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POLARIZATION AT THE CATHODE IN A HIGHLY IONIZED GAS.

By C. A. Skinner.

SOME time ago the writer published a theory of the cathode fall in gases,[1] together with some applications, in which potential measurements in "the Crookes dark space" were made by introducing a wire and observing its potential. The measurements proved to be consistent with the theory.

Basing his conclusions on some experiments of Ashton,[2] H. A. Wilson has recently published a theory of the cathode fall,[3] under the assumption that there is no appreciable difference of potential between the cathode and the adjacent gas—although wire sound measurements indicate it to be about one half the total cathode fall. If Ashton's contention be true my theory is only true in part, and the derived magnitudes very much in error. On the other hand, if the wire sound measurements are reliable, Wilson's theory covers but half the observed cathode fall and his derived magnitudes accordingly incorrect. The relative merits of the two theories are otherwise insignificant compared with the main question: *Is there a large and relatively abrupt difference of potential between the cathode and adjacent gas, or is there none?*

In the writer's opinion the experiments described below give incontestable evidence that there is not only a distinct polarization P.D. between the cathode and the gas, but that it is of the magnitude indicated by the wire sound measurements.

Ashton's Experiments.—Skepticism as to the *accuracy* of wire sound measurements of potential in Crookes dark space is based on reasonable

[1] C. A. Skinner, Phys. Rev., N. S., Vol. V., p. 483, 1915; Vol. VI., p. 158, 1915.
[2] F. W. Ashton, Proc. Roy. Soc., A 84, p. 526, 1911.
[3] H. A. Wilson, Phys. Rev., N. S., Vol. VIII., p. 227, October, 1916.

considerations. But to assume that the error is, for example, eighty volts when an aluminium cathode is used and one hundred fifty volts when a steel cathode is used, seems hardly justified, when the experiments supporting the assumption are carefully considered.

Ashton measured the electric intensity in his Crookes dark space by directing a fine stream of cathode rays—from a branch tube—parallel to the face of a rather large disk cathode and noting the deflection produced. He had to deal with the average value of the field through which his stream passed. From this deflection he calculated the potential, and concluded that there was no appreciable P.D. between the conducting gas and the electrodes.

There are several reasons for questioning the validity of Ashton's conclusions. He worked with his cathode ray tube in open connection with the one under investigation, so that it was necessary to have the gas pressure low enough to give a cathode ray stream. It is a well-known fact, however, that the normal markings of the glow current disappear at pressures giving cathode ray streams satisfactory to work with. With the disappearance of the customary glow the current condenses largely into a bundle of cathode rays proceeding from a small area on the cathode, and little if anything definite can be concluded as to the current density or electric intensity at points off side.

That Ashton was not working under normal glow conditions seems evident from the results of some tests made by the writer under the conditions given by Ashton. With hydrogen at a pressure giving a Crookes dark space 32 millimeters in extent, Ashton had only 265 volts between his electrodes. With a smaller tube (3 cm. in diam.) having as cathode an aluminium disk and as anode a wire extending into the negative glow I have been unable to maintain at such pressures any discharge whatsoever with a P.D. less than about 650 volts. The highest P.D. Ashton records for hydrogen was 610 volts, and this was used at a pressure giving a dark space 28 millimeters in extent.

With an applied P.D. of about 700 volts the current in the present case was started at a higher gas pressure and the change observed as the pressure was gradually reduced. With a dark space about 25 millimeters in extent its boundary began to fade out rapidly, the ordinary current going over into a small bundle of cathode rays proceeding from the center of the cathode. At the pressure which should have given a dark space of 32 millimeters, there was no trace of such a boundary left. At that pressure which should give 38 millimeters dark space the P.D. of 700 volts could not maintain any discharge through the tube.

Ashton's results should not therefore be taken as representing normal glow current conditions.

Polarization at a Cathode Placed in the Negative Glow of a Separate Circuit.

