Long-term stability of quantum resources through gravitational cat states in a dissipative Markovian environment

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(Received 17 September 2025; accepted 22 June 2026; published 15 July 2026)

The paradigm in which gravity can generate entanglement between distant masses represents a profound shift in our understanding of quantum-gravitational interfaces and is still among the active and debated topics. Recent work [A. ur Rahman *et al.*, Phys. Rev. D **111**, 064077 (2025)] established gravitational cat states as robust carriers of quantum correlations under thermal and classical fields. In our work, we demonstrate their capacity to stabilize quantum resources under a dissipative environment, which is the dominant decoherence mechanism in engineered platforms such as superconducting qubits. By incorporating both gravitational-like and spin-orbit couplings in a Markovian open system, we analytically demonstrate that quantum correlations attain nonvanishing steady-state values, and the stationary fidelity is enhanced by increasing the gravitational-like coupling. The spin-orbit interaction drives Rabi-like oscillations that counteract monotonic decay, and the gravitational coupling mimics non-Markovian memory, enabling long-term information stability. These mechanisms position gravity-inspired interactions not merely as entanglement generators, but as active stabilizers of quantumness in realistic dissipative environments.

DOI: 10.1103/68pc-tskl

I. INTRODUCTION

The unification of quantum mechanics and general relativity remains one of the central challenges in theoretical physics [1,2]. Recent advances in quantum control have enabled tabletop experiments to test models of gravitational decoherence [3,4]. Despite ongoing research, the understanding of how gravitation and quantum matter interact remains limited [5]. Especially intriguing is the unlocking potential of gravitational matter as a resource within the dynamic world of open quantum systems and quantum information science, which calls for a deeper and more detailed exploration.

The development of quantum information technology and the conceptual understanding of the quantum world depend heavily on nonclassical correlations (NCCs), which are among the most fundamental features of quantum mechanics. To characterize the nonclassical behavior of the system, we monitor the dynamics of quantum resources using standard quantifiers such as entanglement via negativity, quantum discord via trace distance, and quantum coherence via quantum Jensen-Shannon divergence, which are widely used measures of nonclassicality in composite quantum systems [6–11]. Due to the extremely weak gravitational coupling between small masses, its observable consequences depend crucially on its competition with environmental decoherence. In this situation, quantum correlations offer a sensitive means of tracking whether the interaction produces genuinely nonclassical features and whether such features can persist over experimentally relevant timescales. Similar measures have been employed in gravity-mediated entanglement proposals to identify nonclassical signatures of the interaction [3,4].

In a quantum description of matter, Anastopoulos and Hu [12] have shown that a single stationary massive particle can exist in a superposition of two distinct locations, known as a Schrödinger cat state. The term "gravcat" refers to such superpositions for gravitating objects, each of which can be modeled as a qubit [5]. Gravitational interactions introduce coupling between these qubits, leading to gravity-induced Rabi oscillations and entanglement of energy eigenstates [5]. Nevertheless, it is essential to highlight that the gravitational-like coupling addressed here fundamentally differs from the models of gravitationally induced decoherence proposed by Penrose and Diósi [1,2]. In the latter, gravity is viewed as a source of intrinsic wave function collapse, leading to the suppression of quantum superpositions. In contrast, our framework adopts the semiclassical gravcat approach proposed by Anastopoulos and Hu [5]. In this approach, gravity acts as a coherent Newtonian potential that mediates an effective Ising-like interaction among spatially delocalized qubits. Within this paradigm, gravity does not induce decoherence; rather, it serves as a unitary coupling capable of generating and sustaining quantum correlations. This distinction is critical for understanding the stabilizing role of gravity in

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our findings. Moreover, the investigation into the effect of gravity on quantum correlations has attracted considerable attention in recent years, as advances in both theoretical and experimental methods have been stimulated [13–20]. However, a key unresolved question is how to distinguish genuine gravitational effects from other interactions, such as spin-orbit coupling.

At this stage, the central question is not merely whether coherent internal couplings can transiently counteract environmental damping but which interaction is responsible for the genuine long-time stabilization of quantum resources. In the present framework, the Dzyaloshinskii-Moriya (DM) spin-orbit coupling mainly acts as a source of oscillatory revivals and transient recovery, whereas the gravcat-induced coupling is the mechanism that can sustain nonvanishing asymptotic plateaus. This distinction is essential for separating delayed decay from true stationary protection in dissipative quantum-gravitational-inspired settings.

In this sense, our contribution lies in enhancing the gravcat states model by incorporating the DM spin-orbit term alongside gravitational coupling. While the DM interaction is not of direct Newtonian origin, its inclusion is physically motivated by two complementary scenarios: (i) as a leading-order antisymmetric correction arising from relativistic spin-gravity couplings in the effective low-energy Hamiltonian [21,22], or (ii) as an engineered interaction in analog quantum simulators (e.g., superconducting circuits or trapped ions) designed to emulate gravitational cat dynamics [23]. The unified Hamiltonian thus captures two coherent mechanisms: a distance-dependent interaction resembling Newtonian gravity, and an antisymmetric exchange from spin-orbit coupling. The first stabilizes quantum correlations, while the second generates Rabi-like revivals that counteract dissipative Markovian decay. This interplay highlights how coherent interactions can reshape dissipative dynamics and promote the persistence of quantum resources over extended timescales in open quantum systems. Similar robustness mechanisms have recently been reported in gravcat-state platforms subjected to noisy environments, where coherence and quantum correlations may survive well beyond naive decoherence expectations [17].

The physical motivation for emphasizing a gravity-inspired interaction in this study aligns closely with the work established by Rahman *et al.* [24]. While foundational proposals, such as those by Anastopoulos-Hu [5] and the Bose-Marletto-Vedral (BMV) setups [3,4], focus on emulating or testing the fundamental quantum characteristics of gravity, Rahman *et al.* demonstrated that the structural form and intrinsic parameters of gravitational cat states can be utilized as a distinctive resource to mitigate environmental noise. This study extends that perspective into a purely dissipative Markovian regime. Specifically, this investigation examines how the distance-dependent Ising-type coupling characteristic of gravitational configurations competes with local amplitude damping, which is the predominant decoherence mechanism in superconducting platforms [25,26]. The findings reveal that this interaction does not merely emulate a general coupling but instead offers a unique mechanism whereby the gravitational-like term prevents total information loss by inducing nonvanishing asymptotic plateaus. This feature suggests a qualitative distinction from standard isotropic cross-couplings operating under independent dissipation channels.

The paper is structured into five main sections. Following the introduction, Sec. II presents the physical model as an open quantum system, including the Hamiltonian system and its solution. The Lindblad master equation is then used to describe the effect of a dissipative, Markovian environment on the quantum system. The exact solution is provided in Appendix A. Section III provides numerical results of quantum measures, the definitions of which are provided in Appendix C. Section IV presents fidelity as a dynamical stability indicator, showing its stationary behavior as a function of the main parameters of the system to quantify the gap between the initial and instantaneous states in a dissipative environment. We have also provided the details of the analytical steady-state solution in Appendix B. The conclusion and experimental considerations are summarized in Sec. V.

