one finds that the value K/β^+ for Cu^{61} should be

$$(K/\beta^+)_{61}=2.32/5.5=0.42.$$

The value of 5.5 however is probably about ten percent too high, since the experiment which determined this value used the annihilation radiation as a measure of the positron intensity. Since, in such absorption measurements, the 655-kev gamma-radiation[4,5] is indistinguishable from the annihilation radiation, it will contribute approximately ten percent of the measured intensity. Taking this into account, the experimental value of K/β^+ for Cu^{61} becomes

$$(K/\beta^+)_{61}=2.32/(0.9)(5.5)=0.47$$

in excellent agreement with the anticipated theoretical value of 0.52 as determined from the disintegration scheme proposed for Cu^{61} from work performed in this laboratory.[5]

* Assisted by the joint program of the ONR and AEC.

[1] R. Bouchez and G. Kayas, J. phys. et radium **10**, 110 (1949); Huber, Ruetsche, and Scherrer, Helv. Phys. Acta **22**, 375 (1949); Bouchez, deGroot, Nataf, and Tolhoek, Physica **15**, 863 (1949); J. phys. et radium **11**, 105 (1950).
[2] J. H. Reynolds, Phys. Rev. **79**, 789 (1950).
[3] Bradt, Gugelot, Huber, Medicus, Preiswerk, and Scherrer, Helv. Phys. Acta **18**, 252 (1945).
[4] Boehm, Blaser, Marmier, and Preiswerk, Phys. Rev. **77**, 295 (1950).
[5] Owen, Cook, and Owen, Phys. Rev. **78**, 686 (1950).

Calculation of Nuclear Magnetic Moments

Masataka Mizushima and Minoru Umezawa
Physics Department, Faculty of Science, Tokyo University, Tokyo, Japan
(Received May 21, 1951)

The magnetic moments of some nuclei were calculated by means of the $j-j$ coupling shell model. The calculated moments agree very well with experimental values, and some conclusions about nuclear forces are obtained.

In the case of nuclei with closed shells ± 1 nucleon, the calculated magnetic moments are the same for both the $j-j$ coupling shell model and the single-particle shell model, and they lie on the Schmidt lines.[1] The calculated values are compared with experiment in Table I. They agree except for B^{11}, Al^{27}, and P^{31}.

In the other cases, the magnetic moments predicted by the $j-j$ coupling shell model are not the same as those predicted by the single-particle shell model. The magnetic moments of nuclei with closed shells ± 3 nucleons are calculated under the following assumptions: (i) the succession of energy levels is that indicated by Mayer,[2] (ii) isotopic spin multiplicity is a good quantum number; i.e., $[H, T]=0$ (T: total isotopic spin).

The calculations were made by constructing the isotopic spin part and the ordinary rotational part (including spin) of the wave function separately from the individual isotopic spin and rotational wave functions. They are then symmetrized by the group theoretical method, and coupled one to one, so that the total wave function satisfy Pauli's principle. The results are shown in Table II.

Table I. Magnetic moments of nuclei with closed shells ± 1 nucleon.

	Configuration	T_ζ	μ_{calc}	μ_{exp}[a]
H^3	$(s)^{-1}$	$+1/2$	2.79	2.978643 ± 28
He^3	$(s)^{-1}$	$-1/2$	-1.91	-2.127414 ± 3
B^{11}	$(2p_{3/2})^{-1}$	$+1/2$	3.79	2.68858 ± 28
C^{13}	$(2p_{1/2})^{1}$	$+1/2$	0.64	0.70225
N^{15}	$(2p_{1/2})^{-1}$	$+1/2$	-0.27	-0.28299 ± 3
O^{17}	$(2s_{1/2})^{1}$	$+1/2$	-1.91	-1.8928 ± 19
F^{19}	$(2s_{1/2})^{-1}$?	$+1/2$	2.79	2.6285 ± 7
Al^{27}	$(3d_{5/2})^{-1}$?	$+1/2$	4.79	3.6408
P^{31}	$(2s_{1/2})^{-1}$?	$+1/2$	2.79	1.13165
S^{33}	$(3d_{3/2})^{1}$	$+1/2$	1.14	0.9
K^{39}	$(3d_{3/2})^{-1}$	$+1/2$	0.12	0.391 ± 1

[a] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950); except O^{17} [F. Alder and F. C. Yu, Phys. Rev. **81**, 1067 (1951)].

