

Fast topological photon transport via optimized coupling modulations approaching the adiabaticity control limit

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Achieving adiabatic high-fidelity state transfer on photonic chips is essential for scalable quantum information processing, but remains challenging due to the trade-off between robustness and device compactness. The recently reported adiabaticity control limit (ACL) overcomes this challenge by inverse designing compact couplers. However, the numerical nature and example restriction of the ACL to three-waveguide systems hinder generally analytical insight and scalability. Here we develop an analytically optimized coupling framework that pushes the multiwaveguide topological pumping toward the ACL. By generalizing the conventional trigonometric modulations to a power-law profile in a Su-Schrieffer-Heeger lattice, guided by a global adiabatic criterion to minimize spatial nonadiabatic fluctuations, we suppress the nonadiabatic transitions even in dramatically shortened devices. Numerical simulations demonstrate a fourfold reduction in the required device length while maintaining a transfer fidelity exceeding 95%. Furthermore, we showcase the versatility of this approach by designing a compact topological photonic splitter for applications in functional photonic devices. Our work establishes a practical paradigm for fast, high-fidelity topological state transfer, bridging the critical gap between analytic design and the ACL in scalable photonic networks.

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I. INTRODUCTION

The realization of large-scale, fault-tolerant quantum information processing on the integrated photonic platforms critically depends on the capacity of reliably generating, manipulating, and transporting quantum states on a chip [1–4], where a paramount challenge is to achieve high-fidelity state transfer between distant nodes while mitigating inevitable fabrication disorders and environmental noise [5–8]. Topological photonics has emerged as a powerful framework to address this challenge, offering transport mechanisms that are inherently robust against scattering and imperfections due to the topological protection of edge states [9–16]. The Su-Schrieffer-Heeger (SSH) model, a paradigmatic one-dimensional topological system, provides a versatile platform for such topologically protected adiabatic pumping, enabling the robust and directional transfer of quantum states across lattice sites [17–26].

Recent experimental advances have successfully demonstrated the topological pumping of classical light and even single-photon states in SSH-based platforms, showcasing a near-perfect transfer efficiency [27–36]. Despite these advances, a critical challenge remains: The adiabatic condition required for high-fidelity transfer typically demands long adiabatic evolution lengths, directly conflicting with the miniaturization requirements of integrated photonic circuits

[37–39]. This establishes a fundamental trade-off between robustness and device compactness [40]. To suppress nonadiabatic transitions, the conventional approaches extend the modulation period, leading to impractical device footprints. Alternative strategies such as shortcuts to adiabaticity techniques [41–44] or non-Hermitian engineering [45–48] often introduce additional complexity, limited operational bandwidth, or heightened sensitivity to imperfections.

The adiabaticity control limit (ACL), as a recent conceptual breakthrough, provides a promising path to transcend this trade-off by inverse designing coupling profiles, which approach optimal light evolution [49]. As an example, this approach has been successfully implemented for fast adiabatic transfer in a three-waveguide system via stimulated Raman adiabatic passage. However, being based on numerical optimization for a specific geometry, its generalization to multiwaveguide topological systems and the quest for analytic control schemes remain open and challenging. Therefore, a general design principle that fundamentally resolves the adiabaticity-compactness trade-off for scalable topological photonic devices is urgently needed.

Here we overcome this limitation by introducing an optimized coupling framework to push the topological pumping toward the ACL. We address this goal by moving beyond the conventional trigonometric modulations and designing optimized analytic coupling profiles guided by a global adiabatic criterion for SSH waveguide arrays. Our approach systematically generalizes adiabatic topological pumping to a power-law coupling scheme, which minimizes the spatial variance of nonadiabaticity and dramatically suppresses nonadiabatic transitions in shortened devices. Numerical simulations

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confirm a fourfold reduction in the required pumping length compared to conventional designs while maintaining a photon transport fidelity exceeding 95%. Furthermore, we extend this strategy to implement a compact topological photonic splitter based on an interfaced SSH lattice, demonstrating its utility as a building block for reconfigurable or functional circuits. This work bridges a critical gap in approaching the ACL for multiwaveguide systems via analytic modulations, establishing a versatile and practical paradigm for fast, high-fidelity topological state transfer on integrated photonic chips.

II. MODEL AND SYSTEM

In the context of our multiwaveguide SSH system, the ACL corresponds to the theoretical bound on the minimal nonadiabatic leakage achievable under the given device-length and coupling-strength constraints. Our optimized coupling framework approaches this limit by systematically suppressing nonadiabatic transitions via optimally designed and analytic coupling profiles. We establish a design framework for coupling configurations that can be broadly applied to a diverse system exhibiting topological transports governed by adiabatic evolution. To illustrate the underlying principle, we begin with the well-known dimerized SSH model. Figure 1(a) depicts a SSH photonic lattice comprising N waveguides. This waveguide array system with length L , applied for the topological pumping of photons, is constrained to the nearest-neighbor coupling Hamiltonian in the z -direction propagation as

$$H(z) = \sum_{j=1}^{(N-1)/2} [J_1(z)|2j-1\rangle + J_2(z)|2j+1\rangle]\langle 2j| + \text{H.c.}, \quad (1)$$

with the staggered coupling strengths J_1 (intracell) and J_2 (intercell) being spatially modulated in the z direction. Under the paraxial approximation within the tight-binding model, we describe the state of the waveguide system by the vector $|\Psi(z)\rangle = \sum_{n=1}^N C_n(z)|n\rangle$, with $C_n(z)$ denoting the complex probability amplitude in the n th waveguide basis state $|n\rangle$ and satisfying the normalization condition $\sum_n |C_n(z)|^2 = 1$. The evolution of $|\Psi(z)\rangle$ is governed by the Schrödinger-like coupled-mode equation (CME) $H(z)|\Psi(z)\rangle = -i\partial_z|\Psi(z)\rangle$. When the coupling in the waveguide array is spatially modulated adiabatically, the input light can be transformed from one edge to the other edge of the waveguide array through topological pumping, following the spatial profiles of the zero-energy gap modes of the photonic system [21,28,32,33,36]. The gap mode reads (see Appendix A) [21,51]

$$|G(z)\rangle = N_{\text{or}} \sum_{j=1}^{(N+1)/2} \beta^j(z)|2j-1\rangle, \quad (2)$$

where $\beta(z) \equiv -J_1(z)/J_2(z)$ is the ratio of coupling strengths, N_{or} is the normalization constant, and j is the power exponent of β in β^j . This gap mode exhibits the pronounced edge state

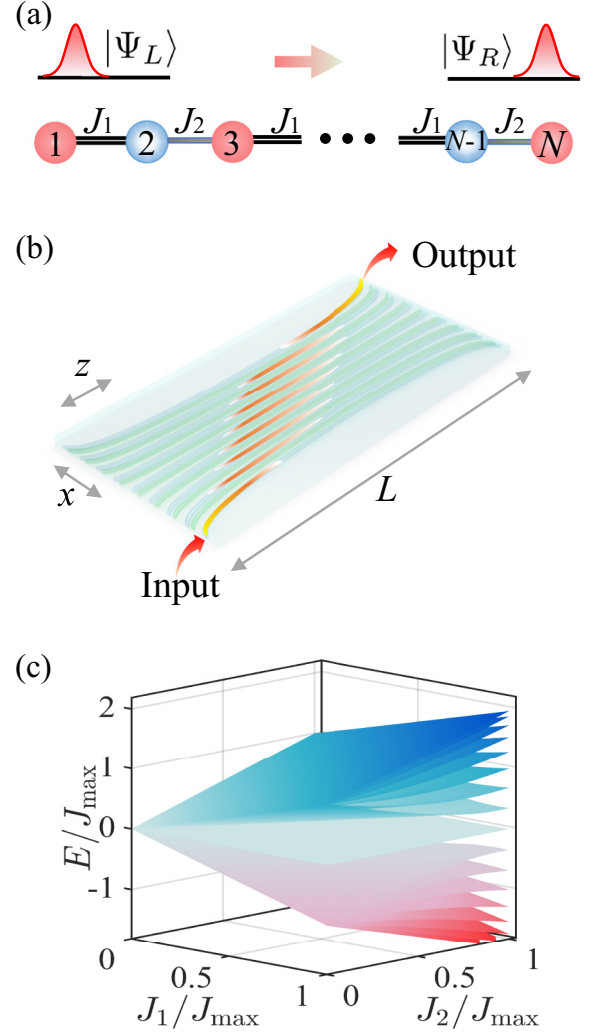


FIG. 1. (a) Schematic of a dimerized SSH model with intracell and intercell coupling strengths J_1 and J_2 , respectively. The diagram illustrates the topological pumping of excitations from the left edge state $|\Psi_L\rangle$ to the right edge state $|\Psi_R\rangle$. (b) Schematic of a spatially modulated SSH lattice implemented by femtosecond-laser-direct-writing photonic waveguides supporting a single x -polarized mode at 808 nm [50]. (c) Dependence of the energy spectrum on the normalized coupling parameters J_1/J_{max} and J_2/J_{max} , where J_{max} denotes the maximum achievable hopping rates.

