on two mechanical modes coupled by interacting with the same static field [12], we use the two parametrically coupled normal modes to construct a Stückelberg interferometer, in which the coupling field is fully tunable.

To overcome the thermal Brownian motion, the system is initialized by piezo-electrically actuating the in-phase mode. When the anticrossing point is traversed nonadiabatically, it functions as a coherent splitter to transfer part of the energy from the in-phase mode to the out-of-phase mode [45]. As the first step of the Stückelberg interferometry, we implement the coherent splitter by applying a frequency-modulated pump pulse with the pump frequency (ω_d) ramped linearly from $\omega_a = 2\pi \times 267$ Hz to $\omega_b = 2\pi \times 667$ Hz in transition time t_{LZ} . The energy splitting ratio, which is measured as the proportion of energy remaining in the in-phase mode after the transition, can be calculated using the LZ formula [48,49]

$$P_{\text{LZ}} = \exp\left(-\frac{\pi\Omega^2}{2v}\right), \quad (2)$$

with $v = [(\omega_a - \omega_b)/t_{\text{LZ}}]$ denoting the speed at which the anticrossing point is traversed. For the parametric coupling strength $\Omega/2\pi = 28.5$ Hz, the oscillation amplitudes of the in-phase (X_-) and out-of-phase (X_+) modes are

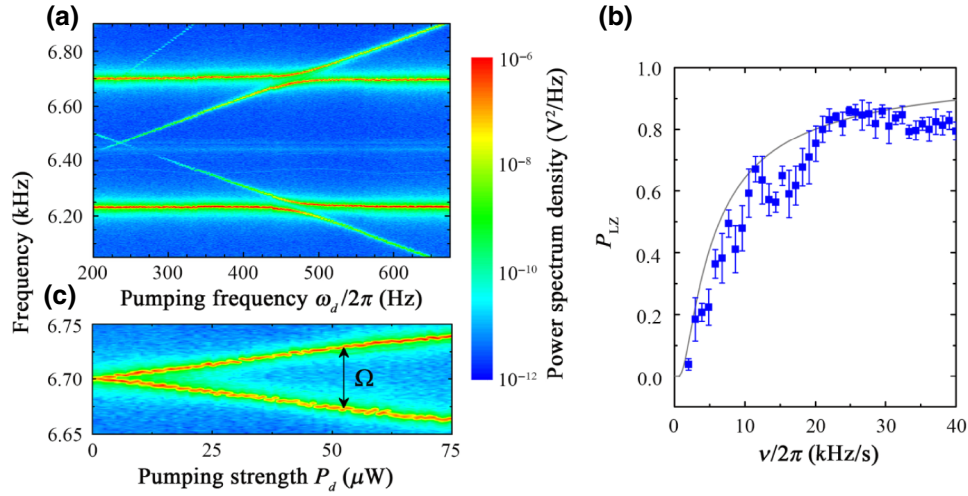


FIG. 1. Two parametrically coupled mechanical modes. (a) The pumping frequency response of the parametrically coupled mechanical modes. The parametric pump is applied by modulating the trapping power at amplitude $P_d = 29 \mu\text{W}$. The thermal-oscillation-power spectral density of cantilever 1 under the parametric pump is measured using an electro-spectra analyzer. (b) The modulating amplitude P_d is dependent on normal-mode splitting at the on-resonance condition. For each modulating amplitude, the strength of parametrical coupling Ω is measured as the anticrossing gap as marked in panel (b). (c) Energy splitting ratio for each transition speed. Here, the energy splitting ratio P_{LZ} , measured as the relative oscillation energy of the in-phase mode after the pump pulse (solid blue dots), $[X_-^2/(X_-^2 + X_+^2)]$, is plotted with the theoretical result (gray line) calculated from Eq. (2).

simultaneously recorded after the pump pulse [45]. As shown in Fig. 1(c), the energy splitting ratio can be tuned according to Eq. (2) by changing the transition time t_{LZ} . Specifically, a 50/50 splitter is achieved with the transition time $t_{LZ} \approx 30$ ms.

