where

$$F_4=\frac{1}{6}\left(\frac{1}{6}-\frac{1}{2}\alpha+\alpha^2+\alpha^2(1+\alpha)\log\frac{1+\alpha}{\alpha}\right);$$

$$F_5=\alpha\left(1+\frac{7}{6}\alpha+\frac{2}{3}\alpha^2+\frac{1}{6}\alpha^3\right)\log\frac{1+\alpha}{\alpha}$$
$$+\frac{1}{3}\log(1+\alpha)-\alpha\left(\frac{10}{9}+\frac{7}{12}\alpha+\frac{1}{6}\alpha^2\right).$$

All of the above results were checked by an independent substitution of auxiliary Feynman variables, leading not only to the same numerical results, but also (after considerable algebra) to the same algebraic form for F_0 to F_5. Hence, the fact that the total fifth-order contribution from virtual π-mesons is negative relative to the third-order term is quite certain. The entirely analogous results obtained for the heavy meson insertions were not checked as carefully, since they only affect the numerical constants in the relationship between the coupling constants and not the fact that cancellation can occur.

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The Ratio of Soft and Hard Particles in Air Showers*

KURT SITTE
Department of Physics, Syracuse University, Syracuse, New York
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While the density spectrum of air showers is well established for energies above 10^{13} ev, doubt has recently been cast upon the customary procedure of identifying the energy spectrum of the shower-initiating particles calculated from this density distribution with the primary cosmic-ray spectrum. These two distributions would differ if the mechanism of energy transfer to the electron component itself varied with energy. As a result, one would then expect to find a variation with energy also in the amount and composition of the penetrating component of extensive showers. Previous investigations of penetrating particles in air showers do not extend below 10^{13} ev because of the bias imposed, in the case of low density showers, by the addition of a shielded tray. An experiment was therefore carried out at Mt. Evans, Colorado, altitude 4300 m, and at Echo Lake, Colorado, altitude 3260 m, in order to compare the penetrating components of showers in the 10^{13}-ev region and in the 10^{14}-ev region at a similar stage of their cascade development. The relative abundance of penetrating particles was $(2.06\pm0.11)\times10^{-2}$ at Mt. Evans, and $(1.20\pm0.11)\times10^{-2}$ at Echo Lake, and the fraction of N-particles among the penetrating component was (62 ± 7) percent for the lower, and (38 ± 6) percent for the higher shower energies. The ratio of μ-mesons to electrons was the same for both energy ranges. The results are therefore consistent with the assumption that the development of the large mixed cascades in air is based on the same processes for all energies above 10^{13} ev, demanding only a slight increase in the multiplicity of meson production with increasing primary energy.

I. INTRODUCTION

THE study of extensive air showers is certainly the most obvious, and hitherto probably the most successful, method of research in the high energy region of cosmic radiation. Unfortunately, though, the results of such a study yield only very indirect evidence on the nature of the primary interactions, since in these experiments a great many steps intervene between the initial acts and the final recording of the fully-grown shower. Thus, for instance, one can hardly expect from an observation on the intensity of the N-component in these showers, to obtain a unique answer as to the multiplicity of the primary nuclear collision. Or, to choose a simpler example, the old idea of deriving the energy spectrum of the primary cosmic radiation from the density spectrum of air showers (e.g., Heisenberg,[1] Hilberry[2]) is correct only if over the entire range of energies considered, the fraction of the primary energy which goes into the electronic component remains constant.

This latter assumption is certainly subject to serious doubt. Recent advances have shown the processes involved to be very complex, and to include the production of a considerable variety of different fundamental particles. It would seem quite likely that the composition of the shower secondaries produced in the initial collision varies with the energy of the primary. As the contribution to the electron cascade differs for the various components originated in the first interaction, a variation with energy of their relative abundance would, or could, also lead to an energy dependence of the abundance of electrons. The power law for the shower energies derived from the density spectrum of the air shower electrons would then no longer be applicable to the primary cosmic radiation.

