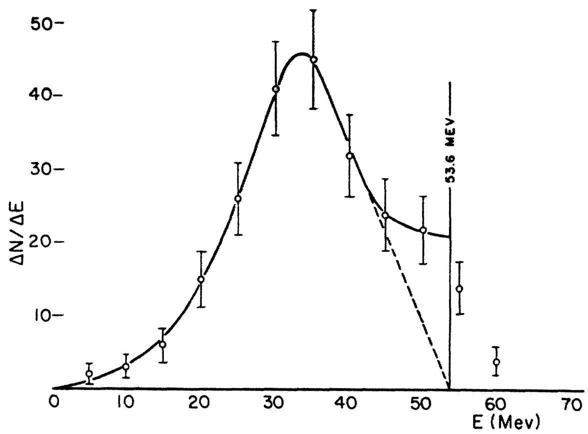

FIG. 3. Spectrum of 117 positrons from μ^+ meson decay. Each point represents the number of positrons per 10-Mev energy interval. The vertical lines indicate the statistical spread. The smooth curve represents the best match to the observed points. The dotted portion represents the best match to the data by a curve approaching zero at the upper energy end.

after leaving the μ-meson. Similar large scatterings (at least eight times the average scattering, including the large scatterings) are found to occur with a frequency of about 10 percent within the first g/cm^2 of emulsion. With a fair bremsstrahlung loss possible in these events, this comprises a possible source of error in techniques employing appreciable "dead" mass to stop the μ-meson. Figure 2(b) shows one of six events in which the decay particle disappears. In view of the uniform minimum ionization achieved, this disappearance is interpreted to be an annihilation process. Hence, in addition to having the same charge as the positron, and a mass not much heavier (if at all),[3] this decay particle is now observed to have a third property in common with the positron, that of annihilation.

The use of nuclear emulsions for investigating the energy spectrum of the positron from μ^+ decay has several advantages. In a few hours of cyclotron running time, $1500\pi\rightarrow\mu\rightarrow\beta$ sequences were obtained. A qualitative examination of the β-tracks can be made at a rate of 40/day/scanner. Positrons of all energies give substantially the same grain density;[4] hence, all energies are recorded with almost equal sensitivity. The complete history of the positron is observed; therefore, there is no possibility of failing to observe appreciable energy losses in any "dead" mass.

However, some disadvantages also exist. The results of multiple scattering measurements must be accurately correlated with energy. A long track is required for good accuracy. Long tracks imply appreciable energy degradation during the measuring process through collision and radiation loss. Long tracks also discriminate against slow positrons because of outscattering. Distortion in the processed emulsion will cause lower estimates of energy. Finally, these scattering measurements are tedious and exacting if precision is desired.

The modification of the stage and drive of a conventional microscope enabled the measurements to be made with greater ease and speed. The effective track length depends upon the shortest "foil" thickness feasible, below which the noise level becomes critical. After some investigation, it was found possible to make Fowler-type measurements of track ordinates with a mean deviation of 0.03 micron. This permitted the use of intervals as short as 50 microns for the fastest particles observed, with a noise error of less than 2 percent in the most unfavorable cases. Therefore, a 1500-micron track length was selected at the minimum acceptable for the desired statistical accuracy: 10 percent standard deviation. Outscattering is then less than 2 percent above 20 Mev, reaching 10 percent at 11 Mev and 38 percent at 5 Mev. The average bremsstrahlung loss is 7 percent for the 2.91-cm radiation length in G5 emulsion. Hence, individual variations from this average loss are much smaller than the statistical uncertainty in energy determination. On all plates no distortion was noted in the central portions for the track lengths used. Taking into account all these sources of error, the standard deviation per track is probably less than 11 percent.

Average multiple scattering angles were calibrated against energy using the Scott-Snyder[5] theory, which from Corson's[6] work appeared to be best. The energy spectrum uncorrected for outscattering resulting from the first 117 decays is shown measured in Fig. 3. The spectrum appears to have a marked non-zero cutoff at the high energy end, as is indicated by both the experimental points and the straggling beyond cutoff (taking the μ-meson mass as 209 electron masses).

It is interesting to note that if (a) the Scott-Snyder-theory provides a perfect calibration and (b) the relativistic momentum of the positron is, on the average, equal to that of each neutrino, then the average energy: 35.76 ± 0.37 Mev, gives the mass of the μ-meson as 209.3 ± 2.2 electron masses. In the absence of an accurate independent verification of the Scott-Snyder theory in this energy region, this excellent agreement with recent values[7] might best be taken as an indication of the high accuracy of this theory as applied here.

We wish to thank Drs. Friedman and Rainwater for kindly lending us their exposure chamber and some plates for our earlier work. We are also grateful to Miss Marjorie Fogarty and Mrs. Carol Major for their invaluable assistance in locating the events, and to Mr. Aurel Seifert for aiding with some of the measurements.