For a round-trip LZ transition, the Stückelberg interferometry of two parametrically coupled mechanical modes can be achieved. The recombination of the mechanical motions after the second LZ transition creates an interference fringe depending on the relative phase of the motions. Generally, the phase acquired between the two consecutive LZ transitions includes both the dynamical phase and the geometric phase that depends solely on the evolution path of the system in state space [50]. In order to realize Stückelberg interferometry based solely on the geometric phase, the dynamical phase is eliminated in our experiment using the Hahn-echo technique. We implement the all-geometric energy transfer through modulating the parametric pump $\varepsilon(t) \rightarrow (1/2)gP_d(t)\cos[\int \omega_d(t')dt' + \theta(t)]$, where the power $P_d(t)$, frequency $\omega_d(t)$, and phase $\theta(t)$ of the pump are controlled as schematically illustrated in Fig. 2(a). Our protocol is similar to that in Ref. [42] except that the pump detuning $\Omega_z \equiv \delta\omega - \omega_d$ is swept via modulating the pump frequency ω_d in this work rather than changing $\delta\omega$ as is done in Ref. [42]. Although our system is intentionally offset from the degenerate point ($\delta\omega \neq \Delta$), we note that this modification is nontrivial because it allows the geometric Stückelberg interferometry to be applied even at the degenerate point, where $\delta\omega$ is most stable against the fluctuation in control parameter, namely, $\partial\delta\omega/\partial P \sim 0$ in our case.

The evolution of the two-mode mechanical system can be described on the Bloch sphere with the longitude field Ω_z and transverse fields $\Omega_x = \Omega \cos(\theta)$ and $\Omega_y = \Omega \sin(\theta)$, respectively. After the initial state of the system is prepared for the in-phase mode, the pump pulse is activated ($t = t_A = 0$). The loop path of the parametric pump is shown in Fig. 2(b). According to the adiabatic-impulse approximation [50], the system evolves adiabatically except at the time instant when the anticrossing point is traversed, where the mechanical motion is split as a consequence of the LZ transition. Therefore, after the first LZ transition at $t = t_C$, the motion of the system can be described as a superposition of two normal modes, which are marked as two orthogonal motion states C and C' on the Bloch sphere [see Fig. 2(c)]. We note that the adiabatic evolution of the system between two LZ transitions can be described by the pair of orthogonal cyclic states C and C' . In the following, we just focus on the evolution path of the state C , which is plotted on the Bloch sphere in Fig. 2(c). The trajectory CE of the evolution adiabatically follows the pump-field trajectory CDE in Fig. 2(b). At the time instant t_E , a π pulse is applied to flip the Bloch vector from point E to point F around the x axis. Then, the phase of the pump is reversed to bring the system back to point C through the trajectory FH on the Bloch sphere. The durations for the evolution including CE and FH are kept equal ($t_{FH} = t_{CE}$) so that the dynamical phase, including both the Stokes phase acquired at each LZ transition and the adiabatic-dynamical phase accumulated during the evolution CE and FH , is completely cancelled out after a full evolution cycle. Consequently, the recombination of

