Direct Photodisintegration Processes in Nuclei*

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The unexpectedly large (γ,p) cross sections found in many moderately heavy nuclei are interpreted in terms of a direct photodisintegration process. The gamma-ray quantum is supposed to be absorbed by one proton in the nucleus, which is then emitted without the formation of an intermediate compound nucleus. The cross section for this process is small compared with that for the formation of a compound nucleus (which would almost always lead to the eventual emission of a neutron), but it has to be only a few millibarns to account for the experimental results. It is shown in this paper that if the nucleus is represented by a square well potential, the calculated cross sections turn out to be smaller than the experimental ones, but much larger than those obtained from the statistical theory.

The angular distribution of the emitted protons should be of the form $A+B\sin^2\theta$, where θ is the angle between the incoming photon and the proton, and the ratio B/A depends on the angular momentum of the proton in the nucleus. This is confirmed by experimental results; from the experimental values of B/A it appears that protons with $l=0$ in the nucleus contribute considerably to the effect in Rh and Ag. Feenberg's "wine bottle" potential can account for this more easily than the square well.

The theory is applicable to fast photoneutron emission as well, and recent experiments by Poss show that fast photoneutrons are indeed emitted anisotropically.

It is suggested that similar direct interaction mechanisms may account for the deviations of (n,p) cross sections from those predicted by the theory, as found recently by Wäffler.

I. INTRODUCTION

ACCORDING to the statistical theory of nuclear reactions[1] neutrons or protons produced in nuclear reactions are "boiled off" from an excited compound nucleus produced by the absorption of the bombarding particle by the target nucleus. The energy distribution of the outgoing particles is governed mainly by the distribution of energy levels in the residual nucleus. Since the level density of the residual nucleus increases rapidly with increasing energy, it follows from statistical considerations that the outgoing nucleus will have something like a maxwell distribution centered about some "nuclear temperature" which is much less than the excitation energy; if the excitation energy is in the region of 10 to 20 Mev, the most probable energy of the emitted particle is only a few Mev.

At these energies the emission of protons is severely inhibited, in all but the lightest nuclei, by the coulomb potential barrier which the proton must penetrate. As a result, the emission of protons may be expected to be much less likely than that of neutrons. For reactions in medium weight nuclei ($A\sim100$) produced by 17.6-Mev gamma-rays from $Li^7(p,\gamma)$ the ratio of proton to neutron emission computed on the basis of this theory turns out to be between 10^{-3} and 10^{-5}.

This prediction is not always confirmed experimentally. Hirzel and Wäffler[2] found that in a number of nuclei with A near 100 the (γ,p) to (γ,n) ratios were consistently several percent, 20 to 1000 times higher than expected from the statistical theory. Schiff[3] tries to explain this by modifying the statistical theory, assuming that the effective level density in the residual nucleus increases much less rapidly with energy than Weisskopf[1] assumed. On the other hand, the author[4] and Jensen[5] attempt to explain the result by postulating that the process is not an evaporation process at all but a direct photoelectric emission process in which the photon interacts directly with one of the protons in the nucleus and gives it enough energy to overcome the potential barrier without the formation of an intermediate "compound nucleus" state. The cross section for this direct process has to be only a few percent of the cross section for formation of a compound nucleus in order to account for the observed (γ,p) cross section. In this paper we shall attempt to show that this is the case.

The angular distribution of the emitted protons should furnish clues to the mechanism of the process. For the case of direct photoelectric emission we expect a maximum of the cross section at 90°. On the other hand, in the evaporation model we expect isotropic distributions, although other angular distributions are also possible.[6] Experimentally, Curtis, Hornbostel, Lee, and Salant[7] and Diven and Almy[8] have found high energy photoprotons produced preferentially at 90°, while Price and Kerst[9] found that photoneutrons are produced isotropically. This lends support to the hypothesis that photoneutrons are generally produced by evaporation while at least a substantial portion of the photoprotons are produced by direct interaction.

II. DIRECT ABSORPTION CROSS SECTIONS

We compute the absorption cross section on the basis of the following model: We describe the nucleus

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[1] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937); V. F. Weisskopf and D. H. Ewing, Phys. Rev. **57**, 472 (1940).
[2] O. Hirzel and H. Wäffler, Helv. Phys. Acta **20**, 373 (1947).
[3] L. I. Schiff, Phys. Rev. **73**, 1311 (1948).
[4] E. D. Courant, Phys. Rev. **74**, 1226A (1948).
[5] P. Jensen, Naturwiss. **35**, 190 (1948).
[6] L. Wolfenstein, Phys. Rev. **78**, 322A (1950).
[7] Curtis, Hornbostel, Lee, and Salant, Phys. Rev. **77**, 290 (1950).
[8] B. C. Diven and G. M. Almy, Phys. Rev. **80**, 407 (1950).
[9] G. A. Price and D. W. Kerst, Phys. Rev. **77**, 806 (1950).

as a square well with Z protons filling the Z lowest states. The interaction between nucleons is supposed to be replaced entirely by this square well potential; thus, each nucleon has its own independent wave function. This approximation is justifiable because, owing to the Pauli principle, the mean free path of a nucleon in nuclear matter is large compared to the nuclear radius in spite of the large free nucleon-nucleon cross sections.[10] We compute the photoelectric absorption matrix element between this state and a final state in which one of the protons is raised into the continuum while the others are undisturbed. For ease of computation we first neglect the coulomb barrier and then multiply the resulting cross section by the barrier penetration probability.