The density spectrum of air showers has been thoroughly investigated. In particular, in the experi-

* Supported in part by the AEC.
[1] W. Heisenberg, *Cosmic Radiation* (Dover Publications, New York, 1946), Chap. I.
[2] N. Hilberry, Phys. Rev. **60**, 1 (1941).

ments of Cocconi and Tongiorgi,[3] and of Cocconi, Tongiorgi, and Greisen,[4] the area of the counter trays was varied between 9 cm^2 and 1560 cm^2, corresponding to a variation in the energies of the recorded showers from about 10^{13} ev to about 10^{15} ev. The result was that over this entire range the well-known power law for the differential density spectrum,

$$s(\Delta)d\Delta = A\Delta^{-\gamma}d\Delta, \tag{1}$$

holds with a $\gamma \approx 2.5$, the exponent varying only slightly both with the density and with the altitude. Their results thus check very well with those obtained in the region of higher energies (e.g., Williams[5]), and with different arrangements (e.g., Singer[6]), and it has been shown by Rossi[7] that all these data, as well as the older ones of Hilberry[2] and the flux measurements of various authors at different latitudes, are well represented by a primary spectrum $s(E)dE = dE/(E+E_0)^{2.75}$, with $E_0 = 5.3$ Bev.

On the other hand, there is serious doubt as to the possible validity of so steep a power spectrum for the primary cosmic-ray particles in the region of 10^{12} ev to 10^{13} ev. It has been pointed out by Schein and Lord[8] that the absolute frequency of extremely energetic nuclear events observed in emulsions is inconsistent with so large an exponent. Similarly, the experiments of the Rochester group[9] yield a lower exponent at least for heavy primaries and for energies per nucleon up to about 4×10^{10} ev. Thus, it would appear that either the primary spectrum undergoes a marked change in the region around 10^{12} ev to 10^{13} ev, or that the balance of the electron-initiating processes changes in that same region, leading in some way to a decreasing fractional transfer of the primary energy to the electron component.

Although the detailed features of the development of the mixed cascades are still not fully understood, a criterion concerning their possible variation with energy can be found in the study of the energy dependence of the ratio of intensities of the soft and the hard shower component. For high energies, this ratio is well known and appears to be remarkably independent of the shower energy;[4] but as it will be shown in the following section, these measurements do not cover the same range as the investigations of the electron density spectrum, and do not extend to energies below 10^{13} ev. It was, therefore, decided to examine this point by measuring the relative abundance of penetrating particles in low energy air showers, and to compare it with values at somewhat higher shower energies.

II. OUTLINE OF THE METHOD

Let us begin with noting two basic facts that will be of importance for the ensuing discussion:

(1) Any shower arrangement consisting of three or more coincidence counters can be used for the partially selective recording of showers of a given density.

This can be easily shown to hold for all possible counter arrangements, even if the area of the counters applied is not uniform. (More accurately, the condition reads $\sum_i n_i > \gamma$, where n_i stands for the number of coincidence counters of the ith type, and γ is again the exponent of the differential density spectrum, assumed to be constant.)[10]

(2) Any shower arrangement which is triggered by showers of a given density Δ will detect mostly events whose *cores* strike nearby, and hence will detect showers with cores of this particular average density. (It is assumed that the triggering arrangement consists of a number of counters, or trays of counters, not too widely separated, a certain coincidence of which will define the "shower" event.)

This point has been discussed at length by various authors (e.g., Ise and Fretter,[11] Singer[6]). It will, therefore, suffice to summarize the results of calculations analogous to theirs, but for a 4-tray arrangement of the kind described below. Using the Bethe approximation[12] for the density distribution in the immediate neighborhood of the shower axis,

$$\Phi(r/r_1) = (C/r_1^2)(r/r_1)^{-1}(1+4r/r_1)\exp[-4(r/r_1)^{\frac{2}{3}}] \tag{2}$$

(r_1 stands for the usual characteristic shower length of the Molière theory[13]), and successive Molière approximations at larger distances,[13,14] one finds that 50 percent of all recorded showers will strike within a radius R of about $0.04r_1 \approx 4$ m from the center of the arrangement. The absolute value of this "half-width" is, of course, somewhat dependent on the approximation applied, but its accuracy is of no particular importance for the argument.

In relating the density, Δ, selected by a counter arrangement to the total particle number, and hence to the energy, of the showers recorded, one may again choose the median as the suitable representative, and, therefore, $N = \Delta/\Phi(R/r_1) \approx \Delta \times 1.19\times10^3$ particles.

A proper arrangement of at least three counters or groups of counters in coincidence can, therefore, be made to select air showers of any required initial energy. The value of the energy is determined by adjusting the size, S, of the counters. For the arrangement

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[3] G. Cocconi and V. C. Tongiorgi, Phys. Rev. **75**, 1058 (1949).