characteristics due to its exponentially decaying profile. In the topologically nontrivial phase ($J_1 < J_2$), the mode is localized at the left edge, while in the trivial phase ($J_1 > J_2$), it is localized at the right edge (see Appendix B), which corresponds to the distributions $|\Psi_{L,R}\rangle$ of schematic wave packet shown in Fig. 1(a). As shown in Figs. 1(a) and 1(b), adiabatic topological pumping, transferring the light from the first waveguide to the last waveguide, is achieved when the waveguide array is structured with the spatial modulation evolving from $J_1(0) \ll J_2(0)$ to $J_1(L) \gg J_2(L)$ in the propagation direction. During the evolution, it should satisfy the adiabatic condition $|E_m(z)| \gg |\langle \Psi_m(z) | \partial_z | G(z) \rangle|$, where $E_m(z)$ is the m th nonzero eigenenergy and $|\Psi_m(z)\rangle$ is its corresponding eigenvector, with $m = \pm 1, \pm 2, \dots, \pm(N-1)/2$.

III. COUPLING ENGINEERING TOWARD THE ACL

To clarify the influence of waveguide couplings on adiabaticity, we calculate the energy spectrum of the SSH waveguide array for varying ratios $J_1/J_{\max}$ and $J_2/J_{\max}$, where $J_{\max}$ denotes the maximum coupling strength, as illustrated in Fig. 1(c). The minimum energy gap between the zero-energy band and each of the nonzero-energy bands occurs along the phase transition points where $J_1 = J_2$ (corresponding to the closing of the band gap in momentum space; see Appendix B), and the minimum energy gap increases with the magnitude of the coupling strengths. To better satisfy the adiabatic condition within a shortened device footprint while maintaining the high-fidelity topological pumping, $J_1(z)$ and $J_2(z)$ are designed such that their intersection point $J_1(z) = J_2(z)$ occurs at the position where both couplings are relatively large, but with a relatively small difference in the absolute slopes; meanwhile, the difference of their slopes can be moderately enlarged in the regions where $|J_1(z) - J_2(z)|$ is large.

The couplings are modulated by adjusting the center-to-center distances between adjacent waveguides, with the minimum and maximum distances set at $d_{\min} = 6 \mu\text{m}$ and $d_{\max} = 15 \mu\text{m}$, respectively (refer to our recent experiment in a femtosecond-laser-direct-writing waveguide platform [50]). To achieve low-loss propagation while preserving waveguide continuity, we specify the coupling strengths shaped by a power-law envelope of a trigonometric function

$$\begin{aligned} J_1(z) &= J_{\max} \sin^\alpha \left(\frac{\pi(z+z_0)}{2L'} \right), \\ J_2(z) &= J_{\max} \cos^\alpha \left(\frac{\pi(z+z_0)}{2L'} \right), \end{aligned} \quad (3)$$

where the parameters $J_{\max} = 1.75 \text{ mm}^{-1}$, $z_0 = 0.005L'$, and $L' = L/0.99$ are explicitly introduced to map the theoretical coupling profiles onto the experimentally achievable waveguide spacing range ($d \in [6, 15] \mu\text{m}$). Due to the exponential dependence of the coupling strength on the spatial separation, a strict zero coupling at the boundaries would mathematically necessitate an infinite waveguide spacing. The offset z_0 addresses this by ensuring that the initial coupling remains marginally positive, which corresponds to a maximum waveguide separation strictly confined within the upper fabrication limit $d_{\max}$. Furthermore, adopting the scaled length $L' = L/0.99$ rather than exactly matching the total length $L = L'$ due to the truncation at both ends, i.e., $L = L' - 2z_0$, this geometrical adjustment provides a critical tolerance margin for inevitable fabrication errors.

The exponent α is a dimensionless adaptive parameter introduced for the trade-off engineering and is the dominant parameter to be optimized to enhance the device performance, i.e., to guarantee better performance of topological state transfer than that based on the conventional topological transport scheme (a special case of $\alpha=2$ indeed [21,22,32,33,52–54]). The coupling strength J between adjacent waveguides depends on the modal overlap and is controlled by their separation d according to the general relation $J = Ae^{-\gamma d}$, with $A = 14.58 \text{ mm}^{-1}$ and $\gamma = 0.353 \mu\text{m}^{-1}$ [50]. In particular, we determine $\alpha = 0.65$ as an optimal parameter for achieving a trade-off in the designed coupling scheme for the case of

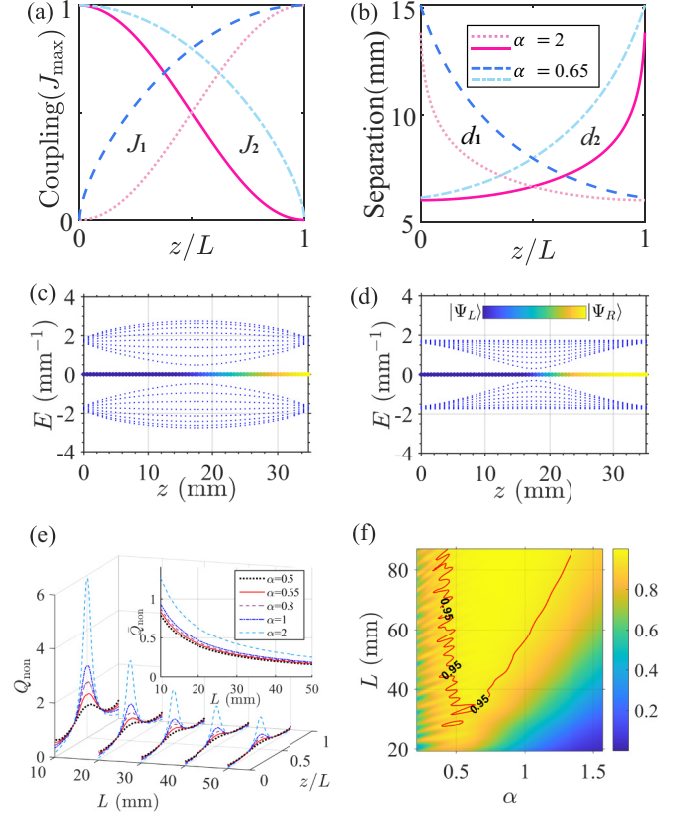


FIG. 2. (a) Envelopes of $J_1(z)$ and $J_2(z)$ and (b) corresponding interwaveguide separations $d_1(z)$ and $d_2(z)$ for the optimized ($\alpha = 0.65$) and conventional ($\alpha = 2$) adiabatic coupling schemes. Also shown is the energy spectrum of (c) the optimized coupling scheme ($\alpha = 0.65$) and (d) the conventional adiabatic scheme ($\alpha = 2$). (e) Evolution of the nonadiabatic factor $Q_{\text{non}}(z)$ and the mean $\bar{Q}_{\text{non}}(z)$ for the conventional scheme and ours with different values of α . (f) Pumping fidelity as a function of device length L for different α values, obtained by solving the CME. The red line marks the 95%-fidelity contour.

waveguide number $N = 17$, for example. Figures 2(a) and 2(b) illustrate the comparisons of envelopes of $J_1(z)$ and $J_2(z)$ and the corresponding interwaveguide separations $d_1(z)$ and $d_2(z)$ under the trade-off coupling engineering ($\alpha = 0.65$) and the conventional adiabatic scheme ($\alpha = 2$), where this systematic optimization of the coupling profile is the core of our strategy toward the ACL for multiwaveguide topological systems.

