

Momentum distributions and probabilities of near-axis superkicks

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We present a purely theoretical, fully analytical momentum-space description of “superkick” momentum transfer to a two-level atom driven by a tightly focused, circularly polarized Laguerre-Gaussian vortex beam. Using the short-time excitation amplitude $\psi_e(\mathbf{r}, t) \propto E_z(\mathbf{r})\psi_g(\mathbf{r})$ and an exact Fourier-transform treatment in the near-focus regime, we derive closed-form expressions for the excited-state wave function $\tilde{\psi}_e(\mathbf{K})$ and its momentum distribution $|\tilde{\psi}_e(\mathbf{K})|^2$. This framework yields direct superkick diagnostics such as peak displacement and transverse-momentum moments, and enables a systematic comparison between opposite-handedness and same-handedness configurations, as well as a paraxial reference case. We show that tight focusing, through the longitudinal-field enhancement, strongly increases the near-axis excitation and makes superkick signatures most prominent in the opposite-handedness configuration, whereas the same-handedness and paraxial reference cases retain related transverse superkick structure but carry much smaller near-axis probability weight.

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I. INTRODUCTION

Optical momentum transfer to atoms is most often discussed against the plane-wave benchmark. Each absorbed photon contributes a momentum $\hbar k$ along the propagation direction, where $\hbar$ is the reduced Planck’s constant and $k = 2\pi/\lambda$ is the optical wave number. Optical vortices depart from this simple picture because their phase is structured. A vortex mode carries a helical factor $e^{i\ell\phi}$, where ϕ is the azimuthal angle about the propagation axis and ℓ is the azimuthal index, producing an on-axis phase singularity and an azimuthal phase winding [1]. As a result, within the paraxial description the optical phase $\Phi(\rho, \phi, z) = \ell\phi + kz$ implies a local wave vector $\mathbf{k}_{\text{loc}} = \nabla\Phi = \ell\hat{\phi}/\rho + k\hat{z}$, so that the transverse component scales as $|k_{\phi}^{\text{loc}}| \sim |\ell|/\rho$ and can greatly exceed the plane-wave wave number k near the vortex core [2].

Barnett and Berry emphasized that this near-axis enhancement enables *superkicks*, i.e., momentum transfers that can exceed the usual $\hbar k$ scale, but also noted a fundamental obstacle: for ideal paraxial vortices the intensity vanishes at the core, so the probability for absorption of a photon with a wavevector large in magnitude $|\mathbf{k}_{\text{loc}}|$ is intrinsically small, making the effect “superweak” in practice [2].

A key route to overcoming this limitation is to move beyond scalar paraxial fields. When a vortex beam is tightly focused, vectorial diffraction generates a longitudinal electric-field component E_z that is absent in the simplest paraxial treatments [3]. Near the optical axis, E_z can become comparable to, or even exceed, the transverse components, producing a nonvanishing near-axis intensity and altering the local momentum and energy flow [4–6]. For polarized structured beams, this behavior is further linked to spin-orbit interaction and the associated redistribution of momentum and energy flow in the focal region [7,8].

Longitudinal-field signatures have been directly observed and quantified in tightly focused beams, establishing E_z as a real, measurable component of the focal field [9–11]. Once present, E_z reshapes light-matter coupling near the optical axis, opening excitation channels and modifying transition behavior according to the local field structure rather than transverse selection rules alone [12,13]. Turning to the related problem of superkick observability, scattering-based studies treat orbital-angular-momentum (OAM)-carrying photons within in-out-state approaches and predict OAM-dependent recoil and transverse-momentum fingerprints, with the proposed observability criteria based on shifts of the $K_{\perp}$ spectrum and on selection-rule restrictions [14–16].

These treatments, however, leave out the focusing-induced longitudinal coupling that is central to the present problem. This point is crucial: tight focusing can make E_z nonzero in the near-axis region, so the atom can be excited in the same spatial domain where the vortex phase gradient reaches its largest values. Thus, while the superkick itself is associated with the large local phase gradient, the focused longitudinal field increases the probability of detecting this effect by providing considerable excitation in the near-axis region.

In a recent work of ours [17], we considered a two-level atom illuminated by a tightly focused circularly polarized vortex beam and isolated the role of the longitudinal field component E_z . To do so, we chose strontium so that the selection rules restrict the coupling in a way that allows the effect of E_z to be identified clearly. Within the resonant scattering-force regime, we showed that the longitudinal field generated by tight focusing can make the near-axis superkick effect experimentally accessible. In that work, we derived analytical expressions for the local wave vector and the scattering-force components at zero detuning, identified the handedness and near-axis conditions under which the azimuthal force becomes strongly enhanced, and demonstrated that the effect is no longer merely “superweak” once the focused longitudinal contribution is taken into account. The present paper

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continues this line of investigation from a complementary perspective. Instead of describing momentum transfer through the radiation-pressure force, we consider the short-time excitation regime, where the atomic momentum change is encoded in the momentum-space wave function and probability distribution. In this way, the present work bridges the remaining gap by extending the same focused-field physics to a fully analytical momentum-space treatment of the superkick signature and its associated probability weight.

Accordingly, we analyze the superkick at the wavefunction level in the short-time excitation regime, where the atomic momentum change is most directly characterized by the momentum-space probability distribution [2]. In this regime the excited component is proportional to the driving field evaluated over the initial wavepacket $\psi_e(\mathbf{r}, t) \propto E_z(\mathbf{r}) \psi_g(\mathbf{r})$, so the superkick mechanism is encoded in the Fourier transform $\tilde{\psi}_e(\mathbf{K})$. Our goal is to derive analytical closed-form expressions for $\tilde{\psi}_e(\mathbf{K})$ and to extract quantitative diagnostics: the peak displacement of the momentum-space probability density, the mean transverse momentum, and the localized transition probability per unit time in the near-axis superkick-relevant region.

Concretely, we work within a controlled quasiparaxial regime characterized by a small diffraction half-angle $\theta_d \simeq \lambda/(\pi w_0) < 1$, for which the paraxial field supplemented by the leading Maxwell-consistent longitudinal correction provides a reliable near-axis description of E_z [18–20]. In this parameter range the perturbative longitudinal term captures the essential focusing-induced brightening of the core region

that controls the superkick weight, while keeping the analysis fully analytical. We further choose w_0 and hence the numerical aperture (NA) so that this quasiparaxial E_z agrees closely with the Richards-Wolf result over the near-axis region of interest [3], and therefore, a full vectorial diffraction treatment is not expected to modify the momentum-space results reported below.

The paper is organized as follows. Section II introduces the focused-vortex field model and the short-time two-level Schrödinger framework, including the near-axis form of the longitudinal component $E_z(\mathbf{r})$. Section III then derives closed-form expressions for the momentum-space excited-state wave function $\tilde{\psi}_e(\mathbf{K})$ and its modulus squared $|\tilde{\psi}_e(\mathbf{K})|^2$ using Fourier-transform rules adapted to the focused-vortex longitudinal field. Section IV quantifies the superkick via momentum-space expectation values and related diagnostics, while Sec. V analyzes the transition probability in a tightly focused optical vortex field and introduces localized probability measures for the near-axis superkick weight. Finally, Sec. VI summarizes the main conclusions.