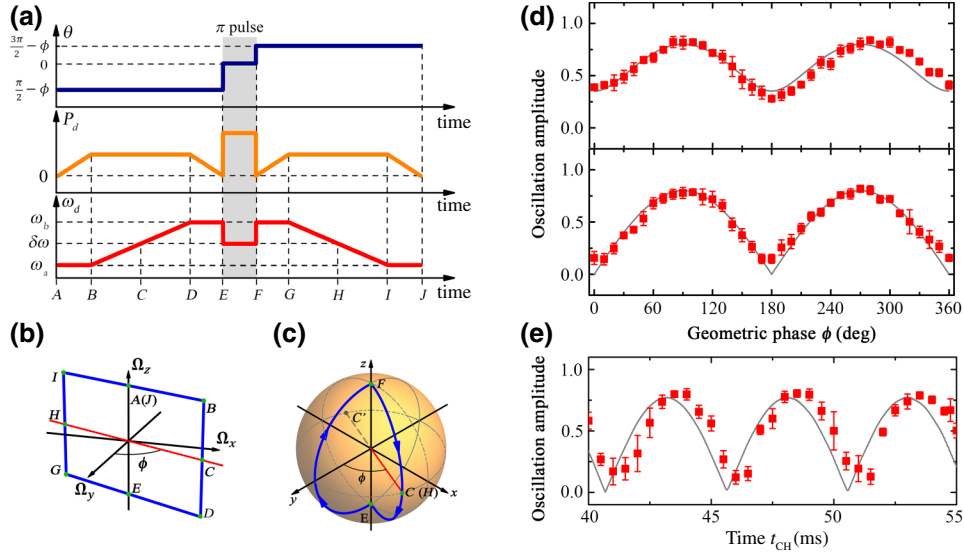


FIG. 2. Stüeckelberg interferometry with two parametrically coupled modes. (a) The control sequence of the pump pulse for the geometric Stüeckelberg interferometry. The strength of parametric coupling is ramped up to $\Omega/2\pi = 28.5$ Hz in $t_{AB} = 5$ ms, and after the pump frequency sweeps from $\omega_a/2\pi = 267$ Hz to $\omega_b/2\pi = 667$ Hz for the first LZ transition, the parametric pump is gradually turned off in $t_{DE} = 5$ ms. After the π pulse, the parametric pump with its phase shifted by 180° is applied for the second LZ transition at the same speed ($t_{BD} = t_{GI}$). The parametric pump is gradually turned on (off) before (after) the transition in $t_{FG} = t_{IJ} = 5$ ms. The width of the π pulse is $t_{EF} \approx 8.6$ ms. (b) The trajectory of the effective parametric field in parametric space. The points marked on the loop path correspond to the parametric pump at the time instants marked in panel (a). (c) The evolution path of the system on the Bloch sphere. The points marked on the Bloch sphere correspond to the motion states of the in-phase mode at time instants marked in panel (a). (d) Geometric Stüeckelberg interference. The final oscillation amplitude of the out-of-phase mode is measured for the transition times $t_{BD} = t_{GI} = 15$ ms (upper panel) and $t_{BD} = t_{GI} = 30$ ms (lower panel), respectively. For each geometric phase ϕ , the oscillation amplitudes of the out-of-phase mode (solid red dots) are plotted with the theoretical results (gray lines) calculated from Eq. (3). (e) Dynamical Stüeckelberg interference. For the transition time $t_{BD} = t_{GI} = 30$ ms, the oscillation amplitudes of the out-of-phase mode, measured after the pump pulse (solid red dots) and calculated from Eq. (1) (gray lines), are plotted against the total evolution time between the two LZ transitions t_{CH} .

the mechanical motions as a result of the second LZ transition at $t = t_H$ creates an interference fringe depending solely on the geometric phase ϕ , which can be calculated as half of the solid angle enclosed by the trajectory $CEFH$ on the Bloch sphere. The oscillation amplitude of the out-of-phase mode after the round-trip transition can be calculated as

$$X_+ = \sqrt{1 - 4P_{LZ}(1 - P_{LZ})\cos^2\phi}. \quad (3)$$

The geometric Stüeckelberg interferometry is highly controllable since the energy splitting ratio P_{LZ} and the geometric phase ϕ can be adjusted independently by controlling the pump field. As shown in Fig. 2(d), an all-geometric energy transfer is achieved by changing the phase ϕ . To enhance the transfer efficiency, the energy splitting ratio is optimized for a 50/50 splitter by adjusting the transition time. In particular, for the parametric coupling strength $\Omega(t_{C,H})/2\pi = 28.5$ Hz, performing the geometric Stüeckelberg interferometry at the transition time $t_{BD} = t_{GI} = 30$ ms yields an interference with its visibility reaching approximately 0.94 [12]. For comparison, a dynamical interference is implemented using a protocol

similar to that illustrated in Fig. 2(a) except that the π pulse is removed in order to preserve the dynamical phase. We control the dynamical phase by adjusting the evolution time $t_{DE} = t_{FG}$ while keeping the geometric phase $\phi = 0$ constant. Generally, a precise control of the dynamical interference can be expected providing the frequencies of the normal modes are accurately measured. Otherwise, the error in controlling the dynamical phase can be accumulated with the evolution time increasing and can lead to the observed mismatch between the experimental and the theoretical results in Fig. 2(e). Here, each data point in Figs. 2(d) and 2(e) is the average result of seven independent measurements using the same parameters. Therefore, for different measurements, the fluctuation of the phase acquired between the two LZ transitions can be reflected as the deviation of these measurements in the figures. It is interesting to see that the effect of decoherence originating from the dephasing is more significant for the case of dynamical interference compared with that of the geometric one.

The improved coherence obtained in the geometric Stüeckelberg interferometry implies that our protocol is more robust against certain noises than the conventional