Apparatus and Method of Experiment.—Entirely within the negative glow of what will be designated the ionizing current, were placed two relatively small plane electrodes A and B (Fig. 1) with their faces parallel to the direction of the ionizing current and their supporting stems sheathed in fine glass tubing. These electrodes, together with the gas of the negative glow, formed part of an otherwise carefully insulated

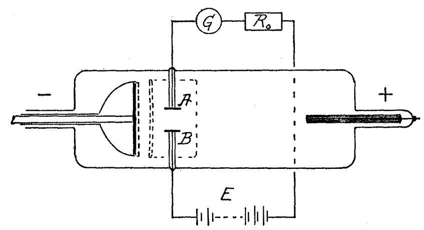

Fig. 1.

circuit, consisting of a storage battery E, a d'Arsonval galvanometer G, and a protecting resistance R_0.[1] The battery E was arranged to furnish readily any P.D. ranging from 2 to 300 volts—as measured by a Weston voltmeter kept in connection during all measurements. The current in the ionizing circuit was furnished by a similar but stronger battery. To keep the electrodes AB in the negative glow under different gas pressures, the cathode of the ionizing circuit was readily shifted along the axis of the tube by a screw device actuated through a ground joint.

Let R_i be the resistance, real or apparent, between A and B inside the discharge tube, and R_0 the resistance of that part of the circuit outside the tube. Let E be the P.D. furnished by the battery, and c the current it produces—referred to in the following as the transverse current.

Then

$$R_i = \frac{E}{c} - R_0,$$

In case, with values of the transverse current that are small compared with the ionizing current, R_i is found to be relatively large, it is to be concluded that it is concentrated between the cathode and the gas. For, the gas in the negative glow is known to be highly conductive; and

[1] H. A. Wilson (Phil. Mag., V., 49, p. 505, 1900) used such a scheme to investigate the relative conductivity of the different parts of the glow current.

no appreciable difference of potential between an anode placed in the negative glow and the gas can be observed.[1]

The following experiments reveal surprisingly large values for R_i, and substantiate in various ways the conclusion that it is located between the cathode and the gas.

Measurements with Transverse Electrodes of Aluminium.—The gas used throughout was hydrogen. It was introduced into the discharge tube by diffusion through a polladium tube heated to redness in a stream of commercial hydrogen. The electrodes serving for the ionizing current were of aluminium—the cathode being a disk (diam. about 2 cm.) with its back enclosed in glass to confine the current to its front face.

Table I. gives a representative series of results obtained with A and B

TABLE I.

Aluminium Electrodes (Area: .25 sq. cm.).

Gas Pressure {before 1.65 mm. / after 1.67 "} Ionizing Current: 1.50 m.a.

V_A-V_B (Volts).	c (Milli-amp.).	R_i (Ohms).	V_A-V_B (Volts).	c (Milli-amps.).	R_i (Ohms).
1.8	.0099	1.8×10^5	77.3	.109	7.1×10^5
3.85	.0165	2.33	81.5	.112	7.3
5.65	.0231	2.45	89.5	.121	7.4
7.5	.0280	2.85	97.5	.129	7.5
9.2	.0405	3.20	108.	.142	7.6
12.9	.0405	3.20	120.	.155	7.7
18.5	.0495	3.75	132.	.174	7.6
27.8	.062	4.50	143.	.192	7.4
39.0	.074	5.25	51.3	.087	5.9
51.5	.086	6.0			
64.1	.096	6.7			

of thin plates of aluminium, rectangular in form (2.25 × 5 mm.), and both faces exposed. Their exposed surface, counting the edges, was very close to .25 sq. cm. The resistance R_0 was practically concentrated in a 20,000-ohm box.

The procedure was to measure by the galvanometer G, the current c when a definite E was applied. From the values of c and R_0 the drop in the external part of the transverse circuit was then calculated, and

[1] The potential gradient in the negative glow under the currents used here is distinctly less than 1 volt/cm. With a transverse current of one-tenth the magnitude of the main current the drop of potential in the gas between the transverse electrodes A and B (3 mm. apart) should therefore be less than one tenth volt. The anode fall in the same gas is not much larger than this. (See Skinner, Phil. Mag.) These values are therefore negligible compared with the P.D. applied between A and B throughout this investigation.

this magnitude—always relatively small—subtracted from E, giving the applied difference of potential ($V_A - V_B$) recorded in the tables.

These values together with the calculated value of R_i are given in each case. This calculated resistance is usually designated as the apparent cathode resistance. The observations are recorded in the sequence taken, those at the end being check observations.