[4] Cocconi, Tongiorgi, and Greisen, Phys. Rev. **75**, 1063 (1949).

[5] R. W. Williams, Phys. Rev. **74**, 1689 (1948).

[6] S. F. Singer, Phys. Rev. **81**, 579 (1951).

[7] B. Rossi, Princeton Conference on High Energy Nuclear Interactions, November, 1950.

[8] M. Schein and J. J. Lord, Phys. Rev. **83**, 198 (1951).

[9] Kaplon, Peters, and Ritson, Phys. Rev. **85**, 900 (1952). I am indebted to Professor B. Peters for kindly communicating these results before publication.

[10] Assume, for instance, that coincidences are formed if of each group or tray, n_i out of N_i counters of area S_i are struck ($i=1, 2\cdots m$). The contribution R to the recorded rate due to showers of the density interval $(\Delta, d\Delta)$ is then

$$Rd\Delta = A\Delta^{-\gamma}d\Delta \sum_{i=1}^{m}\binom{N_i}{n_i}(1-e^{-\Delta S_i})^{n_i}e^{-(N_i-n_i)\Delta S_i},$$

and for constant γ, the condition for the existence of a maximum becomes in view of the monotony character of the functions involved, $(dR/d\Delta)_{\Delta\to 0} > 0$, which can be reduced to

$$\sum_{i=1}^{m} n_i - \gamma > 0.$$

[11] J. Ise, Jr., and W. B. Fretter, Phys. Rev. **76**, 932 (1949).

[12] H. A. Bethe, Phys. Rev. **59**, 684 (1941).

[13] G. Molière, Nature **30**, 87 (1942); *Cosmic Radiation*, edited by W. Heisenberg (Dover Publications, New York, 1946), Chap. 3.

[14] The numerical values are taken from computations of the density distribution of shower electrons by F. E. Froehlich [Atomic Energy Commission Report NYO-796 (1951)].

discussed below, the relative contributions of showers of various size are shown as a function of NS in Fig. 1.

To return now to the main problem: It has been pointed out that in some of the earlier experiments[3,4] air showers of energies around 10^{13} ev have been preferentially recorded. With the same counter sets the relative abundance of hard shower particles was then measured in the usual way, that is, by adding a further shielded counter tray. However, it should be emphasized that in such an experiment, the addition of the shielded tray acts as a strong bias in favor of the detection of showers of higher density, in particular where $\beta S' \ll S$ (β= relative abundance of hard particles, S'= area of the shielded tray, S= area of the unshielded air shower trays). Take, for instance, the arrangement of Cocconi, Tongiorgi, and Greisen: Threefold coincidences of air shower counters were demanded, so that the contribution of showers of density Δ to the recorded events was

$$R(\Delta)=A(1-e^{-\Delta S})^3\Delta^{-\gamma}, \tag{3}$$

while in the case of hard showers the corresponding rate R_h was

$$R_h(\Delta)=A(1-e^{-\Delta S})^3(1-e^{-\beta\Delta S'})\Delta^{-\gamma}. \tag{4}$$

The maximum contributions to these rates are from showers with densities satisfying $(dR/d\Delta)=0$, which gives

$$e^{-\Delta S}(3\Delta S+\gamma)-\gamma=0: \Delta S=0.35 \tag{5}$$

in the first, and from $(dR_h/d\Delta)=0$,

$$e^{-\Delta S}(3\Delta S+\gamma)+e^{-\beta\Delta S'}(\beta\Delta S'+\gamma) - e^{-\Delta(S+\beta S')}(3\Delta S+\beta\Delta S'+\gamma)=0 \tag{6}$$

in the second case. For low density showers one has $\beta S' \ll S$, and (6) can be simplified by expansion of the exponential terms, relying on the fact that only regions where $\Delta S \sim 1$, and hence $\beta\Delta S' \ll 1$, will appreciably contribute. As a lower limit, one finds thus the approximate solution

$$e^{-\Delta S}(3\Delta S+\gamma)-(\gamma-1)=0: \Delta S=1.25, \tag{7}$$

which shows that the maximum contribution comes now from considerably denser showers. The same fact is still more drastically exhibited by a comparison of the average densities of the showers recorded with the two arrangements. For air showers, one has (writing $\Delta S=z$)

$$\langle z_a\rangle=\frac{\int_0^\infty(1-e^{-z})^3z^{-(\gamma-1)}dz}{\int_0^\infty(1-e^{-z})^3z^{-\gamma}dz} =\frac{(-\gamma+1)!\sum_{n=0}^{3}(-1)^n\binom{3}{n}n^{\gamma-2}}{(-\gamma)!\sum_{n=0}^{3}(-1)^n\binom{3}{n}n^{\gamma-1}}, \tag{8}$$