Here we quantify the ACL by estimating the total nonadiabatic leakage from the gap mode $|G(z)\rangle$ to other eigenstates $|\Psi_m(z)\rangle$. We define a dimensionless quantity termed the nonadiabatic factor

$$Q_{\text{non}}(z) = \sum_{m \neq 0} \left| \frac{\langle \Psi_m(z) | \partial_z | G(z) \rangle}{E_m(z)} \right|, \quad (4)$$

which is the direct metric to assess how close the system operates to the ACL. In addition, the evolution of $Q_{\text{non}}(z)$ can further reflect the fidelity and robustness of topological transport. In Fig. 2(e) we compare $Q_{\text{non}}(z)$ between the conventional adiabatic scheme and our coupling engineering

scheme with varying α and find that compared with the conventional one, our scheme significantly suppresses $Q_{\text{non}}(z)$, attributed to the widening of the gap and the flattening of the spectral variation around the point of the minimum energy gap, as shown in Figs. 2(c) and 2(d). Further, the peak value of $Q_{\text{non}}(z)$ decreases, but the bilateral values increase when decreasing α , indicating the existence of an optimal value of α that minimizes the total pumping length while preserving adiabaticity and transport fidelity.

It is worth noting that optimizing the pumping protocol requires a global dynamical criterion rather than merely minimizing the instantaneous nonadiabaticity factor Q_{non} . For instance, while an exponent of $\alpha = 0.5$ yields a lower peak Q_{non} in the central region and a lower mean $\bar{Q}_{\text{non}}$ [inset of Fig. 2(e)], it simultaneously induces overly rapid variations in the coupling strength at the onset and termination of the evolution. In a realistic finite-size system, these abrupt boundary modulations inevitably excite higher-order Bloch modes, leading to modal interference, boundary reflections, and, consequently, pronounced fidelity oscillations [see Figs. 2(e) and 2(f)]. In stark contrast, the optimally selected $\alpha = 0.65$ successfully averts such abrupt modal excitations while maintaining sufficiently suppressed nonadiabatic transitions in the critical region. This global optimization rigorously balances adiabatic stability against finite-size boundary effects, thereby ensuring a robust, high-fidelity state transfer over a broad range of device lengths.

In addition to the minimum value and the mean of $Q_{\text{non}}(z)$, adiabaticity is closely related to the spatial variance. Unlike conventional trigonometric protocols, the power-law profile utilizes the exponent α to tune the modulation steepness. More fundamentally, as detailed by the global adiabatic criterion derived in Appendix C, the total nonadiabatic leakage can be structurally estimated by the spatial variance σ_Q^2 of $Q_{\text{non}}(z)$. This criterion clearly reveals the physical mechanism of the evolution acceleration: To compress the device length L , the spatial evolution must be faster, which inherently increases the average nonadiabaticity $\bar{Q}_{\text{non}}$. In conventional designs where σ_Q^2 is large, the local nonadiabatic peaks act as bottlenecks, forcing the use of a long device to keep the total error within limits. By choosing the optimal α to flatten the nonadiabatic distribution (minimizing σ_Q^2), we eliminate the structural penalty from spatial inhomogeneity. This allows the system to safely tolerate a higher average evolution rate $\bar{Q}_{\text{non}}$, unlocking the ability to drastically shorten the device length L . Specifically, the selection of $\alpha = 0.65$ for our 17-waveguide system is the exact physical regime where the variance σ_Q^2 is globally minimized. Smaller α values induce overly rapid boundary modulations and pronounced fidelity oscillations due to larger spatial inhomogeneity. Conversely, larger exponents ($\alpha \geq 0.8$) demand an undesirable extension of the propagation distance. Thus, $\alpha = 0.65$ provides the optimal and grounded regime for robust, high-fidelity topological pumping within a highly compact geometry.

IV. ACCELERATION PERFORMANCES

We numerically calculate the pumping fidelity $F(z) = |C_N(z)|^2$ across a range of α values and device lengths under the initial condition $C_1(0) = 1$. A systematic analysis

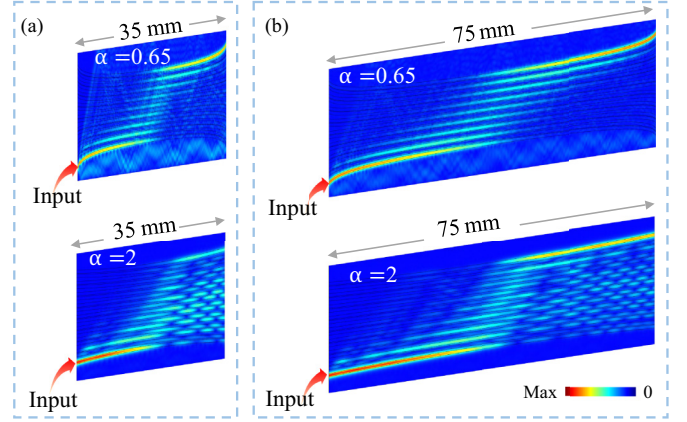


FIG. 3. Comparison of the simulated output intensity distributions for our optimized coupling scheme ($\alpha = 0.65$) and the conventional adiabatic scheme ($\alpha = 2$) at device lengths of (a) $L = 35$ mm and (b) $L = 75$ mm.

of the fidelity-length profiles is conducted by selecting several typical examples $\alpha \in \{0.5, 0.65, 0.8, 1, 2\}$ from continuous α samples, as shown in Fig. 2(f). The results demonstrate that $\alpha = 0.65$ enables the high-fidelity topological pumping together with the significantly reduced device scales. The required length with $\alpha = 0.65$ for obtaining a fidelity $F > 0.95$ is $L_{0.95} = 56$ mm, which is about a quarter of that (i.e., $L_{0.95} = 212$ mm) required for the conventional scheme with $\alpha = 2$. This represents a practical realization of approaching the ACL in a multiwaveguide topological system, achieving near-optimal performance with an analytic control scheme. To further evaluate the optimized-coupling schemes, numerical simulations based on the finite-element method were implemented by using an 808-nm laser source coupled into the first waveguide of each configuration [50]. Figures 3(a) and 3(b) illustrate the simulated light field distributions for different schemes. The results indicate that the light is highly localized at the terminal waveguide in our optimized coupling configurations. In contrast, conventional adiabatic pumping exhibits an improved terminal localization only when increasing the device length. A particularly notable distinction is observed at $L = 35$ mm, where the conventional adiabatic pumping exhibits marked bulk diffusion throughout the waveguide array, while the optimized coupling configuration demonstrates the exceptional terminal localization, exceeding even that of the conventional scheme at $L = 75$ mm. These findings demonstrate that the proposed scheme enables accelerated topological photon state transfer, realizing high-fidelity output localization in substantially shorter waveguide arrays while effectively suppressing delocalization into bulk states.

V. COMPACT TOPOLOGICAL SPLITTER

We finally demonstrate an application of our ACL-oriented optimized coupling approach to create a compact photonic beam splitter [55] by extending the conventional SSH model. The design consists of two interfaced SSH chains in distinct topological phases, with the central lattice site functioning as a tunable defect state. For the interfaced SSH chain