Table I. shows that, with an ionizing current of 1.50 m.a., the resistance R_i between the transverse electrodes rises from 180,000 ohms under a transverse current of .01 m.a., to 770,000 ohms under one of .155 m.a. With a further increase in the transverse current R_i then slowly decreases. In the preliminary experiments no protecting resistance R_0 was introduced. The transverse current increased slowly with E until about 100 volts was reached when a further increase in E produced a sudden rush of current which completely vaporized the transverse electrodes.

Table II. gives a similar series obtained with an ionizing current of

TABLE II.

Aluminium Electrodes (Area: .25 sq. cm.).

Gas Pressure {before 1.57 mm. after 1.59 " Ionizing Current: 2.50 m.a.

$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).	$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).
3.1	.025	1.25×10^5	81.	.200	4.05×10^5
9.7	.068	1.4	85.8	.210	4.1
15.2	.088	1.7	95.0	.225	4.2
22.7	.104	2.2	110.8	.260	4.3
29.6	.118	2.5	126.2	.290	4.35
41.3	.140	2.95	140.6	.320	4.4
53.8	.158	3.4	87.7	.215	4.1
66.7	.180	3.7	10.4	.064	1.6
75.1	.194	3.9			

2.50 m.a. For the same transverse current c the resistance R_i between A and B is distinctly lower for the larger ionizing current, but its variation with c is similar.

Measurements with Platinum Electrodes.—Table III. gives the results obtained when the aluminium electrodes A and B were replaced by platinum, having the same dimensions and at the same distance apart as the aluminium. The two metals are readily compared by referring to their respective curves in Fig. 2, in which, for the same ionizing current, the apparent resistance R_i is plotted against the transverse current c.

The two curves are similar in form but that for platinum distinctly

TABLE III.

Platinum Electrodes (Area: .25 sq. cm.).

Gas Pressure {before 1.77 mm., after 1.91 " } Ionizing Current: 2.50 m.a.

$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).	$V_A - V_B$ (Volts).	c (Milli-amp.).	R_i (Ohms).
3.1	.022	1.4×10^5	80.5	.165	4.9×10^5
6.2	.038	1.65	92.5	.180	5.1
9.7	.049	2.0	105.	.193	5.4
14.1	.061	2.3	117.	.206	5.65
19.1	.073	2.6	128.5	.220	5.85
30.6	.095	3.2	142.5	.233	6.1
43.2	.116	3.7	92.5	.176	5.25
52.5	.130	4.05	52.	.127	4.1
65.	.147	4.4	6.7	.041	1.65
74.	.159	4.65			

higher than that for aluminium. This difference gives support to the argument that R_i cannot be located in the gas between the electrodes,

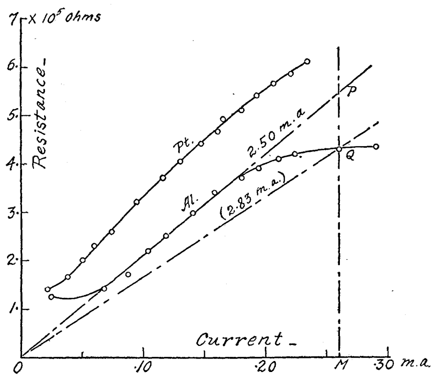

Fig. 2.

for then it should remain unchanged when the materials composing the electrodes are changed.

With the highest values of $(V_A - V_B)$ used in this series, platinum, unlike aluminium, shows no appreciable tendency toward a maximum R_i. Later tests however running to higher potential differences show the maximum to be as clearly marked with platinum as with aluminium.

Measurements with a Platinum-Aluminium Pair.—A more interesting and more accurate comparison of these two metals was made by having one of the transverse electrodes of platinum and the other of aluminium—both having as nearly as possible the same dimensions, and similarly situated in the negative glow. By simply reversing the applied P.D. first the one metal then the other served as cathode under identical conditions.

The results of this simultaneous comparison, for the ionizing currents 2.50 m.a. and 1.50 m.a., are given in Tables IV. and V. respectively.

Table IV.

Simultaneous Comparison of Aluminium and Platinum Cathodes (Area: .25 sq. cm.).

