Therefore it is proposed that the primary interaction takes place among a small, but variable, number of nucleons (most often 2, 3, or 4), as in Menon *et al.*[4] This permits the emission of protons up to 70 Mev with reasonable probability, and protons up to 95 Mev occasionally. These nucleons have a spread of energies which favors the emission of low energy knock-on particles. Since the mean free path of such a nucleon is of the order of the nuclear radius for an element like Ag,[20] each nucleon will collide, on the average, one time inside the nucleus. But most of the knock-on particles will not be able to escape from the nucleus. Qualitatively, therefore, about one extra prong is expected, on the average, to accompany an energetic prong. This figure is consistent with the observed number of star prongs.

A test of this hypothesis would be to calculate the energy and angular distribution of the knock-on particles. The Monte Carlo method of calculation appears to be most suitable, but it will not be attempted in this paper.

TABLE I. Prong spectrum of π^- meson stars.

Number of prongs	1	2	3	4	5	6
Percent of stars	32.4 ± 1.0	32.7 ± 1.0	22.3 ± 0.8	10.5 ± 0.6	2.0 ± 0.2	0.1 ± 0.1

VI. ACKNOWLEDGMENTS

I should like to thank Dr. W. Barkas, Dr. H. Bradner, Dr. C. Richman, and Dr. R. L. Thornton for their guidance and encouragement, and to express my gratitude to the other members of the Film Program at the Radiation Laboratory for their cooperation and for many fruitful discussions. I am also indebted to Mr. A. Oliver for the preparation of Figs. 2 and 3, and to Miss Betty Summers for the other figures.

PHYSICAL REVIEW VOLUME 85, NUMBER 2 JANUARY 15, 1952

Limits of Stability of the Heavy Elements and the Effects of Nuclear Shell Structure*

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(Received September 14, 1951)

The systematics of beta-decay, alpha-emission, and spontaneous fission will be investigated in order: first, to determine the theoretical limits to the periodic table, and secondly, to examine the effects of nuclear shell structure on the end of the periodic table. Curves of beta-stability and lifetimes against alpha-emission and spontaneous fission will be drawn for the heaviest elements and will be extrapolated in order to yield information about the stability or form of instability of the transuranic elements. Nuclear shell structure will be seen to be responsible for the stability of Th^{232} and U^{238} as well as the instability of the group of highly radioactive elements beginning with polonium.

I. INTRODUCTION

URANIUM is the heaviest known naturally occurring element on a macroscopic scale. Presumably, when the earth was formed ($\sim3\times10^9$ years ago), several isotopes, now unknown, were present. The only nuclei still occurring naturally are those whose half-lives are appreciable compared to 10^{8-9} years.

The beta-stable isotopes occur in a relatively narrow band; those nuclei lying outside this band are unstable with respect to positron emission or K-capture on the one side, and negative beta-emission on the other. This band would theoretically continue without limit onward towards higher and higher atomic number. However, the beta-stable transuranic elements do not occur in nature because of instability against alpha-decay or fission.

The manner in which an unstable nucleus is most likely to have decayed is governed by the lifetimes against the three forms of decomposition. At present there is no isotope known which has had fission as its main mode of decay.

Existing data will here be used to draw curves of beta-stability and lifetimes against alpha-decay and fission. These curves will yield information on the effects of nuclear shell structure on the end of the periodic table. To obtain information about the stability or form of instability of the transuranic elements, these curves will be extrapolated. In performing the extrapolations, it is assumed that nuclear shell structure alone is responsible for all major irregularities in the nuclear masses.

II. BETA-STABILITY

As a result of the dependence on the even and odd character of the proton and neutron numbers, all nuclides may be divided into three general classes, those of odd A, those of even A and even Z, and those of even A and odd Z. For each class, the nuclear mass as a function of A and Z lies on a surface having the general shape of a valley. A curve connecting the values of Z_A, the coordinate of the bottom of the "odd A" valley, as a function of A is often called the line of beta-stability. All beta-stable odd-A nuclei lie within $\frac{1}{2}Z$ units of this curve. The region of stability of the even-even nuclides is bounded by limits approximately symmetrical with respect to Z_A. The curves representing Z_A and the region of beta-stability are here indicated on

* Part of a thesis submitted in partial fulfillment of the requirements for the degree of Master of Science in Physics in the Graduate School of Cornell University.

