Polarization of Vector Mesons Produced in Nucleon-Nucleon Collisions

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The polarization of vector mesons produced in free nucleon-nucleon collisions near the threshold is studied. The calculation is made under the Born approximation, assuming a phenomenological nuclear potential obtained from $n-p$ scattering data for the interaction between nucleons. The vector meson produced is shown to be completely polarized (to the order of m/M) parallel to the direction of motion of the incoming nucleon for vector coupling between meson and nucleon, and perpendicular to this direction for tensor coupling; similar to the polarization produced by the interaction between a nucleon and a nuclear force field, as studied by Wentzel. In particular, the same effect is shown to exist for the reactions $p+p\rightarrow p+n+\pi^+$ and $n+n\rightarrow p+n+\pi^-$ when the final proton and neutron fall into ^3s or ^1s deuteron states. The angular distribution of the out-going nucleons in the baricentric system for the inverse reactions can be employed as a means for detecting the polarization of the mesons. Cross sections for these reactions are also estimated.

I. INTRODUCTION

DURING the last year, experiments on artificial mesons began to throw light on the spin and parity of the π-mesons (pions), independent of the Yukawa theory of nuclear forces. For instance, the observed two-photon annihilation of the neutral pion[1] excludes the vector character of this particle. Furthermore, if it is true that the transition

$$\pi^-+\text{deuteron}\rightarrow 2\text{ neutrons}$$

takes place for a π^- captured in a K-orbit, the charged pion cannot be scalar, because otherwise the conservation of angular momentum or parity would be violated. Here, however, some doubts may be raised as to whether the experimental situation has been correctly interpreted. Although rapid progress can be foreseen, the question is by no means settled to-day. In particular, the spin one for the charged pions cannot be ruled out.

This paper attempts to describe a basic feature exhibited by a vector meson because of its spin, which may be subjected to experimental detection. It was shown by Wentzel[2] that a vector meson is polarized when produced by the interaction between a nucleon and a nuclear force field near the threshold, and the direction of polarization depends on the type of coupling between the vector meson and the nucleon, being in the direction of motion of the incoming nucleon if the coupling is of the vector type and perpendicular to this direction if the coupling is of the tensor type. This situation is experimentally approximated when a nucleon, say a proton, is shot at a heavy nucleus. In this paper the same polarization effect is sought in the case of a vector meson produced by collisions of two free nucleons. Experimentally, this is equivalent to shooting protons or neutrons at a hydrogen target. The calculation here, also, is made near the threshold.

Because of the dubious nature of present-day meson theory of nuclear forces, a phenomenological nuclear potential is assumed for the interaction between nucleons. This reduces the problem to a second order perturbation calculation, instead of a third-order perturbation calculation with the nucleonic interaction caused by the exchange of virtual mesons. The nuclear potential is assumed to be central and has the most general form of a static interaction

$$V(\mathbf{r})=v(r)[c_1+c_2\boldsymbol{\sigma}_{(1)}\cdot\boldsymbol{\sigma}_{(2)}+c_3\boldsymbol{\tau}_{(1)}\cdot\boldsymbol{\tau}_{(2)}+c_4\boldsymbol{\sigma}_{(1)}\cdot\boldsymbol{\sigma}_{(2)}\boldsymbol{\tau}_{(1)}\cdot\boldsymbol{\tau}_{(2)}],\quad(1)$$

the tensor forces being neglected because they are of minor importance. The constants c_1, c_2, c_3, c_4, and $v(r)$ may eventually be obtained from scattering experiments, especially the $n-p$ scattering at 90 Mev.[3–5] It will be seen in the discussion later that the Born approximation for the incoming nucleons is not too bad, and a perturbation calculation, unsatisfactory and unsophisticated as it is, seems to be the only manageable way of solution.

In accordance with the charge-independent nuclear potential (1), we have used Kemmer's[6] charge-symmetric theory of the meson field to calculate the production of charged and neutral vector mesons. For neutral mesons, the vector theory is actually in contradiction with the observed two-photon annihilation of π^0. With regard to pions, therefore, the neutral particle production should be disregarded. As a reason for not dropping these terms entirely, it may be mentioned that they may possibly be applied to heavier meson types whose existence has been ascertained lately.[7–10]

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[10] P. M. S. Blackett, *V-Particles and Cosmic Rays*, paper presented at the International Conference on Elementary Particles (Bombay, December, 1950).

Regardless of the applicability to particular meson types, the results of this paper will prove that mesons of spin one, when produced in nucleon-nucleon collisions near the threshold, will in general be strongly polarized.

Recent experiments on pion production in nucleon-nucleon collisions revealed also the significance of the case of deuteron formation[11–13] when the final nucleons are a proton and a neutron at low energies. In this case the discussion of the meson polarization is sketched in the nonrelativistic approximation. The inverse processes to deuteron formation, namely, the absorption of a charged vector meson by a deuteron to form two protons or two neutrons, are also sketchily studied.