II. PHYSICAL MODEL AS AN OPEN QUANTUM SYSTEM

A. Effective two-level description

Each mass is assumed to be confined in a symmetric double-well potential, as in gravity-mediated entanglement proposals. When the barrier height is large, the dynamics are restricted to the two localized position states $|L\rangle$ and $|R\rangle$, which form an effective qubit. In this subspace, the position operator maps onto a Pauli operator ($\hat{x} \propto \hat{s}_z$), while tunneling between the wells generates a transverse term proportional to $\hat{s}_x$. Projecting the Newtonian gravitational interaction onto this reduced subspace yields an effective spin-spin coupling between the two masses. This effective description follows the approach of Bose *et al.* [3] and Marletto-Vedral [4], where spatial superpositions are treated as qubits. Our analysis, however, additionally incorporates environmental decoherence within an open-system framework, in line with the considerations of Anastopoulos and Hu [5].

The model consists of a set of two particles, each of which is considered a one-dimensional double-well potential. The total Hamiltonian of the coupled two-qubit system, which combines gravitational-like and spin-orbit (DM) interactions, is expressed as follows [5,27]:

$$\hat{H} = \sum_{i=1}^{2}\left(\frac{\hbar\omega}{2} + g\mu_B B_z\right)\hat{s}_z^{(i)} - \delta\,\hat{s}_x^{(1)}\hat{s}_x^{(2)} + \lambda\left(\hat{s}_x^{(1)}\hat{s}_y^{(2)} - \hat{s}_y^{(1)}\hat{s}_x^{(2)}\right). \tag{1}$$

In the standard computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the matrix form of the above Hamiltonian reads

$$\hat{H} = \begin{pmatrix} \hbar\omega + 2g\mu_B B_z & 0 & 0 & -\delta \\ 0 & 0 & -\delta + 2i\lambda & 0 \\ 0 & -\delta - 2i\lambda & 0 & 0 \\ -\delta & 0 & 0 & -\hbar\omega - 2g\mu_B B_z \end{pmatrix}. \tag{2}$$

TABLE I. Eigenenergies ξ_i and eigenvectors $|\Psi_i\rangle$ of the system in the computational basis.

State	Eigenenergy ξ_i	Eigenvector $|\psi_i\rangle$
$|\Psi_1\rangle$	$-|\nu|$	$\frac{1}{\sqrt{2}}\left(-\frac{\nu}{|\nu|}\,|01\rangle + |10\rangle\right)$
$|\Psi_2\rangle$	$+|\nu|$	$\frac{1}{\sqrt{2}}\left(\frac{\nu}{|\nu|}\,|01\rangle + |10\rangle\right)$
$|\Psi_3\rangle$	$-|\mu|$	$\frac{1}{\eta_+}\{-[|\mu| + (\mu^2 - \tilde{\delta}^2)^{1/2}]\,|00\rangle + \tilde{\delta}\,|11\rangle\}$
$|\Psi_4\rangle$	$+|\mu|$	$\frac{1}{\eta_-}\{[|\mu| - (\mu^2 - \tilde{\delta}^2)^{1/2}]\,|00\rangle + \tilde{\delta}\,|11\rangle\}$

Here, the parameter ω denotes the intrinsic transition frequency of each qubit, determining the energy gap between its ground and excited states. The term $\sum_{i=1}^{2}\frac{\hbar\omega}{2}\hat{s}_z^{(i)}$ thus accounts for the free evolution of the two noninteracting qubits. The terms $\hat{s}_x$ and $\hat{s}_z$ are Pauli spin-$\frac{1}{2}$ operators. The gravitational-like coupling is defined by $\delta = \frac{Gm^2}{2}(\frac{1}{x} - \frac{1}{x'})$, where m denotes the mass of the particle, G is the universal gravitational constant, and x, x' is the relative distances between the particles. The term $g\mu_B B_z \sum_{i=1}^{2}\hat{s}_z^{(i)}$ represents the Zeeman term, which describes the coupling of each spin to an external magnetic field applied along the z axis. Finally, the parameter λ regulates the spin-orbit interaction strength, representing the antisymmetric DM interaction between the two qubits. We emphasize that the inclusion of λ is not to claim a microscopic spin-gravity derivation of the DM interaction but to introduce a physically motivated reference parameter for coherent coupling in a dissipative open-system model.

In the standard computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the eigenenergies and its corresponding eigenvectors are listed in Table I .

Note that all parameters are expressed in units of $\hbar\omega$. Accordingly, we define the reduced parameters $\tilde{\delta} = \delta/\hbar\omega$, $\tilde{\lambda} = \lambda/\hbar\omega$, and $\tilde{B}_z = g\mu_B B_z/\hbar\omega$ for dimensionless analysis throughout the paper. Moreover, we define the auxiliary variables listed in Table I as $\mu = [\tilde{\delta}^2 + (2\tilde{B}_z + 1)^2]^{1/2}$ and $\nu = (\tilde{\delta}^2 + 4\tilde{\lambda}^2)^{1/2}$, with normalization constants: $\eta_\pm = \{2[\mu^2 \pm \mu(\mu^2 - \tilde{\delta}^2)^{1/2}]\}^{1/2}$. Interestingly, the combination of gravitational-like and DM couplings $(\tilde{\delta}, \tilde{\lambda})$ generates an effective two-level system within the $\{|01\rangle, |10\rangle\}$ subspace, yielding eigenvalues $\pm\nu$. This energy defines a generalized Rabi oscillation, analogous to the driven two-level system but arising from internal coherent coupling rather than external fields. These Rabi-like oscillations govern the coherent exchange of excitation between qubits and underline the nonmonotonic dynamics of quantum features.

B. Lindblad master equation

The two-qubit system is modeled as an open quantum system influenced by a dissipative environment. Its nonunitary dynamics are described by the Lindblad master equation given by [28]

$$\frac{d\hat{\varrho}}{dt} = -\frac{i}{\hbar}[\hat{H}, \hat{\varrho}] + \sum_{k=1}^{2}\gamma_k \mathcal{D}[\hat{s}_-^{(k)}]\hat{\varrho}, \tag{3}$$

where $\mathcal{D}[L]\hat{\varrho} = L\hat{\varrho}L^\dagger - \frac{1}{2}\{L^\dagger L, \hat{\varrho}\}$, the lowering operator in the dissipator is defined as $\hat{s}_-^{(k)} = |0\rangle_k\langle 1| = \frac{1}{2}(\hat{s}_x^{(k)} - i\hat{s}_y^{(k)})$, and γ_k is the decay rate of the kth qubit. The Lindblad dissipator $\mathcal{D}[\hat{s}_-^{(k)}]$ specifically models local amplitude damping, corresponding to energy relaxation from the excited state $|1\rangle$ to the ground state $|0\rangle$ at rate γ. This is the dominant decoherence channel (T_1 process) in superconducting qubit architectures [26,29], where the environment acts as a zero-temperature reservoir absorbing energy from the system. This choice aligns with our focus on engineered platforms capable of simulating gravitational cat dynamics. Additionally, we consider independent and identically distributed local Markovian reservoirs for each qubit ($\gamma_1 = \gamma_2 = \gamma$), which is a valid assumption when the separation between qubits is significantly greater than the correlation length of the surrounding environment. This condition ensures that cross-damping and collective decay effects are negligible, which is relevant to spatially separated mesoscopic systems, such as levitated nanoparticles or superconducting qubits housed in distinct cavities [30,31]. This approach allows for the isolation of the influence of internal coherent couplings $(\tilde{\delta}, \tilde{\lambda})$ from environmental correlations, thus facilitating a clear evaluation of their intrinsic stabilizing effects.