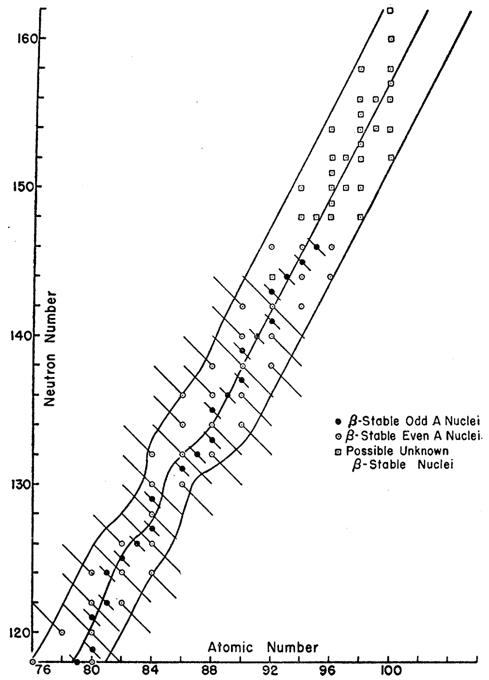

FIG. 1. The limits of beta-stability for the heavy nuclei are plotted on an N–Z plane.

an N–Z plane (Fig. 1), where 45-degree lines represent lines of constant A.

The method used to draw the curves, first suggested by Bohr and Wheeler[1] and later elaborated upon by Kohman,[2] is here carried out for $A>194$. Since the beta-stability of the short-lived alpha-emitters cannot be determined in all cases, the curves are somewhat rougher approximations in this region than they would be for lower A. However, one can get a good idea of the general trends of the curves, and even a sharp indication of the irregularity in the region of 126 neutrons and 82 protons.

In order to make predictions concerning the beta-stability of nuclei with $A \gtrsim 240$, the curves must be extrapolated in some fashion. For $A \lesssim 228$, if possible slight local irregularities are ignored, the curves can be represented by straight lines of the same slope. This slope is slightly greater than the average slope up to this point, because of the tendency towards an increasing N–Z ratio with increasing number of particles due to the Coulomb repulsion of the protons. These lines are continued onward toward higher A with a further slight increase in slope. Any irregularities which would occur in the shape of the curves cannot be foreseen; however, because another completed shell will not occur for a long way off, the irregularities should be of such a nature as not to affect the choice of likely beta-stable nuclides. Possible beta-stable nuclei for $A>242$ are indicated in Fig. 1 as squares and are chosen to stay within the limits of the extrapolated curves. Odd-odd nuclei are excluded, and except where there is considerable doubt as to which of two to pick, only one nucleus for each odd A is included.

III. ALPHA-STABILITY

Most nuclei with $A \gtrsim 140$ are energetically capable of emitting alpha-particles, but are stabilized by the nuclear potential barrier resulting from the high charge. As the decay energy of the alpha-particle becomes small, the lifetime of the alpha-radioactive nucleus becomes extremely large. There is no observed alpha-activity where the kinetic energy of the alpha-particle is $\lesssim 2$ Mev, since the maximum half-life which can be detected by present methods is $\sim 10^{21}$ years.

In order to make predictions concerning the alpha-stability of unknown isotopes of the transuranic elements, one can use existing data on alpha-energies and lifetimes, and examine the trends to make reasonable extrapolations.

There are several ways in which to correlate alpha-decay energies, some of which have already been used.[3] Alpha-energy contours are here drawn on an N–Z plane utilizing data from almost 100 alpha-decaying species.[3–5] Contours corresponding to 4, 5, 6, 7, 8, and 9-Mev alpha-particle energies are drawn (Fig. 2). The contours appear to be essentially parallel straight lines until $N=122$ and $Z=82$, where a very dramatic break begins to occur. Abnormally high alpha-energies are obtained for nuclei with $128 \leq N \leq 134$ and $Z=84$. The configuration of 126 neutrons and 82 protons is revealed as an abnormally stable structure compared with higher neutron and proton numbers.