II. TRANSITION MATRIX ELEMENT NEAR THE THRESHOLD

(A) Perturbation Calculation under Born Approximation

The two relativistically invariant interaction hamiltonians of the first order between a vector meson and a nucleon[14] are

$$\text{(vector)}\quad H_{(v)}'=g_v(\Omega m)^{-\frac{1}{2}}\left(\frac{m}{\omega_k}\right)^{\frac{1}{2}}\mathbf{a}_k^*\cdot\left\{\boldsymbol{\alpha}-\frac{\mathbf{k}}{m}+\left(\frac{\omega_k}{m}-1\right)\frac{(\boldsymbol{\alpha}\cdot\mathbf{k})\mathbf{k}}{k^2}\right\}\tau^{(s)}e^{-i\mathbf{k}\cdot\mathbf{r}}+\text{c.c.},\tag{2}$$

$$\text{(tensor)}\quad H_{(t)}'=g_t(\Omega m)^{-\frac{1}{2}}\left(\frac{\omega_k}{m}\right)^{\frac{1}{2}}\mathbf{a}_k^*\cdot\left\{\boldsymbol{\alpha}-\left(\frac{\mathbf{k}}{\omega_k}\times\boldsymbol{\sigma}\right)+\left(\frac{m}{\omega_k}-1\right)\frac{(\boldsymbol{\alpha}\cdot\mathbf{k})\mathbf{k}}{k^2}\right\}\beta i\tau^{(s)}e^{-i\mathbf{k}\cdot\mathbf{r}}+\text{c.c.},\tag{3}$$

where g_v and g_t are the coupling constants having the dimension of a charge and are assumed to be small in the weak coupling theory; Ω is the normalization volume; m and $\mathbf{k}$ are the rest-mass and the momentum of the meson, and $\omega_k=(m^2+k^2)^{\frac{1}{2}}$ (in the rationalized system of units $\hbar=c=1$); $\mathbf{a}_k^*$ and $\mathbf{a}_k$ are, respectively, the generation and the annihilation operators of a meson of momentum $\mathbf{k}$; $\boldsymbol{\alpha}$ and β are the usual Dirac spin operators of the nucleon; $\boldsymbol{\sigma}$ is the Pauli spin operator; $\tau^{(s)}$ indicates the appropriate isotopic spin operator which in the charge symmetric theory may be conveniently written as

$$\tau^{(s)}=(s^2/\sqrt{2})\tau_1-(is/\sqrt{2})\tau_2+(1-s^2)\tau_3.\tag{4}$$

This expression for $\tau^{(s)}$ can easily be checked by setting $s=+1$ (positive meson), $s=0$ (neutral meson), and $s=-1$ (negative meson). In the barsy (baricentric system), at the threshold $\mathbf{k}=0$, we have for the hamiltonians

$$H_{(v)}'=g_v(\Omega m)^{-\frac{1}{2}}\mathbf{a}_0^*\cdot\boldsymbol{\alpha}\tau^{(s)}+\text{c.c.}\tag{5}$$

$$H_{(t)}'=g_t(\Omega m)^{-\frac{1}{2}}\mathbf{a}_0^*\cdot\boldsymbol{\alpha}\beta i\tau^{(s)}+\text{c.c.}\tag{6}$$

The nuclear potential (1) may be written as

$$V(\mathbf{r})=v(r)c^\alpha J_{ik(1)}{}^\alpha J_{ik(2)}{}^\alpha,\tag{7}$$

where $J_{ik}{}^{(1)}=\delta_{ik}/\sqrt{3}$, $J_{ik}{}^{(2)}=\sigma_i\delta_{ik}$, $J_{ik}{}^{(3)}=\tau_i\delta_{ik}$, $J_{ik}{}^{(4)}=\sigma_i\tau_k$ (not summed). The transition scheme in the barsy at the threshold is

$$\begin{array}{ccccc}
 & \nearrow & (a)\ \begin{pmatrix}0\\ \lambda'\\ \nu'\end{pmatrix}\begin{pmatrix}0\\ \lambda_2\\ \nu_2\end{pmatrix} & \searrow & \\
 & \rightarrow & (b)\ \begin{pmatrix}0\\ \lambda_1\\ \nu_1\end{pmatrix}\begin{pmatrix}0\\ \lambda'\\ \nu'\end{pmatrix} & \rightarrow & \\
(I)\ \begin{pmatrix}\mathbf{p}_0\\ \lambda_{10}\\ \nu_{10}\end{pmatrix}\begin{pmatrix}-\mathbf{p}_0\\ \lambda_{20}\\ \nu_{20}\end{pmatrix} & & & & (F)\ \begin{pmatrix}0\\ \lambda_1\\ \nu_1\end{pmatrix}\begin{pmatrix}0\\ \lambda_2\\ \nu_2\end{pmatrix}\begin{pmatrix}0\\ s\end{pmatrix},\\
 & \rightarrow & (c)\ \begin{pmatrix}\mathbf{p}_0\\ \lambda'\\ \nu'\end{pmatrix}\begin{pmatrix}-\mathbf{p}_0\\ \lambda_{20}\\ \nu_{20}\end{pmatrix}\begin{pmatrix}0\\ s\end{pmatrix} & \rightarrow & \\
 & \searrow & (d)\ \begin{pmatrix}\mathbf{p}_0\\ \lambda_{10}\\ \nu_{10}\end{pmatrix}\begin{pmatrix}-\mathbf{p}_0\\ \lambda'\\ \nu'\end{pmatrix}\begin{pmatrix}0\\ s\end{pmatrix} & \nearrow &
\end{array}\tag{8}$$

where $\mathbf{p}_0$ and $-\mathbf{p}_0$ are the momenta of the incoming nucleons in the barsy, and λ and ν are, respectively, the spin-energy and the charge indices.