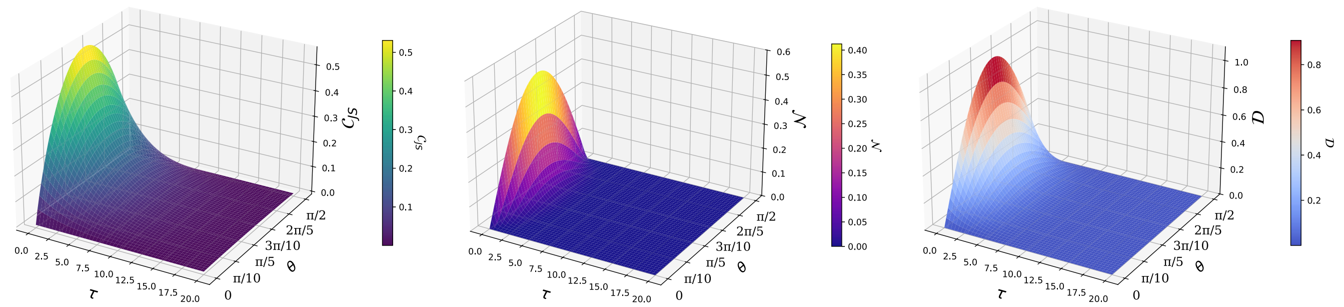
For the sake of dimensionless quantities, by dividing by ω, the reduced decay rate parameter is defined by $\tilde{\gamma} = \gamma/\omega$. Throughout the paper, we consider weak interaction and $\tilde{\gamma} = 0.5$ and we assume the Bell-like state as the initial state given by $|\psi_\theta\rangle = \cos\theta\,|01\rangle + \sin\theta\,|10\rangle$, with $\theta \in [0, \frac{\pi}{2}]$ being its amplitude. In the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the exact solution of Eq. (3) yields an X state (the detailed solution is represented in Appendix A) given as follows:

$$\hat{\varrho}(t) = \begin{pmatrix} \varrho_{11} & 0 & 0 & \varrho_{14} \\ 0 & \varrho_{22} & \varrho_{23} & 0 \\ 0 & \varrho_{32} & \varrho_{33} & 0 \\ \varrho_{41} & 0 & 0 & \varrho_{44} \end{pmatrix}. \tag{4}$$

III. TRANSIENT DYNAMICS AND DECOHERENCE MITIGATION

In what follows, we analyze the time evolution of quantum resources as a function of the dimensionless time $\tau = \omega t$ and physical parameters involved in the Hamiltonian system. At the onset, the observations drawn from Fig. 1 highlight key dynamical behaviors of quantum resources: Jensen-Shannon coherence $\mathcal{C}_{\rm JS}$ [Fig. 1(a)], negativity $\mathcal{N}$ [Fig. 1(b)], and trace distance discord $\mathcal{D}$ [Fig. 1(c)] in the two-qubit open system to a dissipative environment and free from the gravitational, DM spin-orbit interactions, and external magnetic field. The optimal quantum resources occur when $\theta = \pi/4$, which is obvious since this initial state becomes a Bell state corresponding to a maximally entangled state and exhibits strong quantum correlations. All three measures decrease as the dimensionless time τ increases, reflecting the decoherence and dissipation of quantum resources due to interaction with the environment.

Figure 2 illustrates the interaction time dependence of quantum coherence $\mathcal{C}_{\rm JS}$, trace distance discord $\mathcal{D}$, and negativity $\mathcal{N}$ for different values of the spin-orbit interaction strength $\tilde{\lambda}$, while keeping the gravitational coupling $\tilde{\delta} = 0.0$, magnetic field $\tilde{B}_z = 0.5$, and initial state amplitude $\theta = \pi/4$ fixed. In Fig. 2(a), where $\tilde{\lambda} = 0.0$, all resources decay



(a) (b) (c)

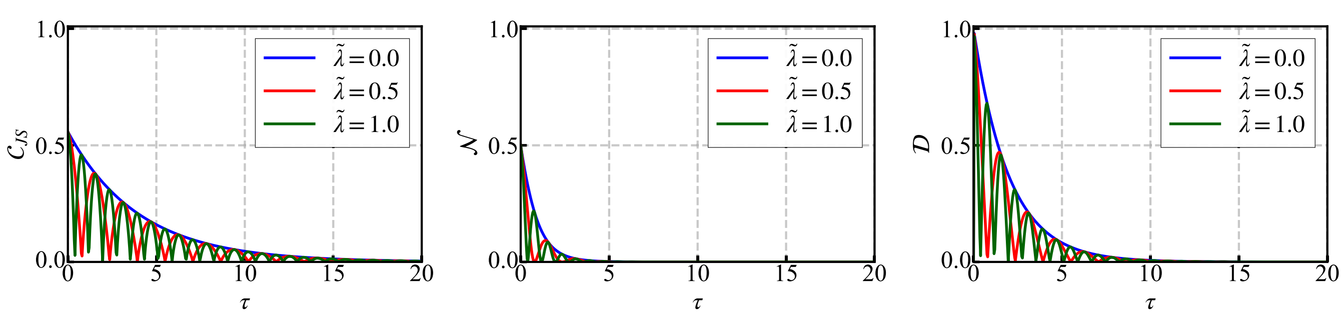
FIG. 1. Three-dimensional (3D) plots of quantum resources [(a) Jensen-Shannon quantum coherence $\mathcal{C}_{\rm JS}$, (b) Negativity $\mathcal{N}$, and (c) Trace Distance Discord $\mathcal{D}$] as a function of dimensionless time τ and initial-state amplitude θ, for $\tilde{\delta} = \tilde{\lambda} = \tilde{B}_z = 0$.

