(here the multipole order of the γ-ray is indicated above the arrow), one obtains

$$W(\vartheta)=1+a_2\cos^2\vartheta$$

with

$$a_2=\frac{-(L^2+L-3)}{3L^2+7L+3}.$$

Since most of the nuclei in which $\gamma-\gamma$-correlations have been measured[1,2] are presumed to have spin zero ground states, we have similarly calculated the coefficients a_{2i} in $W(\vartheta)=1+\Sigma a_{2i}\cos^{2i}\vartheta$ as functions of L for correlations of type:

$$\text{IB}:\quad L\xrightarrow{2^L}1\xrightarrow{2^1}0$$

$$\text{IIA}:\quad L+2\xrightarrow{2^L}2\xrightarrow{2^2}0$$

$$\text{IIB}:\quad L+1\xrightarrow{2^L}2\xrightarrow{2^2}0$$

$$\text{IIIA}:\quad L+3\xrightarrow{2^L}3\xrightarrow{2^3}0$$

$$\text{IIIB}:\quad L+2\xrightarrow{2^L}3\xrightarrow{2^3}0.$$

The results are given in Tables I, II, and III. In all of the above

TABLE I. a_2 in $W(\vartheta)=1+a_2\cos^2\vartheta$ for $\gamma-\gamma$-correlations of types IA and IB.

L	IA a_2	IB a_2
1	+0.076	−0.333
2	−0.103	+0.429
3	−0.176	0.692
4	−0.215	0.810
5	−0.239	0.871
6	−0.255	0.907
7	−0.266	0.930
8	−0.275	0.945

TABLE II. a_2 and a_4 in $W(\vartheta)=1+a_2\cos^2\vartheta+a_4\cos^4\vartheta$ for $\gamma-\gamma$-correlations of types IIA and IIB.

	IIA		IIB	
L	a_2	a_4	a_2	a_4
1	−0.104	0	+0.428	0
2	+0.125	+0.042	0	−0.333
3	0.313	−0.021	−0.455	+0.121
4	0.452	−0.089	−0.721	0.433
5	0.556	−0.149	−0.888	0.640
6	0.635	−0.197	−1.000	0.784
7	0.698	−0.237	−1.079	0.889
8	0.749	−0.271	−1.138	0.969

TABLE III. a_2, a_4, a_6 in $W(\vartheta)$ for $\gamma-\gamma$-correlations of types IIIA and IIIB.

	IIIA			IIIB		
L	a_2	a_4	a_6	a_2	a_4	a_6
1	−0.176	0	0	+0.692	0	0
2	+0.313	−0.024	0	−0.439	0.156	0
3	0.554	−0.014	0.018	−0.441	0.176	−0.161
4	0.676	+0.045	0.001	−0.258	−0.129	−0.023
5	0.742	0.118	−0.030	−0.053	−0.522	+0.206
6	0.779	0.195	−0.067	0.135	−0.908	0.447
7	0.801	0.264	−0.102	0.302	−1.260	0.675

cases the second multipole is the only one allowed by the angular momentum selection rule while the first multipole is taken to be either the maximum allowed (IA, IIA, IIIA) or the next allowed (IB, IIB, IIIB), to cover the case when the former is forbidden by parity.

These tables confirm the expectation that $W(\vartheta)$ should become increasingly anisotropic with higher multipole γ-rays. It is noteworthy that none of these correlation functions can explain the "anomalous"[4] measured correlation in Rh^{106}.

(ii) The most extensive tabulation of angular correlation functions which has appeared so far is that of Gardner,[5] who treated e^--e^- correlations. Tables of equal completeness, i.e., applicable to transitions of any multipole orders, can also be gotten for the $\gamma-\gamma$-correlation. Indeed, one can also use the same methods of Racah,[6] which Gardner used, to obtain directly the $\gamma-\gamma$-correlation functions. However, this method, though very elegant and more wieldy than Hamilton's, would involve an unnecessary duplication of effort. For it has been shown[7] that once correlation functions have been tabulated for *any* successive particle emissions, these same tables may be used to obtain $W(\vartheta)$ for other particles. This theorem was applied in detail in reference 7, where Hamilton's $\gamma-\gamma$-tables were adapted to other correlation processes (such as $\beta-\gamma$, internal conversion$-\gamma$, $\alpha-\gamma$, etc.) which involved the same angular momenta. It thus appears that the most economical procedure for systematically obtaining $W(\vartheta)$ for higher angular momenta is to apply similarly the theorems of reference 7 to the tables already worked out by Gardner.

I should like to thank Miss Theresa Danielson, of the computing group at Brookhaven National Laboratory, for doing the numerical work.