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On a One-to-One Correspondence between Infinitesimal Canonical Transformations and Infinitesimal Unitary Transformations

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It is well known that there is an analogy between the group of the canonical transformations of classical mechanics and the group of the unitary transformations of quantum mechanics, insofar as in the latter we exclude degrees of freedom without classical analog.[1] Until now, however, this analogy has been found to be not very simple; more precisely, no general homomorphism has been given between the two groups.[2,3] Among the reasons for that are the ambiguities in the definition of the quantum-mechanical function $F(\mathbf{p}, \mathbf{q})$ of the noncommutable operators $\mathbf{p}$, $\mathbf{q}$ with

$$[\mathbf{p}, \mathbf{q}] = -i\hbar \mathbf{1}, \tag{1}$$

corresponding to the classical function $F^c(p, q)$ of the commutable variables[4] p, q. In order to have an actual significance, such a correspondence must be independent of the choice of the variables p, q, at least within a certain subset of the group of canonical transformations.

The first step in finding a correspondence between canonical transformations and unitary transformations is to give this correspondence between $F^c(p, q)$ and $F(\mathbf{p}, \mathbf{q})$. We propose the following one: to every classical real function $F^c(p, q)$ which may be written as a fourier integral

$$F^c(p, q) = \int dx \int dy f^c(x, y) e^{i(px+qy)}, \quad \bar{f}^c(-x, -y) = f^c(x, y) \tag{2}$$

($^-$ means imaginary conjugate) corresponds the quantum-mechanical function

$$F(\mathbf{p}, \mathbf{q}) = \int dx \int dy f(x, y) e^{i\mathbf{p}x} \cdot e^{i\mathbf{q}y},$$
$$f(x, y) = \tfrac{1}{2}(1+e^{-i\hbar xy}) f^c(x, y). \tag{3}$$

The operator $F(\mathbf{p}, \mathbf{q})$ has the following properties: (1) it is hermitian; (2) it is "well ordered," i.e., it may be written as a sum of terms where the $\mathbf{p}$ are on the left of the $\mathbf{q}$; (3) for $\hbar$ going to 0, F is reduced to F^c, which expresses the condition of correspondence. All these properties are invariant under the operations of derivation or integration.[5]

The infinitesimal canonical transformation generated by the real function $F^c(p, q)$ is described by the following equations:

$$\begin{cases} p' = p + \alpha\{F^c(p, q), p\} = p - \alpha \partial F^c/\partial q & (\alpha \text{ real}), \\ q' = q + \alpha\{F^c(p, q), q\} = q + \alpha \partial F^c/\partial p. \end{cases} \tag{4}$$

By application of Taylor's formula, we get the transform $G^c(p', q')$ of any real function $G^c(p, q)$, neglecting terms in $\alpha^2, \alpha^3, \cdots,$

$$G^c(p', q') = G^c(p, q) + \alpha\{F^c(p, q), G^c(p, q)\} \tag{4a}$$

with the definition for the poisson bracket

$$\{F^c(p, q), G^c(p, q)\} = \frac{\partial F^c}{\partial p}\frac{\partial G^c}{\partial q} - \frac{\partial G^c}{\partial p}\frac{\partial F^c}{\partial q}.$$

Now we want to extend the notion of a canonical infinitesimal transformation to the case where the commutable variables p, q are replaced by the noncommutables $\mathbf{p}, \mathbf{q}$, and, at the same time, the functions $G^c(p, q)$, $F^c(p, q)$, etc., replaced by $G(\mathbf{p}, \mathbf{q})$, $F(\mathbf{p}, \mathbf{q}), \cdots,$ etc. This extension must satisfy the following conditions: (1) the new variables $\mathbf{p}', \mathbf{q}'$ must be hermitian; (2) the correspondence $F^c(p, q) \to F(\mathbf{p}, \mathbf{q})$ must be the same as the correspondence $F^c(p', q') \to F(\mathbf{p}', \mathbf{q}')$ (invariance of the law of correspondence (2), (3) with respect to a canonical infinitesimal transformation). The only possible extension of (4) satisfying these conditions is

$$\begin{cases} \mathbf{p}' = \mathbf{p} - \alpha \partial F/\partial \mathbf{q}, \\ \mathbf{q}' = \mathbf{q} + \alpha \partial F/\partial \mathbf{p}. \end{cases} \tag{5}$$

The condition (2) gives the transform $G(\mathbf{p}', \mathbf{q}')$ of $G^c(p', q')$:

$$\begin{aligned} G(\mathbf{p}', \mathbf{q}') &= \int dx \int dy g(x, y) \exp\{i\mathbf{p}'x\} \exp\{i\mathbf{q}'y\} \\ &= \int dx \int dy g(x, y) \exp\{ix(\mathbf{p} - \alpha\partial F/\partial \mathbf{q})\} \\ &\qquad \times \exp\{iy(\mathbf{q} - \alpha \partial F/\partial \mathbf{p})\}. \end{aligned} \tag{6}$$

Easy calculation gives, neglecting terms in $\alpha^2, \alpha^3, \cdots,$

$$\begin{aligned} \exp\{ix(\mathbf{p} - \alpha\partial F/\partial\mathbf{q})\} &= \exp\{ix\mathbf{p}\} + \alpha(i/\hbar)\exp\{i\mathbf{p}x\} \\ &\qquad \times(\exp\{-x\hbar\partial/\partial\mathbf{q}\} - 1)F(\mathbf{p}, \mathbf{q}) \cdots. \end{aligned}$$