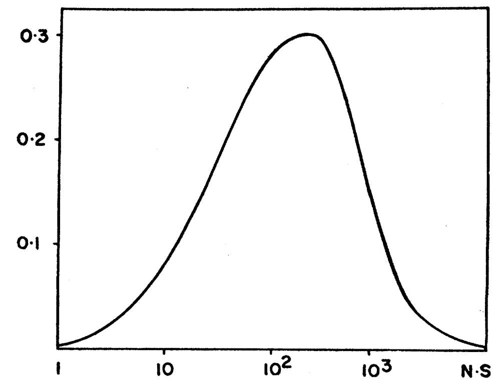

FIG. 1. Contribution to coincidences of 3 out of 4 counters, of showers of various total numbers N of electrons striking at all distances from the center of the arrangement (S in m², R in arbitrary units).

while the average density of the hard showers is (with $\beta S'/S=\alpha$)

$$\langle z_h\rangle=\frac{\int_0^\infty(1-e^{-z})^3(1-e^{-\alpha z})z^{-(\gamma-1)}dz}{\int_0^\infty(1-e^{-z})^3(1-e^{-\alpha z})z^{-\gamma}dz} =\frac{(-\gamma+1)!\sum_{n=0}^{3}(-1)^n\binom{3}{n}\{n^{\gamma-2}-(n+\alpha)^{\gamma-2}\}}{(-\gamma)!\sum_{n=0}^{3}(-1)^n\binom{3}{n}\{n^{\gamma-1}-(n+\alpha)^{\gamma-1}\}}. \tag{9}$$

For showers of low density, one has usually $\alpha \ll 1$, and the ratio of the average densities of the two types of recorded showers becomes

$$\frac{\langle z_h\rangle}{\langle z_a\rangle}=\frac{\sum_{n=1}^{3}(-1)^n\binom{3}{n}n^{\gamma-1}}{\sum_{n=1}^{3}(-1)^n\binom{3}{n}n^{\gamma-2}} \cdot\frac{[\alpha^{\gamma-3}+(\gamma-2)\sum_{n=1}^{3}(-1)^n\binom{3}{n}n^{\gamma-3}]}{[\alpha^{\gamma-2}+(\gamma-1)\sum_{n=1}^{3}(-1)^n\binom{3}{n}n^{\gamma-2}]}. \tag{10}$$

In most experiments α is of the order 10^{-2}, so that $\langle z_h\rangle/\langle z_a\rangle \sim 10$. Even in the experiment of Cocconi, Tongiorgi, and Greisen,[4] where the area of the shielded tray S' was exceptionally large, the ratio (10) is still about 5. Of course, the discrepancy between the two average densities becomes still more pronounced if showers containing more than one hard particle incident from the air are recorded.

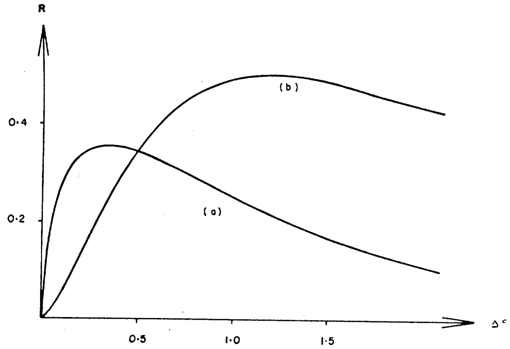

FIG. 2. Contribution to the recorded shower rates as a function of the particle density: (a) triple coincidences of unshielded counters; (b) the same with additional shielded counters ($\beta S'/S = 0.04$).