Gas Pressure {before 1.72 mm. after 1.76 " Ionizing Current: 2.50 m.a.

$V_A - V_B$ (Volts).		c (Milli-amp.).		R_i (Ohms).	
Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.
0.9	2.8	.0043	.0132	2.1×10^5	2.1×10^5
4.6	6.6	.0296	.032	1.55	2.05
10.1	11.9	.063	.058	1.6	2.05
17.7	20.	.089	.077	2.0	2.6
27.3	29.5	.109	.099	2.5	3.0
37.4	39.7	.132	.117	2.8	3.4
49.5	52.	.151	.138	3.3	3.8
63.5	66.	.175	.160	3.6	4.1
77.5	80.5	.196	.181	3.95	4.45
94.5	97.	.225	.206	4.2	4.7
112.	114.	.260	.234	4.3	4.9
131.	134.	.295	.260	4.45	5.15
155.	158.	.346	.295	4.45	5.35
191.	195.	.475	.365	4.0	5.3
17.8	20.	.088	.079	2.05	2.55

The existence of a contact potential between A and B was revealed and its magnitude approximately determined from the applied P.D. necessary to produce zero transverse current. For Table IV. this was about 1 volt, and for V. 1.2 volts. The values given under $(V_A - V_B)$ are therefore the applied P.D. corrected by this amount. Inasmuch as this correction was not large anyway, the possible error, arising from the tacit assumption that it was the same at all values of the transverse current, is of no real moment.

Inspection of these tables and the corresponding curves of Fig. 3, show as expected, a distinct difference in the magnitude of R_i on simply reversing the direction of the transverse current. These results locate the resistance at the cathode; for, with aluminium as cathode and platinum as anode the value of R_i is the same as when both electrodes were aluminium. With platinum as cathode and aluminium as anode they are distinctly higher, though not quite so large as in the previous experiment with both electrodes of platinum.

TABLE V.

Simultaneous Comparison of Aluminium and Platinum Cathodes (Area: .25 sq. cm.).

Gas Pressure {before 1.68 mm. / after 1.71 " Ionizing Current: 1.50 m.a.

$V_A - V_B$ (Volts).		c (Milli-amp.).		R_i (Ohms).	
Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.	Al. Cathode.	Pt. Cathode.
.7	3.0	.002	.007	3.5×10^5	4.3×10^5
4.5	6.9	.0152	.0178	3.0	3.9
10.1	12.5	.0335	.033	3.0	3.8
18.2	20.5	.047	.0485	3.85	4.2
25.7	28.1	.056	.054	4.6	5.2
36.0	38.0	.068	.065	5.3	5.85
48.0	50.0	.079	.075	6.1	6.7
62.0	64.0	.091	.088	6.8	7.3
76.5	78.5	.102	.098	7.5	8.0
94.0	96.0	.119	.111	7.9	8.6
112.0	114.0	.136	.124	8.2	9.2
130.5	133.0	.157	.138	8.3	9.6
150.	153.	.186	.158	8.1	9.7
191.	193.	.266	.200	7.2	9.6
76.5	78.5	.102	.097	7.5	7.9

These results should remove all doubt as to the existence in general of a polarization at the cathode when an electric current flows through an ionized gas. For it is found to exist here under circumstances most favorable to a vanishing magnitude of the electrode fall; namely, in a gas so highly ionized that the potential gradient necessary to maintain

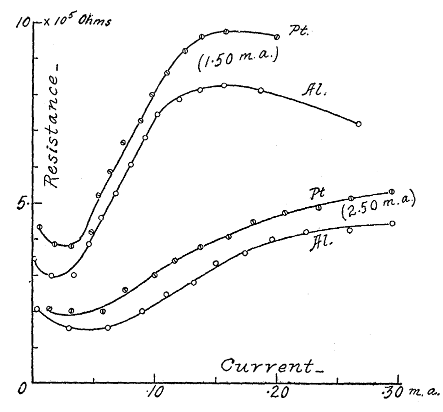

Fig. 3.

a current through it is too small to be measurable. Under many of the working conditions this polarization P.D. may not have an appreciable value, but the probability of its existence in general requires that it be taken into account.