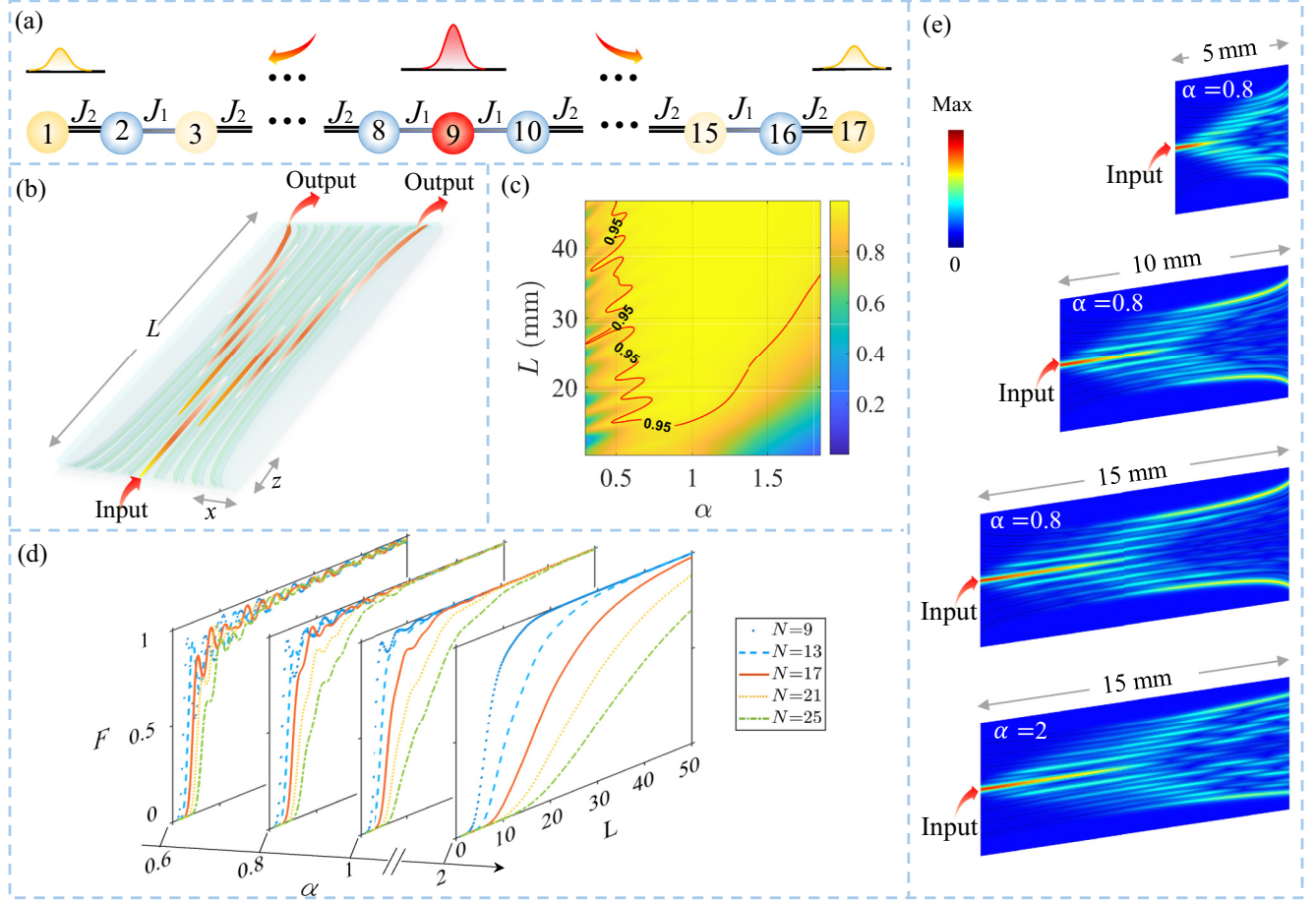


FIG. 4. (a) Schematic of an interfaced SSH configuration comprising two chains with distinct topological phases. The central waveguide acts as a defect site coupled to the input excitation. (b) Schematic of the interfaced SSH lattice implemented by a photonic waveguide array. (c) Simulated pumping fidelity map for the topological photonic splitter as a function of evolution length L and parameter α . The red line marks the 95%-fidelity contour. (d) Evolution of transfer fidelity under distinct modulation indices α and varying system sizes. (e) Comparison of simulated output diffraction patterns between three optimized ($\alpha = 0.8$) configurations with $L = 5, 10$, and 15 mm, respectively, and one conventional ($\alpha = 2$) configuration with $L = 15$ mm.

shown in Fig. 4(a) taking 17 waveguides as an example, the Hamiltonian is given by

$$\begin{aligned} \hat{H}_2(z) = & \sum_{j=1}^4 [J_2(z)|2j-1\rangle + J_1(z)|2j+1\rangle]\langle 2j| \\ & + [J_1(z)|2(j+4)-1\rangle + J_2(z)|2(j+4)+1\rangle] \\ & \times \langle 2(j+4)| + \text{H.c.} \end{aligned} \quad (5)$$

Figure 4(b) illustrates the photonic waveguide array implementing this interfaced SSH model, where the topological interface is realized through the precise engineering of interwaveguide coupling coefficients. Under the adiabatic approximation, the input light propagates along the zero-energy defect mode of the photonic lattice (see Appendix D)

$$|D(z)\rangle = N'_{\text{or}} \sum_{j=0}^4 \beta^j(z) (|9-2j\rangle + |9+2j\rangle), \quad (6)$$

where N'_{or} denotes the normalization constant.

Here the topological beam splitter is defined as a functional waveguide array that adiabatically transports an input beam into two spatially separated output ports via robust topological edge states. To rigorously evaluate the performance of this splitting mechanism, the pump fidelity is defined as $F(z) = |C_1(z)|^2 + |C_{17}(z)|^2$. Because the localized guided modes at the two distant output ends are strictly orthogonal, the total routed power must be calculated by summing their individual modal intensities. This formulation accurately captures the spatial macroscopic separation of the target ports and explicitly prevents any mathematical artifacts associated with artificial modal overlaps. In Fig. 4(c) we compare the fidelity performance across different pumping lengths for both conventional adiabatic and our optimized coupling engineering schemes with the sampled exponents $\alpha = 0.6, 0.8$, and 1 . It is found that the optimized-coupling topological pumping enables a high-fidelity photonic splitting behavior in significantly shorter device lengths compared to conventional scheme (fixed $\alpha = 2$). The optimal performance is achieved at $\alpha = 0.8$. To maintain the fidelity $F > 0.95$, the required length is obtained as $L = 8.5$ mm for $\alpha = 0.8$ and

41 mm for $\alpha = 2$. Simulations are also performed for three configurations using optimized-coupling topological pumping ($\alpha = 0.8$) at lengths $L = 5, 10$, and 15 mm, along with a reference configuration employing the conventional adiabatic pumping ($\alpha = 2$) at $L = 15$ mm.

To evaluate the influence of the system dimension and the parameter α on the state transfer fidelity, a systematic parametric mapping analysis was performed. The fidelity profiles are governed by the interplay between nonadiabatic transitions and boundary effects caused by the finite system size. For smaller α values, the rapid modulation of the coupling coefficients excites bulk supermodes [see Fig. 4(d)]. The coherent interference among these modes causes intrinsic spatial modulations in the fidelity. Additionally, for smaller arrays, the transverse expansion of the optical field during the intermediate evolution stage leads to significant boundary reflections which severely distort the fidelity curves. Increasing the system dimension effectively suppresses these boundary effects and stabilizes the topological transport. The exponent $\alpha = 0.8$ was specifically identified as the optimized parameter because it guarantees a target transfer fidelity above 0.95 while significantly minimizing the required propagation length. A comparative analysis confirms that the configuration with an α of 0.8 and 17 waveguides optimally balances a compact physical footprint and a high transfer efficiency, validating the structural robustness of the proposed protocol. As illustrated in Fig. 4(e), when the device length exceeds the critical threshold of approximately 7 mm [Fig. 4(c)] for the optimized-coupling topological pumping ($\alpha = 0.8$), as exemplified by the cases $L = 10$ and 15 mm, the output light field becomes strongly localized at both terminal waveguides, exhibiting a nearly balanced intensity distribution with approximately 50% intensity at each output port. In contrast, the conventional adiabatic topological pumping at $L = 15$ mm exhibits considerable bulk diffusion across the waveguide array. These results demonstrate that our optimized coupling scheme outperforms its conventional counterpart by enabling high-fidelity beam splitting in substantially shorter device lengths while suppressing undesired bulk-state delocalization. This functional device exemplifies how the ACL design principle can be generalized to the complex, multiport photonic circuits.

In addition, the structural robustness of the proposed rapid topological pumping protocol was systematically investigated against intrinsic propagation losses and stochastic dephasing effects, validating the practical viability of our protocol for actual integrated photonic architectures. Detailed analyses and full-wave simulations for both the unidirectional SSH model and the topological beam splitter are presented in Appendix E.

VI. CONCLUSION

We have proposed and numerically validated an optimized framework that overcomes the adiabaticity-compactness trade-off in topological photonic devices. By transcending conventional trigonometric modulations and introducing a power-law coupling profile in Su-Schrieffer-Heeger waveguide arrays, we significantly suppressed the nonadiabatic transitions by globally minimizing their spatial fluctuations.