The situation is somewhat, but not radically, improved if the contributions of the denser showers are cut down by an anticoincidence arrangement demanding, for instance, that exactly n out of N counters be struck. Thus, triple coincidences in anticoincidence with a fourth counter shift the maximum contribution to the recorded rate, computed as in (5), to $\Delta S = 0.19$, and for hard showers to $\Delta S = 0.61$. While the ratio of these figures differs only little from the one obtained above for pure coincidences, the results for the average densities recorded become much better; instead of a factor 5–10, one finds a factor 2–3 between the two averages. The conditions are illustrated in Fig. 2, where the relative contributions to the counting rate of showers of various densities are plotted for a triple-coincidence experiment, and in Fig. 3, where the contributions are shown for triple coincidences in anticoincidence with a fourth counter of equal size. In both cases, a value of $\alpha = 0.04$ has been chosen, corresponding closely to the conditions and the results of the Cocconi-Tongiorgi-Greisen experiment.[4]

Thus, it appears that the range of densities over which the ratio of penetrating particles to electrons has been observed is actually considerably smaller than it has been tacitly assumed. In particular, the investigations do not reach down to the region below 10^{13} ev. It seemed, therefore, worthwhile to carry out an experiment properly designed to fill this gap.

III. THE EXPERIMENTAL ARRANGEMENT

An arrangement suitable for the determination of the relative abundance β of penetrating particles in air showers must satisfy the following conditions:

(1) Its counter area should yield maximum contributions from showers with less than 10^4 particles (at the cascade maximum);

(2) in order to minimize possible errors due to a variation of γ with the density Δ, it should be triggered only by showers over a comparatively narrow density range;

(3) "soft" air showers and air showers containing penetrating particles that are recorded should have on the average equal densities.

To satisfy (1) it becomes advantageous to run the experiment at an altitude as close as possible to the cascade maximum. Of the two convenient locations, Echo Lake and Mt. Evans, the latter was chosen; its atmospheric depth, taken for an average shower origin at one interaction mean free path in air from the top of the atmosphere, is 510 g/cm^2 or about 14 cascade units, which is only slightly past the maximum for a 10^{13} ev shower.

At 14 cascade units, a shower of 10^{13}-ev initial energy contains about 4.9×10^3 electrons;[15] its detection would thus require a counter arrangement set to select densities of about 3–4 particles/m^2. This would not be too difficult and has actually been achieved in several experiments (Broadbent and Jánossy,[16] McCusker and Millar[17]), but a most inconveniently large counter area would be required to select the corresponding hard showers. The condition was therefore relaxed and the arrangement set for shower densities of about 10 particles/m^2.

Condition (2) is sufficiently satisfied by an anticoincidence arrangement triggered if any three out of four counters are struck. Condition (3), however, demands the use of different sets of unshielded counters for the detection of "soft" and of "hard" air showers. The suitable counter sizes can now be determined.

As the maximum contribution to the registered air showers comes from a density $(\Delta S) = 0.19$, a counter area of 155 cm^2 (two standard 1-in.$\times$12-in. counters in parallel) for the unshielded trays will select densities of about 12 particles/m^2, or shower energies around 2.5

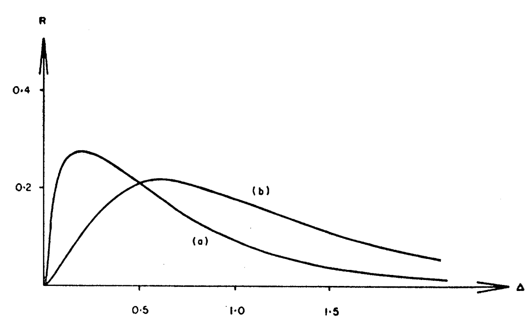

FIG. 3. Contribution to the recorded shower rates as a function of the particle density: (a) coincidences of three out of four unshielded counters; (b) the same with additional shielded counters ($\beta S'/S = 0.04$).

[15] For the determination of the total number of shower electrons, the "Tables of cascade functions" by Leonie Jánossy and H. Messel [Proc. Roy. Irish Acad. **A54**, 217 (1951)] have been used, and corrections have been made both for the ionization loss—following the procedure of these authors—and for the extension of the electron spectrum down to about one-tenth of the critical energy in air, in order to include all shower electrons capable of penetrating the counter walls.

[16] D. Broadbent and L. Jánossy, Proc. Roy. Soc. (London) **A190**, 497 (1947).

[17] C. B. A. McCusker and D. D. Millar, Proc. Phys. Soc. (London) **A64**, 915 (1951).

$\times 10^{13}$ ev. At least a considerable fraction of the recorded events belongs, therefore, to the energy region between 10^{12} ev and 10^{13} ev at which this study was aimed.

