A Theoretical Calculation of the Inelastic Scattering of 90-Mev Neutrons by Deuterons*

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The number and distribution of disintegration protons to be expected from the $H^2(n, p)$ reaction at 90-Mev incident neutron energy is calculated phenomenologically. No assumption about the character of nuclear forces is required, the impulse approximation allowing direct use of high energy $n-p$ and $p-p$ scattering data. Agreement with experiment is satisfactory, there being no evidence for a difference between $n-n$ and $p-p$ interactions.

INTRODUCTION

In a previous paper,[1] hereafter referred to as I, a method has been set up for treating high energy $n-d$ inelastic scattering. The method, called the impulse approximation, requires a knowledge of the $n-p$ and $n-n$ scattering amplitudes as well as the deuteron wave functions, continuum and discrete. It does not, however, require a knowledge of the nature of nuclear forces. The first experiment on the $H^2(n, p)$ reaction at high energy has now been carried out at Berkeley by Powell and associates,[2] using the 90-Mev neutron beam from stripped deuterons. Since information is also available on $n-p$ scattering at this same energy,[3] it is possible to test some of the predictions of the impulse approximation. Such a comparison of theory with the 90-Mev experiment is carried out in this paper as a sequel to I. In part I it was also pointed out that if one is confident of the accuracy of the impulse approximation, one might derive the magnitude of the $n-n$ amplitude from the $H^2(n, p)$ experiment. This derivation will be performed here.

It is worth explaining qualitatively why the inelastic $n-d$ scattering is a more useful application of the impulse approximation than the elastic, when both are theoretically tractable by the method. The difference lies in the relative importance of interference effects. Elastic scattering is, so to speak, completely coherent. It has its origin mainly in small momentum transfers and no observation can be made to tell whether the neutron or the proton in the initial deuteron was the struck particle. Consequently there is interference between $n-p$ and $n-n$ scattered amplitudes and a knowledge of the phases is necessary to make a prediction. Such a knowledge cannot be derived directly from two-body scattering experiments, and one would have to rely on a very shaky theory. Inelastic scattering, on the other hand, tends to result from large momentum transfers, and the struck particle can often be identified because it has the higher energy. Interference effects are then much smaller, as will be seen, and the phases not important.

Gluckstern and Bethe[4] have shown by a closure argument that if one knows the fundamental forces acting, then the evaluation of the total $n-d$ cross section (elastic plus inelastic) can be carried out rather accurately without explicitly using the continuum two-particle wave functions. This simplifies the calculation enormously, but it does not circumvent the necessity of knowing the phases. These authors also discuss in a qualitative way the inelastic part of the problem. We shall often have occasion to refer to their results, many of which are independent of the nature of nuclear forces.

The formulas given in I are not evaluated exactly in this paper because of the excessive amount of computation required. The exact calculation could be performed with the aid of a computing machine if the accuracy of a future experiment should warrant it. Here varying approximations have been made which will be individually discussed as the need for them arises.

Before plunging into details, a brief discussion will first be given of those features of the problem which are understandable without reference to the theory.

A QUALITATIVE DISCUSSION OF THE INELASTIC SCATTERING PROBLEM

In most experiments on the $H^2(n, p)$ reaction, the final proton will be detected because of its charge, whereas the two neutrons will slip away unnoticed. It is therefore reasonable to categorize the possible final states according to the energy and angle of the proton. Following the classical idea that the incoming neutron may hit either the neutron or the proton making up the deuteron, but not both, it is to be expected that there will be two fairly well defined proton groups, for a fixed incident neutron energy. If the proton is hit it will tend to carry away in a predominantly forward direction an energy of the order of magnitude of the incident energy. If the neutron is hit, the proton is likely to be left behind with only the kinetic energy of zero point motion which it happened to possess at the instant of impact. This is only a few Mev and the angular distribution of these protons will tend to be spherically symmetric. The neutron-neutron collision probability therefore is largely responsible for the slow protons ob-

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[1] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[2] W. M. Powell, Phys. Rev. (to be published).
[3] Hadley, Kelley, Leith, Segrè, Wiegand, and York, Phys. Rev. **75**, 351 (1949).
[4] R. L. Gluckstern and H. A. Bethe, Phys. Rev. **81**, 761 (1951).

served. One might think that neutron-proton collisions in which only a small amount of momentum is transferred might contribute to the slow proton yield. They can, but only occasionally, because the deuteron tends not to disintegrate unless either a big kick is given to one of its two particles or the spin state is changed from triplet to singlet (in which it is unstable). It will be shown that the latter is unlikely as a result of $n-p$ collisions.

The fast group of protons will be even less beset by ambiguities. If one considers only those with energy above 20 Mev, there is little need to worry that they may be the residue of an $n-n$ collision, since the deuteron contains such momenta with a very small probability. Off hand one might expect these fast protons to be distributed like the recoil protons from free neutron-proton scattering. The slightly surprising result of this paper is that at small angles the intensity is sharply reduced. The reason is that exchange collisions which project the proton forward with high energy leave the incident neutron with relatively little energy. It then may find itself in a region of phase space already occupied by the second neutron, a situation forbidden by the Pauli principle.

