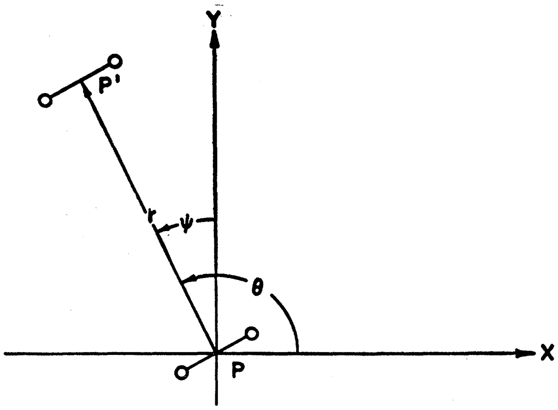

FIG. 2. Interacting pairs of dislocations. The glide planes are parallel to the x axis.

locations of the pair and the slip plane is determined ideally by the applied stress in the neighborhood. If there were a group of such pairs in any local region of a crystal they would then all have the same orientation angle, ϕ. It is the purpose of this note to show that, if the components of the pairs were edge type dislocations, such a group could form a stable array and that this array would have a property which has been observed for deformation bands in metal crystals deformed in tension.

Consider the interaction of two similarly oriented pairs, P and P', such that the spacing, r, between them is large compared to the spacing, ρ, of the component dislocations of either pair. The configuration is shown in Fig. 2 where the x direction is chosen to be the slip direction, P is taken to be at the origin, and θ is the angle between the positive x direction and the line connecting P and P'. The force per unit length, F, exerted on P' by P is readily found to be given by

$$F=Gbb'(\rho\rho'/r^3)\{\alpha^2[\cos 3\theta-3\cos 5\theta]+3\beta^2[\cos 3\theta+\cos 5\theta]-2\alpha\beta[\sin 3\theta+3\sin 5\theta]\},$$

where $\alpha=\cos\phi$, $\beta=\sin\phi$, and b and b' are the component dislocation strengths of P and P', respectively. The equilibrium orientation of the array is given by the value of θ for which $F=0$. It is seen immediately that the equilibrium value of θ is dependent only on the magnitude of ϕ, and so the orientation of a group of more than two pairs is the same as for two.

The variation of the equilibrium orientation angle, ψ (equal to $\theta-90°$), against ξ (equal to $\cot\phi$) is shown in Fig. 3. If an array of this sort is taken as a model of a deformation band, the angle ψ is the angle between the deformation band and the normal to the slip

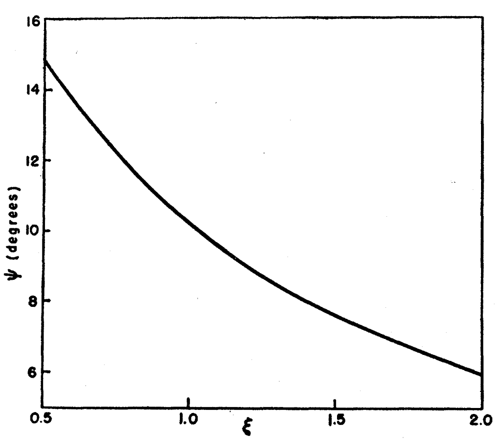

FIG. 3. Static equilibrium configurations for interacting dislocation pairs.

plane. Cahn[2] reported measurements of this angle (it is the angle designated θ in Cahn's paper) for ten specimens. His results, tabulated in Tables I and II of reference 2, comprise a range from 11° to 21°. The range of values of ξ which would correspond to these angles is reasonable. When r is equal to or smaller than ρ, the result above does not hold, and the angle ψ becomes much smaller.

It seems reasonable then to infer that deformation bands as observed in aluminum could be composed of widely spread pair elements of the sort treated here. This would be in agreement with the explanation of asterism proposed by Burgers,[3] based on only one pair. Although the components of the pairs have been represented here as single dislocations, they can equally well be groups of dislocations piled up at the end of slip regions so long as the total dislocation strength of each component in a pair is the same as that of the component with which it is paired.

[1] See, for example, A. H. Cottrell, *Progress in Metal Physics* (Interscience Publishers, Inc., New York, 1949), p. 103.
[2] R. W. Cahn, J. Inst. Metals **79**, 129 (1951).
[3] J. M. Burgers, Proc. Phys. Soc. (London) **52**, 23 (1940).