The shielded tray consisted of eight counters 1 in. $\times$20 in. As it was also desired to get some evidence on the nature of the penetrating particle, these counters were separated from each other by $\frac{1}{2}$-in. lead strips, so that most of the events in which more than one shielded counter was struck can be attributed to either a local penetrating shower, or more than one incident penetrating particle. The shielding consisted of 8 in. of lead on top of the tray, and an equal amount on all sides, so that undoubtedly even the long-range soft γ-radiation, the importance of which was pointed out by Greisen,[18] is practically eliminated.

One can now evaluate the area S of an unshielded air shower tray which in coincidence with the shielded tray of area $S'=1030$ cm^2 will record showers of the same average density as the tray of area $S_0=155$ cm^2 used to select the 2.5×10^{13}-ev air showers. At Mt. Evans, the fraction β is approximately 0.02; this leads to a ratio $S/S_0=6$. Therefore, the large air shower trays were made up of three counters 2 in.$\times$24 in. in parallel, giving $S=930$ cm^2. Again coincidences of three such groups in anticoincidence with the fourth defined the recorded events. A schematic diagram of the arrangement is shown in Fig. 4.

In view of the high individual counting rates, great care had to be taken to isolate the counters in the trays from each other by feeding the pulses through a diode mixing circuit onto the grid of the output cathode follower, all these units being mounted in the counter trays. The pulses from the air shower trays were then clipped to equal size, and fed into an integrating circuit and to the discriminators which selected the desired multiplicities. The anticoincidences were formed in standard Rossi circuits. In the shielded tray, the individual counter pulses were clipped and fed through isolating diodes to the integrating circuit and the output cathode follower. Discriminators set to pass single or double pulses then selected the events which, together with the triple coincidences of the unshielded trays, defined the "hard" air showers.

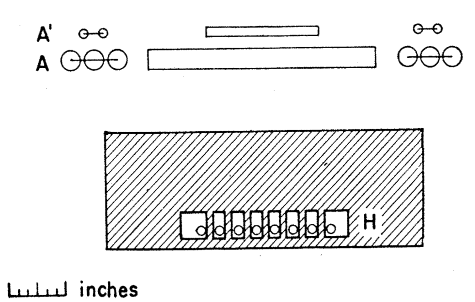

FIG. 4. Schematic diagram of the experimental arrangement (Mt. Evans experiment). A: four unshielded trays, 930 cm^2; A': four unshielded trays, 155 cm^2; H: shielded tray, 1030 cm^2.

[18] K. I. Greisen, Phys. Rev. **75**, 1071 (1949).

TABLE I. Number and rate of recorded "soft" showers A_3 and A_3', and of "hard" showers with one, or more than one, penetrating particle (A_3+H^1), (A_3+H^2), at Mt. Evans. The hard-shower rates are corrected for chance coincidences. Area of unshielded counters: 155 cm^2 and 930 cm^2; area of shielded counters: 1030 cm^2.

Event	Number of showers	Time (min)	Rate (min^{-1})
(A_3')	1867	3959	$0.472\ \pm 0.011$
(A_3)	26,278	3739	$7.03\ \pm 0.04$
(A_3+H^1)	429	4061	$0.0971 \pm 0.004_7$
(A_3+H^2)	189	4061	$0.0466 \pm 0.003_4$

Four kinds of showers were registered:

(1) three out of the four groups of area S_0 struck: "A_3'-showers";
(2) three out of the four groups of area S struck: "A_3-showers";
(3) A_3-showers with at least one hard particle on S': (A_3+H^1)-showers;
(4) A_3-showers with at least two hard particles on S': (A_3+H^2)-showers

(a subscript n denotes exactly n counters or groups of counters struck, a superscript n or more). The ratio of the rates A_3'/A_3 serves as a check on the exponent γ, the ratio $(A_3+H^2)/(A_3+H^1)$ is a measure of the fraction of N-particles among the penetrating air shower particles.

As a check on its performance, it was decided to operate the arrangement also at Echo Lake, and in a density range where previous results permit a comparison. For this purpose, it is advantageous to select showers which are in the same stage of cascade development at Echo Lake, as the 2.5×10^{13}-ev showers are at Mt. Evans. As the altitude difference between the two stations corresponds to about 2.5 cascade units in air, the Echo Lake showers should have about 12-times higher initial energies than the ones recorded at Mt. Evans. Under the conditions of this experiment, the detection of such showers would require counter areas of about 13 cm^2, and counters of this small size were not available. But with counters of 39 cm^2 ($\frac{1}{2}$ in.$\times$6 in.), the maximum contribution comes from showers of initial energy around 1.2×10^{14} ev, and the bias in favor of high densities for the hard-shower set is still rather small: It raises the most efficiently registered shower energies to about 2×10^{14} ev. As in this region there is ample experimental evidence on both the exponent of the power law of the density distribution,[3,5,6,11] and the relative abundance of penetrating particles[4,16,17] it was not deemed necessary for the present study to extend the operations at Echo Lake beyond a redetermination of β as a comparison and control run.