This exclusion effect, although it is easily observable, does not produce a major decrease in the over-all inelastic cross section, being restricted to a small solid angle. A much more serious decrease in the inelastic cross section results from the reluctance of the deuteron to disintegrate when only a small amount of momentum is transferred to it either by an $n-n$ or by an $n-p$ collision.

THE ACCURACY OF THE IMPULSE APPROXIMATION

As shown by Wick and Chew,[5] the order of magnitude of the error in the impulse approximation is given by $(a/R)(1/kR)$, where a is the two particle scattering amplitude, k is the relative neutron wave number and R the deuteron "radius."† The largest and most important amplitude in this problem belongs to the triplet neutron-proton system and can be estimated from experiment[3] to be $\sim\frac{1}{4}R$. The neutron wave number is $\sim\frac{1}{4}R^{-1}$, so the order of magnitude of the error is only 6 percent. We shall, however, make approximations in the evaluation of the impulse formulae which introduce somewhat larger errors than this.

THE IMPORTANCE OF INTERFERENCE EFFECTS

Gluckstern and Bethe[4] have calculated the contribution of interference terms to the total 90-Mev neutron-deuteron cross section, using Born approximation with various conventional static potentials to obtain the phases. The most reasonable of their potentials is the half ordinary-half exchange or Serber potential for both the $n-p$ and $n-n$ interactions. This choice fits fairly well the high energy neutron-proton scattering,[3] but fails completely with the proton-proton.[6] In any case, with this assumption, they find that 16 mb out of the total of 117 mb measured by McMillan, *et al.*[7] are due to interference between neutron-neutron and neutron-proton scattered amplitudes. The elastic part of the total cross section had previously been calculated by Chew[8] with nearly identical assumptions, and he found that out of 59 mb the interference contribution was 15 mb. Too close a comparison of these numerical results is unwarranted, but it empirically apparent that the great majority of the interference contribution is associated with the elastic scattering and only a small part with the inelastic. This is not at all surprising from the qualitative arguments given in the introduction. The explanation becomes quantitative if one observes that as a function of the momentum transferred in the collision the interference term in the elastic cross section is very nearly identical to the interference term in the total cross section. Ignoring the Pauli principle and the spin dependence of the scattering, and using the closure result of Gluckstern and Bethe,[4] one finds that the interference term in the total cross section is (in the notation of I) proportional to

$$2\,\mathrm{Re}\, r_{1p}^{*} r_{12} F(q) \tag{1}$$

where

$$F(q)=\int d\mathbf{r}e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_0^2(r), \tag{2}$$

the symbol $\mathbf{q}$ representing the momentum transfer, $\mathbf{k}_1-\mathbf{k}_0$. On the other hand, the interference term in the elastic part alone is proportional (with the same constant of proportionality) to

$$2\,\mathrm{Re}\, r_{1p}^{*} r_{12} S(q), \tag{3}$$

where $S(q)$ is the "sticking" factor defined in reference 8. It is given by

$$S(q)=F^2(q/2). \tag{4}$$

The functions $S(q)$ and $F(q)$ are numerically very similar, both having the maximum value unity at $q=0$ and falling off with an approximate width of 2α, if $\alpha=1/R$. For a well-known form of the deuteron wave function,[8,9]

$$\varphi_0(r)=\left[\frac{\alpha\beta(\beta+\alpha)}{2\pi(\beta-\alpha)^2}\right]^{\frac{1}{2}}\frac{(e^{-\alpha r}-e^{-\beta r})}{r},\quad (\beta=6\alpha), \tag{5}$$

the functions F and S are plotted in Fig. 1 as a function of $q/2\alpha$. The fact that for small q, S is actually somewhat larger than F seems to be a real effect; but within the accuracy of our method, we are justified in ignoring the difference between the two functions and

[5] G. C. Wick and G. F. Chew, to be published.

† By the "radius" is meant the average nucleon separation distance, 3.2×10^{-13} cm.

[6] Chamberlain, Segrè, and Wiegand, Phys. Rev. **83**, 923 (1951).

[7] Cook, McMillan, Peterson, and Sewell, Phys. Rev. **75**, 7 (1949).

[8] G. F. Chew, Phys. Rev. **74**, 809 (1948).

[9] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950).

therefore the interference effects in the inelastic part of the cross section.

This argument of course fails if by a spin change the deuteron is thrown into the singlet state, in which it cannot be bound. Statistical considerations give this possibility a weight of only $\frac{1}{6}$ and a more detailed consideration of the probable spin dependence of the $n-p$ interaction reveals that the chance of a final singlet state should be less than 10 percent. It therefore seems consistent, in view of experimental uncertainties,[2] to neglect all interference terms in calculating the inelastic part of the cross section. This practice will be followed here and obviates the necessity of knowing anything but the magnitudes of the $n-n$ and $n-p$ scattered amplitudes.

