Conductivity of Thin Films in a Longitudinal Magnetic Field*

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An integral expression for the electrical conductivity of thin films in a longitudinal magnetic field is derived by an analysis of free electron trajectories in the film. This is evaluated in the limit of strong magnetic fields.

IT is well known that a free electron moving in parallel electric and magnetic fields will follow a helical trajectory with the axis of the helix parallel to the field. The projection of an element of the helix in the plane normal to the axis is an arc of a circle. If ψ is the arc angle in the plane normal to the axis, then the arc length of the element of the helix is $r_0\psi$, where r_0 is the radius of curvature of the electron path in a plane normal to magnetic field $H(r_0=mvc/eH)$. The radius of curvature in the plane normal to the axis of the helix is $r=r_0\sin\delta$, where δ is the angle made by the electron velocity and the field direction.

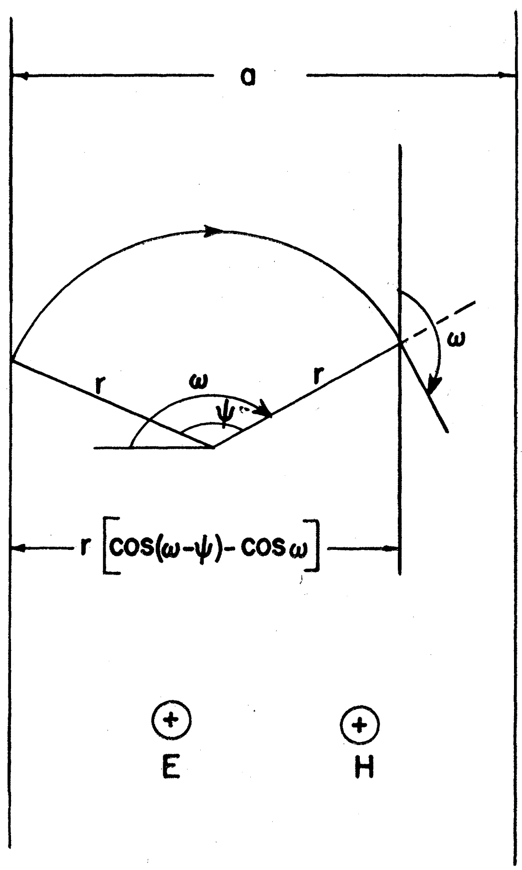

FIG. 1. Projection of an electron trajectory in the plane normal to the field direction, and geometric construction to obtain X; $0<\omega<\pi$.

Chambers[1] has shown that if one assumes diffuse scattering by the walls, the conductivity σ of a thin metal sample in a longitudinal magnetic field may be expressed by

$$\frac{\sigma}{\sigma_0}=1-\frac{3}{4\pi S}\int_S dS\int_0^{2\pi} d\omega\int_0^{\pi} d\delta \sin\delta \cos^2\delta e^{-\psi r_0/l}, \quad (1)$$

where σ_0 is the bulk conductivity, S is the cross-sectional area normal to the field direction, and l is the mean free path. ω is the angle between the projection of the velocity vector in the plane normal to the field and some fixed direction in that plane, dS is the differential cross-sectional area which includes the projection of the electron position in this plane, and ψ is the angle of arc in this plane which a trajectory has traversed since striking the wall and is, in general, a function of δ and ω as well as of position. The term to be integrated represents the effect of the shortened free path for those electrons which collide with the walls of the sample.

Chambers[1] evaluated Eq. (1) for thin *wires* in a longitudinal magnetic field by using geometric arguments. We use similar arguments for the conductivity of thin metal *films* in a longitudinal field.

For thin films dS/S can be replaced by $dX(r,\omega,\psi)/a$, where $X(r,\omega,\psi)$ is the perpendicular distance from the film wall to the electron position and a is the film thickness. Figure 1 shows the geometrical construction which gives $X=r[\cos(\omega-\psi)-\cos\omega]$. If the trajectory is shifted to the right, then, for a given ω, ψ increases; shifting the trajectory to the left will decrease ψ. By differentiation we obtain, for a given value of ω,

$$dS/S=dX/a=r\sin(\omega-\psi)d\psi/a. \quad (2)$$

By symmetry the scattering effects will be the same for both walls. We need only deal with one wall and multiply the integrand by two. Similarly we have symmetry for the effects of the polar angle δ, so we integrate between the limits zero and $\pi/2$ and multiply by two. Equation (1) then becomes

$$\frac{\sigma}{\sigma_0}=1-\frac{3r_0}{\pi a}\int_0^{\pi/2} d\delta \sin^2\delta \cos^2\delta A(\delta), \quad (3)$$

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[1] R. G. Chambers, Proc. Roy. Soc. (London) A202, 378 (1950).