For $N \gtrsim 140$, the contours again appear as parallel straight lines, but are displaced from the position they had before the break. In keeping with the assumption that completed shells are responsible for all major irregularities and that another completed shell will not occur for a long way off, the contours are extrapolated toward higher A by continuing the straight lines onward with unchanged slope.

The energy contours obtained with the use of the Weizäcker-Fermi semi-empirical mass formula[6] are also drawn in Fig. 2. These contours are a series of parallel straight lines, parallel to the experimental curves until the closed-shell interruption, after which ($N \gtrsim 140$) the two sets of curves again resume an almost parallel course. Thus, except for the sudden shell structure irregularities, the mass formula predicts the general behavior of the alpha-energy contours in the region where some data exists, and acts as a further justification for the extrapolation performed where there is no data.

[1] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[2] T. P. Kohman, Phys. Rev. **73**, 16 (1948).
[3] Perlman, Ghiorso, and Seaborg, Phys. Rev. **77**, 26 (1950).
[4] G. T. Seaborg and I. Perlman, Revs. Modern Phys. **20**, 585 (1948).
[5] W. H. Sullivan, *Trilinear Chart of Nuclear Species* (John Wiley and Sons, Inc., New York, 1949).
[6] M. O. Stern, Revs. Modern Phys. **21**, 316 (1949).

In order to predict the stability against alpha-decay of a given nucleus one must be able to determine the order of magnitude of its lifetime against alpha-emission. Thus, what is needed is a series of alpha-lifetime contours which can be extrapolated towards higher A.

If one plots the log of the half-life against alpha-decay *versus* the energy of the emitted alpha-particle, one obtains a family of curves, the parameter Z being held constant for each curve. Unfortunately, these curves are defined only for the even-even nuclei, all other nuclear species falling off the curves, and showing, in general, abnormally long lifetimes.[3]

Using these curves and the energy contours of Fig. 2, lifetime contours for the even-even nuclei are drawn on an N–Z plane in Fig. 3. Contours corresponding to half-lives of 10^{-1}, 10^2, 10^5, 10^7, and 10^9 years are drawn for $Z \geq 92$.

IV. SPONTANEOUS FISSION

The phenomenon of fission is much more difficult to deal with than either alpha- or beta-decay. There exists little experimental data and no theory which fits that experimental information which is known. To some extent, the phenomenon of fission permits a classical interpretation in terms of the liquid drop model of the nucleus.[1,7] The theory is inadequate in many respects. Fission is, in general, not into symmetric fragments, and the fission thresholds do not vary smoothly. It seems that the best one can do at present in order to make any predictions concerning the order of magnitude of fission lifetimes is to draw upon the general aspects of the theory in order to make qualitative predictions, and fit the numbers as well as one can to experimental lifetimes.

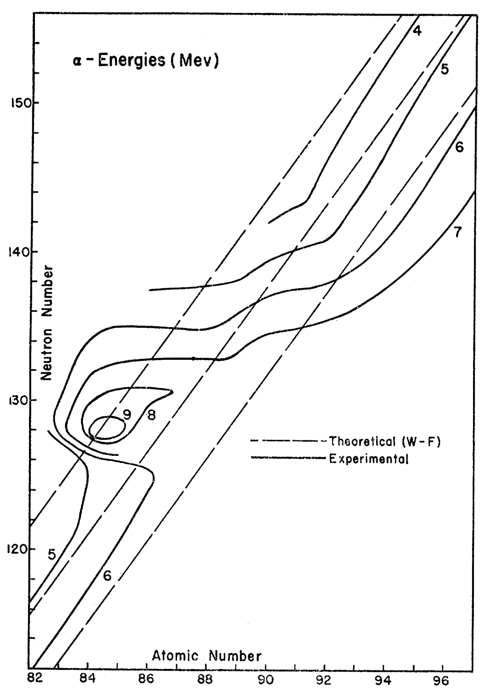

FIG. 2. Contours representing the total energies available to emitted alpha-particles are plotted on an N–Z plane. The heavy lines are based on experimental energies, while the dotted line contours are obtained by computing alpha-energies from the Weizsäcker-Fermi mass formula.