THE TOTAL YIELD OF FAST AND SLOW PROTONS

To calculate the total inelastic cross section, we may avail ourselves of the method of Gluckstern and Bethe.[4] These authors, by a closure argument, show that replacement of the actual final two-body wave functions by plane waves is approximately equivalent to calculating the sum of elastic and inelastic scattering. We shall therefore employ formula (21a) from I, which follows from the use of plane waves, and substitute into I-(15) and I-(16) to obtain the total cross section. The elastic part alone will then be calculated and the inelastic obtained by taking the difference. In making the subtraction it will be assumed that all interference terms (between $n-n$ and $n-p$ scattering) cancel out, the argument for this being given above. The reason for such a roundabout procedure is to avoid the two-body continuum functions, which are difficult to handle analytically.

In the notation of I, then, the total cross section is

$$\begin{aligned}\sigma_{\text{total}}=&\int\int d\mathbf{k}_p d\mathbf{k}_{12}\frac{2\pi}{\hbar v_0}\{[\tfrac{3}{4}|r_{1p}{}^t|^2+\tfrac{1}{4}|r_{1p}{}^s|^2]g_0{}^2(\mathbf{k}_2)\\&+[\tfrac{3}{4}|r_{2p}{}^t|^2+\tfrac{1}{4}|r_{2p}{}^s|^2]g_0{}^2(\mathbf{k}_1)-2\,\mathrm{Re}g_0(\mathbf{k}_1)g_0(\mathbf{k}_2)\\&\times[\tfrac{5}{8}(r_{1p}{}^t)^*(r_{2p}{}^t)+\tfrac{1}{8}(r_{1p}{}^s)^*(r_{2p}{}^s)-\tfrac{1}{8}(r_{1p}{}^t)^*(r_{2p}{}^s)\\&-\tfrac{1}{8}(r_{1p}{}^s)^*(r_{2p}{}^t)]+[\tfrac{3}{4}|r_{12}{}^t-r_{21}{}^t|^2\\&+\tfrac{1}{4}|r_{12}{}^s+r_{21}{}^s|^2]g_0{}^2(\mathbf{k}_p)+\text{interference terms}\}\\&\times\delta(E_1+E_2+E_p-E_0),\quad(6)\end{aligned}$$

with the integration to be extended over only one hemisphere of the $\mathbf{k}_{12}$ space.

If one writes down the formula for $n-p$ scattering at the same energy,

$$d\sigma_{np}/d\mathbf{k}_{1p}=\frac{2\pi}{\hbar v_0}\{\tfrac{3}{4}|r_{1p}{}^t|^2+\tfrac{1}{4}|r_{1p}{}^s|^2\}\delta(E_1+E_p-E_0)\quad(7)$$

and the formula for $n-n$ scattering,

$$d\sigma_{nn}/d\mathbf{k}_{12}=\frac{2\pi}{\hbar v_0}\{\tfrac{3}{4}|r_{12}{}^t-r_{21}{}^t|^2+\tfrac{1}{4}|r_{12}{}^s+r_{21}{}^s|^2\}\times\delta(E_1+E_2-E_0)\quad(8)$$

then one sees that except in the third term of (6), the same combination of $n-n$ and $n-p$ scattering amplitudes occur. This third term tends to inhibit the emission of fast protons forward and represents the effect of the exclusion principle as explained in the qualitative discussion above. Its effect on the total cross section will be shown later to be about 15 percent.

Formula (6) may be rewritten in an even more significant form if a change of variables is made for the term arising from $n-p$ collisions:

$$\begin{aligned}\sigma_{\text{total}}=&\frac{2\pi}{\hbar v_0}\int d\mathbf{k}_2 g_0{}^2(\mathbf{k}_2)\left(\frac{mk_{1p}}{2\hbar^2}\right)\int d\omega_{1p}[\tfrac{3}{4}|r_{1p}{}^t|^2\\&+\tfrac{1}{4}|r_{1p}{}^s|^2]+\frac{2\pi}{\hbar v_0}\int d\mathbf{k}_p g_0{}^2(\mathbf{k}_p)\left(\frac{mk_{12}}{4\hbar^2}\right)\int d\omega_{12}\\&\times[\tfrac{3}{4}|r_{12}{}^t-r_{21}{}^t|^2+\tfrac{1}{4}|r_{12}{}^s+r_{21}{}^s|^2]-\text{exclusion}\\&\text{principle term}+\text{interference terms.}\quad(9)\end{aligned}$$

Carrying out the analogous operations with (7) and (8), one gets

$$\sigma_{np}{}^{\text{total}}=\frac{2\pi}{\hbar v_0}\left(\frac{mk_0}{4\hbar^2}\right)\int d\omega_{1p}[\tfrac{3}{4}|r_{1p}{}^t|^2+\tfrac{1}{4}|r_{1p}{}^s|^2]\quad(10)$$

$$\sigma_{nn}{}^{\text{total}}=\frac{2\pi}{\hbar v_0}\left(\frac{mk_0}{8\hbar^2}\right)\int d\omega_{12}[\tfrac{3}{4}|r_{12}{}^t-r_{21}{}^t|^2+\tfrac{1}{4}|r_{12}{}^s+r_{21}{}^s|^2].\quad(11)$$