where

$$A(\delta)=\int_0^{2\pi}d\omega\int_{\psi}d\psi\,\sin(\omega-\psi)e^{-\psi r_0/l}. \tag{3a}$$

To establish boundary conditions for the ψ-integration we distinguish two cases:

$$(1)\quad a\geq 2r=2r_0\sin\delta, \tag{4a}$$

$$(2)\quad a\leq 2r. \tag{4b}$$

Case (1)

The limits on ψ (for a given ω) are obtained by shifting the trajectories of Figs. 1 and 2 as far as possible to the right and to the left, keeping the endpoints of the trajectories within the walls. Thus for $0<\omega<\pi$ (see Fig. 1) we obtain the lower limit on ψ when the trajectory leaves the wall at an angle ω and the upper limit when the trajectory is tangent to the left wall. For $\pi<\omega<2\pi$ (Fig. 2) we obtain the lower limit when the trajectory returns to the left wall at an angle ω and the upper limit when the trajectory is tangent to the left wall. The limiting conditions are

$$\begin{aligned} &0<\psi<\omega &&\text{for}\quad 0<\omega<\pi,\\ &2(\omega-\pi)<\psi<\omega &&\text{for}\quad \pi<\omega<2\pi. \end{aligned} \tag{5}$$

Using the conditions (5) in Eq. (3a) we obtain

$$A(\delta)=\frac{l}{r_0}\left[2-\frac{l^2}{4r_0^2+l^2}(1+e^{-2\pi r_0/l})\right]. \tag{6}$$

Case (2)

This case has more complicated upper limiting values for ψ because some trajectories can strike both walls (without an intermediate collision). The possible values of ψ for a given ω are limited by the relations

$$\begin{aligned} &r[\cos(\omega-\psi)-\cos\omega]<a &&\text{for}\quad 0<\omega<\pi,\\ &r[\cos(\omega-\psi)+1]<a &&\text{for}\quad \pi<\omega<2\pi, \end{aligned}$$

as is evident from the figures. Unfortunately, we have not been able to express $A(\delta)$ in closed form for this case. $A(\delta)$ and the contribution of this case to the conductivity must be obtained by numerical integration.

If $a\geq 2r_0$, then the integrand is only the type discussed in Case 1. The conductivity, using Sondheimer's[2] notation, is then given by

$$\frac{\sigma}{\sigma_0}=1-\frac{3}{16\kappa}\left[2-\frac{\beta^2}{4\kappa^2+\beta^2}(1+e^{-2\pi\kappa/\beta})\right], \tag{7}$$

where $\kappa=a/l$ and $\beta=a/r_0(\beta\geq 2)$. In the limit $\kappa/\beta\gg 1$, Eq. (7) becomes

$$\sigma/\sigma_0=1-(3/8\kappa), \tag{8}$$

which is Fuchs'[3] result for thick films in zero magnetic field. If $\kappa/\beta\ll 1$, Eq. (7) becomes

$$\sigma/\sigma_0=1-(3\pi/8\beta), \tag{9}$$

independent of the ratio of film thickness to mean free path.

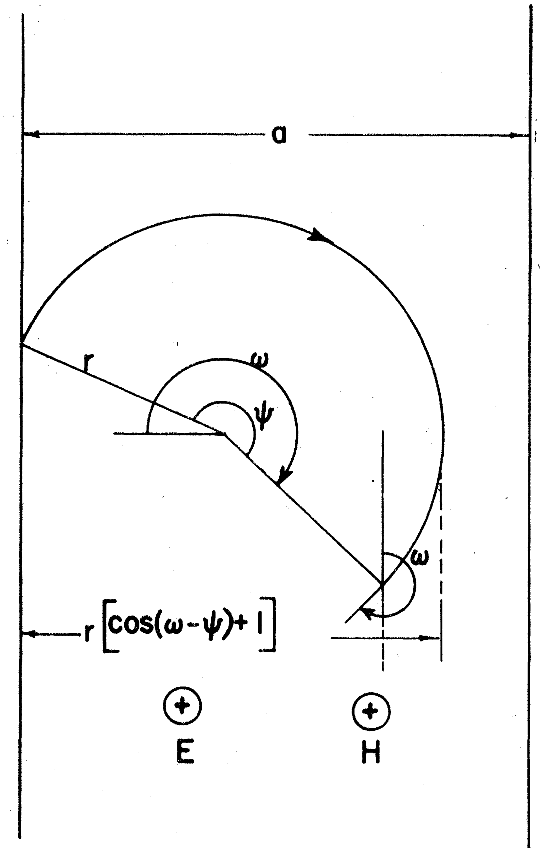

FIG. 2. Projection of an electron trajectory in the plane normal to the field direction; $\pi<\omega<2\pi$.

[2] E. H. Sondheimer, Advances in Phys. 1, 1 (1952).

[3] K. Fuchs, Proc. Cambridge Phil. Soc. 34, 100 (1938).