We will later assume that near the threshold the transition matrix element varies only slowly and will find that, for energies of the initial nucleons up to 50 Mev above the threshold, this assumption is valid

[11] Peterson, Iloff, and Sherman, Phys. Rev. **81**, 647 (1951).
[12] Cartwright, Richman, Whitehead, and Wilcox, Phys. Rev. **81**, 652 (1951).
[13] Crawford, Crowe, and Stevenson, Phys. Rev. **82**, 97 (1951).
[14] See, e.g., N. Kemmer, Proc. Roy. Soc. (London) **A166**, 127 (1938).

and the value of the transition matrix element given by the aforementioned transition scheme at the threshold is a good approximation.

For transitions (a) and (b) the interaction between the incoming nucleons with large momenta $\mathbf{p}_0$ and $-\mathbf{p}_0$ is much weaker compared with that between the slow outgoing nucleons for transitions (c) and (d). Actually, in cases (c) and (d), when the outgoing nucleons are a neutron and a proton, the interaction between them is so strong that they may combine to form a deuteron. In this case the Born approximation is, of course, no longer valid; and a consistent nonrelativistic discussion of the polarization effect using deuteron wave functions for the final nucleons is given later.

With vector interaction (5) between meson and nucleon, the complete transition matrix element from (I) to (F) is

$$\begin{aligned} H_{IF(v)}' = \frac{g_v c^\alpha V_{\mathbf{p}_0}}{m^{\frac{3}{2}}\Omega^{\frac{3}{2}}} [& \langle 1|J^\alpha|10\rangle\langle 2|K_-^\alpha|20\rangle \\ & -\langle 1|J^\alpha|20\rangle\langle 2|K_+^\alpha|10\rangle \\ & -\langle 2|J^\alpha|10\rangle\langle 1|K_-^\alpha|20\rangle \\ & +\langle 2|J^\alpha|20\rangle\langle 1|K_+^\alpha|10\rangle], \end{aligned} \tag{9}$$

where

$$K_\pm^\alpha = \frac{1}{2M+m}[\mathbf{a}_0^* \cdot \boldsymbol{\alpha}\tau^{(s)}(M+m+\beta M)J^\alpha] - \frac{1}{2E_0-m}[J^\alpha(E_0-m\pm\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)\mathbf{a}_0^*\cdot\boldsymbol{\alpha}\tau^{(s)}],$$

$$V_\mathbf{p} = \int v(r)e^{i\mathbf{p}\cdot\mathbf{r}}d^3\mathbf{r} = V_{-\mathbf{p}},$$

M is the mass of the nucleon; and 10 and 1, and 20 and 2 denote, respectively, the initial and the final states of the nucleons. In obtaining Eq. (9) we have written for the energy denominators following from Eq. (8):

$$1/(M+m)\mp M = (M+m+\beta M)/(2M+m)m, \quad \text{for (a), (b);}$$

$$1/(E_0-m)\mp E_0 = (E_0-m+\boldsymbol{\alpha}\cdot\mathbf{p}'+\beta M)/-(2E_0-m)m \quad \text{for (c), (d),}$$

with $\mathbf{p}'=\mathbf{p}_0$ or $\mathbf{p}'=-\mathbf{p}_0$ for the first and second nucleon, respectively. Then the summation over all intermediate states follows the usual scheme. Here, we have taken a one-particle formulation with positive and negative energies for the nucleon states. The calculation in the hole-theory formulation of the nucleons with positive definite energy was also made, and the result agrees with that given here. We now square $H_{IF(v)}'$ and sum through only the positive energy substates of the final state by the introduction of the projection operator $(1+\beta)/2$ to operate on the final states with zero momentum. Introducing also the isotopic spin projection operator $(1+\nu\tau_3)/2$ to operate on the initial states, which eliminates the neutron state when $\nu=+1$ and the proton state when $\nu=-1$, and averaging over the positive energy substates of the initial state, we get (since $J^{\alpha\dagger}=J^\alpha$)

$$\begin{aligned} |H_{IF}'|_{(v)}^2 = g_v^2 V_{\mathbf{p}_0}^2 c^\alpha c^\beta \Omega^{-3} m^{-3} E_0^{-2} \\ \times[P^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20})+Q^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20}) \\ -R^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20})-S^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20}) \\ +P^{\alpha\beta}(-\mathbf{p}_0,\nu_{20},\nu_{10})+Q^{\alpha\beta}(-\mathbf{p}_0,\nu_{20},\nu_{10}) \\ -R^{\alpha\beta}(-\mathbf{p}_0,\nu_{20},\nu_{10})-S^{\alpha\beta}(-\mathbf{p}_0,\nu_{20},\nu_{10})], \end{aligned} \tag{10}$$