This method is predicated on the assumption that the independent particle approximation is valid for the final state as well as the initial state. The argument for this is much weaker than that for the ground state,[10] since the Pauli principle no longer forbids all collisions. However, a quantitative comparison with the theory of Goldberger[11] shows that the mean free path of a proton of energy about 10 Mev (kinetic energy about 30 to 40 Mev inside the nucleus) in nuclear matter is at least of the order of, and probably somewhat larger than the nuclear diameter. This calculation should, therefore, lead to results of the right order of magnitude.

Our model implies that the entire photon absorption cross section is due to a combination of absorption of individual neutrons and protons. The proton emission cross section is a small part of this, because when a proton absorbs a photon it may not only be emitted, but it may instead radiate (especially for low energy photons) or collide with other nucleons, owing to the presence of nucleon-nucleon interaction. The latter process will result in the formation of a compound nucleus with subsequent decay as expected from the statistical model.

It is not expected that the shape of the effective nuclear potential will influence the order of magnitude of the results. The square well is chosen for ease of calculation.

If the nuclear radius is R and the depth of the well W, the initial wave function is

$$\begin{aligned}\psi_0 &= A_l j_l(ar)\Upsilon_l^m(\theta,\phi), && r<R\\ &= A_l\frac{j_l(aR)}{k_l(bR)}k_l(br)\Upsilon_l^m(\theta,\phi), && r>R,\end{aligned}$$

where $a^2=(2M/\hbar^2)(W-|E_0|)$; $b^2=2M|E_0|/\hbar^2$; the eigenvalue E_0 is determined by the continuity condition

$$-bk_{l-1}(bR)/k_l(bR)=aj_{l-1}(aR)/j_l(aR);$$

$\Upsilon_l^m(\theta,\phi)$ are the spherical harmonics normalized so as to make $\int|\Upsilon_l^m(\theta,\phi)|^2d\Omega=1$; A_l is determined by the normalization condition $\int|\psi|^2dV=1$, where the integral is taken over all space, and we employ the "spherical bessel functions"

$$j_l(x)=(\pi/2x)^{\frac{1}{2}}J_{l+\frac{1}{2}}(x),\quad n_l(x)=(-1)^{l+1}(\pi/2x)^{\frac{1}{2}}J_{-l-\frac{1}{2}}(x),$$
$$k_l(x)=(2/\pi x)^{\frac{1}{2}}K_{l+\frac{1}{2}}(x).$$

The final state wave function is of the form

$$\begin{aligned}\psi_1 &= C_{l'}j_{l'}(cr)\Upsilon_{l'}^{m'}(\theta,\phi), && r<R,\\ &= C_{l'}\frac{j_{l'}(cR)}{z_{l'}(dR)}z_{l'}(dr),\Upsilon_{l'}^{m'}(\theta,\phi), && r>R,\end{aligned}$$

where $c^2=a^2+2M\hbar\omega/\hbar^2$, $d^2=-b^2+2M\hbar\omega/\hbar^2$, $\hbar\omega$ is the energy of the absorbed quantum, and $z_{l'}(dr)$ is that particular solution of the radial wave equation which satisfies the continuity condition

$$dz_{l'-1}(dR)/z_{l'}(dR)=cj_{l'-1}(cR)/j_{l'}(cR).$$

The normalization constant $C_{l'}$ is chosen such that $r\phi_1$ is a sine wave of unit amplitude at infinity, where ϕ_1 is the radial part of the wave function.

Let us confine ourselves to light incident in the z-direction and circularly polarized in the $(x+iy)$ direction. Cross sections and angular distributions for $(x-iy)$ polarization will be the same. The cross section for the dipole absorption of a photon of energy $\hbar\omega$ is then

$$\sigma=\frac{e'^2}{\hbar c}\frac{4\pi M\omega}{\hbar d}\left|\int\psi_0\left(\frac{x+iy}{\sqrt{2}}\right)\psi_1 dV\right|^2. \tag{1}$$

The effective charge e' here is eN/A because of the motion of the center of mass of the nucleus.[12] For the same reason dipole absorption by a neutron is possible, with the neutron having the effective charge $-eZ/A$.