This enabled a high-fidelity topological state transfer in the devices four times shorter than conventional adiabatic designs. Furthermore, we extended this strategy to design a compact topological photonic splitter based on an interfaced SSH lattice, demonstrating its capability for the efficient and balanced beam splitting in drastically shortened footprints. This illustrates the practical potential of our framework for building multifunctional and scalable photonic circuits. Our work bridges a critical gap in approaching the adiabaticity control limit by providing analytic and multiwaveguide control schemes, which is a key advance beyond existing numerical and geometrically restricted inverse-design approaches. By reconciling the conflicting demands of adiabatic fidelity and device miniaturization, this strategy paves the way toward high-density, topologically protected integrated photonic platforms, where fast, reliable, and coherent on-chip state manipulation is crucial for advancing quantum information processing and classical integrated photonics alike.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: DERIVATION OF ZERO-ENERGY GAP MODE

Consider a general SSH chain described by the Hamiltonian

$$\hat{H}(z) = \sum_{j=1} [J_1(z)|2j-1\rangle + J_2(z)|2j+1\rangle][\langle 2j| + \text{H.c.}] \quad (\text{A1})$$

Based on the properties of the system, the gap mode can be expressed with the ansatz [17,21,51,56,57]

$$|G(z)\rangle \sim \sum_{j=1} \beta^j(z)(m|2j-1\rangle + n|2j\rangle), \quad (\text{A2})$$

where the symbol $\sim$ indicates that the normalization factor is omitted. The probability amplitude on the j th site decays exponentially for $\beta < 1$ (corresponding to a left edge state) and increases exponentially for $\beta > 1$ (corresponding to a right edge state). Through the eigenvalue equation $E_0|G(z)\rangle = \hat{H}(z)|G(z)\rangle$,

one can obtain

$$\begin{aligned}
 E_0 \sum_{j=1} \beta^j(z) (m|2j-1\rangle + n|2j\rangle) &= \left(\sum_{j=1} [J_1(z)|2j-1\rangle + J_2(z)|2j+1\rangle] \langle 2j| + |2j\rangle [J_1(z)\langle 2j-1| + J_2(z)\langle 2j+1|] \right) \\
 &\quad \times \left(\sum_{j'=1} \beta^{j'}(z) (m|2j'-1\rangle + n|2j'\rangle) \right) \\
 &= \sum_{j'=j} \beta^{j'}(z) n [J_1(z)|2j-1\rangle + J_2(z)|2j+1\rangle] \langle 2j|2j'\rangle + \sum_{j'=j} \beta^{j'} J_1(z) m |2j\rangle \langle 2j-1|2j'-1\rangle \\
 &\quad + \sum_{j'=j+1} \beta^{j'} J_2(z) m |2j\rangle \langle 2j+1|2j'-1\rangle \\
 &= \sum_{j=1} \{J_1(z)\beta^j(z)(n|2j-1\rangle + m|2j\rangle) + J_2(z)[\beta^j(z)n|2j+1\rangle + \beta^{j+1}(z)m|2j\rangle]\}. \quad (\text{A3})
 \end{aligned}$$

When $n = 0$ ($m = 0$), Eq. (A2) specifies that the odd-numbered (even-numbered) waveguides are occupied by the edge state with an eigenenergy of $E_0 = 0$. For the odd-sized SSH chain considered in this work, both the left and right edge states reside on the odd-numbered waveguides. This condition requires $m = 1$ and $n = 0$ in Eq. (A3), leading to the relation $J_1 + J_2\beta = 0$. Consequently, the edge state localized on the odd-numbered waveguides can be derived as

$$|G(z)\rangle \sim \sum_{j=1}^{(N+1)/2} \left(-\frac{J_1(z)}{J_2(z)} \right)^j |2j-1\rangle. \quad (\text{A4})$$

APPENDIX B: SSH MODEL IN TOPOLOGICALLY NONTRIVIAL AND TRIVIAL PHASES

To investigate the topological properties of an SSH chain described by Eq. (A1), we apply the Fourier transformations $|A_k\rangle = \frac{1}{\sqrt{M}} \sum_j e^{ikj} |2j-1\rangle$ and $|B_k\rangle = \frac{1}{\sqrt{M}} \sum_j e^{ikj} |2j\rangle$ with the wave number $k \in [-\pi, \pi]$ to transform the Hamiltonian as

$$\hat{H} = \sum_k \begin{pmatrix} |A_k\rangle & |B_k\rangle \end{pmatrix} \hat{H}_k \begin{pmatrix} |A_k\rangle \\ |B_k\rangle \end{pmatrix}. \quad (\text{B1})$$

Under the periodic boundary condition, the bulk momentum-space Hamiltonian takes the matrix form

$$\hat{H}_k = \begin{pmatrix} 0 & J_1 + J_2 e^{-ik} \\ J_1 + J_2 e^{ik} & 0 \end{pmatrix}. \quad (\text{B2})$$

The eigenenergies and corresponding eigenstates of this momentum-space Hamiltonian are derived as

$$E_{\pm}(k) = \pm \sqrt{J_1^2 + J_2^2 + 2J_1 J_2 \cos(k)} \quad (\text{B3})$$

and

$$|\varphi_{\pm}(k)\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi(k)} \\ \pm 1 \end{pmatrix}, \quad (\text{B4})$$

where the phase $\phi(k)$ is defined by $\cot \phi(k) = -\beta(z)/\sin k + \cot k$. To facilitate the topological characterization, we decompose the Hamiltonian H_k of Eq. (B2) into the vector

form as

$$H(k) = \mathbf{d} \cdot \boldsymbol{\sigma}, \quad (\text{B5})$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ denotes the vector of Pauli matrices and $\mathbf{d} = (d_x, d_y, d_z) = (J_1 + J_2 \cos k, J_2 \sin k, 0)$ represents the corresponding Hamiltonian vector. By defining the unit vector $\hat{\mathbf{d}} = \mathbf{d}/|\mathbf{d}|$, the winding number w geometrically counts the number of times that the vector field winds around the origin of the parameter space. The winding number w is defined as

$$w = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\hat{\mathbf{d}} \times \frac{d\hat{\mathbf{d}}}{dk} \right) dk. \quad (\text{B6})$$

Physically, this model falls into the BDI symmetry class [58]. A topological phase transition occurs at $J_1 = J_2$, where the band gap closes at the Brillouin zone boundary: $k = \pm\pi$ and $E_{\pm}(k = \pm\pi) = 0$. The regimes $J_1 > J_2$ and $J_1 < J_2$ correspond to the topologically distinct phases. For $J_1 < J_2$, the trajectory of $\mathbf{d}(k)$ encloses the origin, yielding $w = 1$. According to the bulk-boundary correspondence, this signifies a topologically nontrivial phase hosting the protected zero-energy edge states. For $J_1 > J_2$, the trajectory excludes the origin, resulting in $w = 0$, which corresponds to a trivial phase.

APPENDIX C: DETERMINING OPTIMAL α BY THE GLOBAL ADIABATIC CRITERION

In this Appendix we establish the global adiabatic criterion to explain the physical mechanism behind the optimized power-law coupling scheme and the selection of the parameter α . The evolution of the system is governed by the CME $H(z)|\Psi(z)\rangle = -i\partial_z|\Psi(z)\rangle$. By expanding the state vector in the instantaneous eigenbasis $|\Psi(z)\rangle = \sum_n a_n(z)|\Psi_n(z)\rangle$, where $|\Psi_0(z)\rangle \equiv |G(z)\rangle$ is the target zero-energy mode, the total nonadiabatic transition probability (infidelity) to all other modes at the output $z = L$ is $\varepsilon = \sum_{m \neq 0} |a_m(L)|^2$.

Under the adiabatic assumption [$|a_0(z)| \approx 1$], solving the first-order differential equations for $a_m(z)$ and applying the triangle inequality yields

$$\varepsilon \leq \left(\sum_{m \neq 0} \int_0^L |\langle \Psi_m(z) | \partial_z G(z) \rangle| dz \right)^2. \quad (\text{C1})$$

Recalling the definition of the instantaneous nonadiabaticity factor $Q_{\text{non}}(z)$ in Eq. (4), we can relate this to the energy gaps $|E_m(z)|$. Using the global minimum energy gap $\Delta E_{\text{min}} \equiv \min_{z,m} |E_m(z)|$, we consider the worst-case scenario where all nonadiabatic couplings act constructively and the energy gap is as small as possible. To obtain a sufficient condition in a physically conservative sense, this leads to an estimate (not a rigorous mathematical bound) of the infidelity

$$\varepsilon \leq \left(\Delta E_{\text{min}} \int_0^L Q_{\text{non}}(z) dz \right)^2. \quad (\text{C2})$$

While this estimate is derived from a worst-case heuristic, it accurately captures the essential dependence on the statistical properties of $Q_{\text{non}}(z)$, serving as a reliable design principle.