Unified Field Theory and Born-Infeld Electrodynamics

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IN a recent paper by one of us,[1] a new version of Einstein's generalized theory of gravitation was proposed. In this note, we show that in the limit of a point charge distribution of an "electron," this theory reduces to that of Born-Infeld electrodynamics.[2]

The Lagrangian of the proposed theory was given as

$$2p^2\mathfrak{L}=\mathfrak{g}^{\alpha\beta}(R_{\alpha\beta}-ip^2B_{\alpha\beta})+2p^2\{\sqrt{(-g)}-\sqrt{(-b)}\},\tag{1}$$

where

$$B_{\alpha\beta}=\partial_\alpha B_\beta-\partial_\beta B_\alpha\tag{2}$$

is an auxiliary field variable that can be eliminated from the field equations, and the metric of the unified space-time is given by

$$ds^2=b_{\alpha\beta}dx^\alpha dx^\beta,\tag{3}$$

$$\mathfrak{b}^{\alpha\beta}=\mathfrak{g}^{\alpha\beta}/[\sqrt{(-\mathrm{Det}\,\mathfrak{g}^{\alpha\beta})}],\tag{4}$$

$$b_{\alpha\beta}b^{\beta\gamma}=\delta_\alpha{}^\gamma.\tag{5}$$

If we let $p\rightarrow\infty$, the Lagrangian (1) reduces to

$$\mathfrak{L}'=-\tfrac{1}{2}i\mathfrak{g}^{\alpha\beta}B_{\alpha\beta}+\{\sqrt{(-g)}-\sqrt{(-b)}\}.\tag{6}$$

The variations of the action, with $\mathfrak{L}'$ as the action-density, with respect to the field variables $\mathfrak{g}^{\alpha\beta}$ and B_α of the unified field theory lead to the field equations,

$$a_{\alpha\beta}=b_{\alpha\beta},\tag{7}$$

$$\phi_{\alpha\beta}=B_{\alpha\beta},\tag{8}$$

$$\mathfrak{g}^{\alpha\beta}{}_{,\beta}=0.\tag{9}$$

Now Eqs. (7), (8), and (9) are the field equations of unified field theory in the limit $p\rightarrow\infty$, in which case the field $\phi_{\alpha\beta}$ degenerates into the curl of a four-vector B_α. By using (7) and (8) in (6), we obtain a different action-density,

$$L=\tfrac{1}{2}\mathfrak{g}^{\alpha\beta}\phi_{\alpha\beta}+\{\sqrt{(-g)}-\sqrt{(-b)}\}\tag{10}$$

$$=\sqrt{(-a)}(\Omega+2\Lambda^2)(1-\Omega-\Lambda^2)^{-\frac{1}{2}}+\{\sqrt{(-g)}-\sqrt{(-a)}\}\tag{11}$$

$$=-\sqrt{(-g)}+\sqrt{(-a)}(1+\Lambda^2)(1-\Omega-\Lambda^2)^{-\frac{1}{2}}+\{\sqrt{(-g)}-\sqrt{(-a)}\}$$

$$=\sqrt{(-a)}\left\{\frac{1+\Lambda^2}{\sqrt{(1-\Omega-\Lambda)^2}}-1\right\}.\tag{12}$$

We define the tensor

$$q_{\alpha\beta}=a_{\alpha\beta}+iF_{\alpha\beta},\qquad F_{\alpha\beta}=-\tfrac{1}{2}\epsilon_{\alpha\beta\mu\nu}\mathfrak{g}^{\underset{\vee}{\mu\nu}}. \tag{13}$$

Then, from (9), it follows that F can be written

$$F_{\alpha\beta}=\partial_\alpha A_\beta-\partial_\beta A_\alpha. \tag{14}$$

The determinant of (13) is

$$q=a\{1-Q-\Lambda^2\}=a(1+\Lambda^2)^2/(1-\Omega-\Lambda^2),\qquad Q=\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}. \tag{15}$$

From (15), it follows that

$$L=\sqrt{(-a)}\{\sqrt{(1-Q-\Lambda^2)}-1\}, \tag{16}$$

which is the Lagrangian of the Born-Infeld theory.

It was shown in reference 1 that $\sqrt{2}p=\kappa$ is the reciprocal of the range of charge distribution in the linear approximation. Hence, we infer that in the limit $p\rightarrow\infty$ the charge distribution tends to a point. This comparison suggests that $F_{\alpha\beta}$ is the electromagnetic field tensor, while B_α is the antipotential defined by Born and Infeld.

As in the Maxwell-Lorentz electrodynamics, the Born-Infeld theory does not prescribe the law of distribution of the charges, but introduces this law as an extra postulate. On the contrary, unified field theory implies this law in the field equations, and this must be regarded as an important consequence of the unification of physical fields.

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[1] B. Kurşunoğlu, Phys. Rev. **88**, 1369 (1952).
[2] M. Born and L. Infeld, Proc. Roy. Soc. (London) **A144**, 425 (1934).

Second-Sound Velocity below 1°K as a Function of Pressure*

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The publication of Gorter's recent letter[1] has prompted us to submit the following, which is essentially the text of a paper delivered at the ONR–GE Cryogenics Conference at Schenectady on October 6, 1952.