where

$$P^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20}) = \tfrac{1}{8}\,\mathrm{sp}[J^\beta(1+\beta)J^\alpha(E_0+\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{10}\tau_3)]\tfrac{1}{8}\,\mathrm{sp}[K_-^{\beta\dagger}(1+\beta)K_-^\alpha(E_0-\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{20}\tau_3)],$$

$$Q^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20}) = \tfrac{1}{8}\,\mathrm{sp}[K_+^{\beta\dagger}(1+\beta)J^\alpha(E_0+\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{10}\tau_3)]\tfrac{1}{8}\,\mathrm{sp}[J^\beta(1+\beta)K_-^\alpha(E_0-\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{20}\tau_3)],$$

$$R^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20}) = \tfrac{1}{64}\,\mathrm{sp}[J^\beta(1+\beta)J^\alpha(E_0+\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{10}\tau_3)K_+^{\beta\dagger}(1+\beta)K_-^\alpha(E_0-\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{20}\tau_3)],$$

$$S^{\alpha\beta}(\mathbf{p}_0,\nu_{10},\nu_{20}) = \tfrac{1}{64}\,\mathrm{sp}[K_-^{\beta\dagger}(1+\beta)J^\alpha(E_0+\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{10}\tau_3)J^\beta(1+\beta)K_-^\alpha(E_0-\boldsymbol{\alpha}\cdot\mathbf{p}_0+\beta M)(1+\nu_{20}\tau_3)].$$

The evaluation of the Spurs is lengthy but straightforward. The resulting complete $|H_{IF}'|_{(v)}^2$ shows that the contribution from transitions (a) and (b) is of the order m/M smaller than that from transitions (c) and (d), and so is the contribution from transitions to negative energy substates of the intermediate states. These terms are neglected; contributions are retained from transitions to positive energy substates of the intermediate states (c) and (d) alone. Only in this approximation is it legitimate to use the nuclear potential obtained from scattering experiments, i.e., transitions from positive energy states to positive energy states, and to neglect possible "non-static" corrections to $V(\mathbf{r})$, for example terms of the form $v(r)c_5\boldsymbol{\alpha}_{(1)}\cdot\boldsymbol{\alpha}_{(2)}$. The resulting $|H_{IF}'|_{(v)}^2$ and the corresponding squared transition matrix element with tensor interaction between meson and nucleon, $|H_{IF}'|_{(t)}^2$, are

$$|H_{IF}'|_{(v)}^2 = \frac{g_v^2 V_{\mathbf{p}_0}^2 p_0^2}{\Omega^3 m^3 E_0^2}\left(\frac{2E_0+2M-m}{2E_0-m}\right)^2 T_{(v)}\cos^2\theta, \tag{11}$$

$$|H_{IF}'|_{(t)}^2 = \frac{g_t^2 V_{\mathbf{p}_0}^2 p_0^2}{\Omega^3 m^3 E_0^2}\left(\frac{2E_0+2M-m}{2E_0-m}\right)^2 T_{(t)}\sin^2\theta, \tag{12}$$

where θ is the angle between $\mathbf{a}_0^*$ and $\mathbf{p}_0$ and the T functions—$T_{(v)}$ and $T_{(t)}$—are

$$T_{(v)}=\tfrac{3}{2}[2+s(\nu_{10}+\nu_{20})](c_1+c_2-3c_3-3c_4)^2 -3(1-s^2)\nu_{10}\nu_{20}(c_1+c_2-3c_3-3c_4)^2 -(1-\nu_{10}\nu_{20})(c_1^2-3c_2^2+13c_3^2+9c_4^2+6c_1c_2 -10c_1c_3-6c_1c_4-6c_2c_3-18c_2c_4+30c_3c_4),$$

$$T_{(t)}=\tfrac{1}{2}[2+s(\nu_{10}+\nu_{20})](3c_1^2+11c_2^2+19c_3^2+27c_4^2 -2c_1c_2-10c_1c_3-18c_1c_4-18c_2c_3+6c_2c_4+30c_3c_4) -(1-s^2)\nu_{10}\nu_{20}(c_1^2-7c_2^2+17c_3^2+9c_4^2+10c_1c_2 -14c_1c_3-6c_1c_4-6c_2c_3-30c_2c_4+42c_3c_4) -(1-\nu_{10}\nu_{20})(c_1+c_2-3c_3-3c_4)^2.$$

(B) Deuteron Formation

For the reactions

$$p+p\to\pi^++n+p, \tag{13}$$

and

$$n+n\to\pi^-+n+p \tag{14}$$

near the threshold, as mentioned in the Introduction, the final neutron and proton having very small momenta interact so strongly that they may fall into deuteron states. The Born approximation of plane waves for the final nucleons is, of course, no longer valid; instead an appropriate "deuteron" wave function should be used. Since under the assumption of charge symmetry, and neglecting coulomb effects, the two reactions (13) and (14) are obviously equivalent, we will fix our attention on the first reaction (13). As to the final states of the proton and neutron, we shall consider only ^3s and ^1s states, because for p and higher states there is no resonance with bound or low-lying virtual states and their contribution will be much smaller near the threshold.