Applying the Cauchy-Schwarz inequality to the integral of $Q_{\text{non}}(z)$, we obtain $\int_0^L Q_{\text{non}}(z) dz \leq \sqrt{L \int_0^L Q_{\text{non}}^2(z) dz}$. We then express the second moment of $Q_{\text{non}}(z)$ in terms of its spatial mean $\bar{Q}_{\text{non}} = \frac{1}{L} \int_0^L Q_{\text{non}}(z) dz$ and its spatial variance $\sigma_Q^2 = \frac{1}{L} \int_0^L [Q_{\text{non}}(z) - \bar{Q}_{\text{non}}]^2 dz$. Substituting this result yields the global adiabatic criterion

$$\varepsilon \leq (\Delta E_{\text{min}} L \sqrt{\bar{Q}_{\text{non}}^2 + \sigma_Q^2})^2. \quad (\text{C3})$$

Equation (C3) reveals crucial structural insight: The infidelity scales with $L^2(\bar{Q}_{\text{non}}^2 + \sigma_Q^2)$, and spatial fluctuations σ_Q^2 contribute directly to this scaling. To achieve fast (small L) yet high-fidelity evolution, the optimal strategy is to minimize the variance σ_Q^2 . The power-law modulation in our framework provides the exact degree of freedom α needed to homogenize $Q_{\text{non}}(z)$ along the waveguide array. As shown in Fig. 5, calculating the variance σ_Q^2 across the α - L parameter space demonstrates that σ_Q^2 reaches its physical minimum around $\alpha = 0.65$. This analytically confirms that the chosen exponent is the optimal value that suppresses spatial nonadiabatic fluctuations, thereby bridging the theoretical ACL and practical compact device design.

APPENDIX D: DEFECT STATE OF THE INTERFACED SSH CHAIN

Considering the interfaced SSH chain depicted in Fig. 4(a), we further analyze the nature of system by introducing a basis of nonlocal normal modes $\{|\phi_{j,18-j}^\pm\rangle\}$, formed by the waveguide pairs that are symmetric with respect to the central (ninth) waveguide as

$$|\phi_{j,18-j}^\pm\rangle = \frac{1}{\sqrt{2}}(|j\rangle \pm |18-j\rangle), \quad j = 1, 2, \dots, 8. \quad (\text{D1})$$

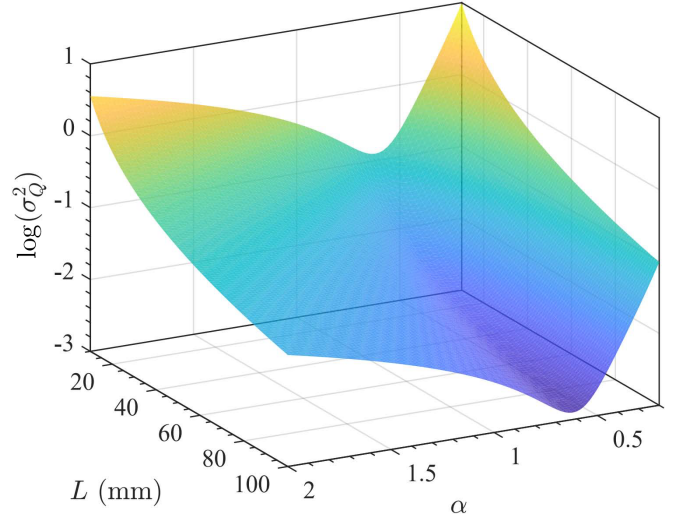


FIG. 5. Logarithmic plot of the variance σ_Q^2 in the α - L plane. It confirms that the minimum σ_Q^2 appears around $\alpha = 0.65$, minimizing nonadiabatic fluctuations.

Substituting these modes into Eq. (5), the Hamiltonian transforms into

$$\begin{aligned} \hat{H}'_2(z) = & \sqrt{2}J_1(z)|9\rangle\langle\phi_{8,10}^+| + \frac{J_2(z)}{2} \sum_{p=\pm} \sum_{q=1}^4 (|\phi_{2q,18-2q}^+ \rangle \\ & + p|\phi_{2q,18-2q}^- \rangle)(\langle\phi_{2q-1,19-2q}^+| + p\langle\phi_{2q-1,19-2q}^-|) \\ & + \frac{J_1(z)}{2} \sum_{p=\pm} \sum_{q=1}^3 (|\phi_{2q+1,17-2q}^+ \rangle + p|\phi_{2q+1,17-2q}^- \rangle) \\ & \times (\langle\phi_{2q,18-2q}^+| + p\langle\phi_{2q,18-2q}^-|) + \text{H.c.} \end{aligned} \quad (\text{D2})$$

Given the mirror symmetry of the waveguide system, the odd-parity normal modes (denoted by the minus sign superscript) decouple from the dynamics. Consequently, the 17-waveguide system is transformed into a nine-mode system governed by the Hamiltonian

$$\begin{aligned} \hat{H}'_2(z) = & \sqrt{2}J_1(z)|9\rangle\langle\phi_{8,10}^+| + J_2(z)|\phi_{8,10}^+\rangle\langle\phi_{7,11}^+| \\ & + J_1(z)|\phi_{7,11}^+\rangle\langle\phi_{6,12}^+| + J_2(z)|\phi_{6,12}^+\rangle\langle\phi_{5,13}^+| \\ & + J_1(z)|\phi_{5,13}^+\rangle\langle\phi_{4,14}^+| + J_2(z)|\phi_{4,14}^+\rangle\langle\phi_{3,15}^+| \\ & + J_1(z)|\phi_{3,15}^+\rangle\langle\phi_{2,16}^+| + J_2(z)|\phi_{2,16}^+\rangle\langle\phi_{1,17}^+| \\ & + \text{H.c.} \end{aligned} \quad (\text{D3})$$

Apart from the prefactor of $\sqrt{2}$ in the first term, it is exactly a standard nine-site SSH chain. In our waveguide system, the coupling strength $J_1(z)$ in the first term can be reduced by a factor of $\sqrt{2}$ to rigorously map the system onto a standard SSH chain. Alternatively, this factor can be neglected due to the inherent topological protection of the SSH model, which ensures the robustness against disorder in coupling strengths. Given the above analysis, the zero-energy mode corresponding to Eq. (6) can be obtained via a derivation similar to Eq. (A4).

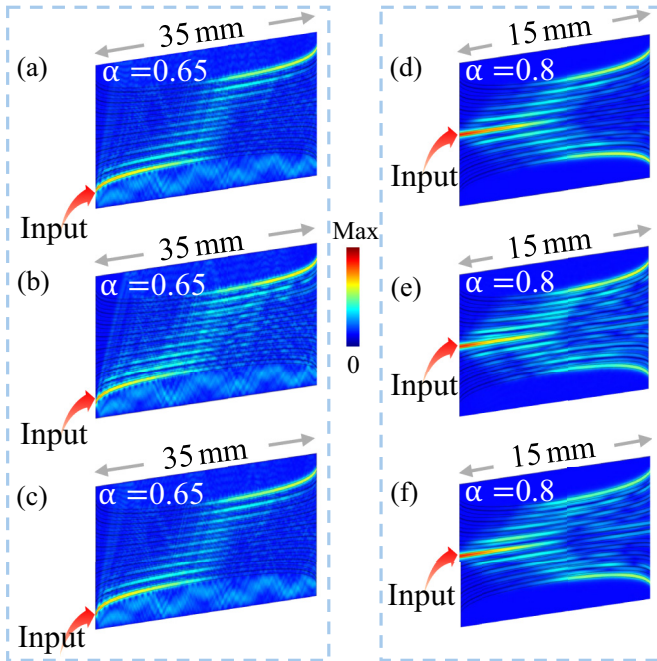


FIG. 6. Comparison of the simulated output intensity distributions for (a)–(c) the optimized coupling scheme ($\alpha = 0.65$ and $L = 35$ mm) and (d)–(f) the beam splitting model ($\alpha = 0.8$ and $L = 15$ mm). Within each sequence, the three panels represent systems subjected solely to loss ($\Gamma = 1 \times 10^{-6}$), solely to dephasing ($\Delta = 1 \times 10^{-4}$), and to both perturbations concurrently.