II. MODEL AND TWO-LEVEL SCHRÖDINGER SOLUTION

We model the driving light as a circularly polarized, tightly focused Laguerre-Gaussian (LG) beam propagating along $+z$, and we evaluate the field in the focal region at the atomic position. Within a controlled quasiparaxial treatment, the focused field is written as a transverse circular component together with the leading longitudinal correction [1], so we use

$$\mathbf{E}(\rho, \phi, z, t) = \frac{E_0 w_0}{2 w(z) \sqrt{|\ell|!}} \left(\frac{\sqrt{2} \rho}{w(z)} \right)^{|\ell|} \left[\frac{\hat{\mathbf{x}} + i\sigma \hat{\mathbf{y}}}{\sqrt{2}} + \frac{i e^{i\sigma\phi}}{\sqrt{2} \rho k} \left(|\ell| - \sigma\ell + \frac{2\rho^2}{w^2(z)} \left(i \frac{z}{z_R} - 1 \right) \right) \hat{\mathbf{z}} \right] e^{i(kz + \theta - \omega t)} e^{-\rho^2/w^2(z)}. \quad (1)$$

Here (ρ, ϕ, z) denote cylindrical coordinates, while $\hat{\mathbf{x}}, \hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are Cartesian unit vectors, and E_0 is the reference electric-field amplitude. The beam waist is w_0 , the Rayleigh range is $z_R = \pi w_0^2/\lambda$, and the z -dependent spot size is $w(z) = w_0 \sqrt{1 + z^2/z_R^2}$. The parameter $\sigma = \pm 1$ specifies the circular polarization helicity and hence the spin angular momentum $\sigma \hbar$ per photon. The phase $\theta(\rho, \phi, z)$ contains the azimuthal vortex phase together with the Gouy and wavefront-curvature terms

$$\theta(\rho, \phi, z) = \ell\phi - (|\ell| + 1) \tan^{-1} \left(\frac{z}{z_R} \right) + \frac{kz \rho^2}{2(z^2 + z_R^2)}. \quad (2)$$

The field expression used here is written for the fundamental radial member of the Laguerre-Gaussian family, namely, $p = 0$, where $p = 0, 1, 2, \dots$, denotes the radial index in LG_p^ℓ . From Eq. (1), the longitudinal contribution contains a helicity-related azimuthal factor $e^{i\sigma\phi}$, in addition to the usual optical-vortex phase $e^{i\ell\phi}$. The phase structure of E_z is therefore governed by the combined factor $e^{i(\ell+\sigma)\phi}$, showing that the longitudinal field effectively carries a helicity-shifted topological charge. This modification is a manifestation of

spin-orbit coupling in a tightly focused circularly polarized vortex beam, where the spin degree of freedom associated with σ becomes coupled to the orbital winding ℓ . The same equation also shows that the strength of the longitudinal component is controlled by the helicity-OAM factor $|\ell| - \sigma\ell$. This factor separates the problem into two distinct cases used throughout this work: the *same-handedness* (SH) configuration, defined by $\sigma\ell = |\ell|$, and the *opposite-handedness* (OH) configuration, defined by $\sigma\ell = -|\ell|$. Close to the beam axis ($\rho \rightarrow 0$), the behavior of E_z is therefore strongly helicity dependent: the SH configuration is suppressed by this prefactor, whereas the OH configuration is favored. As a result, the longitudinal near-axis intensity is larger for OH than for SH.

We model the atom as a two-level system with internal states $|g\rangle$ and $|e\rangle$, having energies E_g and E_e , respectively. The corresponding transition frequency is $\omega_0 = (E_e - E_g)/\hbar$. The driving laser is assumed to be a highly occupied coherent mode, allowing the optical field to be represented by a classical field amplitude while the internal atomic dynamics are still treated quantum mechanically. This enables us to use the standard *semiclassical* treatment of the atom-field interaction [21,22].

The free atomic Hamiltonian, including both the center-of-mass kinetic energy and the internal energies, is

$$\hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2M} + E_e |e\rangle\langle e| + E_g |g\rangle\langle g|, \quad (3)$$

where M is the atomic mass and $\hat{\mathbf{p}}$ is the center-of-mass momentum operator. In the electric-dipole approximation, the interaction Hamiltonian is

$$\hat{H}_{\text{int}}(t) = -\hat{\boldsymbol{\mu}} \cdot \mathbf{E}(\hat{\mathbf{r}}, t), \quad (4)$$

where $\hat{\boldsymbol{\mu}}$ and $\hat{\mathbf{r}}$ are the atomic dipole and position operators, respectively.

The interaction-picture potential is defined by

$$\hat{V}(t) = e^{i\hat{H}_0 t/\hbar} \hat{H}_{\text{int}}(t) e^{-i\hat{H}_0 t/\hbar}. \quad (5)$$

We assume that the atom is sufficiently massive and slowly moving, and that the interaction time is sufficiently short, so that its displacement and free motional evolution during the pulse can be neglected. Under this approximation, the kinetic-energy contribution to the interaction-picture transformation is negligible during the interaction, and the atomic position is treated as fixed in the internal two-level dynamics. After applying the rotating-wave approximation (RWA), the rapidly oscillating terms are neglected, and the interaction-picture Hamiltonian reduces to

$$\hat{V}(t) = -\hbar[\Omega(\mathbf{r})e^{-i\Delta t}|e\rangle\langle g| + \Omega^*(\mathbf{r})e^{i\Delta t}|g\rangle\langle e|], \quad (6)$$

where $\Delta = \omega - \omega_0$ is the laser detuning,

$$\Omega(\mathbf{r}) = \boldsymbol{\mu} \cdot \mathbf{E}(\mathbf{r})/\hbar \quad (7)$$

is the Rabi frequency, and $\hbar$ is the reduced Planck's constant.

The interaction-picture Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{V}(t) |\psi(t)\rangle, \quad (8)$$

is solved by writing the two-level state as

$$|\psi(t)\rangle = \psi_g(t) |g\rangle + \psi_e(t) |e\rangle. \quad (9)$$

Substituting the two-level expansion into the interaction-picture Schrödinger equation yields coupled equations for $\psi_g(t)$ and $\psi_e(t)$. For an atom initially in the ground state, with $\psi_e(0) = 0$, the excited-state amplitude takes the standard Rabi form

$$\psi_e(t) = i e^{-i\Delta t/2} \frac{2\Omega(\mathbf{r})}{\Omega_R} \sin\left(\frac{\Omega_R t}{2}\right) \psi_g(0), \quad (10)$$

where $\Omega_R = \sqrt{4|\Omega(\mathbf{r})|^2 + \Delta^2}$ is the generalized Rabi frequency.

Although Barnett and Berry used first-order time-dependent perturbation theory, we solve the reduced internal two-level Schrödinger equation at a fixed atomic position. The resulting internal solution reduces to theirs in the appropriate limit. In particular, we set the atom on resonance, $\Delta = 0$, and consider a very short interaction time $\Omega_R t/2 \ll 1$ for which $\sin(\Omega_R t/2) \approx \Omega_R t/2$. Under these conditions

$$|\Psi(t)\rangle \simeq \psi_g(0) |g\rangle + i\Omega(\mathbf{r})t \psi_g(0) |e\rangle, \quad (11)$$

which recovers the first-order excited-state amplitude of the Barnett-Berry approach. Writing the state in position space

and denoting the initial motional wavepacket by $\psi_{\text{init}}(\mathbf{r})$, we obtain

$$\langle \mathbf{r} | \Psi(t) \rangle \simeq \psi_{\text{init}}(\mathbf{r}) |g\rangle + i\Omega(\mathbf{r})t \psi_{\text{init}}(\mathbf{r}) |e\rangle. \quad (12)$$

Thus, Eq. (12), which is equivalent to the first-order Barnett-Berry result, is the starting point for all subsequent momentum-space calculations.