The following calculation is made in the barsy at the threshold. The velocity of the incoming nucleon is approximately $v_0\approx[(4M+m)m]^{\frac{1}{2}}/(2M+m)=0.38$ for $M/m=6.4$. Therefore a nonrelativistic calculation is tolerable even for the initial states. The properly antisymmetrized wave function of the incoming nucleons is then either

$$i(\sqrt{2}/\Omega)T_+(1)T_+(2)\;{}^3\Sigma\,\sin\mathbf{p}_0\cdot\mathbf{r} \tag{15}$$

or

$$(\sqrt{2}/\Omega)T_+(1)T_+(2)\;{}^1\Sigma\,\cos\mathbf{p}_0\cdot\mathbf{r}, \tag{16}$$

where $T_\pm$ is the isotopic spin state function; T_+ for proton, and T_- for neutron. $^3\Sigma$ and $^1\Sigma$ are, respectively, the symmetric (triplet) and antisymmetric (singlet) spin state functions of the two nucleons. The numbers 1 and 2 in $T_+(1)T_+(2)$ designate nucleons No. 1 and No. 2, and $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. The incoming nucleons are assumed to be free, since the contribution from transitions (a) and (b) in Eq. (8) was only of the order m/M. The wave functions of the "deuteron" in the ^3s and the ^1s states are, respectively,

$$(1/\sqrt{2})[T_+(1)T_-(2)-T_-(1)T_+(2)]\;{}^3\Sigma D_{(^3s)}(r) \tag{17}$$

and

$$(1/\sqrt{2})[T_+(1)T_-(2)+T_-(1)T_+(2)]\;{}^1\Sigma D_{(^1s)}(r). \tag{18}$$

The symmetrized interaction hamiltonians between the pion and the nucleons at the threshold in the nonrelativistic approximation become

$$H_{(v)}'=(g_v/iM)(\Omega m)^{-\frac{1}{2}}[\tau_+(1)-\tau_+(2)]\mathbf{a}_0^*\cdot\boldsymbol{\nabla}, \tag{19}$$

$$H_{(t)}'=(g_t/iM)(\Omega m)^{-\frac{1}{2}}[\boldsymbol{\sigma}(1)\tau_+(1)-\boldsymbol{\sigma}(2)\tau_+(2)]\cdot\mathbf{a}_0^*\times\boldsymbol{\nabla}, \tag{20}$$

where $\boldsymbol{\nabla}$ is the differential operator "del" with respect to $\mathbf{r}$ and $\tau_+=(\tau_1-i\tau_2)/\sqrt{2}$ destroys one positive charge. Since the spatial parts of the "deuteron" wave functions (17) and (18) have even parity and those of the interaction hamiltonians (19) and (20) have odd parity, to get non-zero transition matrix elements, we must use the triplet wave function (15) of the incoming nucleons having an odd spatial part. The transition matrix elements are then: for a "deuteron" formed in the ^3s state

$$H_{IF(D)(v)}'=-\frac{2}{\sqrt{m}}\frac{g_vp_0}{\Omega M}\langle{}^3\Sigma|{}^3\Sigma\rangle \times\left[\int D_{(^3s)}(r)\cos\mathbf{p}_0\cdot\mathbf{r}d^3\mathbf{r}\right]\cos\theta, \tag{21}$$

$$H_{IF(D)(t)}'=-\frac{1}{\sqrt{m}}\frac{g_tp_0}{\Omega M}\langle{}^3\Sigma|\sigma_z(1)+\sigma_z(2)|{}^3\Sigma\rangle \times\left[\int D_{(^3s)}(r)\cos\mathbf{p}_0\cdot\mathbf{r}d^3\mathbf{r}\right]\sin\theta; \tag{22}$$

for a "deuteron" formed in the ^1s state

$$H_{IF(D)(v)}'\equiv 0, \tag{23}$$

$$H_{IF(D)(t)}'=\frac{1}{\sqrt{m}}\frac{g_tp_0}{\Omega M}\langle{}^1\Sigma|\sigma_z(1)-\sigma_z(2)|{}^3\Sigma\rangle \times\left[\int D_{(^1s)}(r)\cos\mathbf{p}_0\cdot\mathbf{r}d^3\mathbf{r}\right]\sin\theta, \tag{24}$$

where θ, as before, is the angle between $\mathbf{a}_0^*$ and $\mathbf{p}_0$; and σ_z is the component of $\boldsymbol{\sigma}$ in the direction of $\mathbf{a}_0^*\times\mathbf{p}_0$. Squaring the matrix elements, summing over the final spin states, and averaging over the initial spin states we get, for ^3s deuteron state,

$$|H_{IF(D)}'|_{(v)}^2=\frac{9}{m}\frac{g_v^2p_0^2}{\Omega^2M^2} \times\left[\int D_{(^3s)}(r)[\cos\mathbf{p}_0\cdot\mathbf{r}d^3\mathbf{r}\right]^2\cos^2\theta, \tag{25}$$

$$|H_{IF(D)}'|_{(t)}^2=\frac{6}{m}\frac{g_t^2p_0^2}{\Omega^2M^2} \times\left[\int D_{(^3s)}(r)\cos\mathbf{p}_0\cdot\mathbf{r}d^3\mathbf{r}\right]^2\sin^2\theta; \tag{26}$$

for ${}^1s$ deuteron state,

$$|H_{IF(D)}'|_{(v)}^2 \equiv 0, \tag{27}$$

$$|H_{IF(D)}'|_{(t)}^2 = \frac{3}{m}\frac{g_t^2 p_0^2}{\Omega^2 M^2} \times \left[\int D_{(^1s)}(r)\cos \mathbf{p}_0\cdot\mathbf{r}d^3\mathbf{r}\right]^2 \sin^2\theta. \tag{28}$$

For the inverse processes the transition matrix elements are evidently the same as those given.