monotonically, with the robustness hierarchy $\mathcal{C}_{\rm JS} \supset \mathcal{D} \supset \mathcal{N}$ preserved throughout the evolution. This behavior confirms that quantum coherence remains nonvanishing for longer times compared with $\mathcal{N}$ and $\mathcal{D}$. So quantum coherence is a more fundamental resource, as entanglement $\mathcal{N}$ and geometric discord $\mathcal{D}$ generally require coherence for their generation, but not vice versa [32]. When $\tilde{\lambda}$ increases to 0.5 and 1.0 in Figs. 2(b) and 2(c), the emergence of Rabi-like oscillations, with frequency $\nu = (\tilde{\delta}^2 + 4\tilde{\lambda}^2)^{1/2}$, drives coherent revivals in coherence and discord with partial recovery of entanglement, partially counteracting dissipative decay. The persistence of $\mathcal{C}_{\rm JS}$ under decoherence, even in the absence of gravitational coupling ($\tilde{\delta} = 0.0$), highlights its robustness as a resource, while the rapid degradation of $\mathcal{N}$ reflects the fragility of entanglement. These results demonstrate that the spin-orbit interaction $\tilde{\lambda}$, acting as an internal coherent driving term, can effectively protect or revive quantum coherence. In contrast, the emergence of nonlocal quantum correlations remains more limited. This confirms that spin-orbit coupling alone acts as a coherent driving mechanism but cannot stabilize quantum resources without additional interactions. This temporal profile establishes the relation $\mathcal{D} \supset \mathcal{N}$ within the hierarchy. As shown by the blue curves ($\tilde{\lambda} = 0.0$), $\mathcal{N}(\tau)$ vanishes abruptly at $\tau \approx 4$, whereas $\mathcal{D}(\tau)$ decays purely asymptotically and remains strictly nonzero without experiencing sudden collapse, demonstrating that quantum discord captures quantum correlations beyond entanglement [33,34].

Figure 3 explores the effect of gravitational-like coupling $\tilde{\delta}$ on quantum resources at fixed $\tilde{\lambda} = 1.0$, $\tilde{B}_z = 0.5$, and $\theta = \pi/4$. All three quantifiers exhibit damped oscillations followed by saturation at nonvanishing steady-state values. Increasing $\tilde{\delta}$ systematically raises the plateau values of $\mathcal{C}_{\rm JS}$ and $\mathcal{D}$, while $\mathcal{N}$ shows a more sensitive dependence, reviving after initial collapse for sufficiently strong coupling. These results, consistent with the analytical steady-state expressions [Eqs. (B1)–(B4)], demonstrate that $\tilde{\delta}$ mimics effective non-Markovian memory, establishing a dynamical equilibrium between coherent evolution and local damping.

Interestingly, the comparison between Figs. 2 and 3 isolates the distinct roles of the two coherent couplings. For $\tilde{\delta} = 0$ [Fig. 2], spin-orbit interaction $\tilde{\lambda}$ induces Rabi-like oscillations that produce transient revivals, yet all resources decay asymptotically to zero, consistent with prior studies of gravcats under thermal or dephasing noise [15–17]. In contrast, for $\tilde{\delta} > 0$ [Fig. 3], gravitation preserves quantum resources. The analytical steady-state expressions [Eqs. (B1)–(B4)] confirm that these plateaus depend solely on $\tilde{\delta}$, $\tilde{\gamma}$, and $\tilde{B}_z$, providing a falsifiable protocol for engineered platforms. Thus, while $\tilde{\lambda}$ offers transient protection, $\tilde{\delta}$ enables genuine long-term stabilization, positioning gravity-like interactions as active stabilizers rather than mere entanglement generators.

Figure 4 presents the parametric dependence of quantum coherence $\mathcal{C}_{\rm JS}$, negativity $\mathcal{N}$, and trace distance discord $\mathcal{D}$ on the magnetic field $\tilde{B}_z$ and gravitational-like interaction $\tilde{\delta}$, under fixed conditions of $\tau = 10$ and $\theta = \pi/4$. $\mathcal{C}_{\rm JS}$ shows a monotonic enhancement as $\tilde{\delta}$ increases (for higher values of $\tilde{B}_z$), indicating its structural resilience to decoherence [Eq. (B4)]. Conversely, the negativity $\mathcal{N}$ is observed only



(a) (b) (c)

FIG. 2. Time evolution of (a) Jensen-Shannon quantum coherence $\mathcal{C}_{\rm JS}$, (b) Negativity $\mathcal{N}$, and (c) Trace Distance Discord $\mathcal{D}$ for three values of the DM coupling: $\tilde{\lambda} = 0,\ 0.5,$ and 1.0, at fixed $\tilde{\delta} = 0$, $\tilde{B}_z = 0.5$, and initial Bell state $\theta = \pi/4$.

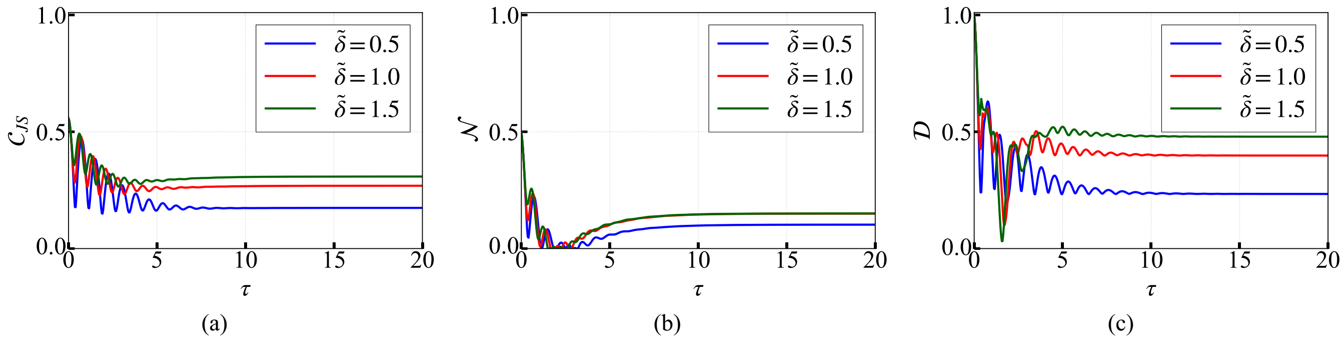


FIG. 3. Time evolution of (a) Jensen-Shannon quantum coherence $C_{\rm JS}$, (b) Negativity $\mathcal{N}$, and (c) Trace Distance Discord $\mathcal{D}$ for three values of the gravitational-like coupling: $\tilde{\delta} = 0.5,\ 1.0,$ and 1.5, at fixed $\tilde{\lambda} = 1.0$, $\tilde{B}_z = 0.5$, and initial Bell state $\theta = \pi/4$.

within narrow and isolated regions of the parameter space, highlighting the pronounced fragility of entanglement. Notably, lower values of $\tilde{B}_z$ may entirely suppress entanglement, suggesting that the magnetic field does not confer uniform benefits. In contrast, the discord $\mathcal{D}$ exhibits a wider and more sustained distribution, indicating that quantum correlations extending beyond entanglement exhibit considerable robustness. In the limiting case of $\tilde{\delta} = 0$, all quantum resources disappear uniformly, irrespective of magnetic-field strength. This finding underscores the necessity of gravitational-like coupling for the emergence and maintenance of nonclassical correlations and quantum coherence. Also, the present analysis underscores the distinct stability profiles of quantum resources and demonstrates that the coordinated manipulation of $\tilde{B}_z$ and $\tilde{\delta}$ facilitates the selective preservation of nonentangled quantum correlations within open quantum systems. Notably, the hierarchy $\mathcal{D} \supset N$ outlined in the description of Fig. 2 is further corroborated by the parametric analysis in Figs. 4(b) and 4(c). While the negativity $\mathcal{N}$ is strictly confined to narrow, isolated regions and easily destroyed by the Zeeman field, the trace distance discord $\mathcal{D}$ maintains a wide and resilient distribution on the $(\tilde{\delta}, \tilde{B}_z)$ plane.