IV. RESULTS AND DISCUSSION

A summary of the results of the Mt. Evans run is given in Table I. As in the following only relative

TABLE II. Number and rate of "soft" showers A_3, and of "hard" showers with one, or more than one, penetrating particle (A_3+H^1), (A_3+H^2), at Echo Lake. The hard-shower rates are corrected for chance coincidences. Area of unshielded counters: 39 cm²; of shielded counters: 1030 cm².

Event	Number of showers	Time (min)	Rate (hr⁻¹)
(A_3)	604	17,462	2.08 ±0.09
(A_3+H^1)	293	12,877	1.37 ±0.08
(A_3+H^2)	93	11,080	0.504±0.052

values are of importance, no corrections have been applied for the inefficiency of the counters and for the barometric effect. The correction for random coincidences turns out to be negligible for all but the $(A_3'+H^1)$-events.

The observed ratio of the air shower rates with different counter areas, A_3'/A_3, yields for the exponent of the power law $\gamma=2.51\pm0.02$, in good agreement with earlier observations at the same altitude (e.g., Treat and Greisen[19]). Using this value, one can then determine the relative abundance β of the penetrating particles in the air showers. As $\alpha=\beta S'/S\ll1$, one has for the ratio of the rates of "hard" showers N_h and of "soft" showers N_s,

$$\frac{N_h}{N_s}=-(\gamma-1)\alpha\sum_{n=0}^{3}(-1)^n\binom{3}{n}(n+1)^{\gamma-2}\Big/\sum_{n=0}^{3}(-1)^n\binom{3}{n}(n+1)^{\gamma-1}. \quad (11)$$

From the observed rates, one thus obtains $\beta=(2.06\pm0.11)\times10^{-2}$, or one penetrating particle to 49 electrons.

The showers (A_3+H^2) are mainly due to three kinds of events: (1) "hard" showers in which the penetrating particle is accompanied by a knock-on electron discharging the second counter of the H-tray; (2) "hard" showers with two random incident penetrating particles, and (3) "hard" showers in which the penetrating particle belongs to the N-component and has initiated a local shower in the 8-in. lead absorber on top of the shielded tray. The knock-on contribution was determined by taking the rates (H^1) and (H^2) separately; the first is the total flux of penetrating particles through the tray, the second essentially the flux of mesons accompanied by knock-on electrons, if corrections are applied for the chance coincidences of single mesons. The result of this test run shows that, on the average, 2.1 percent of the mesons give rise to recorded knock-on events. As the energy spectrum of the mesons in air showers is probably not much different from that of single mesons, the ratio of H^2/H^1 can also be used for an estimate of the knock-on accompaniment of the single hard particles in the shower.

[19] J. E. Treat and K. Greisen, Phys. Rev. **74**, 414 (1948).

The correction for random pairs of incident particles follows the established procedure (e.g., Auger and Daudin,[20] Broadbent and Jánossy[21]). The calculation indicates a contribution of 2.8 percent of the rate (A_3+H^1), so that the total correction for non-nucleonic events amounts to 4.9 percent. The observed fraction of 47 percent has therefore to be reduced to 42 percent. On the other hand, an appreciable fraction of the N-particles will not contribute to the (A_3+H^2) showers, either because they failed to interact in the lead absorber (about 25 percent of all particles for a mean free path of 160 g/cm²), or because the local showers are absorbed or not dense enough to strike two counters. If previous hodoscope experiments are used as a guide,[22] this latter fraction may be estimated as about 20 percent. Thus the recorded nucleonic events derive from only 60 percent of all N-particles, and if a neutron/proton ratio of 0.8 is assumed (Walker,[23] Greisen *et al.*[24]), the true fraction of the N-component among the penetrating particles is (62±7) percent. The error stated includes only the statistical deviation. This rather large value is in good agreement with the data of Greisen, Walker, and Walker,[24] observed at the same altitude. However, in view of the many differences between their experiment and the present one in the location of the shower cores and in the shower energies involved, this agreement may well be fortuitous. A comparison with the data taken with the same equipment at Echo Lake promises more reliable conclusions.