III. CROSS SECTION

(A) Nucleonic Inelastic Scattering

Near the threshold we can consider the transition matrix elements as energy independent, having their respective values at the threshold. (In the discussion we will see the condition for the validity of this assumption.) Since in the barsy the momenta of the two outgoing nucleons and the meson produced are small, we can use the nonrelativistic expression for the final energies. The energy density of final states is then

$$d^6n/dE_f = [\Omega^2/2(2\pi)^6]Mpd\Omega_p k^2 dk d\Omega_k, \tag{29}$$

where $\mathbf{k}$ is the momentum of the meson and $\mathbf{p}$ is half of the difference of the momenta of the outgoing nucleons, with its magnitude given by

$$p = \{M[2\epsilon - (2M+m)k^2/4Mm]\}^{\frac{1}{2}},$$

$2\epsilon = 2(E_i - E_0)$ being the total excess energy above the threshold.

The differential cross section for the vector meson production is then

$$d^5\sigma = (\Omega^3 ME_0/4(2\pi)^5 p_0)|H_{IF}'|^2 pd\Omega_p k^2 dk d\Omega_k \tag{30}$$

where we have put $v_0 = p_0/E_0$ as a good approximation for the velocity of the incoming nucleons. The total cross section is given by integration

$$\sigma = \frac{\Omega^3 M^3}{(2\pi)^2}\left(\frac{m}{2M+m}\right)^{\frac{3}{2}}\frac{E_0}{p_0}|H_{IF}'|^2\epsilon^2. \tag{31}$$

(B) Deuteron Formation

For the cross section of deuteron formation we consider only the case when the deuteron is formed in the bound ${}^3s$ state. The momenta $\mathbf{p}_D$ of the final deuteron and $\mathbf{k}$ of the meson produced are related through momentum conservation by $\mathbf{p}_D = -\mathbf{k}$. Near the threshold energy conservation gives

$$2\epsilon_D \equiv 2E_i - M_D - m = \tfrac{1}{2}[(1/M_D)+1/m]p_D^2, \tag{32}$$

where M_D is the rest-mass of the deuteron. The energy density of final states is

$$\left(\frac{d^3n}{dE_f}\right)_D = \frac{\Omega}{4\pi^3}\frac{\epsilon_D^{\frac{1}{2}}}{[(1/M_D)+1/m]^{\frac{3}{2}}}d\Omega_{p_D}. \tag{33}$$

The differential cross section is then

$$d^2\sigma_D = \frac{\Omega^2}{2\pi^2}\frac{E_{0D}}{p_{0D}}\frac{\epsilon_D^{\frac{1}{2}}}{[(1/M_D)+1/m]^{\frac{3}{2}}}|H_{IF(D)}'|^2 d\Omega_{p_D}, \tag{34}$$

and the total cross section is

$$\sigma_D = \frac{2\Omega^2}{\pi}M_D^{\frac{3}{2}}\left\{\frac{m^3}{[(M_D+m)^2-4M^2](M_D+m)}\right\}^{\frac{1}{2}} \times |H_{IF(D)}'|^2\epsilon_D^{\frac{1}{2}} \tag{35}$$

where we have put in the values

$$E_{0D} = \tfrac{1}{2}(M_D+m), \quad p_{0D} = \tfrac{1}{2}[(M_D+m)^2-4M^2]^{\frac{1}{2}}$$

for the energy and momentum of the incoming nucleon at the threshold.

(C) Absorption of Charged Meson by Deuteron

For the inverse process, namely, the absorption of a charged vector meson by a deuteron, the total cross section σ_A can be obtained from σ_D immediately by detailed balancing.

$$\sigma_A = \frac{2}{3}\left(\frac{p_i^2}{p_D^2}\right)\sigma_D$$

$$= \frac{2\Omega^2}{3\pi}[M_D m(M_D+m)]^{\frac{1}{2}}\frac{M}{[(M_D+m)^2-4M^2]^{\frac{1}{2}}} \times \left[\epsilon_D^{\frac{1}{2}} + \frac{M_D+m-2M}{2\epsilon_D^{\frac{1}{2}}}\right]|H_{IF(D)}'|^2. \tag{36}$$

As before, $|H_{IF(D)}'|^2$ is given by Eqs. (25), (26), (27), and (28), where θ now is the angle in the barsy between the direction of polarization of the incoming linearly polarized meson and the direction of the outgoing nucleons, and ϵ_D defined by Eq. (32) is now to be interpreted as half of the total kinetic energy of the incoming meson and the deuteron. When ϵ_D is small, the term proportional to $\epsilon_D^{-\frac{1}{2}}$ is the dominating term in the bracket which exhibits essentially the $1/v$ law.