IV. QUANTUM FIDELITY AS A DYNAMICAL STABILITY INDICATOR

In this section, we aim to evaluate the dynamical stability of an initial quantum state under open-system evolution. We analyze the quantum fidelity, which is defined as follows [35]

$$\mathcal{F}(\tau) = \mathrm{Tr}[\hat{\chi}(0)\,\hat{\varrho}(\tau)], \tag{5}$$

where $\hat{\chi}(0) = |\Psi^+\rangle\langle\Psi^+|$ represents the pure initial state, and $\hat{\varrho}(\tau)$ corresponds to the evolved density matrix of Eq. (4) (where $t \rightarrow \frac{\tau}{\omega}$) governed by the dissipative dynamics of the system. We choose $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$, which serves as the reference state for fidelity computation. Hence, the general expression of fidelity [Eq. (5)] simplifies to

$$\mathcal{F}(\tau) = \tfrac{1}{2}[\varrho_{22}(\tau) + \varrho_{33}(\tau)] + \mathrm{Re}[\varrho_{23}(\tau)]. \tag{6}$$

By construction, $\mathcal{F}(0) = 1$, and the subsequent decay of $\mathcal{F}(\tau)$ quantifies the loss of overlap between the instantaneous state $\hat{\varrho}(\tau)$ and the initial entangled configuration $\hat{\chi}(0)$. Thus, $\mathcal{F}(\tau)$ serves as a direct measure of dynamical stability under dissipative evolution.

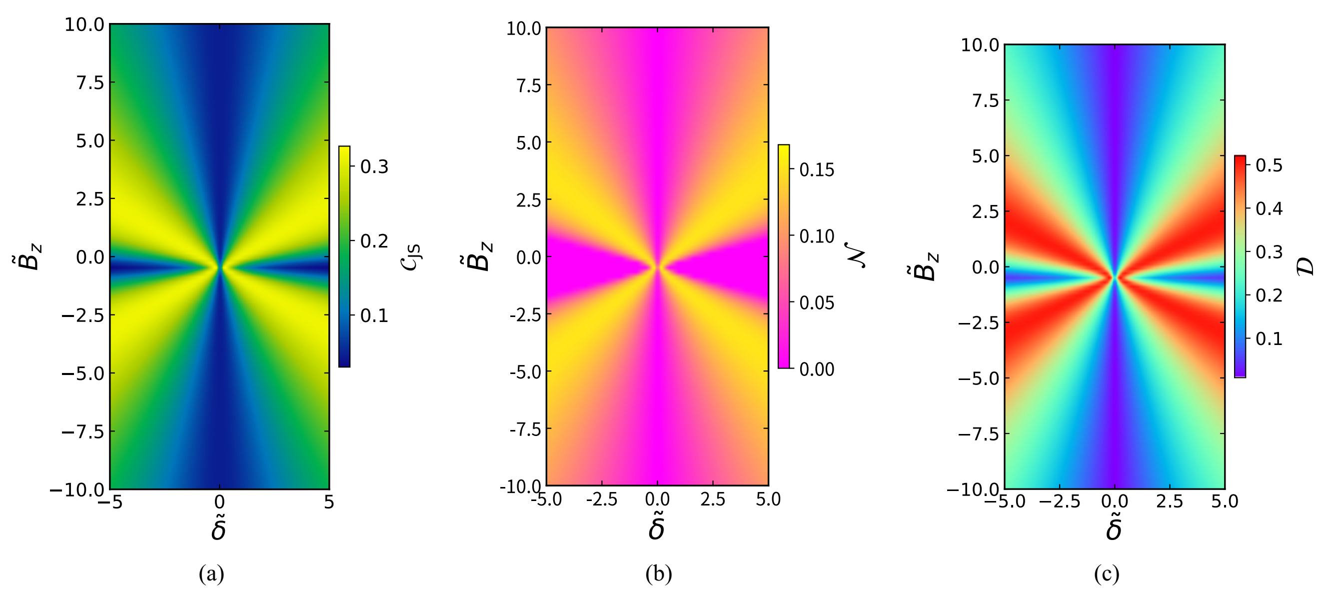


FIG. 4. Density plots of quantum resources [(a) Jensen-Shannon quantum coherence $C_{\rm JS}$, (b) Negativity $\mathcal{N}$, and (c) Trace Distance Discord $\mathcal{D}$] as a function of the external magnetic field $\tilde{B}_z$ and the gravitational-like coupling $\tilde{\delta}$, for initial Bell state $\theta = \pi/4$, and $\tilde{\lambda} = 0.0$, $\tau = 10$.

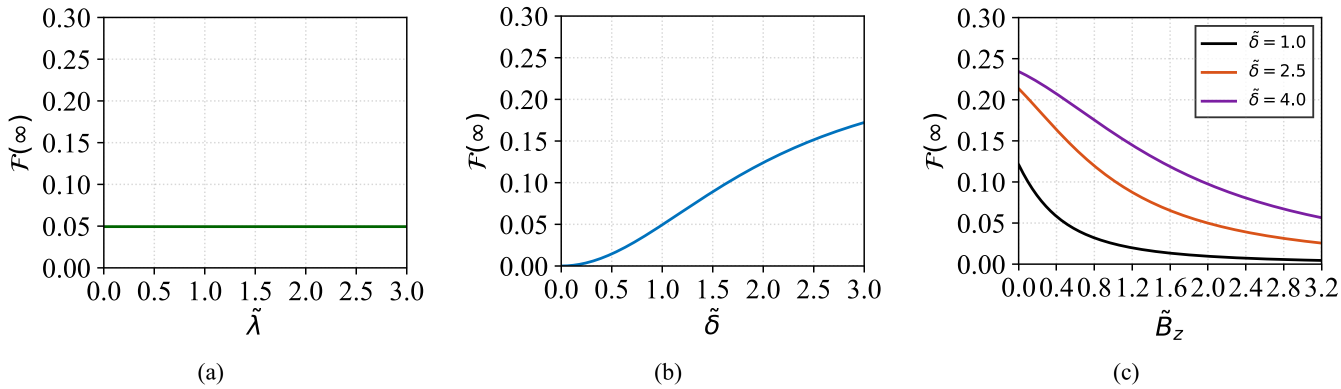


FIG. 5. Stationary fidelity $\mathcal{F}(\infty)$ as a function of (a) spin-orbit coupling $\tilde{\lambda}$ ($\tilde{\delta} = 1.0$ and $\tilde{B}_z = 0.5$), (b) gravitational-like coupling $\tilde{\delta}$ ($\tilde{B}_z = 0.5$), and (c) magnetic field $\tilde{B}_z$ for selected values of $\tilde{\delta}$. In all panels, we fix $\theta = \pi/4$.