Because the energy-momentum conditions in the two-body problem differ from that in the three, the scattering amplitudes in (10) and (11) are, strictly speaking, not to be evaluated for the same magnitudes of initial and final momenta as in (9). In (10) and (11) the magnitude of both initial and final relative momenta is $k_0/2$. In (9) the initial and final relative momenta depend on the variable of integration, either k_p or k_2, but the integrands are strongly peaked around zero and at that point both initial and final relative momenta take on the value $k_0/2$. It is well within the accuracy of our method to neglect the variation of the amplitudes, giving them the value which they take on at the maximum of the integrand. If we do this and simultaneously replace k_{1p} and k_{12} by $k_0/2$, we arrive at the following very simple result, also found by previous workers[4,10] who used the Born approximation,

$$\sigma_{\text{total}}=(1-\epsilon)\sigma_{np}{}^{\text{total}}+\sigma_{nn}{}^{\text{total}}+I,\quad(12)$$

where $\epsilon\sigma_{np}{}^{\text{total}}$ is the exclusion principle correction and I represents interference terms.

The corresponding calculation of the elastic part of the cross section is made by using the deuteron wave function as both initial and final state in the overlap integrals and by calculating the density of final states for a two-body configuration rather than a three. Also

[10] F. De Hoffman, Phys. Rev. 76, 217 (1950).

the spin states must be chosen as in reference 8 rather than as in reference 1. Carrying out this program, one finds

$$\sigma_{\mathrm{el}}=\left(\frac{2\pi}{\hbar v_0}\right)\left(\frac{\pi m}{\hbar^2 k_0}\right)\int_0^{(16/9)k_0^2} dq^2 S(q)\left\{\frac{33}{48}|r_{1p}{}^t|^2\right.$$

$$+\tfrac{3}{16}|r_{1p}{}^s|^2+\tfrac{1}{8}\,\mathrm{Re}(r_{1p}{}^s)^*(r_{1p}{}^t)+\frac{33}{48}|r_{12}{}^t-r_{21}{}^t|^2$$

$$\left.+\tfrac{3}{16}|r_{12}{}^s+r_{21}{}^s|^2+\tfrac{1}{8}\,\mathrm{Re}(r_{12}{}^s+r_{21}{}^s)^*(r_{12}{}^t-r_{21}{}^t)\right\}$$

$$+\text{interference terms}-\text{exclusion principle correction}$$

$$+\text{pickup terms.} \quad (13)$$

The interference term we shall take to be equal to I in formula (12), as explained at the beginning of this section. The exclusion principle correction is considerably smaller here than in the total cross section, since the final momenta of the two neutrons are very rarely the same when one of them is bound in a deuteron and the other free. Exclusion principle terms will be ignored. The so-called pickup terms are discussed both in reference 8 and in a paper by Goldberger and Chew.[9] They contribute only a small fraction to the elastic cross section and are experimentally distinguishable, since they are important only for values of q greater than k_0, corresponding to deuteron angles less than 41° in the laboratory system. If we agree to separate these deuterons from the rest, thus reducing the experimental elastic and total cross sections by about 4 mb, the comparison with theory becomes more correct. Henceforth the term "elastic" scattering will refer only to deuterons recoiling at angles greater than 41°. The upper limit of the integral (13) is then to be reduced to k_0^2.

The $n-p$ and $n-n$ cross sections per unit interval of q^2 are

$$d\sigma_{np}/dq^2=\left(\frac{2\pi}{\hbar v_0}\right)\left(\frac{\pi m}{\hbar^2 k_0}\right)\left[\tfrac{3}{4}|r_{1p}{}^t|^2+\tfrac{1}{4}|r_{1p}{}^s|^2\right] \quad (14)$$

$$d\sigma_{nn}/dq^2=\left(\frac{2\pi}{\hbar v_0}\right)\left(\frac{\pi m}{\hbar^2 k_0}\right)\left[\tfrac{3}{4}|r_{12}{}^t-r_{21}{}^t|^2+\tfrac{1}{4}|r_{12}{}^s+r_{21}{}^s|^2\right], \quad (15)$$

so that the combination of scattering amplitudes which appears is not quite the same as in formula (13). However, the ambiguity is not great. It is almost certain that for $n-p$ scattering the triplet dominates; so that whatever the relative phases of singlet and triplet amplitudes may be, the term $(33/48)|r_{1p}{}^t|^2$ in (13) will be by far the largest. Correspondingly in (14), the term $\frac{3}{4}|r_{1p}{}^t|^2$ is most important, so in line with our previous approximations the difference between the $n-p$ terms in (13) and in (14) can be ignored. (This procedure amounts to neglecting the possibility of a

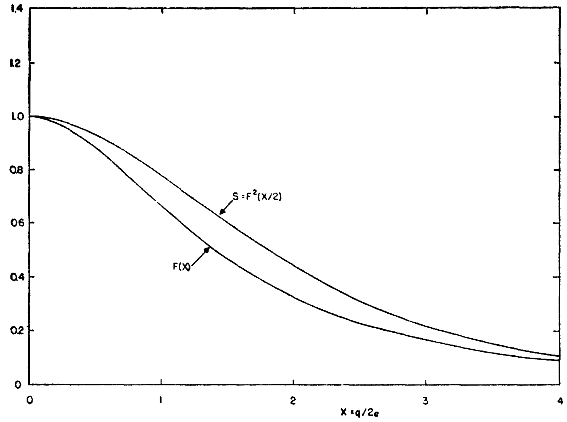

FIG. 1. The "sticking" factor S compared to the interference factor F, as a function of the momentum transfer, q.

final singlet deuteron state as a result of an $n-p$ collision. It would be exact if the forces were spin independent. The maximum reasonable dependence can be estimated by setting $r_{1p}{}^s=0$. In this case one would get a final singlet state one-twelfth of the time.)