IV. DISCUSSION OF RESULTS

(A) Polarization near the Threshold

The resulting transition matrix elements (11) and (12), and Eqs. (25), (26), (27), and (28) show complete polarization (to the order of m/M) of the vector meson produced in either an inelastic nucleon collision process or a deuteron formation process in both the ${}^3s$ and ${}^1s$ states (with the unessential exception of $|H_{IF(D)}'|_{(v)}^2 \equiv 0$ for ${}^1s$ deuteron state). It is polarized parallel to the direction of motion of the incoming nucleon for vector coupling between the vector meson and the nucleon, and perpendicular to this direction for tensor coupling. This is exactly the same as the polarization of the vector meson produced by the interaction between a nucleon and a nuclear field, the case treated by Wentzel.[2] This effect, if it exists, should be detectable experimentally and offers a basic and clear-cut argument for or against the vector theory of charged pions.

As for the methods of detection of this polarization effect, we may mention the following two:

(1) It was shown by Wentzel[2] that the $\pi-\mu$ decay of polarized vector pions should lead to an angular distribution of the μ-meson (muon) anisotropic in the pion rest frame of reference. This anisotropy should be empirically detectable.

(2) The inverse processes of deuteron formation, namely, the absorption of charged pions by a deuteron to form two identical nucleons, may be employed as a means for detecting the polarization of the charged vector pions. For low kinetic energy of the pion and the deuteron in the barsy, the final nucleons will have velocities mainly in the direction of polarization of the incoming pion if the coupling is purely vectorial, and perpendicular to this direction if the coupling is purely tensorial. Since the mass of a deuteron is much larger than that of a pion, the aforementioned effect should be easy to detect in the labsy (laboratory system) when a pion beam of low energy is shot at a liquid deuterium target. The cross section of this process will be calculated in the next section and is of the order 10^{-27} cm^2 for 100-Mev incoming pions. This effect was first qualitatively discussed by Wentzel.[15,16]

(B) Total Cross Section

From neutron-proton scattering experiments at 40 Mev, 90 Mev, and 260 Mev done in Berkeley,[3–5] the best fit to the experimental results, neglecting tensor forces, is obtained by a central force potential having an exchange property of a 45–55 mixture of ordinary-exchange forces and a Yukawa-type radial dependence with a range of 1.18×10^{-13} cm. For this potential the total cross sections obtained from Eqs. (11), (12), and (31) are

$$\sigma_{(v)}/(g_v^2/\hbar c)=1.8\times10^{-25}T_{(v)}\cos^2\theta(\epsilon/Mc^2)^2\ \text{cm}^2, \quad (37)$$

$$\sigma_{(t)}/(g_t^2/\hbar c)=1.8\times10^{-25}T_{(t)}\sin^2\theta(\epsilon/Mc^2)^2\ \text{cm}^2, \quad (38)$$

where

$T_{(v)}=$	$\nu_{10}=1,\ \nu_{20}=1$	$\nu_{10}=1,\ \nu_{20}=-1$	$\nu_{10}=-1,\ \nu_{20}=-1$
$s=1$	0.98	0.08	0
$s=0$	0	0.57	0
$s=-1$	0	0.08	0.98

$T_{(t)}=$	$\nu_{10}=1,\ \nu_{20}=1$	$\nu_{10}=1,\ \nu_{20}=-1$	$\nu_{10}=-1,\ \nu_{20}=-1$
$s=1$	0.81	0.08	0
$s=0$	0.16	0.33	0.16
$s=-1$	0	0.08	0.81

Since $g^2/\hbar c$ is of the order of unity, we see that the value obtained for the total cross section agrees in the order of magnitude with that calculated for the pseudoscalar theory by Marshak and Foldy,[17] also under the Born approximation. Both results are necessarily underestimates, because the Born approximation ignores resonances with deuteron ^3s and ^1s states.

In the case of deuteron formation, we will calculate the total cross section only when the deuteron is formed in the bound ^3s state. To get an analytical form for $D_{(^3s)}(r)$ we assume a Hulthén potential of range[18] 1.18×10^{-13} cm and take the binding energy of deuteron ^3s state as 2.23 Mev. The total cross sections as given by Eqs. (25), (26), and (35) are

$$\sigma_{D(v)}/(g_v^2/\hbar c)=1.3\times10^{-26}\cos^2\theta(\epsilon_D/Mc^2)^{\frac{1}{2}}\ \text{cm}^2, \quad (39)$$

$$\sigma_{D(t)}/(g_t^2/\hbar c)=0.86\times10^{-26}\sin^2\theta(\epsilon_D/Mc^2)^{\frac{1}{2}}\ \text{cm}^2. \quad (40)$$

With the Hulthén potential, the total cross section is very sensitive to the range. Calculations with different ranges show that doubling the range will reduce the cross section by a factor of as much as 10^{-2}. Considering the possible error in the value 1.18×10^{-13} cm for the range and all the approximations made in the calculation, therefore, we should not put more than order-of-magnitude emphasis on the numerical values of the total cross sections given above. With a square well potential, however, the values obtained for the total cross section are consistently smaller than the values quoted above by a factor of about 2×10^{-3}. This is due to the absence of the $1/r$ infinity at the origin and the consequent lack of fourier components of higher wave numbers.