For the present model, we consider the strontium transition $5s^2 {}^1S_0 \rightarrow 5s5p {}^3P_1$ with the quantization axis along z . The $\Delta m = 0$ electric-dipole π transition is therefore driven by the longitudinal field component, so that $\Omega(\mathbf{r}) = \mu_z E_z(\mathbf{r})/\hbar$. Although electric-quadrupole couplings may become relevant near optical vortices for suitable transitions, they do not contribute here because the chosen transition changes parity and connects $J = 0$ to $J = 1$, both of which exclude an electric-quadrupole ($E2$) transition. Higher-multipole effects may nevertheless be important for other atomic transitions.

III. EXCITED-STATE WAVE FUNCTION IN MOMENTUM SPACE

The fully focused field in Eq. (1) is algebraically too involved to allow a simple closed-form Fourier transform. As shown in Ref. [17], the longitudinal component E_z is strongly localized around the vortex core and the focal plane. We therefore focus on the near-axis region, which is the relevant domain for the superkick signal, and evaluate the field close to the focus $|z|/z_R \ll 1$. In this regime, the transverse components are negligible for the excitation channel considered here, and the atom is driven primarily by the longitudinal component E_z , as discussed in Ref. [17]. For the derivation below, we use the Cartesian forms of E_z for the two handedness configurations

$$E_z^{\text{OH}}(x, y, z, t) = \frac{iE_0}{2kw_0\sqrt{|\ell|!}} \left(\frac{\sqrt{2}}{w_0}\right)^{|\ell|-1} (x \pm iy)^{|\ell|-1} \times \left(2|\ell| - \frac{2(x^2 + y^2)}{w_0^2}\right) \times e^{-\frac{x^2 + y^2}{w_0^2}} e^{i(k - \frac{|\ell|+1}{z_R})z} e^{-i\omega t}, \quad (13a)$$

$$E_z^{\text{SH}}(x, y, z, t) = -\frac{iE_0}{2kw_0\sqrt{|\ell|!}} \left(\frac{\sqrt{2}}{w_0}\right)^{|\ell|-1} \frac{2}{w_0^2} (x \pm iy)^{|\ell|+1} \times e^{-\frac{x^2 + y^2}{w_0^2}} e^{i(k - \frac{|\ell|+1}{z_R})z} e^{-i\omega t}. \quad (13b)$$

The upper sign corresponds to a positive azimuthal index $\ell > 0$, whereas the lower sign corresponds to $\ell < 0$. Because the OH configuration provides the dominant near-axis longitudinal excitation, we carry out the derivation explicitly for OH. The SH result is obtained in the same way, and its final form is included in Eq. (19).

Take the initial atomic wavepacket to be a Gaussian of spatial width σ_s , centered at $(x_0, 0, 0)$:

$$\psi_{\text{init}}(\mathbf{r}) = \left(\frac{1}{\pi\sigma_s^2}\right)^{3/4} \exp\left[-\frac{(x - x_0)^2 + y^2 + z^2}{2\sigma_s^2}\right]. \quad (14)$$

Substituting Eqs. (13a) and (14) into the excited-state part of Eq. (12) gives

$$\psi_e^{\text{OH}}(\mathbf{r}) = \mathcal{N} (x \pm iy)^{|\ell|-1} \left(2|\ell| - \frac{2(x^2 + y^2)}{w_0^2} \right) \times e^{-ax^2+bx} e^{-ay^2} e^{-cz^2+idz}, \quad (15)$$

where

$$a = \frac{1}{w_0^2} + \frac{1}{2\sigma_s^2}, \quad b = \frac{x_0}{\sigma_s^2}, \quad c = \frac{1}{2\sigma_s^2}, \quad d = k - \frac{|\ell| + 1}{z_R}, \quad (16)$$

with

$$\mathcal{N} = -\frac{\Omega_0 t}{2k\sqrt{2}|\ell|!} \left(\frac{1}{\pi\sigma_s^2} \right)^{3/4} \left(\frac{\sqrt{2}}{w_0} \right)^{|\ell|} e^{-\frac{b^2}{4c}} e^{-i\omega t}, \quad (17)$$

where $\Omega_0 = \frac{\mu_z E_0}{\hbar}$.

At this point, the wave function separates into a transverse part (x, y) and a longitudinal part z , allowing direct application of the Fourier transform. Thus, denoting the atomic momentum-space wave vector by $\mathbf{K} = (K_x, K_y, K_z)$, the momentum-space wave function is

$$\tilde{\psi}_e^{\text{OH}}(\mathbf{K}) = \mathcal{N} \frac{\pi^{3/2}}{a\sqrt{c}} \left(\frac{\pm K_y - iK_x + b}{2a} \right)^{|\ell|-1} \times \left[2|\ell| + \frac{2}{w_0^2} \left(\frac{(K_x + ib)^2 + K_y^2}{4a^2} - \frac{|\ell|}{a} \right) \right] \times \exp \left[-\frac{(K_x + ib)^2}{4a} - \frac{K_y^2}{4a} \right] \exp \left[-\frac{(K_z - d)^2}{4c} \right]. \quad (18)$$

The observable momentum distribution is obtained from the modulus squared of Eq. (18). It is convenient to express the result in the unified form

$$|\tilde{\psi}^{\chi}(\mathbf{K})|^2 = |\mathcal{N}_K|^2 (K_x^2 + (\pm K_y + b)^2)^{|\ell|+i_\chi} W_\chi(K_x, K_y) \times \exp \left[-\frac{K_x^2 + K_y^2}{2a} \right] \exp \left[-\frac{(K_z - d)^2}{2c} \right], \quad (19)$$

where

$$|\mathcal{N}_K|^2 = \pi^{3/2} \sqrt{2c} \frac{\Omega_0^2 t^2}{k^2 |\ell|!} (2a^2 w_0^2)^{-|\ell|} \exp \left[\frac{b^2}{2} \left(\frac{1}{a} - \frac{1}{c} \right) \right], \quad (20)$$

and $\chi \in \{\text{OH}, \text{SH}, \text{BB}\}$ labels the three cases, corresponding to $i_\chi = -1, +1, 0$, respectively. The label BB refers to the Barnett-Berry paraxial reference case. The sign convention remains that introduced above: the upper sign is used for $\ell > 0$, and the lower sign for $\ell < 0$. The explicit form of the case-dependent factor $W_\chi(K_x, K_y)$ for each configuration is given in Table I.

In the weak-focusing limit $z_R \rightarrow \infty$, the factor $W_\chi(K_x, K_y)$ becomes independent of K_x and K_y , and therefore reduces to a constant prefactor that can be absorbed into the overall normalization. Moreover, $a \rightarrow 1/(2\sigma_s^2)$ and $d \rightarrow k$. Under

TABLE I. Correction factor $W_\chi(K_x, K_y)$ for opposite handedness OH, same handedness SH, and the Barnett-Berry (BB) case.