In these last tests the similarity in form of the resistance curves for both metals is clearly evident. Both show a minimum for low values of c and a maximum at higher values of c. The minimum and maximum both become larger and more abrupt with decreased ionizing current. It was suspected at first that the evidence of an appreciable value of R_i for a zero value of c arose from extraneous conditions, but its persistence in all tests renders such a conclusion at least questionable.

Variation in the Apparent Cathode Resistance with Ionizing Current.— A complete investigation of this problem would consist in obtaining the family of (R_i, c) curves for various ionizing currents and different gas pressures. The curves of Fig. 3 suggest the general change in form with change in ionizing current. Such an investigation requires the metal tested to be in use so long that its condition is likely to undergo important changes before the series of measurements is complete. A simpler plan was devised. This consisted in making a study of R_i under a definite applied P.D. and different ionizing currents—at various gas pressures.

For this the transverse electrodes were again of aluminium—the same as used in Tables I. and II., their surfaces having been carefully cleaned by scraping. By reversing the applied P.D. first the one electrode, then the other, was used as cathode. In this way the one served to check the results obtained from the other. They were found to differ throughout by about five per cent., probably arising from a slight difference in surface.

Tables VI. and VII. give an uninterrupted series of measurements all with an applied P.D. of about 80 volts between the transverse electrodes. Each group of measurements consisted in observing, with a given gas pressure, the transverse current c with various ionizing currents. Then another group was taken with fresh gas, and so on. The pressure recorded at the top of each group was the initial; that at bottom, the final. The slight increase recorded did not, as shown, appreciably change the results.

The magnitude of the applied P.D. was chosen at 80 volts for various reasons. It was large enough to eliminate the possibility of disturbance arising from contact potential, resistance in the gas, anode fall, and the like. It is distinctly below the magnitude which causes, as explained later, a disturbing ionization by the electrons freed from the transverse cathode. Finally, it is of the magnitude which the wire sound indicates as present between gas and cathode under "normal" cathode current density; so that a comparison could be made between the resistance of the cathode placed in the negative glow and that of the main cathode working under normal conditions.

TABLE VI.

Aluminium Cathodes—(Area: .25 sq. cm.)—Various Ionizing Currents.

Gas Press. (mm.).	Ioniz. Current (m. a.).	V_A-V_B (Volts).	c (m. a.).		R_i (Ohms).		Prod. of R_i by Ionizing Current.	
			Cathode A.	Cathode B.	Cathode A.	Cathode B.	Cathode A.	Cathode B.
1.63	1.50	79.5	.107	.102	7.4×10^5	7.8×10^5	11.1×10^5	11.7×10^5
	2.00	78.5	.161	.152	4.9	5.2	9.8	10.4
	2.50	79.5	.210	.200	3.8	4.0	9.5	10.0
	3.00	78.5	.255	.243	3.1	3.2	9.3	9.6
	3.50	80.0	.305	.290	2.6	2.75	9.1	9.6
	4.00	79.5	.335	.325	2.4	2.45	9.6	9.8
1.67	1.50	79.0	.106	.102	7.5	7.8	11.2	11.7
1.39	1.00	79.5	.070	.066	11.3	12.0	11.3	12.0
	1.50	78.5	.120	.115	6.5	6.8	9.8	10.2
	2.00	77.5	.174	.167	4.45	4.65	8.9	9.3
	2.50	78.0	.225	.213	3.5	3.7	8.8	9.2
	3.00	77.0	.270	.255	2.85	3.0	8.5	9.0
	3.50	78.5	.315	.300	2.50	2.60	8.8	9.1
	4.00	77.5	.350	.335	2.22	2.32	8.9	9.3
1.43	1.00	79.0	.072	.069	11.0	11.5	11.0	11.5
1.15	.75	79.5	.057	.054	13.9	14.7	10.4	11.0
	1.00	79.0	.086	.082	9.2	9.6	9.2	9.6
	1.50	77.5	.143	.137	5.4	5.6	8.1	8.4
	2.00	78.5	.187	.178	4.2	4.4	8.4	8.8
	2.50	78.0	.230	.220	3.4	3.55	8.5	8.9
	3.00	77.5	.270	.255	2.9	3.05	8.7	9.1
	3.50	78.5	.310	.300	2.53	2.62	8.9	9.2
1.20	.75	79.5	.058	.056	13.6	14.2	10.2	10.6