The total cross section for the absorption of a pion by a deuteron is given by Eq. (36), together with Eqs. (25) and (26). Assuming the same Hulthén potential as above, we get

$$\sigma_{A(v)}/(g_v^2/\hbar c)=2.2\times10^{-27}[13.6(\epsilon_D/Mc^2)^{\frac{1}{2}}+(\epsilon_D/Mc^2)^{-\frac{1}{2}}]\cos^2\theta\ \text{cm}^2, \quad (41)$$

$$\sigma_{A(t)}/(g_t^2/\hbar c)=1.5\times10^{-27}[13.6(\epsilon_D/Mc^2)^{\frac{1}{2}}+(\epsilon_D/Mc^2)^{-\frac{1}{2}}]\sin^2\theta\ \text{cm}^2, \quad (42)$$

where ϵ_D as mentioned before is half of the total kinetic energy of the incoming pion and deuteron in the barsy.

These values of the total cross sections of the reversible reaction $\pi^++d\rightleftarrows p+p$ agree in order of magnitude with those obtained theoretically by Marty and Prentki[19] and by Cheston.[20] They also agree in order of magnitude with the experimental result of 1.4×10^{-28} cm^2 for the reaction $p+p\rightarrow d+\pi^+$ for 340-Mev incident protons obtained in Berkeley and the order of magni-

[15] G. Wentzel, Helv. Phys. Acta **22**, 101 (1949).

[16] R. E. Marshak and W. B. Cheston have suggested that the spin of the pion be determined from the ratio of the total cross sections of the two inverse processes $p+p\rightleftarrows d+\pi^+$. The anisotropic distribution of the protons for the case of polarized vector pions should not be overlooked in evaluating the total cross section from experiments selecting certain proton directions.

[17] R. E. Marshak and L. L. Foldy, Phys. Rev. **75**, 1493 (1949).

[18] Quantity defined as κ^{-1} in L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, Inc., New York, 1949), p. 67.

[19] C. Marty and J. Prentki, J. phys. et radium (Sér. 8) **10**, 156 (1949).

[20] W. B. Cheston, *On the Reactions* $\pi^++d\rightleftarrows p+p$ (unpublished paper, Rochester, New York).

tude of 10^{-27} cm^2 for the total cross section of the reaction $\pi^+ + d \rightarrow p + p$ obtained by Roberts in Rochester.

(C) Condition for the Validity of the Results

In the complete interaction hamiltonians (2) and (3) there occur terms dependent on $\mathbf{k}$, proportional to $\mathbf{k}/m$. The matrix element of $\boldsymbol{\alpha}$ gives the velocity of the nucleons v_n, and the matrix element of $\mathbf{k}/m$ is a quantity of the order of magnitude of the velocity of the meson produced v_π. Therefore, when terms of the second order in $\mathbf{k}/m$ are neglected, the matrix elements of the complete hamiltonians near the threshold are of the form (in the barsy)

$$H_{IF}' \cong A v_n + B v_\pi \cong A v_0 + B(4\epsilon/m)^{\frac{1}{2}},$$

where A and B are of the same order of magnitude, since the coefficients of $\boldsymbol{\alpha}$ and $\mathbf{k}/m$ in the hamiltonians are of the same order of magnitude. Hence, to neglect $\mathbf{k}/m$ compared with $\boldsymbol{\alpha}$, we must have

$$(4\epsilon/m)^{\frac{1}{2}} \ll v_0 = 0.38,$$

which gives

$$(\epsilon_l/Mc^2) \ll 2.4 \text{ percent}, \tag{43}$$

where ϵ_l is the energy above threshold in the labsy. To obtain a better estimate, the square of the transition matrix element for the production of neutral meson during neutron proton collision with only the ordinary force term in the nuclear potential was worked out with the complete interaction hamiltonians. The condition obtained there for neglecting resulting terms in k is

$$(\epsilon_l/Mc^2) \ll 14.4 \text{ percent}. \tag{44}$$

This condition agrees in order of magnitude with the one given above, but is looser. The argument above would necessarily lead to a too stringent condition, because we substituted for v_π the too large value $(4\epsilon/m)^{\frac{1}{2}}$ which is actually forbidden by momentum conservation, and for v_n the minimum value v_0. Take $Mc^2 = 938$ Mev; then, it is safe to assume the results to be valid for

$$\epsilon_l \ll 100 \text{ Mev}. \tag{45}$$

The condition for the validity of the application of the Born approximation[21] is not very well satisfied; but it is well known that the Born approximation usually gives sensible results even when the criterion, which is too stringent, is not well satisfied or barely satisfied as it is in our case.

The validity of these calculations is, of course, further impaired by the limitations of meson theories in general and the unreliability of a perturbation calculation.

I would like to express my deepest indebtedness and thanks to Professor Gregor Wentzel for suggesting this problem and for his constant help and encouragement during the course of development of this work.

[21] See, e.g., L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 168.