For the $n-n$ terms we are on less sure ground. Even assuming that high energy $n-n$ and $p-p$ forces are equal, one still is uncertain about the relative importance of singlet and triplet scattering and it is possible that the cross term in (13) (between singlet and triplet $n-n$ amplitudes) might make a difference. In the frame work of our calculation, however, which assumes independent conservation of spin and orbital angular momentum, there is only one way to represent the observed isotropic nature of high energy $p-p$ scattering[6,11] and that is by a singlet interaction alone. If this is done the $n-n$ terms appearing in (13) will give three-fourths the contribution of those in (15), since the triplet scattering disappears, as does the cross term.

To make a preliminary evaluation of the integral in (13) we shall adopt what at first sight appears a rash procedure; that is, to treat the contents of the bracket as a constant. This leads to

$$\sigma_{\mathrm{el}}=\left[\sigma_{np}{}^{\mathrm{total}}+(\tfrac{3}{2})\sigma_{nn}{}^{\mathrm{total}}\right]\bar{S}+I, \quad (16)$$

where $\bar{S}$ is the average of the sticking factor over the interval in q^2 from 0 to k_0^2. From Fig. 1, $\bar{S}$ for 90 Mev can be computed to be 0.28 so that

$$\sigma_{\mathrm{el}}=0.28\sigma_{np}{}^{\mathrm{total}}+0.42\sigma_{nn}{}^{\mathrm{total}}+I. \quad (17)$$

If $n-n$ scattering is indeed equal to $p-p$, then the above seems as sensible a procedure as any at the present time to obtain the coefficient of $\sigma_{nn}{}^{\mathrm{total}}$, since high energy $p-p$ scattering is not only isotropic but also energy independent.[6,11] On the other hand, $n-p$ scattering is well known to have a rather strong energy and angular dependence.[3] Representing this dependence as a function of q^2, however, one finds $\bar{S}=0.29$, so the effect is very small. A final check can be obtained from

[11] R. Birge, Phys. Rev. 80, 490 (1950).

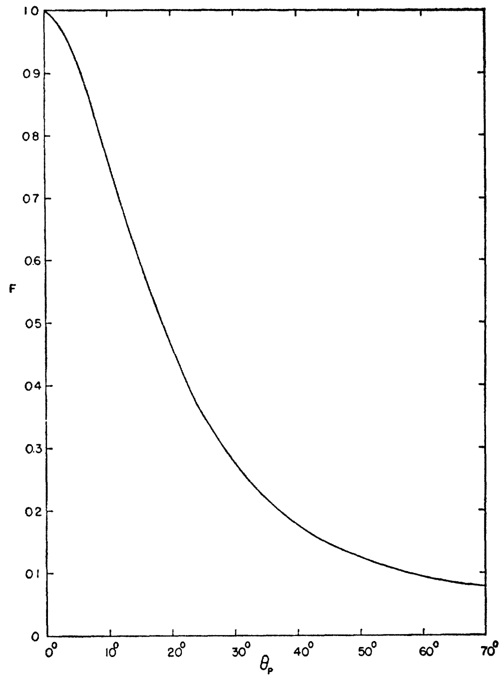

FIG. 2. The function $F(k_0 \sin\theta_p)$, which determines the reduction in the fast proton cross section owing to the exclusion principle.

the Born approximation calculation with the Serber force,[8] which also very closely gives this same result. The reason for the validity of the procedure which neglects the variation of the cross section is that most of the $n-p$ cross section is relatively isotropic, even at 90 Mev, the large variations occurring over only a small interval of q^2.

By taking the difference of formulas (12) and (17) the inelastic cross section is obtained

$$\sigma_{\text{in}} = (0.72-\epsilon)\sigma_{np}{}^{\text{total}} + 0.58\sigma_{nn}{}^{\text{total}}. \tag{18}$$

To the extent to which a group of fast protons can be experimentally distinguished from a slow one, this result can be further broken down. For if

$$\sigma_{\text{in}} = \sigma_{\text{fast}} + \sigma_{\text{slow}}, \tag{19}$$

then

$$\sigma_{\text{fast}} = (0.72-\epsilon)\sigma_{np}{}^{\text{total}}, \tag{20}$$

and

$$\sigma_{\text{slow}} = 0.58\sigma_{nn}{}^{\text{total}}, \tag{21}$$

according to the argument given in the introduction.