Case χ	$W_\chi(K_x, K_y)$
OH	$(2 \ell + \frac{K_x^2 + K_y^2 - b^2 - 4a \ell }{2a^2 w_0^2})^2 + (\frac{bK_y}{a^2 w_0^2})^2$
SH	$\frac{1}{4a^4 w_0^4}$
BB	$\frac{k^2}{2a^2}$

these conditions, the momentum distribution reduces to

$$|\tilde{\psi}^{\chi}(\mathbf{K})|^2 \propto (K_x^2 + (\pm K_y + x_0/\sigma_s^2)^2)^{|\ell|+i_\chi} \times \exp [-(K_x^2 + K_y^2)\sigma_s^2] \exp [-(K_z - k)^2\sigma_s^2]. \quad (21)$$

If the helicity-dependent circular-polarization contribution encoded in i_χ is neglected (i.e., by setting $i_\chi = 0$), Eq. (21) reproduces the scalar momentum distribution obtained in Ref. [23] [cf. Barnett-Berry Eq. (3.5)] for $\ell > 0$, and extends it straightforwardly to $\ell < 0$ through the $\pm K_y$ dependence.

The momentum-space probability density is represented by $|\tilde{\psi}^{\chi}(\mathbf{K})|^2$ whose transverse profile reveals the recoil structure. A transverse kick is identified by a shift of the probability maximum away from $(K_x, K_y) = (0, 0)$. Here $K_\perp = \sqrt{K_x^2 + K_y^2}$ denotes the transverse atomic wave vector, and we classify the momentum transfer as a superkick when $K_\perp > k$. In the parameter range considered here, the photon boundary k is much smaller than the transverse scale shown in the plots; hence any resolved nonzero peak displacement effectively lies in the superkick regime. Physically, this behavior originates from the near-axis phase structure of the vortex field: at the displaced atomic position x_0 , the local transverse wave vector can exceed the photon wave number $|k_\phi^{\text{loc}}| > k$, allowing population of transverse-momentum components beyond the usual recoil scale.

Following previous work [17], the local wave vector of a tightly focused circularly polarized LG beam has the azimuthal component $k_\phi^{\text{loc}}(\rho) = \frac{\sigma + \ell}{\rho}$. Therefore, a superoscillatory near-axis region is defined by $|k_\phi^{\text{loc}}| > k$, i.e.,

$$\rho < \rho_{\text{sup}}, \quad \rho_{\text{sup}} = \frac{|\sigma + \ell|}{k}. \quad (22)$$

Accordingly, we choose x_0 and σ_s such that $x_0, \sigma_s \ll \rho_{\text{sup}}$, ensuring that the atomic wave packet is localized inside the superoscillatory core.

In Fig. 1, we plot $|\tilde{\psi}^{\text{OH}}(\mathbf{K})|^2$ at $K_z = d$ for several values of ℓ . Rabi frequencies of order 100 MHz have been considered for the same 689-nm strontium transition [24]. Consistent with this scale, we use $\Omega_0/2\pi = 75$ MHz and the free-space spontaneous decay rate $\Gamma/2\pi = 7.5$ kHz. For this transition, $|\mu_z| \simeq 7.4 \times 10^{-31}$ C m, giving $E_0 \simeq 6.7 \times 10^4$ V m $^{-1}$ and an optical power of approximately 1.5 μ W for $w_0 = 1.15\lambda$. For this waist, the focusing-angle estimate is $\theta_d \approx 0.277$, indicating a moderately small divergence angle consistent with our quasiparaxial treatment. We set $x_0 = 0.01w_0$ and $\sigma_s = 2x_0$, corresponding to the near-axis region where the superkick effect is predicted to be most pronounced [17]. We take an

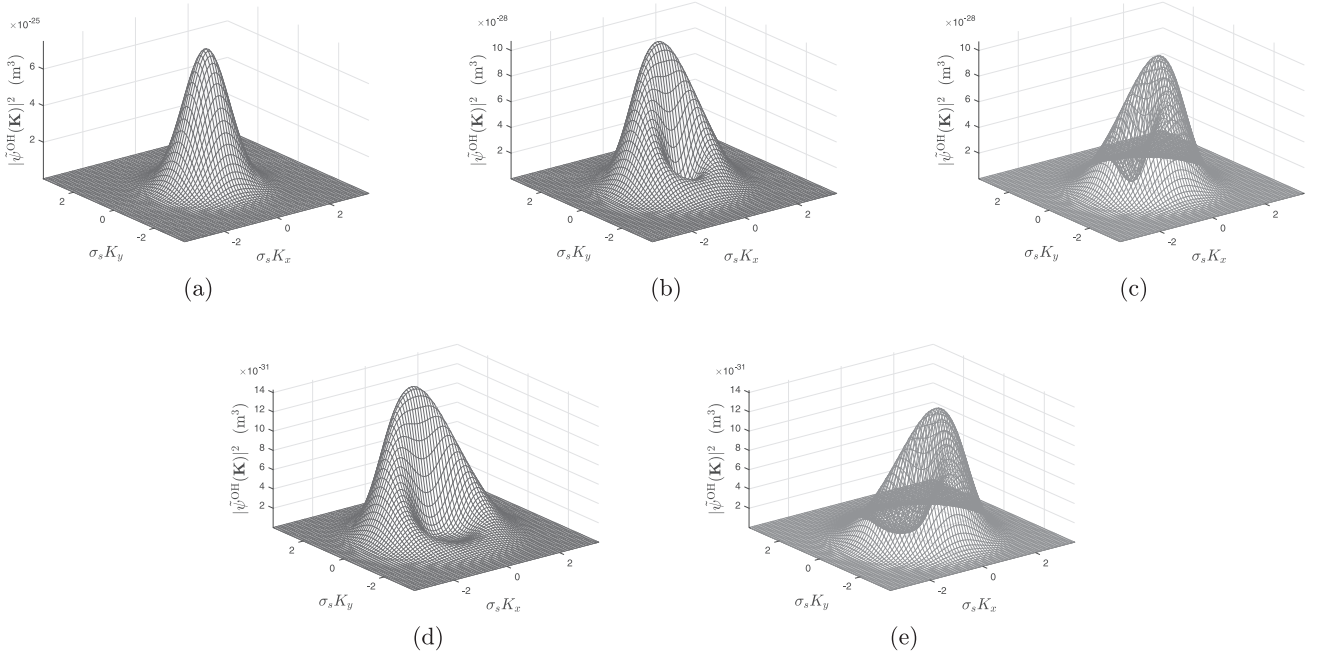


FIG. 1. Modulus squared of the momentum-space wavefunction $|\tilde{\psi}^{\text{OH}}(\mathbf{K})|^2$ as a function of the dimensionless transverse wave-vector components $\sigma_s K_x$ and $\sigma_s K_y$, at fixed $x_0 = 0.01w_0$ for (a) $\ell = \pm 1$ (same plot), (b) $\ell = 2$, (c) $\ell = -2$, (d) $\ell = 3$, and (e) $\ell = -3$, with $\sigma_s = 0.02w_0$.

interaction time of $t = 1$ ns, for which the local condition $|\Omega(\mathbf{r})|t \ll 1$ is satisfied for all values of ℓ considered. Since the plotted axes are the dimensionless variables $K_x \sigma_s$ and $K_y \sigma_s$, the photon boundary is $k\sigma_s \approx 0.1445$. Therefore, consistent with the criterion introduced above, peaks with $K_\perp \sigma_s > 0.1445$ are identified as superkicks.