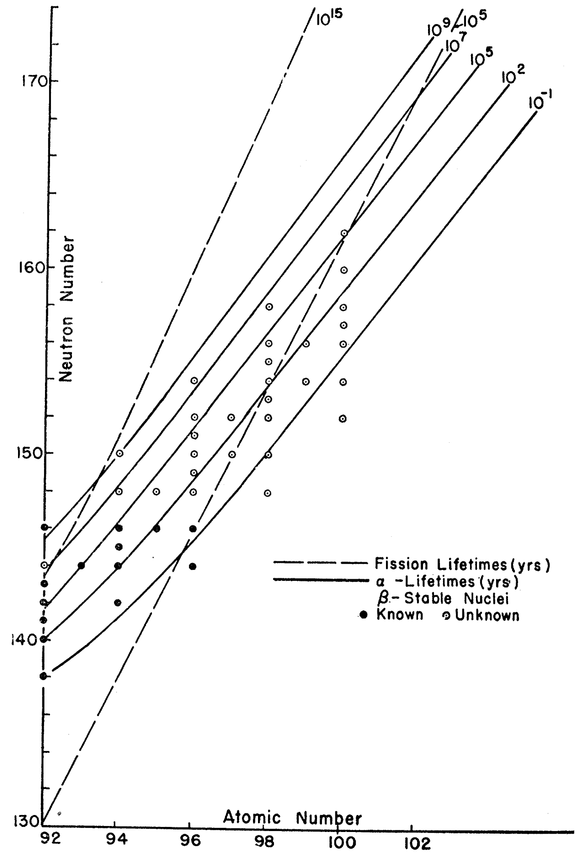

FIG. 3. Known and possible unknown beta-stable nuclei and contours of lifetimes against alpha-emission (defined only for the even-even nuclei) and spontaneous fission for the transuranic nuclei.

According to the theory, except for slight variations with A, contour lines of constant Z^2/A should be constant fission lifetime contours.

An estimated lower limit of the half-life of U^{235} ($Z^2/A=36$) for spontaneous fission is about 10^{15} years.[8] The spontaneous fission half-life of Cm^{242} ($Z^2/A=38.1$) is about 10^5 years.[9] Contour lines are drawn (Fig. 3) for $Z^2/A=36$ and $Z^2/A=38.1$ with the assigned lifetimes of 10^{15} and 10^5 years, respectively.

V. CONCLUSION

In Fig. 3, the known and possible unknown beta-stable nuclei are plotted as closed and open circles, and the alpha- and fission lifetimes are represented by

[7] S. Frankel and N. Metropolis, Phys. Rev. **72**, 914 (1947).

[8] W. E. Stephens, *Nuclear Fission and Atomic Energy* (The Science Press, Lancaster, Pennsylvania, 1948), p. 12.

[9] Hanna, Harvey, Moss, and Tunnicliffe, Phys. Rev. **81**, 466 (1951).

solid and broken lines, respectively. This diagram is used as the basis for the predictions made in Table I concerning the stability or form of instability of the unknown transuranic nuclei.

In considering possibly geologically stable nuclei, one must first stay within the confines of the band of beta-stability. It can be seen from Fig. 3, that for $Z>92$ the slopes of the various curves are such as to bring the beta-stable isotopes to lower and lower alpha- and fission lifetimes as Z increases. Thus, the list of stable isotopes is brought to an end long before the possible occurrence of another completed shell to alter the main features of the stability curves.

The natural periodic table on earth probably ends with uranium. Since spontaneous fission does not act as a possible competitor to alpha-decay for atomic numbers less than about 100, it is alpha-instability that causes the ending of the list of geologically stable isotopes.