Our result for the total inelastic scattering is ambiguous only as to the magnitude of ϵ, the exclusion principle correction. If it is possible to distinguish clearly between a group of fast and a group of slow protons, the size of the slow group is connected directly by (21) to the neutron-neutron cross section. For example, Powell[2] gives 20 mb as the slow proton cross section, using 10 Mev as the dividing energy between groups. This implies a neutron-neutron cross section equal to 20 mb/0.58=35 mb±10 mb, where the uncertainty reflects theoretical as well as experimental difficulties. Such a figure is well within the limits required by equality of $n-n$ and $p-p$ interactions.[6,11]

We may at this time also compare theory with experiment for the fast group of protons. Powell gives 51 mb as the fast proton cross section, while the total neutron-proton cross section is 83 mb.[7] In the next section the exclusion principle correction ϵ will be estimated to be 0.15. Formula (20) then predicts that $\sigma_{\text{fast}} = (0.72-0.15)\times 83$ mb $=47$ mb. Agreement with experiment for both slow and fast integrated cross sections is therefore satisfactory.

If one is skeptical of the possibility of separating the fast and slow groups in this particular experiment, one may start from Powell's total inelastic cross section of 71 mb and equate this to (18). Using the known value of $\sigma_{np}{}^{\text{total}}$, one finds $\sigma_{nn}{}^{\text{total}} = [(71-47)/0.58]$ mb $=41$ mb.

Powell's measurements, however, allow a more detailed test of the theory, since they contain information about the angular distributions of the protons. Let us consider next what the impulse approximation has to say about this aspect of the experiment.

THE ANGULAR DISTRIBUTION OF THE FAST PROTONS

If one is interested only in those final states for which the proton has a high energy then the pair of particles which most strongly interact after the collision is the pair of neutrons. Therefore in formula (I-21) one must choose the upper alternative, that is, the one containing the overlap integrals $I_{21}{}^t$ and $I_{21}{}^s$. We need keep only the terms generated by $n-p$ scattering, and substituting into (I-15) and (I-16) one finds the following formula for the cross section for emission of a fast proton into $d\mathbf{k}_p$.

$$\begin{aligned} d\sigma_f/d\mathbf{k}_p = (2\pi/\hbar v_0)\int d\mathbf{k}_{12}\{ &|r_{1p}{}^t|^2[(33/48)|I_{21}{}^t|^2 \\ &+\tfrac{1}{16}|I_{21}{}^s|^2] + |r_{1p}{}^s|^2[\tfrac{3}{16}|I_{21}{}^t|^2+\tfrac{1}{16}|I_{21}{}^s|^2] \\ &-\mathrm{Re}[(r_{1p}{}^t)^*(r_{2p}{}^t)\{(33/48)I_{21}{}^{s*}I_{12}{}^t-\tfrac{1}{16}I_{21}{}^{s*}I_{12}{}^s\} \\ &+(r_{1p}{}^s)^*(r_{2p}{}^s)(\tfrac{3}{16}I_{21}{}^{t*}I_{12}{}^t-\tfrac{1}{16}I_{21}{}^{s*}I_{12}{}^s) \\ &+\tfrac{1}{16}(r_{1p}{}^{t*}r_{2p}{}^s+r_{1p}{}^{s*}r_{2p}{}^t)(I_{21}{}^{s*}I_{12}{}^s+I_{21}{}^{t*}I_{12}{}^t)] \\ &+\tfrac{1}{8}\,\mathrm{Re}(r_{1p}{}^t)^*(r_{1p}{}^s)[|I_{21}{}^s|^2-|I_{21}{}^t|^2]\} \\ &\times\delta(E_1+E_2+E_p-E_0). \end{aligned} \tag{22}$$

The triplet interaction between the neutrons is certainly sufficiently weak that one can replace $I_{21}{}^t$ by $g_0(k_2)$ but the singlet interaction in the s state is by no means weak. Indeed the virtual resonance, as pointed out by Gluckstern and Bethe,[4] will reflect itself by a sharp peak in the proton spectrum near the upper limit. At a much lower energy, with a clean spectrum of incident neutrons, such a peak ought to be detectable and will be of interest as a check on the neutron-neutron scattering length. In the Berkeley experiment, however, the neutron beam has an energy width of about 30 Mev, so that such details are entirely obscured. The shape of the proton spectrum is largely a reflection of the incident neutron spectrum.

If one does not attempt to calculate the detailed spectrum but only the total fast proton intensity at a fixed laboratory angle, then a rather simple result can be obtained, again by the use of closure. One finds for the cross section per unit solid angle in the laboratory system

$$\sigma_f(\theta_p)=\sigma_{np}(\theta_p)-F(k_0\sin\theta_p)\sigma_{\mathrm{ex}}(\theta_p) \tag{23}$$

where

$$\sigma_{np}(\theta_p)=\frac{2\pi}{\hbar v_0}\left(\frac{mk_0}{4\hbar^2}\right)\left[\tfrac{3}{4}|r_{1p}{}^t|^2+\tfrac{1}{4}|r_{1p}{}^s|^2\right]4\cos\theta_p$$

$$\sigma_{\mathrm{ex}}(\theta_p)=\frac{2\pi}{\hbar v_0}\left(\frac{mk_0}{4\hbar^2}\right)\left[\tfrac{5}{8}|r_{1p}{}^t|^2+\tfrac{1}{8}|r_{1p}{}^s|^2-\tfrac{1}{4}\,\mathrm{Re}\,r_{1p}{}^{t*}r_{1p}{}^s\right]4\cos\theta_p. \tag{24}$$

In obtaining (23) the same sort of approximations were made as in obtaining (12). These approximations are harder to justify for the second term of (23) than for the first because the peak of the integrand in (22) is less sharp for the cross terms than for the direct. However, the second term represents only a relatively small correction to the first, and in any case we shall have difficulty in knowing the exact value of $\sigma_{\mathrm{ex}}(\theta_p)$. This quantity is the cross section for the incident neutron to end up with its spin parallel to that of the other neutron. If that happens the process tends to be forbidden by the Pauli principle, the amount of "forbiddenness" depending on the momentum retained by the incident neutron, which is roughly $k_0\sin\theta_p$.

