Letters to the Editor

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Mass Corrections to Hyperfine Structure

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A. Hydrogen.—An accurate value for the fine structure constant α is obtained from precision measurements of the hyperfine structure splitting (hfs) of the ground state of hydrogen. Using the radiative corrections[1] of relative order α and α^2 and the simple reduced mass factor $(1-3m/M_P)$ to the elementary Fermi formula for the hfs of hydrogen (for an infinitely heavy nucleus), one finds[2]

$$1/\alpha=(137.0365\pm0.0006)(1+\tfrac{1}{2}F)(1+\tfrac{1}{2}\delta), \tag{1}$$

where F and δ are further correction terms.

Nuclear motion contributes mass corrections to hfs of relative order $\alpha(m/M_P)$, in addition to the factor $(1-3m/M_P)$, where m and M_P are electron and proton mass, respectively. In a previous note[3] it was pointed out that a simple approximation for these corrections can be obtained from orthodox quantum electrodynamics and perturbation theory, but the result reported there was incorrect. Two types of correction terms arise, (1) one involving the exchange of a single transverse photon between the electron and proton, (2) the other the exchange of two transverse photons. The complete results are as follows.

(1) For photon momenta k large compared with the Bohr momentum, the Fourier components of large momentum for the Coulomb potential of the hydrogen atom can also be treated as a perturbation. A contribution to hfs is then obtained by third-order perturbation theory (using Dirac hole theory for both electron and proton), involving the emission of the photon by the one particle, absorption by the other, and one Coulomb scattering (to conserve momentum approximately). For $k\ll m$ *no* corrections of order $\alpha(m/M_P)$ are obtained. For $m\ll k\ll M_P$ we expand in powers of (k/M_P) and consider the proton to have both its Dirac magnetic moment and an anomalous moment of the Pauli type. The Dirac and Pauli moments are treated separately and both positive and negative energy intermediate states for the electron and proton contribute.

(2) The exchange of two photons, neglected completely in the Fermi formula for hfs, also contributes terms of order $\alpha(m/M_P)$ (hfs), which can be calculated by means of fourth-order perturbation theory.

Integrating the approximations obtained (strictly valid only for $m\ll k\ll M_P$) over k with m and M_P as lower and upper limits, respectively, we get an addition to the hfs of F (hfs), where

$$F=-\{4\mu_P+(1/4)[1-2\mu_P-3\mu_P^2]\}(\alpha m/\pi\mu_P M_P)\ln(M_P/m)=-1.4\times10^{-5}, \tag{2}$$

and μ_P is the total magnetic moment of the proton in nuclear magnetons. In Eq. (2) the first term in the curly brackets is a single photon term, while the remaining ones are double photon terms.

The approximations leading to Eq. (2) involve the neglect of terms of order unity compared with $\log(M_P/m)$. For a nucleus consisting of a point particle of Fermi-Dirac type with no anomalous magnetic moment, the corrections to order $\alpha(m/M_P)$ (hfs) can be calculated exactly.[4,5] But the structure effects of the proton magnetic moment (which cannot be calculated reliably at present) will also alter the hfs by an additional fraction δ, which might be more than $\pm2\times10^{-5}$. Using Eq. (2) we then get

$$1/\alpha=(137.0355\pm0.0010)(1+\tfrac{1}{2}\delta). \tag{3}$$

B. Deuterium.—The mass correction to hfs, equivalent to Eq. (2), is markedly different for deuterium because of the relatively loose structure of the deuteron. The perturbation treatment used for hydrogen has to be modified by introducing sums over all states of the neutron-proton system for the intermediate states. If nuclear exchange forces and the percentage of D state are neglected, simple expressions are obtained in the limiting cases of $kc\ll E_D$ and $kc\gg E_D$, where E_D is the deuteron binding energy. Instead of Eq. (2) we then obtain

$$F=\{[2+8\mu_P-4\mu_D-(3/4)(2\mu_P^2+2\mu_N^2-\mu_D^2)]\ln(W/mc^2)-[\tfrac{1}{2}+7\mu_P-(3/2)(\mu_P^2+\mu_N^2)]\ln(M_P/m)\}(\alpha m/\pi\mu_D M_D), \tag{4}$$

where μ_N and μ_P are the neutron and proton moment, respectively (in nuclear magnetons), and W is an average excitation energy of the order of magnitude of E_D. Putting (arbitrarily) $W=E_D$, Eq. (4) reduces to[6]

$$F=-(1\pm3)\times10^{-5}. \tag{4a}$$

Part of the corrections, Eq. (4), have already been included in Low's[7] calculations of the structure corrections to the hfs of deuterium (the term called ϵ_N). Using Eq. (4a) we get for the discrepancy between the calculated and experimental values for (ν_D/ν_H) (the ratio of deuterium to hydrogen hfs), instead of Eq. (6) of reference 3, the value

$$(+0.6\pm0.4)\times10^{-4}. \tag{5}$$

The large uncertainty in this result could be reduced greatly by explicitly calculating W and the effect of nuclear exchange forces. At the moment the discrepancy, Eq. (5), is not known with sufficient accuracy to furnish any conclusive evidence for a large spread of the magnetic moment of a nucleon or a large error in the percentage D state assumed in reference 7.

1 N. M. Kroll and F. Pollock, Phys. Rev. **84**, 594 (1951); Karplus, Klein and Schwinger, Phys. Rev. **84**, 597 (1951).
2 N. M. Kroll and F. Pollock, Phys. Rev. **86**, 876 (1952). The probable error in Eq. (1) includes an estimate for the error due to radiative corrections of order α^3 and higher, which have not been calculated yet.
3 F. E. Low and E. E. Salpeter, Phys. Rev. **83**, 478 (1951).
4 Arnowitt, Karplus, and Klein (to be published).
5 W. A. Newcomb and E. E. Salpeter (to be published). Using a four-dimensional, Lorentz invariant method described by E. E. Salpeter [Phys. Rev. **86**, 595(A) (1952)], all the terms involving $\log(M_P/m)$ were found to be identical with Eq. (2) of the present paper.
6 The numerical value for F is small, due to some fortuitous cancellations.
7 F. E. Low, Phys. Rev. **77**, 361 (1950).

Internal Conversion of γ-Rays with Pair Production

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THIS problem has been given an "exact" treatment by Jaeger and Hulme[1] using the Dirac equation. The γ-radiation was considered as a perturbation in the Coulomb field round the nucleus, arising from an electrical dipole or quadrupole.

Oppenheimer and Nedelsky[2] have applied the Born approximation to both cases, assuming $Z=0$. The Born approximation has also been used by Rose,[3] who extended the calculation to an arbitrary radiating 2^l-pole (electric as well as magnetic) and suggested the region of validity to be $Z\lesssim40$. Numerical results were given for $l=1-5$, with γ-energies in the region 2.5–20 Mev.

Finally, Rose and Uhlenbeck[4] have used the semi-relativistic Schrödinger approximation of the Dirac wave functions in order to solve the problem. Besides giving the cross section for energies exceeding $2mc^2$ by only a small amount, they also made a comparison between the results of the two approximation methods mentioned here and those of the "exact" theory.