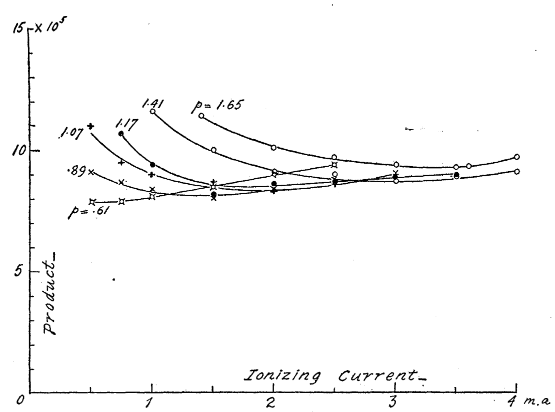

Fig. 4.

An interesting relation is brought out in these tables (VI. and VII.), and shown also in the corresponding curves of Fig. 4, where the products of cathode resistance by the ionizing current are plotted against ionizing currents.

TABLE VII.

Continuation of Table VI.

Gas Pres. (mm.)	Ioniz. Current. (m. a.)	$V_A - V_B$ (Volts.)	c (m. a.).		R_i (Ohms).		Prod. of R_i by Ionizing Current.	
			Cathode A.	Cathode B.	Cathode A.	Cathode B.	Cathode A.	Cathode B.
1.05	.50	80.0	.038	.035	21.0×10^5	23.0×10^5	10.5×10^5	11.5×10^5
	.75	79.0	.061	.061	12.5	12.9	9.4	9.7
	1.00	79.0	.091	.085	8.7	9.3	8.7	9.3
	1.50	78.0	.136	.132	5.7	5.9	8.6	8.8
	2.00	78.0	.188	.185	4.15	4.2	8.3	8.4
	2.60	77.5	.230	.217	3.35	3.55	8.4	8.9
	3.00	77.0	.265	.250	2.9	3.1	8.7	9.3
1.10	.75	79.0	.063	.059	12.5	13.3	9.4	10.0
.87	.50	78.5	.0435	.040	18.0	18.6	9.0	9.3
	.75	78.0	.069	.065	11.3	12.0	8.5	9.0
	1.00	77.5	.094	.090	8.2	8.6	8.2	8.6
	1.50	76.5	.146	.138	5.2	5.5	7.8	8.2
	2.00	78.5	.192	.183	4.1	4.3	8.2	8.6
	2.50	78.0	.225	.215	3.45	3.65	8.6	9.1
.90	3.00	77.5	.260	.250	3.0	3.1	9.0	9.3
.60	.50	79.0	.051	.0485	15.5	16.3	7.8	8.1
	.75	78.5	.076	.073	10.3	10.8	7.7	8.1
	1.00	78.0	.099	.094	7.9	8.3	7.9	8.3
	1.50	77.5	.140	.134	5.5	5.8	8.3	8.7
	2.00	79.0	.180	.173	4.4	4.6	8.8	9.2
	2.50	78.5	.213	.203	3.7	3.85	9.2	9.6
.63	.50	78.5	.051	.049	15.4	16.0	7.7	8.0
.40	.50	79.5	.055	.051	14.5	15.6	7.2	7.8
	.75	79.0	.077	.073	10.2	10.8	7.7	8.1
	1.00	78.5	.095	.091	8.3	8.6	8.3	8.6
	1.50	78.0	.131	.124	6.0	6.3	9.0	9.4
	2.00	79.5	.168	.158	4.75	5.0	9.5	10.0
.43	.50	78.5	.053	.050	14.8	15.7	7.4	7.8

Assuming that the transverse P.D. is concentrated at the cathode we find: *for a given polarization P.D. at the transverse cathode, its apparent resistance is nearly inversely proportional to the ionizing current, and nearly independent of the gas pressure.*

This observed relation appears to have a simple explanation. Whatever may be the exact nature of the apparent cathode resistance it must be separately encountered by each discharging ion. The current should therefore be proportional to the number of ions simultaneously engaged in discharging, and this number—other conditions remaining the same—should be proportional to the number of positive ions per c.c. in the gas surrounding the cathode. To have this explanation fit the case it would require that the number density of ions in the negative glow be nearly proportional to the current density of the ionizing current. General considerations indicate this to be true. One of the unique features of the glow current is that the potential gradient changes but very slowly with increasing current density, meaning that the number of ions per c.c. carrying the current must be nearly proportional to the current density.