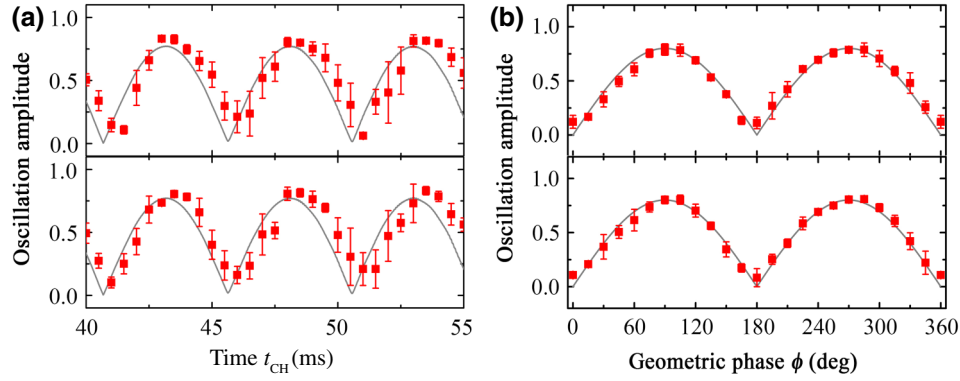


FIG. 3. Effect of the noises on the Stückelberg interference. (a) Dynamical Stückelberg interference and (b) geometric Stückelberg interference in the presence of random noises with bandwidths of 1000 Hz (upper panel) and 10 Hz (lower panel). The white noise is added in the pump power to create random fluctuations on the strength of the longitude field with the amplitude approximately 3 Hz in peak. Similar pump pulses used in Figs. 2(d) and 2(e) are applied, respectively, to realize the geometric and dynamical Stückelberg interferometry with the parametric coupling strength at each LZ transition $\Omega(t_{CH})/2\pi = 28.5$ Hz and the transition time $t_{BD} = t_{GI} = 30$ ms. The oscillation amplitudes of the out-of-phase mode measured after the operation (solid red dots) are plotted. The corresponding numerical results are also given (gray lines) according to Eq. (1).

dynamical one. To investigate the effect of noise on the operation coherence, two kinds of white noises of the same amplitude are intentionally added in the trapping power to create random fluctuations in the pump power with bandwidths of 10 and 1000 Hz, respectively. For the noise with a bandwidth of 10 Hz, although the pump field can be treated as approximately uniform during each measurement, it causes a slow fluctuation of the pump field compared with the adiabatic evolution time scale t_{CH} so that the pump field is inhomogeneous for different measurements. By increasing the noise bandwidth to 1000 Hz, we also include noise at a high frequency to induce a fast fluctuation of the pump field during each measurement. As shown in Fig. 3(a), both noises can increase decoherence in the dynamical interference. Moreover, the noise-induced phase fluctuation can be accumulated and the effect of dephasing is amplified with the increase of operation time. From the interference fringes, we can obtain the standard deviation of phase, $\delta\phi$, accumulated between the two consecutive LZ transitions. The average values of $\delta\phi$ for the adiabatic evolution time between $t_{CH} = 45$ ms and $t_{CH} = 51$ ms are calculated to be 0.83 and 0.86 rad for noises with bandwidths of 10 and 1000 Hz, respectively. In contrast, a noise-resilient energy transfer is achieved using geometric Stückelberg interferometry [Fig. 3(b)]. For the measurements with a noise bandwidth of 10 Hz, we note that the decoherence induced by the slow fluctuation of the pump field is significantly reduced by the Hahn-echo technique, which is used to cancel the dynamical phase accumulated between two consecutive LZ transitions. More remarkably, it is interesting to see that a well-preserved coherence can also be obtained under the influence of the noise with a bandwidth of 1000 Hz. Although the low-frequency noise in this bandwidth can be effectively

suppressed using the Hahn echo, the resilience to the noise that also includes high-frequency components should not be completely attributed to the use of a Hahn echo. The well-preserved coherence obtained even in the presence of the fast fluctuating pump field offers clear evidence that the geometric phase is robust against local fluctuation on the evolution path. The phase deviations as calculated from the geometric interference fringe are nearly equal for the two noises, with an average value $\delta\phi \approx 0.10$ rad.

IV. CONCLUSIONS

In summary, we present the geometric control of energy transfer in the Stückelberg interferometer consisting of two parametrically coupled mechanical modes. When the parametric pump is on-resonance, an anticrossing phenomenon is observed, which allows for energy transfer between the mechanical modes. Our experiments demonstrate that the nonadiabatic transition of the anticrossing point via modulating the pump frequency leads to coherent splitting of the mechanical motion following the LZ model. For a round-trip LZ transition, the all-geometric control of energy transfer based on the Stückelberg interferometry is implemented by eliminating the dynamical phase using the Hahn-echo technique. We note that the speed of the geometric operation is comparable with the typical coupling strength ($2\pi/t_{AJ} \sim \Omega$). Moreover, we show that our protocol integrating the merits of both the geometric operation, which is inherently robust against local fluctuations on the control parameters, and the Hahn-echo technique, which is capable of canceling the slow fluctuations, can achieve a highly efficient transfer of mechanical energy with a well-preserved phase coherence even in the presence of certain noises. Therefore, we conclude

that our protocol offers a fast and highly controllable method for noise-resilient control of mechanical energy transfer, which can be generally implemented in most electromechanical and optomechanical systems using currently available technology.

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