There are two undiscovered transuranic isotopes, Pu^{244} and Cm^{250}, which might possibly exist in nature. They are both certainly stable against fission. The probable alpha-lifetimes are 10^9 years for Pu^{244}, and 10^{7-8} years for Cm^{250}. However, since they are both just on the border of the beta-stable band, their beta-stability is in doubt.

Spontaneous fission is not likely as the principal mode of decay for atomic numbers less than $Z=100$. However, as Z increases past $Z=100$, it becomes increasingly likely that many isotopes decay mainly by fission.

The alpha-stability of the heavy elements is very strongly affected by the phenomenon of shell structure, which results in completed shells with 82 or 126 particles. It is this phenomenon which is responsible, first, for the sudden appearance of a group of highly radioactive elements beginning with polonium ($Z=84$) and

TABLE I. The unknown beta-stable transuranic isotopes, their probable main mode of decay and log of estimated lifetimes (years) against this form of decay.

Z	A	Main decay form	Log lifetime
94	242	alpha	7
	244	beta, alpha	?, 9
95	243	alpha	4
96	244	alpha	1
	245	alpha	2
	246	alpha	3.5
	247	alpha	5
	248	alpha	6
	250	beta, alpha	?, 7.5
97	247	alpha	1
	249	alpha	3
98	246, 248 250–254 256	alpha	−3 to 6
99	253	alpha	0
	255	alpha	2
100	252, 254, 256 257, 258, 260 262	fission, alpha	−4 to 5

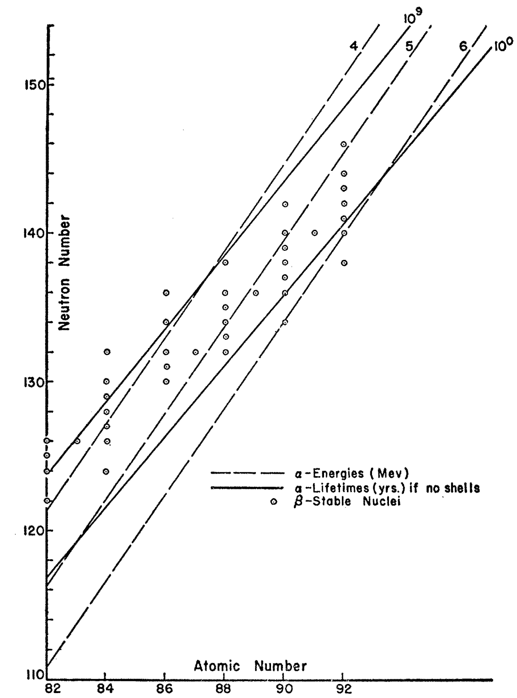

FIG. 4. Contours of energies of emitted alpha-particles and lifetimes against alpha-emission for the elements at the high end of the periodic table as they would be if there were no shell structure.

secondly, for the stability of thorium and uranium isotopes, since the alpha-energy contours are displaced after the interruption in such a fashion as to favor stability. Thus, while there are no stable nuclei for $84\leq Z\leq 89$, there are two stable isotopes of thorium and uranium (Th^{232} and U^{238}).

Curves representing probable alpha-energies and lifetimes for isotopes with $82\leq Z\leq 92$, if there had been no completed shells with 82 or 126 particles, are drawn in Fig. 4. In drawing the curves the straight line contours which exist for $Z\lesssim 83$ (Fig. 2) are extended onward with unchanged slope, parallel, therefore, to the Weizsäcker curves. This is done for energies of 4, 5, and 6 Mev, and lifetimes of 1 and 10^9 years. Beta-stable nuclei are indicated as open circles.

If there were no shells, alpha-activity would appear gradually with increasing atomic number. There would be geologically stable isotopes of polonium, radon, and possibly one for radium. However, for $Z>88$, there would be no such stable nuclei, the periodic table thus ending with either Rn or Ra instead of U. Moreover, the possibility of fission would be completely obscured by the short alpha-lifetimes of the heavy elements.

The author wishes to thank Professor Philip Morrison of Cornell University, who suggested the problem, for his continued assistance and encouragement.