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On Soft Photon Emission in Radiation Processes*

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IN the present note we have investigated the intensity of the soft photon spectrum emitted in ordinary hard photon radiation processes: e.g., x-ray emission and absorption, pair annihilation, etc.[1] The motivation for the work was the reported possibility of a discrepancy between the wavelength of the electron-positron annihilation gamma-ray, λ_{ann}, and h/mc: $\lambda_{ann}-h/mc \approx 10^{-3}\ h/mc$, a discrepancy[2] which could conceivably be interpreted on the basis of an extended energy conservation relation: $2mc^2=$energy of the two hard annihilation photons+energy of all the accompanying soft photons.

We first considered the equivalence of the two possible descriptions of the annihilation process: the configuration space treatment of the electron and positron in the sense of Fock,[3] and the customary one-particle treatment.[4] The two descriptions are essentially identical except for the impossibility of the inclusion of the electron-positron coulomb interaction in the latter. For the soft photon emission problem, the coulomb interaction has the role of determining the velocity spectrum of the annihilating particles; however, an appropriate velocity spectrum can always be assigned to the initial and final electron states in the one particle description. The two modes of treatment then become exactly equivalent and in particular, the intensity of the soft photon energy spectrum turns out to be determined by the average relative velocity v of the electron and positron. Quantitatively, the average energy $\bar{E}_{soft}$ carried off by this spectrum (per individual pair recombination) is approximately given by

$$(2/3\pi)(1/137)(v/c)^2\ 2\ mc^2 \approx 10^{-7}\ mc^2$$

for v/c values appropriate to the annihilation of slowed down positrons in matter. It is seen, therefore, that the soft photon effect is much too small to account for a discrepancy between hc/λ and mc^2 of the order of magnitude reported, even if the corresponding shift of the intensity maximum in the wavelength distribution of the annihilation line were as large as $\bar{E}_{soft}$ (see below). It is thus very satisfactory that the most recent precise absolute measurements of the energy of the 0.51-Mev ThC″ γ-ray line by Lindström,[5] taken together with the recent precise measurements by Hedgran[6] of the ratio of the energy of this line to that of the annihilation line, yield exact equality of λ_{ann} and h/mc within an experimental error of 3×10^{-4}.

In x-ray emission and absorption the average energy of the accompanying soft photon spectrum (per individual hard photon process) is again given by:

$$\bar{E}_{soft} \approx (2/3\pi)(1/137)(v/c)^2 E_{max}, \tag{1}$$

where E_{max} is the maximum available energy for the soft photons in a single hard photon process and v is the electron velocity appropriately averaged over the initial and final orbits. ($E_{max}\approx h\nu_{hard}$, $v/c\approx Z/137$ for mission; $E_{max}\approx$photoelectron kinetic energy, $v\approx$photoelectron velocity, for absorption.) It might therefore be thought that deviations of 1/10 to 1/100 percent could be observed from the Bohr frequency rule, the Ritz combination principle, and the Einstein photoelectric equation in such hard photon emission and absorption. The emitted soft photon spectrum however, is described by the almost flat intensity distribution:

$$\left(\frac{\bar{E}_{soft}}{E_{max}}\right)\left(\frac{E_{soft}}{E_{max}}\right)^{\bar{E}_{soft}/E_{max}} dE_{soft} \tag{2}$$

with the consequence that in the case of emission, for example, the usual hard photon line shape (derived with neglect of the soft photon effect)[7] is multiplied by a correction factor given approximately by

$$1-\pi(\bar{E}_{soft}/E_{max})[(E_{hard}-E_0)/\Gamma]. \tag{3}$$

Here, E_0 is the energy difference between the initial and final states involved in the hard photon emission, and Γ the hard photon natural line breadth. The energy of an emitted hard photon (or in a similar way, the kinetic energy of an ejected photoelectron) suffers, therefore, a most probable net displacement in the direction of lower energy of only

$$\frac{\pi}{8}\frac{\Gamma}{E_{max}}\bar{E}_{soft} \approx \frac{\pi}{8}(\bar{E}_{soft}/E_0)^2 E_0 \approx \frac{1}{50}\left(\frac{1}{137}\right)^2\left(\frac{v}{c}\right)^4 E_0, \tag{4}$$

Γ in the atomic x-ray region being $\approx(1/10)(1/137)(Z/137)^2E_0 \approx \bar{E}_{soft}$. The smallness of this net displacement as compared with $\bar{E}_{soft}$ is due to the extreme flatness of the soft photon energy spectrum.

Equations (3) and (4) show that any measurement of the emitted hard photon or ejected photoelectron energy, by extrapolation of the corresponding line shape on the high energy side, will be independent of the actual presence of the soft photon emission to about one part per million. Essential agreement with the usual energy conservation relations (neglecting soft photons) is thus always to be expected within this precision.

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The Isomeric Level of Cd^{111m}

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IN a previous investigation of the radioactivity of the 111-isobars,[1] spin and parity were assigned to the several levels, with the result that eight of the nine assignments agreed with the predictions of the shell model.[2] The 48-min isomeric level of Cd^{111} at 396 kev was the exception, being given spin 13/2 and even parity rather than the predicted $h_{11/2}$. This assignment was made on the basis of the usual γ-ray half-life formula,[3] row 3, Table I, and the ratio of 149/247 internal conversion electrons, row 4.

TABLE I. Data for assigning spin and parity to the 396-kev level of Cd^{111}.

	Theory			Expt.
396-kev level	13/2 even	9/2 odd	11/2 odd	
149-kev γ-ray	elec. 4	mag. 2	elec. 3	
149-kev γ, $T_{\frac{1}{2}}$ (sec)	117	8×10^{-4}	8×10^{-4}	2916
149/247 conv. e^-	15.7	10.2	11.4	14.5±1
149/247 γ-rays	0.085	0.43	0.35	0.33±0.06
In^{111} decay ratio	1.9×10^{-8}	2.9×10^{-4}	2.9×10^{-4}	1×10^{-4}