Stationary fidelity $\mathcal{F}(\infty)$, obtained analytically [see Eq. (7)], as a function of the normalized system parameters. The steady-state fidelity is fully determined by the competition between the gravitation-like parameter $\tilde{\delta}$, the magnetic field $\tilde{B}_z$, and the dissipation rate $\tilde{\gamma}$:

$$\mathcal{F}(\infty) = \frac{\tilde{\delta}^2}{\tilde{\gamma}^2 + 4[\tilde{\delta}^2 + (2\tilde{B}_z + 1)^2]} \times [\text{derived from Eqs. (6), (B1), (B2)}]. \tag{7}$$

To gain deeper insight into the mechanisms governing the long-time preservation of the initially prepared gravitational cat state, we analyze the asymptotic fidelity $\mathcal{F}(\infty)$ as a function of the relevant system parameters. We first examine the role of the DM spin-orbit interaction, since this coupling governs the coherent exchange dynamics between the two gravcats. As illustrated in Fig. 5(a), the stationary fidelity remains unchanged over the entire range of the spin-orbit coupling strength $\tilde{\lambda}$. This result is fully consistent with the analytical expression given in Eq. (7), where $\tilde{\lambda}$ does not explicitly appear. Therefore, while the DM interaction can strongly influence the transient oscillatory dynamics, it has no impact on the asymptotic fidelity and does not contribute to the long-time stabilization of the gravitational cat state in the present Markovian dissipative regime.

We next turn to the gravity-inspired coupling $\tilde{\delta}$, whose contribution to the stationary solution suggests a direct influence on the asymptotic behavior of the system. The dependence of $\mathcal{F}(\infty)$ on $\tilde{\delta}$ is displayed in Fig. 5(b) for $\tilde{B}_z = 0.5$. As the coupling strength increases, the asymptotic fidelity grows monotonically before gradually approaching a saturation regime. This behavior indicates that stronger gravity-inspired interactions enhance the overlap between the steady state and the initially prepared gravitational cat state, thereby improving its resilience against environmental dissipation. The observed saturation further suggests that beyond a certain coupling strength, the stabilizing effect becomes less sensitive to additional increases in $\tilde{\delta}$.

Finally, we investigate the influence of the external magnetic field $\tilde{B}_z$, which is shown in Fig. 5(c) for different values of the gravity-inspired coupling. In contrast with the behavior induced by $\tilde{\delta}$, increasing the magnetic field leads to a systematic reduction of the stationary fidelity. Physically, the Zeeman contribution introduces an increasing energy imbalance that competes with the coherent interaction responsible for maintaining the gravitational cat state. As a consequence, the overlap between the asymptotic and initial states progressively decreases as $\tilde{B}_z$ becomes larger. Taken together, these results demonstrate that the long-time preservation of the gravitational cat state is primarily governed by the competition between dissipation, the gravity-inspired coupling, and the external magnetic field, whereas the DM interaction mainly affects the transient dynamical regime.

V. CONCLUSION AND EXPERIMENTAL CONSIDERATIONS

In sum, we have derived closed-form analytical solutions to the Lindblad master equation and quantified the behavior of various quantum resources and fidelity as a dynamical indicator of quantum state stabilization. The quantum resources include entanglement (via negativity), quantum coherence (via Jensen-Shannon divergence), and geometric quantum discord (via trace distance discord).

Our findings demonstrate that a gravitational-like coupling, assisted by the DM interaction, can stabilize quantum resources over long times in a Markovian dissipative environment. By deriving exact analytical solutions and the full steady-state density matrix, we identify a clear mechanism through which coherent interactions generate nonvanishing stationary quantum resources. This approach complements the broader noise analyses of gravitational cat states performed by other studies [15–17,24] and provides an analytically controlled model together with a concrete, experimentally testable protocol for current superconducting-circuit platforms. Our finding that stronger gravitational-like coupling favors long-time preservation does not contradict Rahman *et al.* [24], given the different dynamical regimes and notions of preservation. In this dissipative setting, a larger coupling counteracts amplitude damping, enhancing nonvanishing asymptotic quantum correlations.

Moreover, our findings should not be interpreted as contradicting gravitationally induced decoherence scenarios such as those of Diósi and Penrose [1,2]; rather, they explore a complementary regime inspired by semiclassical gravitational models [5], where gravity acts as a coherent interaction channel between spatially delocalized qubits. This highlights the model-dependence of the role of gravity in quantum dynamics and underscores the importance of distinguishing between gravity as a source of intrinsic noise and as a mediator of quantum resources [3,4,20]. This distinction lies at the heart of recent proposals to test the quantum nature of gravity in laboratory settings [3,4]. Indeed, if gravity can generate entanglement between distant masses, as argued in the BMV framework [3,4], then it must possess quantum features, and its effective description as a unitary coupling (as adopted here) becomes physically meaningful even if engineered or simulated. Our results thus align with this emerging paradigm, in which gravitational-like coupling is harnessed as a resource for mitigating information loss and preserving nonclassical correlations, rather than being viewed solely as a decoherence mechanism.

The existing framework does not constitute a first-principles microscopic derivation of a gravity-induced interaction. It rather establishes a controlled open-system environment that contrasts a gravity-inspired coherent coupling with nongravitational exchange under conditions of dissipation. In this particular setting, spin-orbit coupling results in only transient revivals, while the gravitational-like coupling serves as the mechanism that sustains nonvanishing long-time steady-state quantum resources. The long-time nonvanishing steady-state values of quantum resources [Eqs. (B1)–(B4)] result from the interplay between the gravcat-derived coupling $-\delta\,\hat{s}_x^{(1)}\hat{s}_x^{(2)}$ and local Markovian dissipation ($\mathcal{D}[\hat{s}_-^{(k)}]$). While the Newtonian origin $\delta = Gm^2/r$ is not yet accessible for genuine massive superpositions, the dynamical mechanism can be tested in engineered quantum platforms. Superconducting circuits, for instance, support tunable qubit-qubit couplings and local amplitude damping, the latter being the dominant decoherence channel in transmon devices [29]. Moreover, Rabi-like revivals are not driven by external microwave fields but emerge intrinsically from engineered spin-orbit couplings, offering a dissipation-resilient alternative to conventional coherent control. The initial Bell state $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ can be prepared with high fidelity using standard gate sets [36]. Although our paper does not implement gravity, the effective interaction $\hat{s}_x^{(1)}\hat{s}_x^{(2)}$ can be synthesized in superconducting circuits using parametric modulation techniques, as demonstrated in recent experiments on tunable qubit couplings [29]. Such platforms operate under local amplitude damping, consistent with the independent reservoir model of open quantum systems [30,31]. Performing quantum state tomography at times $t \gg 1/\gamma$ would allow direct verification that $\varrho_{14}(\infty)$ saturates to nonzero values only when a $\hat{s}_x^{(1)}\hat{s}_x^{(2)}$ coupling is present, as predicted analytically. The insensitivity of the steady state to the initial amplitude θ and spin-orbit strength $\tilde{\lambda}$ [Eqs. (B1)–(B4)] ensures robustness. This provides a direct and falsifiable protocol for testing the proposed stabilization mechanism using current quantum hardware.