If this explanation be accepted the above results are easily interpreted. The curves all show a minimum product at some definite ionizing current. Here the ratio of number of positive ions per c.c. to the ionizing current is a maximum, and the number density proportional to the current. As the gas pressure is lowered, this ratio slowly increases.

For the purpose of later reference we will express the relation in mathematical form:

$$R_i \propto \frac{1}{n} \quad \text{for} \quad (V_A - V_B) = \text{const.} \tag{1}$$

where R_i is the apparent cathode resistance, n the number of positive ions per c.c. and $(V_A - V_B)$ the polarization P.D. at the cathode.

Resistance at the Transverse Cathode Compared with that at the Main Cathode.—This comparison is especially valuable because it proves that wire sound measurements in Crookes dark space are reliable; and, it is from such measurements that knowledge of conditions from point to point in the gas is obtained.

The problem to solve is: Given the same polarization P.D. at both cathodes, to calculate the apparent resistance per sq. cm. at each when the current density in the tube is the same.

In a previous article on the cathode fall[1] are given potential measurements taken with a wire sound at different distances from an aluminium cathode in hydrogen. From these the magnitude of the polarization P.D. at the cathode was obtained by theoretical extrapolation—practically linear. Two of the gas pressures given there are the same as two given in Table VI., namely: 1.6 mm. and 1.1 mm. For these, at "normal" cathode current density, the average polarization P.D. at the cathode was 78 volts. The "normal" current density for the first pressure was .192 m.a./cm^2.; for the second, .091 m.a./cm^2. The apparent resistance per sq. cm. for the first was therefore

$$\frac{78}{.192 \times 10^{-3}} = 4.1 \times 10^5 \text{ ohms}$$

for the second

$$\frac{78}{.091 \times 10^{-3}} = 8.6 \times 10^5 \text{ ohms.}$$

[1] C. A. Skinner, PHYS. REV., N. S., VI., pp. 160, 161.

In Table VI. the minimum product of R_i by the ionizing current, for a pressure of 1.63 mm., was 9.3×10^5. This was from a transverse cathode having an area of .25 cm². It is also with the total ionizing current which was spread over a cross-section of about 3 cm².—the area of the main cathode. To obtain the product in terms of cathode resistance *per square centimeter*, and *ionizing current density*, we have

$$\frac{.25}{3} \times 9.3 \times 10^5 = 7.8 \times 10^4.$$

From this, the resistance per sq. cm. of the transverse cathode for "normal" current density at the main cathode at the same pressure, may be calculated if the product be assumed to remain constant. We have then as calculated resistance for the first case:

$$\frac{7.8 \times 10^4}{.192} = 4.1 \times 10^5 \text{ ohms},$$

similarly, for the second case:

$$\frac{7.0 \times 10^4}{.091} = 7.7 \times 10^5 \text{ ohms},$$

as compared with the corresponding magnitudes at the main cathode of 4.1 and 8.6×10^5 ohms. The average agreement is unexpectedly good, and *proves that the wire sound measurements in Crookes dark space are reliable.*

The Form of the Resistance Curve.—The curve for aluminium in Fig. 2 may be taken as typical of the (R_i, c) curves. At the lower magnitudes of c the values are rather unexpected but they are regularly present. Beyond this region however the curve assumes almost a straight line direction from the origin, and holds it for some distance. This straight line gives a simple law for the cathode resistance so long as the transverse current remains between two fairly well separated magnitudes. The deviation of the curve from the straight line, at the higher values of c, is probably caused by the increased density of ionization which the electrons from the transverse cathode produce in the surrounding gas.

Assuming this we calculate the resistance at Q (Fig. 2) as that which would be obtained if an ionizing current of 2.83 m.a. were used instead of 2.50 and there were no superposed ionization arising from the electron current from the transverse cathode.