Substituting in (6) and using Fourier's development of $F(\mathbf{p}, \mathbf{q})$, we have

$$\begin{aligned} G(\mathbf{p}', \mathbf{q}') = G(\mathbf{p}, \mathbf{q}) &+ \alpha(i/\hbar) \int dx \int dy \int dx_1 \int dy_1 g(x, y) f(x_1 y_1) \\ &\times \begin{bmatrix} \exp\{-i\hbar x y_1 + i(x+x_1)\mathbf{p}\} \exp\{i(y+y_1)\mathbf{q}\} \\ -\exp\{-i\hbar x_1 y + i(x+x_1)\mathbf{p}\} \exp\{i(y+y_1)\mathbf{q}\} \end{bmatrix} \end{aligned}$$

or, finally, still neglecting terms on $\alpha^2, \alpha^3, \cdots,$ we have

$$G(\mathbf{p}', \mathbf{q}') = G(\mathbf{p}, \mathbf{q}) + \alpha(i/\hbar)[F(\mathbf{p}, \mathbf{q}), G(\mathbf{p}, \mathbf{q})], \quad [A, B] = AB - BA. \tag{7}$$

In other words, the extension of the infinitesimal canonical transformation (4, 4a) generated by the classical function $F^c(p, q)$ is the infinitesimal unitary transformation

$$\begin{cases} \mathbf{p}' = \mathbf{p} + \alpha(i/\hbar)[F(\mathbf{p}, \mathbf{q}), p] = U\mathbf{p}U^{-1}, \\ \mathbf{q}' = \mathbf{q} + \alpha(i/\hbar)[F(\mathbf{p}, \mathbf{q}), q] = U\mathbf{q}U^{-1}, \end{cases} \tag{5'}$$

$$G(\mathbf{p}', \mathbf{q}') = G(\mathbf{p}, \mathbf{q}) + \alpha(i/\hbar)[F(\mathbf{p}, \mathbf{q}), G(\mathbf{p}, \mathbf{q})] = UG(\mathbf{p}, \mathbf{q})U^{-1} \tag{5a'}$$

"generated" by the hermitian operator $F(\mathbf{p}, \mathbf{q})$ or also performed by the unitary operator $U = 1 + \alpha(i/\hbar)F(\mathbf{p}, \mathbf{q})$.

On the whole, the infinitesimal unitary transformations appear here as a suitable generalization of the infinitesimal canonical transformations for noncommutable variables, preserving chiefly the hermitian character of the suitably defined function of these noncommutable variables. Obviously, there is a one-to-one correspondence between an infinitesimal canonical transformation and its unitary extension.

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4 To simplify the notation, we consider only the case of two canonical variables; generalization to $2n$ variables causes no trouble.

5 The correspondence given here between F^c and F is a slight modification of that proposed by H. Weyl (reference 6), using the notion of well-ordered function introduced by P. Jordan (see reference 2).

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Neutron Crystal Monochromators*

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A POLYCHROMATIC beam of thermal neutrons from the Brookhaven reactor, collimated to about one-minute divergence (slit geometry), was diffracted from a single crystal and the contour of the diffracted beam was determined with a slit mounted on a comparator in front of a BF_3 counter. Coherent regions of the crystal tilted with respect to one another will diffract through slightly different angles according to the Bragg law. It was found that the diffracted beams invariably exhibited several maxima for crystals grown from the melt, and these have been attributed to the lineage structure of such crystals. Lineages are large regions of the crystal tilted with respect to one another but not to such an extent as to call them grains.[1] Within each lineage region there are coherent domains (commonly called mosaic blocks) tilted with respect to one another to a lesser degree than lineages and about 5000A in size[2] in annealed specimens. The existence of lineage is

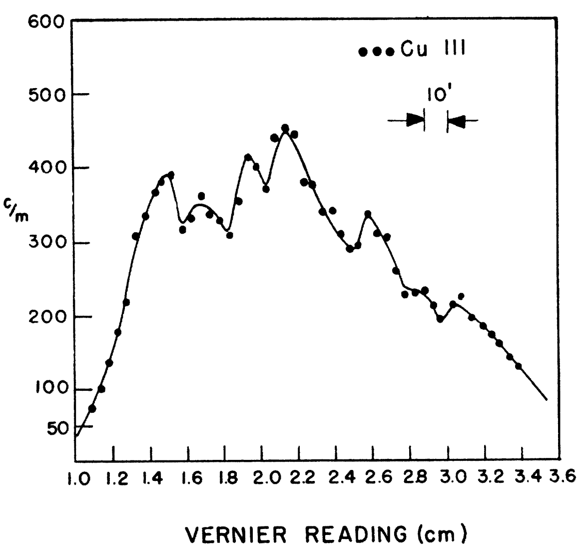

FIG. 1. Contour of Bragg peak from a diffracting plane perpendicular to growth direction, showing lineage.