Beyond the specific gravity-inspired framework considered here, the present analytical approach may be extended to multipartite systems, structured environments, and experimentally motivated quantum simulators. Such directions are closely connected to ongoing efforts aimed at understanding the interplay between coherent interactions, environmental noise, and the preservation of quantum resources in gravity-inspired quantum systems [3,4,20,23].

DATA AVAILABILITY

All data needed to evaluate the conclusions in the paper are present in the paper.

APPENDIX A: EXACT SOLUTION OF DIFFERENTIAL MASTER EQUATION

Introducing the dimensionless time $\tau = \omega t$ and the reduced decay rate $\tilde{\gamma} = \gamma/\omega$, the Lindblad master equation in dimensionless form (after dividing by ω) reads

$$\frac{d\hat{\varrho}}{d\tau} = -i[\hat{H}, \hat{\varrho}] + \tilde{\gamma}\sum_{k=1}^{2}\mathcal{D}[\hat{s}_-^{(k)}]\hat{\varrho}, \tag{A1}$$

where $\mathcal{D}[L]\hat{\varrho} = L\hat{\varrho}L^\dagger - \frac{1}{2}\{L^\dagger L, \hat{\varrho}\}$.

Expanding Eq. (A1) for the X-shape density matrix [Eq. (4)] yields the following system of differential equations in τ:

$$\frac{d\varrho_{11}}{d\tau} = -2\tilde{\gamma}\varrho_{11} - i\tilde{\delta}(\varrho_{14} - \varrho_{41}), \tag{A2}$$

$$\frac{d\varrho_{14}}{d\tau} = -[\tilde{\gamma} + 2i(1 + 2\tilde{B}_z)]\varrho_{14} - i\tilde{\delta}(\varrho_{11} - \varrho_{44}), \tag{A3}$$

$$\frac{d\varrho_{22}}{d\tau} = \tilde{\gamma}(\varrho_{11} - \varrho_{22}) + (2\tilde{\lambda} - i\tilde{\delta})\varrho_{23} + (2\tilde{\lambda} + i\tilde{\delta})\varrho_{32}, \tag{A4}$$

$$\frac{d\varrho_{23}}{d\tau} = -\tilde{\gamma}\varrho_{23} - i(\tilde{\delta} - 2i\tilde{\lambda})(\varrho_{22} - \varrho_{33}), \tag{A5}$$

$$\frac{d\varrho_{33}}{d\tau} = \tilde{\gamma}(\varrho_{11} - \varrho_{33}) + i(\tilde{\delta} + 2i\tilde{\lambda})\varrho_{23} + (-2\tilde{\lambda} - i\tilde{\delta})\varrho_{32}, \tag{A6}$$

$$\frac{d\varrho_{44}}{d\tau} = \tilde{\gamma}(\varrho_{22} + \varrho_{33}) + i\tilde{\delta}(\varrho_{14} - \varrho_{41}), \tag{A7}$$

with $\varrho_{14} = \varrho_{41}^*$ and $\varrho_{23} = \varrho_{32}^*$.

All reduced variables follow the definitions given in Sec. II.

For the initial Bell-like state $|\psi_\theta\rangle = \cos\theta|01\rangle + \sin\theta|10\rangle$, the exact time-dependent solution in τ reads

$$\varrho_{11}(\tau) = \frac{\tilde{\delta}^2 e^{-\tilde{\gamma}\tau}[2\mu\sinh(\tilde{\gamma}\tau) - \tilde{\gamma}\sin(2\mu\tau)]}{\mu(\tilde{\gamma}^2 + 4\mu^2)}, \tag{A8}$$

$$\begin{aligned}\varrho_{14}(\tau) = {}& \frac{\sqrt{i}\tilde{\delta}e^{-\tilde{\gamma}\tau}}{4\mu^3(\tilde{\gamma}^2 + 4\mu^2)}\big[-32i\tilde{B}_z^3[\tilde{\gamma}\sin(2\mu\tau) - 2\mu(e^{\tilde{\gamma}\tau} - 1)] + 8\tilde{B}_z^2[\tilde{\gamma}(\tilde{\gamma} - 6i)\sin(2\mu\tau) + 12i\mu(e^{\tilde{\gamma}\tau} - 1)\\ &+ 2\tilde{\gamma}\mu(\cos(2\mu\tau) - e^{\tilde{\gamma}\tau})] + 4i\tilde{B}_z(\mu\{\tilde{\gamma}^2[\cos(2\mu\tau) - 1] + 4(\tilde{\delta}^2 + 3)(e^{\tilde{\gamma}\tau} - 1) - 4i\tilde{\gamma}[\cos(2\mu\tau) - e^{\tilde{\gamma}\tau}]\}\\ &- 2i\tilde{\gamma}\sin(2\mu\tau)(\tilde{\gamma} - 3i) - i\tilde{\delta}^2) + 2i\mu\{\tilde{\gamma}^2[\cos(2\mu\tau) - 1] - 2i\tilde{\gamma}(\tilde{\delta}^2 + 1)[\cos(2\mu\tau) - e^{\tilde{\gamma}\tau}]\\ &+ 4(\tilde{\delta}^2 + 1)(e^{\tilde{\gamma}\tau} - 1)\} + 2\tilde{\gamma}(\tilde{\gamma} - 2i)(\tilde{\delta}^2 + 1)\sin(2\mu\tau)\big],\end{aligned} \tag{A9}$$

$$\varrho_{22}(\tau)=\frac{1}{2}e^{-\tilde{\gamma}\tau}\left(\frac{\tilde{\delta}^2[4\mu^2\cosh(\tilde{\gamma}\tau)+\tilde{\gamma}^2\cos(2\mu\tau)]}{\mu^2(\tilde{\gamma}^2+4\mu^2)}+\frac{2\tilde{\lambda}\sin(2\theta)\sin(2\nu\tau)}{\nu}+\cos(2\theta)\cos(2\nu\tau)-\frac{\tilde{\delta}^2}{\mu^2}+1\right), \tag{A10}$$

$$\varrho_{23}(\tau)=\frac{e^{-(\tilde{\gamma}+2i\nu)\tau}}{4(\tilde{\delta}+2i\tilde{\lambda})}(2\sin(2\theta)e^{2i\nu\tau}[\tilde{\delta}+2i\tilde{\lambda}\cos(2\nu\tau)]+\nu\cos(2\theta)(1-e^{4i\nu\tau})), \tag{A11}$$

$$\varrho_{33}(\tau)=\frac{1}{2}e^{-\tilde{\gamma}\tau}\left(\frac{\tilde{\delta}^2[4\mu^2\cosh(\tilde{\gamma}\tau)+\tilde{\gamma}^2\cos(2\mu\tau)]}{\mu^2(\tilde{\gamma}^2+4\mu^2)}-\frac{2\tilde{\lambda}\sin(2\theta)\sin(2\nu\tau)}{\nu}-\cos(2\theta)\cos(2\nu\tau)-\frac{\tilde{\delta}^2}{\mu^2}+1\right), \tag{A12}$$

$$\begin{aligned}\varrho_{44}(\tau)=&\frac{e^{-2\tilde{\gamma}\tau}}{\mu^2(\tilde{\gamma}^2+4\mu^2)}(\mu^2e^{2\tilde{\gamma}\tau}[4(2\tilde{B}_z+1)^2+\tilde{\gamma}^2+\tilde{\delta}^2]\\&-e^{\tilde{\gamma}\tau}\{(2\tilde{B}_z+1)^2(\tilde{\gamma}^2+4\mu^2)-\tilde{\gamma}\tilde{\delta}^2[\mu\sin(2\mu\tau)+\tilde{\gamma}\cos(2\mu\tau)]\}-\tilde{\delta}^2\mu^2).\end{aligned} \tag{A13}$$

One verifies that the trace is preserved, $\mathrm{Tr}\,\hat{\varrho}(\tau)=1$, for all τ.