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On the Decay of μ-Mesons

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RECENTLY Ogawa and Kamefuchi[1] called attention to an interesting mode of decay of μ-mesons in matter, *viz.*, the transmutation of a μ-meson into one electron (or positron) only. This should be possible e.g. in the coulomb field of a nucleus if we adopt the scheme $\mu\rightarrow e+\nu+\nu^\dagger$ with charge exchange[1,2] for the ordinary decay.

Clearly, the same process, with a closed neutrino loop, should then be possible also in the field of an electron.[3] But also the spontaneous decay $\mu\rightarrow e+\gamma$ should occur as a second-order process. With scalar or pseudoscalar coupling its probability w for a meson at rest turns out to be

$$w=(48/\pi)\cdot(e^2/\hbar c)\cdot(A/\mu^3c^3)^2\cdot w_0,$$

where $w_0=(g^2/48\pi^3)(\mu/2)^5(c^4/\hbar^7)$ is the probability of the ordinary first-order decay[4] and $A=\int p^2dp$ comes from the integration over the momentum space of the virtual neutrino. The mass m of the electron has been put equal to zero in the approximation. With cut-off momentum $p=\mu c$ the ratio $R=w/w_0=0.01$.

For μ^+-mesons, especially, we have in addition the annihilation processes $\mu^++e\rightarrow 2\gamma$ (third-order process with a closed neutrino loop) and $\mu^++e\rightarrow\nu+\nu^\dagger$ (or 2ν). The latter process (of the first order) is possible in all three of Wheeler's cases of the theory. As just such an event seems to be rather frequent in the electron sensitive emulsions,[5] we have calculated its probability, w', and the ratio, R', of w' to the corresponding probability of the ordinary spontaneous decay, with all five types of the coupling in each case.

For very slow mesons w' vanishes in the case of charge retention with scalar or pseudovector coupling in the approximation $m=0$. The optimum numerical value of R', for slow mesons, is provided by the theory of charge retention with pseudoscalar coupling and turns out to be (with $m=0$):

$$R'=192\pi^2(\hbar/\mu c)^3N.$$

Here N is the number of electrons per cm^3 of the medium. For $N\sim 10^{24}$/cm^3 the ratio R' is only of the order of magnitude 10^{-11}. Neither can the preparatory formation of the mesotronium yield significantly larger values for R'. The absence of the positron track at the end of some μ^+-meson tracks must therefore be accounted for in another way.[5]

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[3] This second event might also be observable (in spite of having presumably smaller probability than the first one) because it may be technically easier to detect in the photographic emulsion a splitting of the μ-meson track into two electron tracks than to find an unusually fast decay electron.
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A Note on Time Reversal and the Dirac Equation*

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IT has been known for some time that in the customary formulation of the one-particle Dirac theory the bilinear form $\psi^\dagger\psi(\psi^\dagger\equiv i\gamma^4\psi^*)$ is invariant under continuous Lorentz transformations and space inversion but not under time reflection.[1] For a properly quantized field theory, however, $\psi^\dagger\psi$ can be made invariant to time reflection.[2] This behavior causes no difficulty in the one electron theory [it is necessary, in fact, in order that the normalization integral $\int(\psi^\dagger\gamma^4\psi)dx^1dx^2dx^3$ be a scalar under the extended group]. It might be of consequence, however, for interaction with other spin $\frac{1}{2}$ particles (e.g., β-decay).

If the invariance of $\psi^\dagger\psi$ to the extended group is postulated in addition to the usual postulates of the Dirac theory, some generalization must be made. A possible generalization is to consider equations with all real elements in which ψ is an 8-rowed column vector, and the matrices 8×8. Motivation for this is the observation that the nonlinear operation[3] of complex conjugation, K_0, is adjointed to the Dirac matrix operations and plays an intimate role in the usual theory. By adjoining to the 4×4 Dirac matrices (multiplied by i if imaginary) the four matrices

$$I=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad K_0=\begin{pmatrix}1&0\\0&-1\end{pmatrix},\quad K_1=\begin{pmatrix}0&-1\\1&0\end{pmatrix},\quad K_2=\begin{pmatrix}0&1\\1&0\end{pmatrix},$$

we linearize all matrix operations. The algebra generated by this process has 64 elements, is irreducible, and is quite similar to the usual Dirac algebra. It is generated by the commutation relation:

$$\Gamma^\mu\Gamma^\nu+\Gamma^\nu\Gamma^\mu=2g^{\mu\nu}(\mu,\ \nu=0\cdots 5;\quad g^{\mu\nu}=\pm\delta^{\mu\nu},\quad -\text{ for }\mu=0,4,5;\quad +\text{ for }\mu=1,2,3).$$