To emphasize the near-axis scaling behavior, we present the distributions in scaled variables and take x_0 and σ_s proportional to w_0 , keeping x_0/w_0 and σ_s/w_0 fixed. With this convention, the w_0 dependence factors out as an overall prefactor, so the dimensionless momentum-space profiles are independent of the specific value of w_0 over the quasiparaxial range considered here.

Figure 1(a) shows that the $\ell = \pm 1$ distributions coincide and remain sharply centered at $K_x = K_y = 0$, so no resolved transverse superkick is obtained in this case. This behavior follows directly from the analytical OH expression for $|\tilde{\psi}^{\text{OH}}(\mathbf{K})|^2$: the factor $(K_x^2 + (\pm K_y + b)^2)^{|\ell|-1}$ becomes unity when $|\ell| = 1$, and therefore cannot displace the maximum. For $\ell = 2$ [Fig. 1(b)], the peak moves clearly toward positive K_y , indicating a net transverse recoil. For $\ell = 3$ [Fig. 1(d)], the displacement is slightly larger, but the amplitude of $|\tilde{\psi}^{\text{OH}}(\mathbf{K})|^2$ is reduced by roughly three orders of magnitude compared with the $\ell = 2$ case, which strongly lowers the probability of observing large kicks. The negative-charge cases in Figs. 1(c) and 1(e) reproduce the corresponding mirror-reflected patterns for $\ell = -2$ and $\ell = -3$, as expected from reversing the azimuthal phase winding in the OH configuration.

Figure 2 separates the effect of spin-dependent focusing from the spin-independent Barnett-Berry reference by comparing the OH, BB, and SH momentum distributions

for $\ell = 2$, with $x_0 = 0.01w_0$ and $\sigma_s = x_0$. The configuration index i_χ in Eq. (19) encodes the circular-polarization contribution associated with spin-orbit coupling in the focused field, and therefore modifies the near-axis momentum distribution through the longitudinal component. In each case, the distribution is composed of a Gaussian envelope in (K_x, K_y) and a vortex-dependent prefactor $(K_x^2 + (\pm K_y + b)^2)^{|\ell|+i_\chi}$, which suppresses the probability near the vortex point $K_x = 0, K_y = \mp b$, and produces the transverse asymmetry.

The three configurations therefore share the same basic transverse structure, but differ strongly in probability weight. The OH case gives the largest magnitude, the Barnett-Berry reference is substantially weaker, and the SH case is strongly suppressed. Quantitatively, the OH peak is about 10^3 larger than the Barnett-Berry peak and about 10^7 larger than the SH peak. The OH distribution is also more tightly localized in the (K_x, K_y) plane, whereas the Barnett-Berry distribution is broader and the SH distribution is the most diffuse. Thus, although all three cases retain the transverse superkick structure, only the OH configuration provides an appreciable probability for observing large transverse momentum transfer.

IV. MEAN TRANSVERSE MOMENTUM SHIFT AS A SUPERKICK QUANTIFIER

The displaced probability weight in the (K_x, K_y) plane provides a direct momentum-space signature of transverse recoil. To quantify this displacement, we evaluate the momentum expectation values and use $\langle K_y \rangle$ as the main transverse diagnostic. Because $\langle K_y \rangle$ is averaged over the full distribution, it measures the net transverse momentum transferred to the

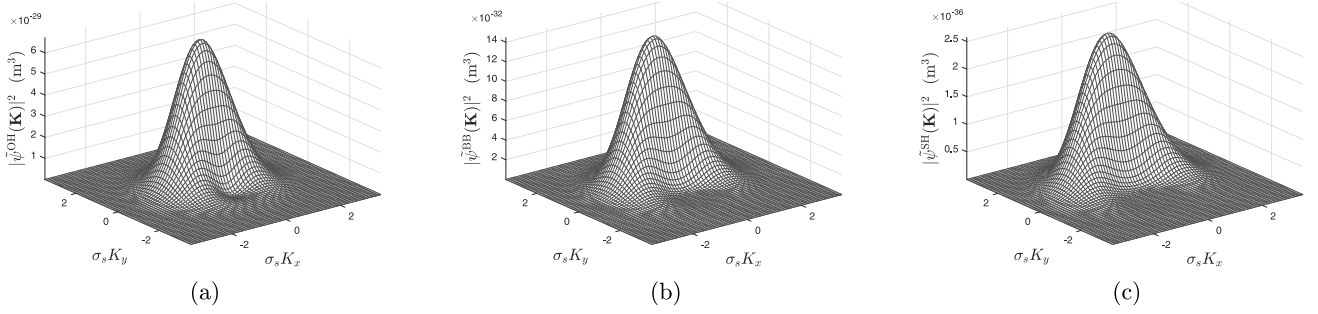


FIG. 2. Modulus squared of the momentum-space wave function $|\tilde{\psi}^{(x)}(\mathbf{K})|^2$ as a function of the dimensionless transverse wave-vector components $\sigma_s K_x$ and $\sigma_s K_y$, at fixed $x_0 = \sigma_s = 0.01 w_0$: (a) OH, (b) BB, and (c) SH.

atom and allows the different configurations to be compared on the same quantitative basis.

For the momentum-space wave function $\tilde{\psi}^{(x)}(\mathbf{K})$, the transverse mean is

$$\langle K_y \rangle_\chi = \frac{\int K_y |\tilde{\psi}^{(x)}(\mathbf{K})|^2 d^3 K}{\int |\tilde{\psi}^{(x)}(\mathbf{K})|^2 d^3 K}. \quad (23)$$

By symmetry one has $\langle K_x \rangle_\chi = 0$. In addition, the K_z dependence is separable and Gaussian, so $\langle K_z \rangle_\chi = d$.

Because the non-Gaussian part of $|\tilde{\psi}^{(x)}(\mathbf{K})|^2$ is polynomial, $\langle K_y \rangle_\chi$ can be evaluated in closed form. We expand the polynomial prefactor using the binomial theorem and reduce the numerator and denominator to Gaussian moments. The required reduction is based on the standard integral

$$\begin{aligned} & \iint_{-\infty}^{\infty} K_x^{2P} K_y^{2Q} \exp \left[-\frac{K_x^2 + K_y^2}{2a} \right] dK_x dK_y \\ &= (2a)^{P+Q+1} \times \Gamma \left(P + \frac{1}{2} \right) \Gamma \left(Q + \frac{1}{2} \right). \end{aligned} \quad (24)$$