APPENDIX B: ANALYTICAL STEADY-STATE SOLUTION

From the above analytical solution, one obtains the asymptotic limit $\tau\to\infty$, and by discarding exponentially decaying transient terms, the stationary dimensionless elements are

$$\varrho_{11}(\infty)=\varrho_{22}(\infty)=\varrho_{33}(\infty)=\frac{\tilde{\delta}^2}{\tilde{\gamma}^2+4\mu^2}, \tag{B1}$$

$$\varrho_{23}(\infty)=0, \tag{B2}$$

$$\varrho_{44}(\infty)=\frac{\tilde{\gamma}^2+4(2\tilde{B}_z+1)^2+\tilde{\delta}^2}{\tilde{\gamma}^2+4\mu^2}, \tag{B3}$$

$$\begin{aligned}\varrho_{14}(\infty)=&\frac{\sqrt{2}\tilde{\delta}}{2(\tilde{\gamma}^2+4\mu^2)}[-(4\tilde{B}_z+\tilde{\gamma}+2)\\&+i(4\tilde{B}_z-\tilde{\gamma}+2)].\end{aligned} \tag{B4}$$

One checks explicitly that $\mathrm{Tr}\,\hat{\varrho}(\infty)=1$. Importantly, these closed-form results show that the steady-state depends only on $\tilde{\delta}$, $\tilde{\gamma}$, and $\tilde{B}_z$ (and not on the initial-state amplitude θ nor on $\tilde{\lambda}$). This explains why certain coherences and correlations persist over long times.

APPENDIX C: QUANTUM RESOURCE MEASURES

For completeness, we recall the standard definitions and provide the analytical expressions of the quantum resource quantifiers employed throughout the main text.

1. Jensen-Shannon coherence

The quantum Jensen-Shannon (JS) coherence is defined as [37]

$$\mathcal{C}_{\mathrm{JS}}=\sqrt{S[\hat{r}(\tau)]-\tfrac{1}{2}S[\hat{\varrho}(\tau)]-\tfrac{1}{2}S[\hat{\varrho}_d(\tau)]}, \tag{C1}$$

where $\hat{r}(\tau)=[\hat{\varrho}(\tau)+\hat{\varrho}_d(\tau)]/2$, and $\hat{\varrho}_d(\tau)=\mathrm{diag}(\varrho_{11}(\tau),\varrho_{22}(\tau),\varrho_{33}(\tau),\varrho_{44}(\tau))$.

For the present X-state density matrix, the eigenvalues of $\hat{\varrho}(\tau)$ are

$$\begin{aligned}e_1^{\pm}(\tau)=&\tfrac{1}{2}[\varrho_{11}(\tau)+\varrho_{44}(\tau)\\&\pm\sqrt{[\varrho_{11}(\tau)-\varrho_{44}(\tau)]^2+4|\varrho_{14}(\tau)|^2}],\end{aligned} \tag{C2}$$

$$\begin{aligned}e_2^{\pm}(\tau)=&\tfrac{1}{2}[\varrho_{22}(\tau)+\varrho_{33}(\tau)\\&\pm\sqrt{[\varrho_{22}(\tau)-\varrho_{33}(\tau)]^2+4|\varrho_{23}(\tau)|^2}].\end{aligned} \tag{C3}$$

Similarly, the eigenvalues of the intermediate state $\hat{r}(\tau)$ are

$$\begin{aligned}q_1^{\pm}(\tau)=&\tfrac{1}{2}[\varrho_{11}(\tau)+\varrho_{44}(\tau)\\&\pm\sqrt{[\varrho_{11}(\tau)-\varrho_{44}(\tau)]^2+|\varrho_{14}(\tau)|^2}],\end{aligned} \tag{C4}$$

$$\begin{aligned}q_2^{\pm}(\tau)=&\tfrac{1}{2}[\varrho_{22}(\tau)+\varrho_{33}(\tau)\\&\pm\sqrt{[\varrho_{22}(\tau)-\varrho_{33}(\tau)]^2+|\varrho_{23}(\tau)|^2}].\end{aligned} \tag{C5}$$

The corresponding entropies are evaluated from the standard von Neumann expression $S(\hat{X})=-\sum_i x\log_2 x_i$, where $\{x_i\}$ denotes the eigenvalue spectrum of the density matrix $\hat{X}$.

2. Negativity

The negativity is defined as [38]

$$\mathcal{N}=\frac{\|\hat{\varrho}^{T_1}\|_1-1}{2}, \tag{C6}$$

where $\|.\|_1$ denotes the trace norm and T_1 denotes the partial transpose with respect to the first subsystem.

For the present X-state density matrix, the partial transpose preserves a block structure, leading to the analytical expression

$$\begin{aligned}\mathcal{N}=\max\Bigg[0,&\sqrt{\left(\frac{\varrho_{22}(\tau)-\varrho_{33}(\tau)}{2}\right)^2+|\varrho_{14}(\tau)|^2}\\&-\frac{\varrho_{22}(\tau)+\varrho_{33}(\tau)}{2}\Bigg].\end{aligned} \tag{C7}$$

3. Trace distance discord

The trace distance discord (TDD) quantifies quantum correlations through the minimum trace distance between the density matrix ϱ and the set of classical-quantum states Ω_0 defined as [39]

$$\mathcal{D}=\min_{\chi\in\Omega_0}\tfrac{1}{2}\|\varrho-\chi\|_1. \tag{C8}$$

For the two-qubit X-state considered in the present work, the analytical expression of the TDD reads [40]

$$\mathcal{D} = \sqrt{\frac{c_1^2 c_{\max} - c_2^2 c_{\min}}{c_{\max} - c_{\min} + c_1^2 - c_2^2}}, \tag{C9}$$

with $c_{1,2} = 2(|\varrho_{23}(\tau)| \pm |\varrho_{14}(\tau)|)$, $c_3 = 1 - 2[\varrho_{22}(\tau) + \varrho_{33}(\tau)]$, and $x_3 = 2[\varrho_{11}(\tau) + \varrho_{22}(\tau)] - 1$.

The auxiliary quantities are given by $c_{\max} = \max\{c_3^2,\ c_2^2 + x_3^2\}$ and $c_{\min} = \min\{c_1^2,\ c_3^2\}$.

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