We will now formulate a tentative law governing the polarization P.D. and test its applicability to the general results.

Let V_b be the polarization P.D. at the cathode—the same as $(V_A - V_B)$ in the tables—j the cathode current density, R_b its apparent resistance per sq. cm., and n the number of positive ions per c.c. in the gas surrounding the cathode. From (1)

$$R_b \propto \frac{1}{n} \text{ for } \quad V_b = \text{const.} \tag{2}$$

From the above considerations

$$R_b \propto j \quad \text{for} \quad n = \text{const.} \tag{3}$$

Equation (2) gives

$$V_b = jR_b$$

$$\propto \frac{j}{n}$$

hence

$$j \propto n \quad \text{for} \quad V_b = \text{const.} \tag{4}$$

Similarly from (3)

$$j \propto V_b^{1/2} \quad \text{for} \quad n = \text{const.} \tag{5}$$

Combining (4) and (5)

$$j \propto nV_b^{\frac{1}{2}}.$$

This gives the simple tentative law for the polarization P.D.

$$V_b = g\left(\frac{j}{n}\right)^2 \tag{6}$$

where g is a constant. Inspection of the experimental results indicate that, with an aluminium cathode in hydrogen, it should hold for values of V_b between about 20 and 80 volts.

Something of the nature of this polarization phenomenon is suggested by substituting the customary relation between j and n. For those conditions where the electron current from the cathode is a negligible part of the total current, we have simply

$$j = neu \tag{7}$$

in which e is the ionic charge and u the velocity of the positive ions where their number density is n. Substituting (7) in (6)

$$V_b = ge^2u^2. \tag{8}$$

In those cases where it is simply necessary to maintain a terminal velocity of the ions, the P.D. necessary is proportional to that velocity. Here, we find it proportional to the square of the velocity, as if it produced an acceleration. This is in keeping with the view that the polarization P.D. is necessary to overcome the kinetic energy of rebound of the discharging ions.

The applicability of (8) to the general results obtained from the cathode will now be tested. The following general relation has been found to hold in the space between the cathode and negative glow: *With the cathode at zero potential, and its current density a definite multiple of the "normal" value, the potential at any definite number of mean free paths from the cathode is the same, over a wide range of gas pressures.*[1] Stated more definitely for the purpose of present application: The polarization P.D. at the cathode remains constant over a wide range of gas pressures so long as the potential gradient per cm. E in the gas just outside the polarization region, is proportional to the gas pressure p. That is, we have the experimental relation

$$V_b = \text{const.} \quad \text{for} \quad E \propto p. \tag{9}$$

From the law of mobility

$$u \propto \frac{E}{p}.$$

Substituting this in (8) gives

$$V_b \propto \left(\frac{E}{p}\right)^2$$

which states that, as in (9), V_b is constant for $E \propto p$. Thus we find our tentative law predicts the general cathode relations.

Summary.

1. Considerations and experimental results are given showing that Ashton's experiments, indicating that there is no difference in potential between a cathode and the conducting gas adjacent to it, were not made under normal glow current conditions, as he assumed.

2. A series of experiments are described which show that a cathode placed in the highly ionized negative glow, where a polarization P.D. would be least expected, shows it present in large magnitude.

3. For a constant polarization P.D. at the cathode, the apparent cathode resistance per sq. cm. is *nearly* inversely proportional to the ionizing current and *nearly* independent of the gas pressure. From these relations it is concluded that the apparent cathode resistance is simply inversely proportional to the density of the positive ions in the gas around the cathode.

This relation is applied to calculate the apparent resistance of the transverse cathode with "normal" cathode current density at the main cathode. The calculated magnitude was the same as that obtained from wire sound measurements at the main cathode, under the same conditions. It therefore proves the reliability of wire sound measurements in Crookes dark space.

[1] C. A. Skinner, *l. c.*; W. L. Cheney, Phys. Rev., N. S., Vol. VII., p. 241, 1916; Wm. Newswanger, id., p. 253.

4. Between two fairly well separated values of the polarization P.D. the apparent cathode resistance is, for constant ionizing current, practically proportional to the current density. The deviation of the resistance curve from a straight line at the higher potentials probably arises from the extra ionization produced by the electrons escaping from the transverse cathode.

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