It is found possible to maintain all the postulates leading to the Dirac theory plus the additional requirement of the invariance of $\psi^\dagger\psi$. The wave equation is

$$(\Gamma^\lambda\partial_\lambda+\kappa)\psi=0(\lambda=0\cdots 3;\quad \partial_\lambda=\partial/\partial x^\lambda;\quad x^0=t;\quad \hbar=c=1);$$

the adjoint equation is $\psi^\dagger(\Gamma^\lambda\partial_\lambda-\kappa)=0$ with the adjoint uniquely determined to be $\psi^\dagger=$ transpose $(\Gamma^0\Gamma^4\Gamma^5\psi)$. To every wave function ψ there correspond three "conjugate" functions generated by the operators $\Gamma^6\Gamma^4$, $\Gamma^6\Gamma^5$, and $\Gamma^4\Gamma^5(\Gamma^6\equiv\Gamma^0\Gamma^1\Gamma^2\Gamma^3\Gamma^4\Gamma^5)$. These conjugates are analogous to the usual charge conjugate solution; interactions with external fields distinguish among them. Three anti-commuting operators are available for interactions $\Gamma^4\Gamma^5$, Γ^4, Γ^5. The proper operator for vector interactions is $\Gamma^4\Gamma^5$, and with this we get gauge invariance and a continuity equation. Gauge invariance (of the first kind) is lost, however, if the operators Γ^4 and/or Γ^5 appear in the wave equation with nonzero mass. The spin of the particles is readily shown to be $\frac{1}{2}$.

The time reversal operator in this formulation is $\Gamma^0\Gamma^4$ (or $\Gamma^0\Gamma^5$ equivalently). In the representation where $\Gamma^4\Gamma^5$ corresponds to i (the usual Dirac formalism), the time reversal operator becomes $i\sigma_yK_0$.[3-5] The reversal in sign of $\psi^\dagger\psi$ under time reflection in the Dirac theory is therefore due to the incorrect identification of $\gamma^1\gamma^2\gamma^3$ as the time reversal operator.

Because transpose $(\psi^\dagger O\psi)=\psi^\dagger O\psi$, the expectation value of many operators is identically zero. In particular, we may have only the S, A, P covariants[5] with I (the identity); V, T with $\Gamma^4\Gamma^5$; and T, A with Γ^4 and Γ^5. As a result, the only nonvanishing vector is $j^\mu=\psi^\dagger\Gamma^4\Gamma^5\Gamma^\mu\psi(\mu=0\cdots 3)$. Since $\Gamma^4\Gamma^5$ is a scalar to all Lorentz transformations except time reversal, where it changes sign, the proper behavior for j^0 is obtained. Considering $\Gamma^4\Gamma^5$ as the charge operator, one is led to the conclusion that changing sign under time reversal is a basic property of charge.

Besides the Dirac equation the proposed generalization includes other equations describing spin $\frac{1}{2}$ particles, the significance of which is yet to be explored.

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Neutron Yield for the Li+T Reactions*

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A PRELIMINARY investigation of neutron production from bombardment of natural metallic lithium by tritons with energies between 0.25 and 2.10 Mev has been made using the Los Alamos 2.5-Mev electrostatic accelerator.

Targets were made by evaporating lithium on thin aluminum or tungsten disks which could be attached to the target end of the accelerating tube. Thicknesses of the lithium films were obtained by measuring the neutron yields from $Li^7(p,n)Be^7$ for 2-Mev protons and using a value[1] of 0.025 barn for the differential cross section at 0°. Targets were about $\sim$50 kev thick for 2-Mev protons.

Neutron yields were measured with a BF_3 "long counter"[2] placed about one meter from the target and at angles of 0° to 135°

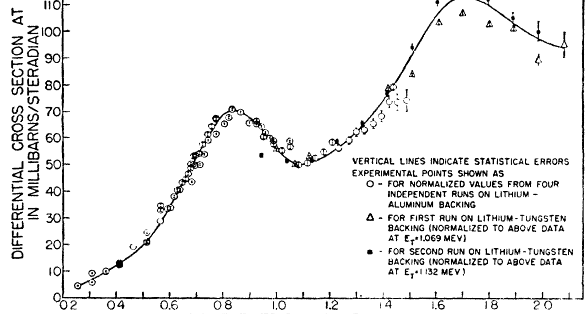

FIG. 1. Neutron yield at 0° in the laboratory system, expressed as a cross section, for the Li+T reactions.