APPENDIX E: ROBUSTNESS UNDER INTRINSIC DISSIPATION AND STOCHASTIC DEPHASING

To evaluate the experimental feasibility of the proposed topological pumping protocol, we simulate its resilience against intrinsic propagation losses and stochastic dephasing. Dissipation is phenomenologically incorporated by adding a complex diagonal term $-i\Gamma$ to the propagation constants, with $\Gamma = 1 \times 10^{-6} \text{ mm}^{-1}$ [59] representing standard dielectric waveguide attenuation. Dephasing effects arising from fabrication tolerances are introduced as random on-site potential detunings $\Delta_n = \Delta\delta$. Here the perturbation strength is $\Delta = 1 \times 10^{-4}$ [60] and δ is a uniformly distributed random variable within the interval $[-1, 1]$. We applied these combined nonideal perturbations to both the unidirectional SSH model ($\alpha = 0.65$ and $L = 35$ mm) and the topological beam splitter ($\alpha = 0.8$ and $L = 15$ mm).

As illustrated in Figs. 6(a)–6(c) for the unidirectional case, the adiabatic pumping trajectory exhibits global stability and preserves its spatial confinement despite the concurrent presence of scalar attenuation and phase noise. Similarly, the beam splitting dynamics shown in Figs. 6(d)–6(f) maintains high-fidelity power division under identical structural imperfections. The preservation of these deterministic evolutionary pathways confirms that the dynamically engineered adiabatic energy gap provides robust topological protection against both localized refractive index fluctuations and material absorption. This inherent structural resilience validates the practical viability of the protocol for actual integrated photonic architectures.

- [1] J. Wang, F. Sciarrino, A. Laing, and M. G. Thompson, Integrated photonic quantum technologies, *Nat. Photon.* **14**, 273 (2020).
- [2] H.-S. Zhong, H. Wang, Y.-H. Deng, M.-C. Chen, L.-C. Peng, Y.-H. Luo, J. Qin, D. Wu, X. Ding, Y. Hu, P. Hu, X.-Y. Yang, W.-J. Zhang, H. Li, Y. Li, X. Jiang, L. Gan, G. Yang, L. You, Z. Wang, *et al.*, Quantum computational advantage using photons, *Science* **370**, 1460 (2020).
- [3] E. Pelucchi, G. Fagas, I. Aharonovich, D. Englund, E. Figueroa, Q. Gong, H. Hannes, J. Liu, C.-Y. Lu, N. Matsuda, J.-W. Pan, F. Schreck, F. Sciarrino, C. Silberhorn, J. Wang, and K. D. Jöns, The potential and global outlook of integrated photonics for quantum technologies, *Nat. Rev. Phys.* **4**, 194 (2022).
- [4] K. Azuma, S. E. Economou, D. Elkouss, P. Hilaire, L. Jiang, H.-K. Lo, and I. Tzitrin, Quantum repeaters: From quantum networks to the quantum internet, *Rev. Mod. Phys.* **95**, 045006 (2023).
- [5] D. Stefanatos and E. Paspalakis, A shortcut tour of quantum control methods for modern quantum technologies, *Europhys. Lett.* **132**, 60001 (2020).
- [6] S. Li, T. Chen, and Z.-Y. Xue, Fast holonomic quantum computation on superconducting circuits with optimal control, *Adv. Quantum Technol.* **3**, 2000001 (2020).
- [7] M. M. Müller, R. S. Said, F. Jelezko, T. Calarco, and S. Montangero, One decade of quantum optimal control in the chopped random basis, *Rep. Prog. Phys.* **85**, 076001 (2022).
- [8] J.-X. Han, J.-L. Wu, Y. Wang, Y. Xia, Y.-Y. Jiang, and J. Song, Efficient large-scale entanglement with domain-divided topological synthetic dimension, *Adv. Quantum Technol.* **8**, e00362 (2025).
- [9] L. Lu, J. D. Joannopoulos, and M. Soljačić, Topological photonics, *Nat. Photonics* **8**, 821 (2014).
- [10] J. Noh, W. A. Benalcazar, S. Huang, M. J. Collins, K. P. Chen, T. L. Hughes, and M. C. Rechtsman, Topological protection of photonic mid-gap defect modes, *Nat. Photon.* **12**, 408 (2018).
- [11] T. Ozawa, H. M. Price, A. Amo, N. Goldman, M. Hafezi, L. Lu, M. C. Rechtsman, D. Schuster, J. Simon, O. Zilberberg, and I. Carusotto, Topological photonics, *Rev. Mod. Phys.* **91**, 015006 (2019).
- [12] X. Ni, S. Yves, A. Krasnok, and A. Alù, Topological metamaterials, *Chem. Rev.* **123**, 7585 (2023).
- [13] A. Szameit and M. C. Rechtsman, Discrete nonlinear topological photonics, *Nat. Phys.* **20**, 905 (2024).
- [14] J. Wang, K. Li, and Z. Quan, Integrated structured light manipulation, *Photonics Insights* **3**, R05 (2024).
- [15] J. Gao, Z.-S. Xu, Z. Yang, V. Zwiller, and A. W. Elshaari, Quantum topological photonics with special focus on waveguide systems, *npj Nanophotonics* **1**, 34 (2024).
- [16] W. Yan, B. Zhang, and F. Chen, Photonic topological insulators in femtosecond laser direct-written waveguides, *npj Nanophotonics* **1**, 40 (2024).