Parity implies that odd total powers of K_y vanish, so only even powers contribute in the denominator, while only odd powers contribute in the numerator. The result is

$$\langle K_y \rangle_\chi^{(\pm)} = \frac{\pm \sum_{j=0}^{|\ell|+i_\chi} \sum_{r=0}^{j-1} \binom{|\ell|+i_\chi}{j} \binom{2j}{2r+1} b^{2r+1} \mathcal{G}_{jr}^{(x)}}{\sum_{j=0}^{|\ell|+i_\chi} \sum_{r=0}^j \binom{|\ell|+i_\chi}{j} \binom{2j}{2r} b^{2r} \mathcal{G}_{jr}^{(x)}}, \quad (25)$$

where the sign convention is the same as in Eq. (19). The functions $\mathcal{G}_{jr}^{(x)}$ are defined by

$$\begin{aligned} \mathcal{G}_{jr}^{(x)} &= (2a)^{|\ell|-r+1+i_\chi} \Gamma(|\ell| - j + \tfrac{1}{2} + i_\chi) \Gamma(j - r + \tfrac{1}{2}) \\ &\times [\delta_{i_\chi, -1} \mathcal{P}_{jr} - (\delta_{i_\chi, -1} - 1)], \end{aligned} \quad (26)$$

where

$$\begin{aligned} \mathcal{P}_{jr} &= 4a^2(|\ell| - r)(|\ell| - r + 1) + 4a h(|\ell| - r) \\ &+ 4a b^2(2(|\ell| - j) - 1) + h^2, \end{aligned} \quad (27)$$

and

$$h = 4a^2 w_0^2 |\ell| - 4a |\ell| - b^2. \quad (28)$$

Using Eq. (25), we evaluate $\langle K_y \rangle_\chi^{(\pm)}$ in closed form for $\ell = 1, 2, 3$; the resulting expressions are collected in Table II. Several features are worth noting. First, for the OH configuration we obtain $\langle K_y \rangle_{\text{OH}} = 0$ at $\ell = 1$, consistent with the

absence of any detectable shifted $|\tilde{\psi}|^2$ signal in Fig. 1(a). Second, in the Barnett-Berry (BB) limit, corresponding to the nonfocusing regime $w_0 \rightarrow \infty$, our expressions reduce exactly to those reported by Barnett and Berry under the same parameter identification [23] [cf. Barnett and Berry, Eq. (3.8)]. Finally, the SH and BB cases are simply related: $\langle K_y \rangle_{\text{SH}}|_\ell = \langle K_y \rangle_{\text{BB}}|_{\ell+1}$; this follows directly from Eqs. (25) and (26) and therefore holds for arbitrary ℓ .

To quantify the presence of a superkick, we compare the *mean* transverse momentum shift $\langle K_y \rangle$ obtained from the closed-form expressions in Table II with the characteristic *local optical-momentum* scale set by the vortex azimuthal phase gradient. Using the local azimuthal wave vector introduced above, $k_\phi^{\text{loc}} = (\ell + \sigma)/\rho$ in the tightly focused circularly polarized case, we define a reference superkick scale by evaluating it at the wavepacket displacement $\rho \simeq x_0$ [17], giving $k_{\text{OH/SH,super}} = |\ell + \sigma|/x_0$. In the paraxial Barnett-Berry (BB) limit, $k_\phi^{\text{loc}} = \ell/\rho$, which yields the corresponding reference $k_{\text{BB,super}} = |\ell|/x_0$.

Accordingly, in Fig. 3 we plot the dimensionless products $\sigma_s \langle K_y \rangle$ (solid curves) and $\sigma_s k_{\text{super}}$ (dashed curves) versus the scaled displacement x_0/σ_s , for a representative wavepacket width $\sigma_s = 0.01 w_0$. We show only $\ell > 0$, since changing the sign of ℓ simply mirror-reflects the momentum distribution.

Figure 3(a) shows the three configurations for $\ell = 2$, each accompanied by its corresponding reference curve. The OH case (red) exhibits the smallest deviation between $\sigma_s \langle K_y \rangle$ and $\sigma_s k_{\text{super}}$, the red solid curve approaches its red dashed reference more rapidly than in the BB (black) and SH (blue) cases. In contrast, the BB configuration shows a larger mismatch, especially around the peak and near the crossing with its dashed reference, while the SH case displays the largest separation from its own reference over the full range of x_0/σ_s , indicating the slowest approach. These trends are summarized in Table III, which lists the peak value $\sigma_s \langle K_y \rangle_{\text{max}}$, the corresponding position $x_{0,\text{max}}/\sigma_s$, and the ratio $R_y = \langle K_y \rangle_{\text{max}}/k_{\text{super}}$, which quantifies how close the mean transverse momentum shift comes to the local optical-momentum scale. For $\ell = 2$, R_y is largest for OH, smaller for BB, and smallest for SH, indicating that in the OH case the peak mean shift $\langle K_y \rangle_{\text{max}}$ lies closest to the reference scale k_{super} . For $\ell = 3$, the same ordering holds but with smaller values of R_y , indicating a weaker mean-based superkick signature for all configurations. This behavior is consistent with the physical picture emphasized by Barnett and Berry. As x_0/σ_s increases, the peak mean transverse

TABLE II. Closed-form expressions for $\langle K_y \rangle_\chi^{(\pm)}$ for $\chi \in \{\text{OH}, \text{BB}, \text{SH}\}$ and $\ell = 1, 2, 3$. The denominator $\mathcal{D}$ is defined as: $\mathcal{D} = 384a^4 + 96a^3(3b^2 + h) + 8a^2(b^2 + h)(7b^2 + h) + 4a(b^6 + b^4h + 2b^2h^2) + b^4h^2$.

ℓ	OH	BB	SH
1	0	$\frac{\pm 2ab}{2a+b^2}$	$\frac{\pm 4ab(4a+b^2)}{8a^2+8ab^2+b^4}$
2	$\frac{\pm 2ab(4a(6a+b^2)+8ah+h^2)}{4a(12a^2+6ab^2+b^4)+4ah(4a+b^2)+h^2(2a+b^2)}$	$\frac{\pm 4ab(4a+b^2)}{8a^2+8ab^2+b^4}$	$\frac{\pm 6ab(24a^2+12ab^2+b^4)}{48a^3+72a^2b^2+18ab^4+b^6}$
3	$\frac{\pm 4ab}{\mathcal{D}} [192a^3 + 48a^2(b^2 + h) + 4a(b^2 + h)^2 + b^2h^2]$	$\frac{\pm 6ab(24a^2+12ab^2+b^4)}{48a^3+72a^2b^2+18ab^4+b^6}$	$\frac{\pm 8ab(192a^3+144a^2b^2+24ab^4+b^6)}{384a^4+768a^3b^2+288a^2b^4+32ab^6+b^8}$

shift $\langle K_y \rangle$ approaches its corresponding reference scale k_{super} , whereas for smaller x_0 it falls short and vanishes as $x_0 \rightarrow 0$. The underlying reason is that, when the atomic wavepacket overlaps both sides of the vortex core, regions where the local azimuthal momentum points in opposite directions contribute with opposite signs, leading to partial cancellation in the average. Consequently, $\langle K_y \rangle$ provides a conservative near-axis signature of superkicks since very large local momenta can

still occur even when the mean shift is reduced by this cancellation.