From (24) it is seen that σ_{ex} is certainly smaller than σ_{np}, but the ratio of the two is sensitive to the size and phase of $r_{1p}{}^s$. For example, if $r_{1p}{}^s=r_{1p}{}^t$ then $\sigma_{\mathrm{ex}}=\frac{1}{2}\sigma_{np}$. On the other hand if $r_{1p}{}^s=0$, then $\sigma_{\mathrm{ex}}=\frac{5}{6}\sigma_{np}$. These are probably the upper and lower limits. Let us assume that $\sigma_{\mathrm{ex}}(\theta_p)=K\sigma_{np}(\theta_p)$, where K is a number which is independent of θ_p and which lies between $\frac{1}{2}$ and $\frac{5}{6}$, so that

$$\sigma_f(\theta_p)=\left[1-KF(k_0\sin\theta_p)\right]\sigma_{np}(\theta_p). \tag{25}$$

The function, $F(k_0\sin\theta_p)$ is plotted in Fig. 2 for a 90-Mev incident neutron energy and in Fig. 3 the fast proton cross section according to (25) with $K=\frac{1}{2}$, $\frac{5}{6}$ is plotted and compared with experiment. The free neutron-proton cross section is also shown ($K=0$). Formula (25) must not be believed for angles greater than 60°, beyond which the proton moves so slowly that one of the neutrons has a good chance to capture it, or at least perturb it strongly. Agreement with experiment within this range is certainly satisfactory.

The value of ϵ, needed for formula (20) may be estimated by integrating the exclusion correction term, $KF(k_0\sin\theta_p)\sigma_{np}(\theta_p)$, over all angles. The result for $K=0.6$, the value which seems indicated by the forward part of the angular distribution, is $\epsilon=0.15$. Most of this (80 percent) is due to angles less than 45°.

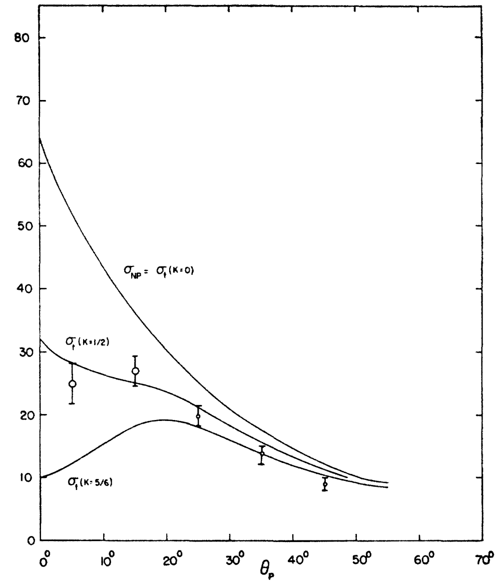

FIG. 3. The theoretical fast proton angular distribution for several different values of K, compared with experiment. (See reference 2.)

If one is optimistic about both theory and experiment, one may say that neutron-deuteron scattering provides a new piece of information about the neutron-proton interaction which cannot be obtained from a free neutron-proton scattering experiment. The new number is K, which as we have seen depends on the ratio of singlet and triplet amplitudes. The theory at 90 Mev is too rough to make the number obtained here of real use in testing theories, but at a higher energy, say 300–400 Mev, this will no longer be the case.

THE ANGULAR DISTRIBUTION OF THE SLOW PROTONS

If the final state of the proton is a low energy one, the most strongly interacting pair of particles after the collision will in almost every case include the proton. Kinematically it is possible for the two neutrons to each have high energy and at the same time a low relative energy with respect to each other. Such a situation corresponds to a pickup process with formation of an unstable di-neutron. It will certainly occur only a very small fraction of the time, since the neutron-proton pickup, which is much more probable, amounts (experimentally) to only 4 mb. Neutron-neutron pickup will be ignored here.

It is necessary now to correct an error in part I, namely the lower alternative of (I-21), which contains terms proportional to I_{p1} and I_{p2} and which was said to be appropriate to the case now being considered.