- [17] W. P. Su, J. R. Schrieffer, and A. J. Heeger, Solitons in polyacetylene, *Phys. Rev. Lett.* **42**, 1698 (1979).
- [18] Y. E. Kraus, Y. Lahini, Z. Ringel, M. Verbin, and O. Zilberberg, Topological states and adiabatic pumping in quasicrystals, *Phys. Rev. Lett.* **109**, 106402 (2012).
- [19] M. Lohse, C. Schweizer, H. M. Price, O. Zilberberg, and I. Bloch, Exploring 4D quantum Hall physics with a 2D topological charge pump, *Nature (London)* **553**, 55 (2018).
- [20] O. Zilberberg, S. Huang, J. Guglielmon, M. Wang, K. P. Chen, Y. E. Kraus, and M. C. Rechtsman, Photonic topological boundary pumping as a probe of 4D quantum Hall physics, *Nature (London)* **553**, 59 (2018).
- [21] F. Mei, G. Chen, L. Tian, S.-L. Zhu, and S. Jia, Robust quantum state transfer via topological edge states in superconducting qubit chains, *Phys. Rev. A* **98**, 012331 (2018).
- [22] M. Jürgensen, S. Mukherjee, and M. C. Rechtsman, Quantized nonlinear Thouless pumping, *Nature (London)* **596**, 63 (2021).
- [23] Y.-K. Sun, X.-L. Zhang, F. Yu, Z.-N. Tian, Q.-D. Chen, and H.-B. Sun, Non-Abelian Thouless pumping in photonic waveguides, *Nat. Phys.* **18**, 1080 (2022).
- [24] Q. Cheng, H. Wang, Y. Ke, T. Chen, Y. Yu, Y. S. Kivshar, C. Lee, and Y. Pan, Asymmetric topological pumping in nonparaxial photonics, *Nat. Commun.* **13**, 249 (2022).
- [25] R. Citro and M. Aidelsburger, Thouless pumping and topology, *Nat. Rev. Phys.* **5**, 87 (2023).
- [26] A.-S. Walter, Z. Zhu, M. Gächter, J. Minguzzi, S. Roschinski, K. Sandholzer, K. Viebahn, and T. Esslinger, Quantization and its breakdown in a Hubbard–Thouless pump, *Nat. Phys.* **19**, 1471 (2023).
- [27] T. Tian, Y. Zhang, L. Zhang, L. Wu, S. Lin, J. Zhou, C.-K. Duan, J.-H. Jiang, and J. Du, Experimental realization of nonreciprocal adiabatic transfer of phonons in a dynamically modulated nanomechanical topological insulator, *Phys. Rev. Lett.* **129**, 215901 (2022).
- [28] W. Liu, C. Wu, Y. Jia, S. Jia, G. Chen, and F. Chen, Observation of edge-to-edge topological transport in a photonic lattice, *Phys. Rev. A* **105**, L061502 (2022).
- [29] J. Deng, H. Dong, C. Zhang, Y. Wu, J. Yuan, X. Zhu, F. Jin, H. Li, Z. Wang, H. Cai, C. Song, H. Wang, J. Q. You, and D.-W. Wang, Observing the quantum topology of light, *Science* **378**, 966 (2022).
- [30] C. Wu, W. Liu, Y. Jia, G. Chen, and F. Chen, Observation of topological pumping of a defect state in a Fock photonic lattice, *Phys. Rev. A* **107**, 033501 (2023).
- [31] X. Liu, Z. Lin, W. Song, J. Sun, C. Huang, S. Wu, X. Xiao, H. Xin, S. Zhu, and T. Li, Perfect excitation of topological states by supersymmetric waveguides, *Phys. Rev. Lett.* **132**, 016601 (2024).
- [32] S. Wu, W. Song, J. Sun, J. Li, Z. Lin, X. Liu, S. Zhu, and T. Li, Approaching the adiabatic infimum of topological pumps on thin-film lithium niobate waveguides, *Nat. Commun.* **15**, 9805 (2024).
- [33] W. Song, O. You, J. Sun, S. Wu, C. Chen, C. Huang, K. Qiu, S. Zhu, S. Zhang, and T. Li, Fast topological pumps via quantum metric engineering on photonic chips, *Sci. Adv.* **10**, eadn5028 (2024).
- [34] T. Tian, H. Cai, L. Zhang, Y. Zhang, C.-K. Duan, and J. Zhou, Nonadiabatic topological transfer in a nanomechanical phononic lattice, *Phys. Rev. B* **109**, 125123 (2024).
- [35] Z. Guan, H. Liu, R. Zheng, J. Liang, M. Ke, J. Lu, W. Deng, X. Huang, and Z. Liu, Topological pumping in acoustic Fock lattices, *Phys. Rev. Appl.* **21**, 064068 (2024).
- [36] J. Yao, S. Tang, C. Lü, J. Zhang, J. Song, and Y. Jiang, Fast energy transfer in an acoustic multicavity coupler based on the Su-Schrieffer-Heeger topological model, *Phys. Rev. Appl.* **22**, 044009 (2024).
- [37] I. L. Garanovich, S. Longhi, A. A. Sukhorukov, and Y. S. Kivshar, Light propagation and localization in modulated photonic lattices and waveguides, *Phys. Rep.* **518**, 1 (2012).
- [38] R. Menchon-Enrich, A. Benseny, V. Ahufinger, A. D. Greentree, T. Busch, and J. Mompart, Spatial adiabatic passage: A review of recent progress, *Rep. Prog. Phys.* **79**, 074401 (2016).
- [39] L. Privitera, A. Russomanno, R. Citro, and G. E. Santoro, Nonadiabatic breaking of topological pumping, *Phys. Rev. Lett.* **120**, 106601 (2018).
- [40] N. V. Vitanov, A. A. Rangelov, B. W. Shore, and K. Bergmann, Stimulated Raman adiabatic passage in physics, chemistry, and beyond, *Rev. Mod. Phys.* **89**, 015006 (2017).
- [41] D. Guéry-Odelin, A. Ruschhaupt, A. Kiely, E. Torrontegui, S. Martínez-Garaot, and J. G. Muga, Shortcuts to adiabaticity: Concepts, methods, and applications, *Rev. Mod. Phys.* **91**, 045001 (2019).
- [42] F. M. D’Angelis, F. A. Pinheiro, D. Guéry-Odelin, S. Longhi, and F. Impens, Fast and robust quantum state transfer in a topological Su-Schrieffer-Heeger chain with next-to-nearest-neighbor interactions, *Phys. Rev. Res.* **2**, 033475 (2020).
- [43] J.-K. Guo, J.-L. Wu, J. Cao, S. Zhang, and S.-L. Su, Shortcut engineering for accelerating topological quantum state transfers in optomechanical lattices, *Phys. Rev. A* **110**, 043510 (2024).
- [44] W. Liu, Y. Ke, and C. Lee, Shortcuts to adiabatic topological pumping, *Phys. Rev. A* **112**, 013317 (2025).
- [45] E. J. Bergholtz, J. C. Budich, and F. K. Kunst, Exceptional topology of non-Hermitian systems, *Rev. Mod. Phys.* **93**, 015005 (2021).
- [46] X. Zhang, F. Zangeneh-Nejad, Z.-G. Chen, M.-H. Lu, and J. Christensen, A second wave of topological phenomena in photonics and acoustics, *Nature (London)* **618**, 687 (2023).
- [47] A. Li, H. Wei, M. Cotrufo, W. Chen, S. Mann, X. Ni, B. Xu, J. Chen, J. Wang, S. Fan, C.-W. Qiu, A. Alù, and L. Chen, Exceptional points and non-Hermitian photonics at the nanoscale, *Nat. Nanotechnol.* **18**, 706 (2023).
- [48] Z.-K. Lin, Q. Wang, Y. Liu, H. Xue, B. Zhang, Y. Chong, and J.-H. Jiang, Topological phenomena at defects in acoustic, photonic and solid-state lattices, *Nat. Rev. Phys.* **5**, 483 (2023).
- [49] X. Liu, W. Song, J. Sun, S. Wu, Y. Zhu, Z. Lin, C. Huang, S. Zhu, and T. Li, Approaching optimal light evolution at adiabaticity control limit in inverse-designed waveguides, *Phys. Rev. Lett.* **135**, 266601 (2025).
- [50] J.-L. Wu, K.-H. Xiao, X. Ni, J.-K. Guo, X.-L. Zhang, Q.-D. Chen, Y. Wang, Z.-Z. Li, S.-L. Su, Z.-N. Tian, and H.-B. Sun, Superadiabatic topological pumping on photonic chips, *Nat. Commun.* **17**, 966 (2026).
- [51] J.-X. Han, J.-L. Wu, Y. Wang, Y. Xia, Y.-Y. Jiang, and J. Song, Large-scale Greenberger-Horne-Zeilinger states through a topologically protected zero-energy mode in a superconducting qutrit-resonator chain, *Phys. Rev. A* **103**, 032402 (2021).

- [52] S. Hu, Y. Ke, and C. Lee, Topological quantum transport and spatial entanglement distribution via a disordered bulk channel, *Phys. Rev. A* **101**, 052323 (2020).
- [53] L. Qi, G.-L. Wang, S. Liu, S. Zhang, and H.-F. Wang, Controllable photonic and phononic topological state transfers in a small optomechanical lattice, *Opt. Lett.* **45**, 2018 (2020).
- [54] L. Qi, G.-L. Wang, S. Liu, S. Zhang, and H.-F. Wang, Engineering the topological state transfer and topological beam splitter in an even-sized Su-Schrieffer-Heeger chain, *Phys. Rev. A* **102**, 022404 (2020).
- [55] L. Du, J.-H. Wu, M. Artoni, and G. C. La Rocca, Phase-dependent topological interface state and spatial adiabatic passage in a generalized Su-Schrieffer-Heeger model, *Phys. Rev. A* **100**, 012112 (2019).
- [56] W. P. Su, J. R. Schrieffer, and A. J. Heeger, Soliton excitations in polyacetylene, *Phys. Rev. B* **22**, 2099 (1980).
- [57] J. K. Asbóth, L. Oroszlány, and A. Pályi, *A Short Course on Topological Insulators: Band Structure and Edge States in One and Two Dimensions*, Lecture Notes in Physics (Springer, Cham, 2016), Vol. 919, pp. 1–22.
- [58] A. Altland and M. R. Zirnbauer, Nonstandard symmetry classes in mesoscopic normal-superconducting hybrid structures, *Phys. Rev. B* **55**, 1142 (1997).
- [59] Z. Liu, Y. Liao, Z. Fang, W. Chu, and Y. Cheng, Suppression of bend loss in writing of three-dimensional optical waveguides with femtosecond laser pulses, *Sci. China Phys. Mech. Astron.* **61**, 70322 (2018).
- [60] H. R. Philipp, Silicon Dioxide (SiO₂) (Glass), in *Handbook of Optical Constants of Solids*, edited by E. D. Palik (Academic Press, New York, 1985), pp. 749–763.