V. TRANSITION PROBABILITY PER UNIT TIME

We next examine which configuration carries the largest excitation weight in the near-axis region. For this purpose, we compare the probability induced by the longitudinal field E_z in the tightly focused OH and SH configurations with the paraxial BB reference. At the level of Fermi's golden-rule scaling, the transition probability per unit time is governed by the spatial overlap between the coupling strength and the initial atomic probability density, namely [25],

$$P = \frac{1}{t} \int d^3r |\langle \mathbf{r}, e | \Psi(t) \rangle|^2 \simeq t \int d^3r |\Omega(\mathbf{r})|^2 |\psi_{\text{init}}(\mathbf{r})|^2, \quad (29)$$

where $\psi_{\text{init}}(\mathbf{r})$ is the initial spatial wave function given in Eq. (14), and $\Omega(\mathbf{r}) = \mu_z E_z(\mathbf{r})/\hbar$ is the position dependent Rabi frequency associated with the π coupling driven by E_z . If ψ_{init} is localized within the near-axis region around $\mathbf{r}_0 = (x_0, 0, 0)$, then

$$\int d^3r |\Omega(\mathbf{r})|^2 |\psi_{\text{init}}(\mathbf{r})|^2 \approx |\Omega(\mathbf{r}_0)|^2, \quad (30)$$

so that the transition probability per unit time is controlled by the product of the interaction time and the local coupling strength at the displaced position. For the tightly focused vortex field, the near-axis longitudinal coupling in the OH and SH configurations, and for the paraxial reference (case BB),

TABLE III. Peak values of the mean transverse momentum shift and the corresponding displacement. Here $\sigma_s \langle K_y \rangle_{\text{max}}$ is evaluated at the maximum, occurring at $x_{0,\text{max}}/\sigma_s$, for the OH, BB, and SH configurations. The dimensionless ratio R_y is defined by $R_y = \langle K_y \rangle_{\text{max}}/k_{\text{super}}$.

Case	Parameter	OH	BB	SH
$\ell = 2$	$\sigma_s \langle K_y \rangle_{\text{max}}$	0.50007	0.85852	1.15103
	$x_{0,\text{max}}/\sigma_s$	1.0000	0.9388	0.9005
	R_y	0.50007	0.40299	0.34550
$\ell = 3$	$\sigma_s \langle K_y \rangle_{\text{max}}$	0.93885	1.15103	1.40379
	$x_{0,\text{max}}/\sigma_s$	0.8585	0.9005	0.8740
	R_y	0.40300	0.34550	0.30673

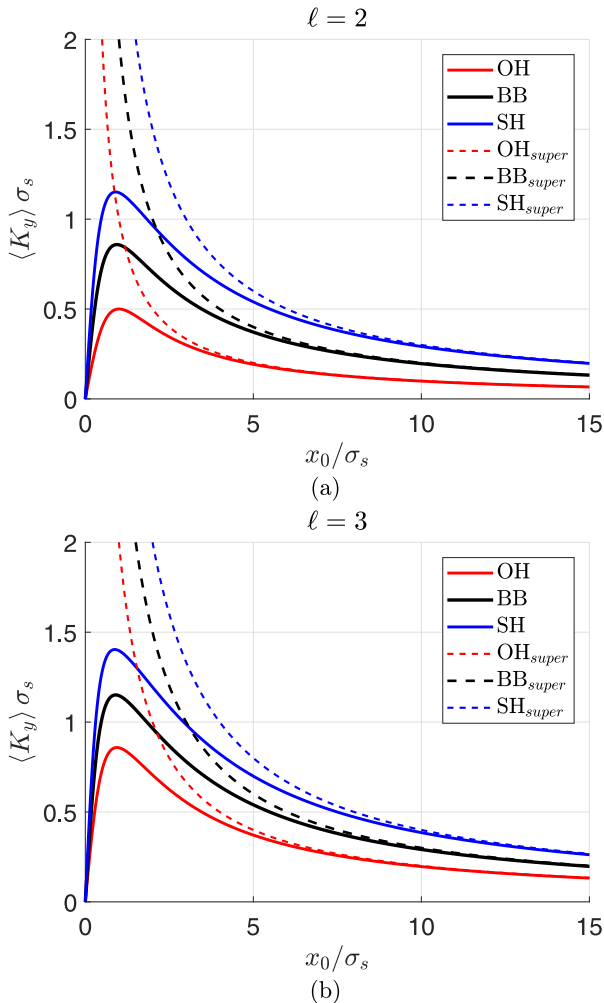


FIG. 3. Dimensionless mean transverse momentum shift $\langle K_y \rangle \sigma_s$ as a function of the dimensionless scaled displacement x_0/σ_s for the OH, BB, and SH configurations. Panels (a) and (b) correspond to $\ell = 2$ and $\ell = 3$, respectively.

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$$\begin{aligned} |\Omega(\rho)|_{\text{OH}}^2 &= C_\ell(\rho) \rho^{2|\ell|-2} \left(2|\ell| - \frac{2\rho^2}{w_0^2} \right)^2, \\ |\Omega(\rho)|_{\text{SH}}^2 &= C_\ell(\rho) \rho^{2|\ell|-2} \left(\frac{2\rho^2}{w_0^2} \right)^2, \\ |\Omega(\rho)|_{\text{BB}}^2 &= 2k^2 C_\ell(\rho) \rho^{2|\ell|}, \end{aligned} \quad (31)$$

with the common factor

$$C_\ell(\rho) = \frac{1}{2^{|\ell|} |\ell|!} \left(\frac{\Omega_0}{2k} \right)^2 \left(\frac{\sqrt{2}}{w_0} \right)^{2|\ell|} \exp \left(-\frac{2\rho^2}{w_0^2} \right). \quad (32)$$

With the wavepacket localized at transverse displacement x_0 from the vortex axis, we evaluate the localized approximation (30) at $\rho = x_0$. The corresponding transition probabilities per unit time are then

$$\begin{aligned} P_{\text{BB}} &\simeq 2k^2 C_\ell(x_0) x_0^{2|\ell|} t, \\ P_{\text{SH}} &\simeq \frac{4 C_\ell(x_0)}{w_0^4} x_0^{2|\ell|+2} t, \\ P_{\text{OH}} &\simeq 4 C_\ell(x_0) t \left(|\ell|^2 x_0^{2|\ell|-2} - \frac{2|\ell|}{w_0^2} x_0^{2|\ell|} + \frac{1}{w_0^4} x_0^{2|\ell|+2} \right). \end{aligned} \quad (33)$$

To determine which configuration provides the largest excitation weight near the vortex core, and to clarify that the present enhancement is not based on superweak-measurement amplification, we plot the localized transition probabilities per unit time P as functions of the normalized displacement x_0/w_0 , taking $\Omega_0 = 10^4 \Gamma$ and $t = 1 \text{ ns}$. The comparison includes the opposite-handedness OH, same-handedness SH, and paraxial BB cases. Figure 4 demonstrates a clear near-axis hierarchy: P_{OH} is dominant, whereas P_{SH} and P_{BB} remain strongly suppressed. The insets highlight this behavior in the small- x_0/w_0 region. The same ordering is obtained for both $\ell = 2$ [Fig. 4(a)] and $\ell = 3$ [Fig. 4(b)]. For $\ell = 2$, the hierarchy is especially pronounced. At $x_0/w_0 = 0.01$, the OH transition probability per unit time exceeds the corresponding BB and SH values by $P_{\text{OH}}/P_{\text{BB}} \sim 10^3$ and $P_{\text{OH}}/P_{\text{SH}} \sim 10^8$, respectively. At the larger displacement $x_0/w_0 = 0.1$, the contrast decreases but remains significant, with $P_{\text{OH}}/P_{\text{BB}} \sim 10$ and $P_{\text{OH}}/P_{\text{SH}} \sim 10^4$. These ratios show that, in the near-axis region relevant to the superkick mechanism, the OH configuration carries the dominant localized transition probability per unit time, while the SH and BB contributions are negligible by comparison. For $\ell = 3$, the same trend remains, although the near-axis contrast is somewhat reduced relative to $\ell = 2$; nevertheless, P_{OH} continues to dominate.