The data of the Echo Lake run are summarized in Table II. For the evaluation of β, the value of $\gamma=1.44$ is taken from the results of Cocconi and Tongiorgi,[3] and as for the Mt. Evans experiment, corrections have again been made only for random coincidences. Obviously the approximation (11) is not applicable, and the determination of β has to be done by the usual graphical method. The result is $\beta=(1.20\pm0.11)\times10^{-2}$, in satisfactory agreement with earlier measurements.[4]

In the analysis of the hard component, both corrections for random pairs and knock-ons are again substantial, and reduce the rate of (A_3+H^2) events initiated by the N-component to 24.5 percent of the (A_3+H^1)-rate. After correcting for undetected N-particles in the same way as before, one thus finds a fraction of (38±6) percent for the N-component in the penetrating shower radiation (the errors shown again include only the statistical deviation).

In comparing the two fractions, a possible different attenuation of the two components must be taken in account. Unfortunately, the theory of nuclear cascades is not yet in a state to permit accurate numerical predictions, as some of the fundamental constants are

[20] P. Auger and J. Daudin, J. phys. et rad. **6**, 233 (1945).
[21] D. Broadbent and L. Jánossy, Proc. Roy. Soc. (London) **A192**, 364 (1948).
[22] K. Sitte, Phys. Rev. **78**, 721 (1950).
[23] W. D. Walker, Phys. Rev. **77**, 686 (1950).
[24] Greisen, Walker, and Walker, Phys. Rev. **80**, 535 and 546 (1950).

still only approximately known. But it is certainly safe to conclude from the results of Messel[25] that the nucleonic component in showers of 10^{13} ev to 10^{14} ev initial energy does not decrease greatly between atmospheric depths of 600 g/cm^2 and 700 g/cm^2 where it is still close to its maximum development. For higher energies it may even increase, a fact which is borne out by the results of McCusker[26] who, working at sea level and hence in the average with more energetic showers, still finds about 30 percent N-particles in his penetrating radiation.

The attenuation of the meson component in the showers is probably not much different from that of single mesons, which according to Rossi's data[27] decrease by about a factor 0.8 between the two levels. If, as it is likely, the attenuation of the N-component is somewhat larger than that of the μ-mesons, the discrepancy between the values derived above is not completely removed: Even if the attenuation for the shower particles were the same as for single nucleons, a (62 ± 7) percent fraction at Mt. Evans would still yield a (50 ± 6) percent N-component at Echo Lake, but this last assumption is certainly an overestimate of the absorption of the shower nucleons.

It should also be noticed that, while possible systematic errors in the estimates of production and detection probabilities of the local showers may have falsified the absolute values given to the N-component above, such an error would change the data obtained at the two stations in the same direction. Thus, one is led to the conclusions that the ratio of N-particles to μ-mesons within the same shower does not change very much between atmospheric depths of 600 g/cm^2 and 700 g/cm^2, and that the observed difference in the composition of the showers of about 10^{13} ev and 10^{14} ev initial energy, though possibly exaggerating the true conditions, is nevertheless of significance.

On the other hand, it is interesting to note that the ratio of μ-mesons to electrons, if taken at the same stage of the electron cascade development—slightly past the cascade maximum—is practically identical for the two types of showers recorded in this experiment: 0.78 percent, or one in 128 particles, for the lower shower energies, and 0.74 percent, or one in 135 particles, for the higher shower energies. It is plausible that such a comparison should be made at the same stage of the cascade development, and preferably at the cascade maximum. Only then the electron shower will be truly representative of the fraction of the primary energy transferred to the electron component.

The results indicate, therefore, that the ratio of charged and neutral mesons—if the latter, in accordance with the current simple picture, are considered as the producers of the bulk of the electron component—does not undergo an appreciable change for collisions over an energy range from 10^{12} ev to 10^{14} ev. If other processes are involved as well, their relative frequencies must also have about reached an equilibrium state. The considerable decrease in the N-component can be understood as caused by a slight increase in the multiplicities of meson production with increasing energy, provided that even for the 10^{14}-ev showers the bulk of the π-mesons originated in the initial collision decays before undergoing a nuclear collision, or that at least the distribution of the energy contained in the charged-meson component, among its two successors (electrons and nucleons), does not change much between primary energies of 10^{12} ev and 10^{14} ev.

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[27] B. Rossi, Revs. Modern Phys. **20**, 537 (1948).