Measurements of the velocity of second sound have contributed significantly to an understanding of the nature of the excitations of liquid helium II. The results of Atkins and Osborne[2] along the vapor pressure line seem to confirm approximately the prediction of Landau[3] that c_2 should approach the constant $c_1/\sqrt{3}$, which results from the proposal that the liquid should approach an ideal phonon gas at low temperatures. Further evidence that the excitations of the liquid are due solely to phonons at low temperatures is given by the specific-heat measurements of Kramers,[4] showing a T^3 dependence for C_v below approximately 0.6°K. There is some discrepancy between the second-sound measurements and the specific-heat measurements quoted above, in that the second-sound velocities depart radically from the phonon-gas model above approximately 0.35°K instead of the 0.6°K of the specific-heat measurements.

Measurements now in progress by the authors, both at vapor pressure and at higher pressure, are being made by the pulse method of Pellam,[5] later used by Maurer and Herlin.[6] A second-sound chamber 3.035 cm long and 3.2 mm in diameter employs carbon-composition films on Bakelite disks for both transmitter and receiver. The transmitted pulse for the measurements reported here was 20 μsec long, of intensity 3.5 mw/cm², and repeated 30 times per second. (The results did not seem to depend appreciably on these parameters.) The received pulse is displayed on a Dumont type 256-D A/R scope; the internal time marker of the scope is used to measure the delay time. Temperatures are measured with an ac mutual inductance bridge, calibrated against the vapor pressure of helium from 1° to 4°. Warm-up times of three to four minutes are typical.

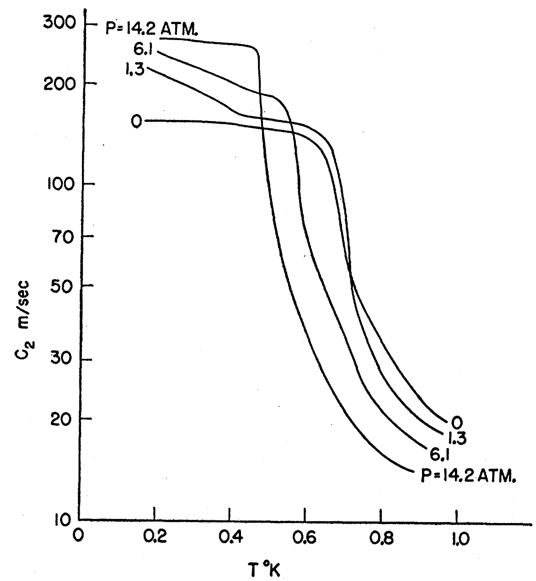

Fig. 1. Second-sound velocity *vs* temperature at various pressures. Experimental points have been omitted for clarity, but the various regions of the curves are unmistakable. Each curve represents twenty to thirty points, obtained from several demagnetizations.

Our results at vapor pressure are in reasonably good agreement with Kramers' specific heat data and with the results of DeKlerk, Hudson, and Pellam,[7] just announced, at least above approximately 0.3°K. At vapor pressure, we find no trace of the rise in velocity that they report at lower temperatures, but this may be due to our more rapid warm-up time. Our extrapolated value for $c_2(0)$ is 155 m/sec, in agreement with Atkins and Osborne, and in reasonably good agreement with the value reported by DeKlerk, Hudson, and Pellam for a temperature just below that of the sharp drop in c_2. Our results at higher pressure, as may be seen in Fig. 1, tend to bear out the supposition that this low temperature rise in velocity is a real phenomenon, becoming more apparent at higher pressure. It will be noted that our values for c_2 at a temperature just below that of the sharp drop in c_2 are, at each pressure, just about 10 percent higher than $c_1/\sqrt{3}$ for that pressure. (In the case of 14.2 atmospheres, the low temperature rise apparently has begun before the sharp rise at intermediate temperature is complete.) At the very lowest temperatures, velocities have risen to near c_1.

A possible explanation of this behavior is as follows:[8] If we consider liquid helium at temperatures below the sharp drop in c_2 to be phonon gas, then, as in the usual picture, heating at some point in the helium will cause a concentration gradient in this gas. This gradient may revert toward equilibrium in one of three ways. First, if the mean free path for inelastic phonon-phonon collisions (such as Umklapp collisions) is short, the phonons will diffuse away and ordinary heat conduction will result. In liquid helium II in the pure phonon-gas region, the temperature is lower, and the inelastic mean free path is long, while the mean free path for elastic collision is still short, so that a compression wave of phonons is propagated, resulting in second sound. At still lower temperatures, the phonons are becoming more rarefied and their total mean free path may become larger than the dimensions of the measuring apparatus, in which case we get a flow of phonons analogous to Knudsen flow of a gas. Some will travel straight from the transmitter to the receiver at their velocity c_1, and others will be reflected from the walls of the tube and eventually reach the receiver, thus building up a received pulse which looks as though it were caused by diffusive heat flow. The foot of this pulse will indicate a velocity which be-