The parameters entering the localized transition-probability analysis must also satisfy the relevant field-strength and timescale constraints. Since $\Omega_0 = |\mu_z| E_0 / \hbar$, the requirement $\Gamma \ll \Omega_0$ gives the lower bound

$$E_0 \gg \frac{\hbar \Gamma}{|\mu_z|}. \quad (34)$$

However, the short-time approximation is governed by the local Rabi frequency, $|\Omega(\mathbf{r})|t \ll 1$. Writing $\Omega(\mathbf{r}) = \Omega_0 f(\mathbf{r})$, where $f(\mathbf{r}) = \Omega(\mathbf{r})/\Omega_0$, this gives the complementary upper

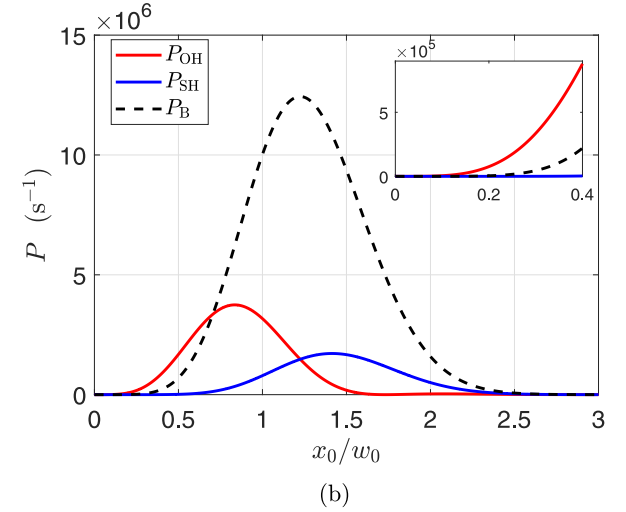
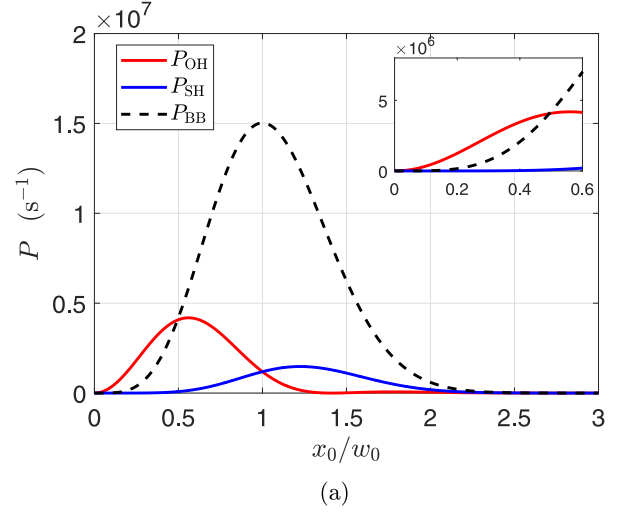


FIG. 4. Localized transition probability per unit time P versus the dimensionless initial displacement x_0/w_0 for the opposite-handedness (OH), same-handedness (SH), and paraxial reference (BB) cases. Panel (a) corresponds to $\ell = 2$ and panel (b) to $\ell = 3$.

constraint

$$E_0 \ll \frac{\hbar}{|\mu_z| t |f(\mathbf{r})|}. \quad (35)$$

For the parameters considered here, $\Omega_0/2\pi = 75 \text{ MHz}$ corresponds to $E_0 \simeq 6.7 \times 10^4 \text{ V m}^{-1}$, while the calculated values of $|\Omega(x_0)|t$ remain much smaller than unity in the near-axis region. Thus, both constraints are simultaneously satisfied.

The localized approximation further requires that the atomic wavepacket remain close to x_0 during the interaction interval. If v_{cm} denotes the velocity of the wavepacket center, its displacement during the interaction is approximately $v_{\text{cm}} t$, and hence localization requires

$$v_{\text{cm}} t \ll \sigma_s. \quad (36)$$

For $\sigma_s = 0.01 w_0$ and $t = 1 \text{ ns}$, this corresponds to $v_{\text{cm}} \ll 7.9 \text{ m s}^{-1}$, which is readily compatible with the slowly moving cold atoms assumed in the present treatment. Therefore, the atomic center of mass remains effectively localized near x_0 over the interaction interval.

VI. CONCLUSION

This paper has established an analytical momentum-space framework for describing near-axis superkick transfer in the near-focus region $|z|/z_R \ll 1$. In this regime, the longitudinal electric field can be expressed in Cartesian variables, allowing the Fourier transform to be evaluated in closed form. This yields explicit analytical expressions for the excited-state momentum-space wave function $\tilde{\psi}_e(\mathbf{K})$ and the corresponding probability distribution $|\tilde{\psi}_e(\mathbf{K})|^2$. The resulting distribution separates into a Gaussian envelope, a vortex-dependent prefactor, and a longitudinal correction factor $W(K_x, K_y)$. Through this structure, the polarization-vorticity configuration (ℓ - σ sign) enters directly into the momentum distribution and provides a unified way to compare the OH, SH, and BB cases. The analysis shows that the combined effect of the vortex prefactor and the W factor determines whether the near-axis distribution supports an observable superkick or remains effectively superweak. These findings are consistent with the force-based picture developed in our previous work [17]: for OH with $\ell = 1$, the transverse peak displacement vanishes, whereas higher azimuthal indices produce clear shifted momentum-space peaks.

To characterize the transverse momentum transfer quantitatively, we derived a closed-form expression for the mean shift $\langle K_y \rangle$ for general ℓ , and evaluated it explicitly for $\ell = 1, 2, 3$ in the OH, SH, and BB configurations. Together with the momentum-distribution plots, these expressions provide a direct comparison of the different cases. When benchmarked against the local superkick scale, the OH configuration remains closest to the expected superkick trend, whereas the SH configuration shows the largest deviation.

We also evaluated the localized transition probability per unit time P by writing the coupling $|\Omega(\rho)|^2$ in terms of a common envelope $C_\ell(\rho)$ and comparing the three configurations under the same near-axis conditions. This analysis shows that the excitation weight is concentrated near the vortex axis and is dominated by the OH configuration

in the superkick-relevant region. The OH advantage increases the detectability of the effect, even though the peak displacement itself becomes less pronounced for larger ℓ .

Overall, the analysis shows that tight focusing mitigates the superweak limitation by enhancing the longitudinal-field transition probability per unit time precisely in the near-axis region where the transverse phase gradient is largest. As a result, the OH configuration converts the near-axis superkick from a nominally weak effect into a momentum-space signature with appreciable probability weight. In contrast to our first paper, where the effect was established through a force-based framework, the present formulation provides directly accessible momentum-space observables: $|\tilde{\psi}_e(\mathbf{K})|^2$, its peak displacement, and $\langle K_y \rangle$ —that can be used to assess and optimize experimental detectability.

Although the present study is restricted to circular polarization, the superkick is not exclusive to this polarization state. Our separate analysis of x - and y -linearly polarized Laguerre-Gaussian beams showed that near-axis superkicks persist, although circular polarization produces more pronounced results. More generally, observable near-axis superkicks may be expected for other polarization states when a large local phase gradient is accompanied by a dominant longitudinal field near the beam axis. A systematic investigation of the polarization dependence of the superkick will be pursued in future work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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