Table II. Magnetic moments of nuclei with closed shells ± 3 nucleons.

	I	Configuration	T_ζ	T	μ_{calc}	μ_{exp}[a]
Li^7	3/2	$(2p_{3/2})^3$	$+1/2$	1/2 3/2	3.07 0	3.26
Be^9	3/2	$(2p_{3/2})^{-3}$	$+1/2$	1/2 3/2	-1.14 1.9	-1.18
Cl^{35}	3/2	$(3d_{3/2})^3$	$+1/2$	1/2 3/2	-0.48 0.80	0.82
Mg^{25}	5/2	$(3d_{5/2})^{-3}$	$+1/2$	1/2 3/2	-1.06 to 2.76 2.57	-0.96

[a] J. E. Mack, Revs. Modern Phys. **22**, 64 (1950).

Assumption (ii) is satisfied when (a) the interaction hamiltonian is τ-independent, or (b) scalar products $(\tau_i\tau_j)$ are the only τ-dependent factors.[3] Condition (a) is the case of neutral meson theory, while (b) is the case of symmetrical meson theory.[4] Thus our result is compatible with neutral and symmetrical meson theory, but charged meson theory is excluded.

From Table II we can conclude that the ground states of all these nuclei except Cl^{35} have $T=\frac{1}{2}$. This is quite natural, since $N=Z\pm 1$ nuclei are much more stable than their neighboring nuclei with the same A. ($T=\frac{3}{2}$ means that the level is common to these nuclei.) This means that the interaction between nucleons can be expressed as $a+bP_\tau$ with positive b. (P_τ is the charge exchange operator.) From our result that T can be assigned definitely, b must not be zero; that is, the symmetrical meson theory is needed to explain the internucleon force.

Table III. Predicted magnetic moments of nuclei with closed shells ± 3 nucleons.

	Assumed spin	Configuration	T_ζ	T	μ_{calc}
Be^7	3/2	$(2p_{3/2})^3$	$-1/2$	1/2 3/2	-1.14 1.9
B^9	3/2	$(2p_{3/2})^{-3}$	$-1/2$	1/2 3/2	3.07 0
$A^{35,37}$	3/2	$(3d_{3/2})^{3,-3}$	$\mp 1/2$	1/2 3/2	1.01 0.46
Al^{25}	5/2	$(3d_{5/2})^{-3}$	$-1/2$	1/2 3/2	3.958 to 0.155 0.33
K^{37}	3/2	$(3d_{3/2})^{-3}$	$-1/2$	1/2 3/2	-0.48 0.80

In the case of Mg^{25} there are two independent states which belong to $T=\frac{1}{2}$, so that the calculated magnetic moment lies between two values. Experiment shows that the moment of the ground state is near the minimum theoretical value. From this fact some characteristics of the interaction hamiltonian must be concluded.

It should be mentioned that the success of Feenberg's approach[5] in the case of odd-odd nuclei is also preserved as such in our theory.

In Table III the magnetic moments are predicted for nuclei on which measurements have not yet been made because of their short life. It is hoped to determine these moments experimentally.

It should be noted that even P-odd N nuclei have quadrupole moments in the present $j-j$ coupling shell model. Investigations in quadrupole moment, β-decay, and excited states from the present standpoint of $j-j$ coupling shell model have been made. A more detailed discussion will be published in the near future.

The authors are grateful to Professors M. Kotani, S. Nakamura, M. Sasaki, M. Taketani, and T. Yamanouchi.

[1] T. Schmidt, Z. Physik **106**, 358 (1937).
[2] M. G. Mayer, Phys. Rev. **78**, 16 (1950).
[3] E. Wigner, Phys. Rev. **51**, 106 (1937).
[4] L. Rosenfeld, *Nuclear Forces* (North-Holland Publishing Company, Amsterdam, 1948), pp. 198, 170.
[5] E. Feenberg, Phys. Rev. **76**, 1275 (1949).