The correct procedure is to use the same formula as for elastic scattering, except that the final state is in the continuum and the singlet possibility must be considered. Thus while the correct antisymmetrized quartet amplitude is still (keeping only terms generated by neutron-neutron scattering)

$$(r_{12}{}^t - r_{21}{}^t) I_{p1}{}^t, \tag{26}$$

the doublet amplitude must be decomposed into that part which leads to a final neutron-proton triplet state, namely

$$\tfrac{1}{4}(r_{12}{}^t - r_{21}{}^t) I_{p1}{}^t + \tfrac{3}{4}(r_{12}{}^s + r_{21}{}^s) I_{p1}{}^t \tag{27}$$

and that part which leads to a final singlet state

$$-(\sqrt{3}/4)(r_{12}{}^t - r_{21}{}^t) I_{p1}{}^s + (\sqrt{3}/4)(r_{12}{}^s + r_{21}{}^s) I_{p1}{}^s. \tag{28}$$

Setting, as before, $r_{12}{}^t = r_{21}{}^t = 0$ and $r_{12}{}^s = r_{21}{}^s = r_{nn}$, we then arrive at a formula for the slow proton distribution:

$$\frac{d\sigma^s}{d\mathbf{k}_p} = \frac{2\pi}{\hbar v_0} \int d\mathbf{k}_{12} \big[\tfrac{3}{4} |I_{p1}{}^t|^2 + \tfrac{1}{4} |I_{p1}{}^s|^2\big] |r_{nn}|^2 \delta(E_1 + E_2 + E_p - E_0), \tag{29}$$

where the integral over $d\mathbf{k}_{12}$ is to be restricted to the region $k_{1p} < k_{2p}$. Closure cannot be applied to this case and while the evaluation of (29) as it stands is a well-defined task, it is very tedious, requiring a double numerical integration for each value of $\mathbf{k}_p$. Thus to find the total number of slow protons at a fixed laboratory angle involves a triple numerical integral! The data now available do not warrant this much labor, and so far only rough approximations to (29) have been attempted. Reference to part I, Eq. (20) shows that the overlap integrals, $I_{p1}{}^t$ and $I_{p1}{}^s$, are functions of the variables, $\frac{1}{2}(\mathbf{k}_p - \mathbf{k}_1)$ and $\frac{1}{2}(\mathbf{k}_p + \mathbf{k}_1)$. Actually the dependence on the first variable is much more sensitive than on the second, and one possible approximation, described by Gluckstern and Bethe,[4] consists of neglecting the dependence on $\frac{1}{2}(\mathbf{k}_p + \mathbf{k}_1)$, giving the integrals the value they take when this vector vanishes. Because of the orthogonality of the continuum triplet $n-p$ states to the ground state, however, such an approximation neglects completely the contribution of the final triplet states. ($I_{p1}{}^t = 0$ when $\frac{1}{2}(\mathbf{k}_p + \mathbf{k}_1) = 0$.) Actually, from the closure results, we know that the triplet states contribute somewhat more than do the singlet to the total number of slow protons; so this approximation is evidently poor.

A better result can be obtained by keeping terms linear in $\frac{1}{2}(\mathbf{k}_p + \mathbf{k}_1)$, although an honest consideration of the problem shows this procedure still to be quantitatively inadequate, except for protons going backward. (From kinematic considerations, backward protons on the average involve small values of $\frac{1}{2}(\mathbf{k}_p + \mathbf{k}_1)$ and large values of $\frac{1}{2}(\mathbf{k}_p - \mathbf{k}_1)$, while the reverse is true for forward protons. This follows from the fact that the neutron, as the struck particle, generally tends to go forward.) The backward protons corresponding to a final triplet state turn out according to our rough method to be almost isotropically distributed, but the intensity increases near 90° and probably has a maximum somewhere between 45° and 90°, where it is easiest kinematically for $\frac{1}{2}(\mathbf{k}_p - \mathbf{k}_1)$ to be small. The exact position and shape of the maximum has not been determined, the approximation not being valid in this region. The protons corresponding to a singlet state are even more concentrated between 45° and 90°, as already shown by Gluckstern and Bethe,[4] so the over-all effect remains that of a maximum in this region with an isotropic backward tail. No statement can be made about the slow protons between 0° and 45° until a method of handling large values of $\frac{1}{2}(\mathbf{k}_p + \mathbf{k}_1)$ has been developed.

The experimental determination of the "slow" proton angular distribution in the forward half plane is difficult because the "fast" protons are so much more numerous there. The poor definition of the incident neutron energy thus causes trouble. In the backward half-plane, to which only $n-n$ collisions can contribute appreciably, Powell[2] finds an approximately isotropic proton distribution, in agreement with our qualitative guess. There can be no doubt from the data that the intensity of slow protons rises sharply in the forward half-plane, but the details of this part of the reaction will remain obscure until a high energy neutron beam of well defined energy is available.

SUMMARY

It has been shown explicitly that by the impulse approximation one may calculate high energy inelastic neutron-deuteron scattering with rather little ambiguity if one knows the nucleon-nucleon cross sections at the same energy. The total number of disintegration protons is particularly easy to obtain because closure may be applied. Also easily found by the same method is the angular distribution of fast forward protons. This differs from free neutron-proton exchange scattering because of the action of the Pauli principle; and the difference might conceivably be used as a measure of the spin dependence of the $n-p$ interaction at high energy. In addition, a formula has been given for the angular distribution of slow protons. This formula is discussed qualitatively but not evaluated. Comparison of the various results with the experiment of Powell at 90-Mev neutron energy shows no serious disagreement, so that one may tentatively conclude the theory to be adequate. The value of the $n-n$ cross section derived here from the experiment is consistent with the directly measured $p-p$ cross section.

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