This corresponds to the classical virial theorem, since in the presence of a magnetic field the right-hand side of (1) is replaced by $\langle\boldsymbol{\alpha}\cdot\boldsymbol{\pi}\rangle$ in the quantum case.

Perhaps the most useful form of the virial theorem is the case corresponding to no magnetic field,

$$-\langle\mathbf{r}\cdot\mathbf{F}\rangle=\langle\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=\langle E-e\phi-\beta\rangle. \quad (6)$$

It is clear that (5) and (6) do not apply for continuum states. For example, for a free particle $\langle\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=\langle p^2/E\rangle$.

Alternatively, for a static potential ϕ, we consider the expectation value for any stationary state of the anticommutator of β and H to obtain

$$\langle\beta(E-e\phi)\rangle=1, \quad (7)$$

(that is, $\langle\beta\boldsymbol{\alpha}\cdot\mathbf{p}\rangle=0$) which gives the well-known result $\langle\beta\rangle=1/E$ for a free particle. However, for a bound state in the Coulomb field, it follows from (6) that

$$\langle\beta\rangle=E, \quad (7a)$$

so that in this case,

$$e\langle\beta\phi\rangle=E^2-1. \quad (7b)$$

The nonrelativistic limit $e\langle\phi\rangle=-2\langle T\rangle$, T the nonrelativistic kinetic energy operator, follows from (7b); and more generally from (6) the usual limiting form $e\langle\mathbf{r}\cdot\nabla\phi\rangle=2\langle T\rangle$ results.

* This paper is based on work performed for the AEC at the Oak Ridge National Laboratory.

Soft and Hard *F*-Centers

JORDAN J. MARKHAM
Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland
(Received March 19, 1952)

OBERLY[1] has suggested that two types of F-centers exist. The soft type can be bleached easily by light, while the hard type is relatively unaffected. The suggestion is based on his photoconductivity measurements and work by Petroff.[2] The author would like to explain the observations in an alternate manner which does not require two types of centers and agrees with the conventional model.[3]

Using x-rayed KBr, Oberly measured at room temperature the changes of $\eta\omega$ (η is the quantum yield and ω is the electron range) caused by irradiation with F-light. He measured $\eta\omega$ as a function of wavelength after a 7- and a 23-minute exposure to F-light. Between 550 mμ and 1000 mμ, $\eta\omega$ decreases between the 7- and the 23-minute measurements while the reverse is true in the violet beyond the F-band.

One interpretation is that η changes because the soft centers disappear. This requires, in the author's opinion, soft and hard F, R, M, and N centers.

The author would like to suggest that ω changes. During the exposure to F-light negative-ion vacancies are formed from those centers which release electrons. We shall assume that negative-ion vacancies remain as incipient vacancies during the course of the experiments. Thus when the crystal is exposed to F-light, the concentration of negative-ion vacancies and the corresponding macroscopic cross section for an electron to be captured by a vacancy increases; the latter in turn decreases ω. Oberly's data is explained as follows: (1) The photoconductivity between 550 mμ and 1200 mμ is due to electrons whose ω decreases during F-light exposure. (2) The photoconductivity between 470 mμ and 550 mμ is due to holes whose ω is not affected by the concentration of negative-ion vacancies. (3) Oberly observed that an exposure to F-light affects photoconductivity measurements more than absorption measurements. The reason seems to be that absorption depends only on the concentration of F-centers, while photoconductivity depends critically on the concentration of negative-ion vacancies as well as the concentration of F-centers.

Another reason which might cause one to believe that there are two types of F-centers is that the half-width of the band varies experimentally.[4] Some F-centers may be surrounded by a perfect lattice, while other F-centers may be near vacancy clusters or dislocations. The centers near a perturbation in the lattice probably have a different half-width than those in a nearly perfect lattice. The method of excitation and the past history of the crystal would seem to determine the location of the F-centers and their half-width. The variation in the F-centers seems to be a matter of degree rather than a matter of having two types with distinct η.

Petroff has suggested two types of F-centers to explain the rate of growth of the M-band. Seitz[3] has proposed that an M-center is an electron attached to two negative and one positive-ion vacancy. It seems probable that the M-band is formed by breaking up vacancy clusters and that the intermediate state may be a vacancy complex which gives no absorption rather than a center hidden under the F-band.

There seems to be the possibility of explaining Oberly's data without changing the present picture of color centers. The author will not attempt to explain some secondary effects observed by Oberly which need further experimental exploration. At present there seems to be no reason to believe that these secondary effects will require a more elaborate model.

The author would like to thank Professor F. Seitz and Mr. W. H. Deurig for helpful discussions of these ideas.

* Supported by the Bureau of Ordnance, U. S. Navy.
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Optical Focusing in Constant Radius Accelerators

DAVID C. DEPACKH
Nucleonics Division, Naval Research Laboratory, Washington, D. C.
(Received March 19, 1952)

THAT the focusing forces are weak at low energies and large radii in electron accelerators with magnetic fields characterized by the usual parameter n can be readily deduced from the definition of this quantity, namely,

$$n\equiv-(R/H_z)(\partial H_z/\partial r)\equiv-(R/H_z)(\partial H_r/\partial z),$$

where R is the equilibrium orbit radius, H_z is the vertical field at the orbit, and H_r is the radial field at the orbit. We find in fact that

$$\Delta z/\Delta H_r=n^{-1}R^2/(RH_z),\quad \Delta r/\Delta H_z=(1-n)^{-1}R^2/(RH_z).$$

The quantity on the left is a direct measure of the orbit shift due to a stray field ΔH_r or ΔH_z and thus of the amount the particle will go astray due to this field. RH_z depends only on the particle energy, which must be considered at its lowest value, namely, the injection energy. The value of n is of course restricted to lie between 0 and 1.

It is evident that for machines of large radius a stronger focusing force at injection is very desirable. Such a force can be supplied by a sequence of magnetic lenses regularly spaced around the acceleration tube. Because of the axial symmetry of this arrangement there is no theoretical limit, short of the fact that the lenses must be discrete, on the amount of focusing force available. It will be assumed that the field strength of the lenses is constant in the time, at least after a certain time. This condition must be fulfilled because of the size limitation of the lenses, which must fit into the available aperture. The calculations show that there is no adiabatic variation of oscillation amplitude with particle acceleration if the lens fields are constant. It may thus be desirable to supply a small increase in the lens field during the early part of the acceleration cycle to insure that the orbits will lie within the acceleration tube.