Integration over the angles yields the angular momentum selection rules $m'=m-1$, $l'=l\pm1$:

$$(l,m|x+iy|l+1,m-1)=\left[\frac{(l-m+1)(l-m+2)}{(2l+1)(2l+3)}\right]^{\frac{1}{2}}(l|r|l+1), \tag{2}$$

$$(l,m|x+iy|l-1,m-1)=-\left[\frac{(l+m-1)(l+m)}{(2l-1)(2l+1)}\right]^{\frac{1}{2}}(l|r|l-1). \tag{3}$$

Since all values of m between $-l$ and $+l$ are equally probable, we must average over m, obtaining

$$\sigma_{l\to l+1}=S\left[\int_0^\infty r^3\phi_0\phi_1dr\right]^2\tfrac{2}{3}(l+1)/(2l+1), \tag{4}$$

$$\sigma_{l\to l-1}=S\left[\int_0^\infty r^3\phi_0\phi_1dr\right]^2\tfrac{2}{3}l/(2l+1), \tag{5}$$

where $S=(e'^2/\hbar c)(2\pi M\hbar\omega/\hbar^2 d)$.

[10] V. F. Weisskopf, Helv. Phys. Acta **23**, 187 (1950).
[11] M. Goldberger, Phys. Rev. **74**, 1267 (1948).
[12] H. A. Bethe, Revs. Modern Phys. **9**, 221 (1937).

Carrying out the integrations for the matrix elements and normalization constants (see Appendix), with due regard for the continuity conditions at $r=R$, we finally obtain the cross section for one proton of angular momentum l:

$$\sigma_l=\frac{16\pi}{3}\frac{e'^2}{\hbar c}\frac{b^2(a^2+b^2)R}{(c^2-a^2)^3d}\cdot\frac{j_l^2(aR)}{-j_{l-1}(aR)j_{l+1}(aR)}\times\left\{\frac{l}{2l+1}C_{l-1}^2j_{l-1}^2(cR)+\frac{l+1}{2l+1}C_{l+1}^2j_{l-1}^2(cR)^2\right\} \quad (6)$$

with

$$C_l^2=(1/R^2)\{[dRj_{l-1}(dR)j_l(cR)-cRj_l(dR)j_{l-1}(cR)]^2+[dRn_{l-1}(dR)j_l(cR)-cRn_l(dR)j_{l-1}(cR)]^2\}^{-1}.$$

This expression may be further simplified by replacing the bessel functions by their asymptotic approximations. The ratio $[j_l(aR)]^2/[-j_{l+1}(aR)j_{l+1}(aR)]$ then equals a^2/b^2 because of the continuity conditions at $r=R$, and the last factor in (6) becomes

$$\frac{d^2\sin^2(cR+\alpha)}{R^2[c^2-(c^2-d^2)\sin^2(cR+\alpha)]}, \quad (7)$$

where α is a phase angle. This is an oscillating function; since the phase of cR is random, we may average over the phase α, obtaining

$$d/[R^2(c+d)]. \quad (8)$$

It should be noted however, that the phase of $(cR+\alpha)$ in (7) changes rather slowly with energy, so that for a particular transition the value of (7) may differ considerably from (8) over an energy interval of several Mev. This may lead to large variations in the cross section from one element to another and from one energy level to another.

If we nevertheless adopt the average (8), we obtain the following approximate expression for the cross section for one proton:

$$\sigma=\frac{16\pi}{3}\frac{e'^2}{\hbar c}\frac{a^2(a^2+b^2)}{(c^2-a^2)^3(c+d)R}=\frac{16\pi}{3}\frac{e^2}{\hbar c}\left(1-\frac{Z}{A}\right)^2\times\frac{TW}{(\hbar\omega)^3[(\hbar\omega+T)^{\frac{1}{2}}+(\hbar\omega+T-W)^{\frac{1}{2}}]}\left(\frac{\hbar^2}{2MR^2}\right)^{\frac{1}{2}}R^2. \quad (9)$$

Here T is the kinetic energy of the proton in the nucleus and W is the depth of the nuclear well. This expression is evidently valid only if the asymptotic expressions for the bessel functions are valid, i.e., if aR, bR, cR, and dR are all greater than l. This requirement is fairly well satisfied, in medium weight nuclei, for those protons whose energy is above or near the top of the coulomb barrier.

TABLE I. Experimental and theoretical (γ,p) cross sections for 17.6-Mev gamma-rays.

Nucleus	σ_{exp} (millibarns)	σ_{theor}	$\sigma_{\alpha\ part}$	$\sigma_{statist}$
$_{34}Se^{77}$	4.8	0.35	1.0	0.14
$_{42}Mo^{98}$	3.5	0.26	0.75	0.12
$_{46}Pd^{106}$	7.3	0.25	0.75	0.07
$_{48}Cd^{111}$	4.4	0.24	0.72	0.026
$_{48}Cd^{112}$	5.3	0.24	0.72	0.09
$_{48}Cd^{113}$	6.0	0.24	0.72	0.05
$_{50}Sn^{117}$	2.9	0.23	0.70	0.013
$_{50}Sn^{118}$	1.5	0.23	0.70	0.017

The actual cross section is further reduced by the coulomb barrier. This may be taken into account by using Weisskopf's[13] tables of penetration cross sections; the penetration probability for a proton of given energy is taken to be Weisskopf's penetration cross section divided by πR^2. Assuming a Fermi distribution inside the nucleus, and taking $\hbar\omega=17.6$ Mev, $T_{max}=22$ Mev, $W=30$ Mev (binding energy 8 Mev), $R=1.5A^{\frac{1}{3}}\times10^{-13}$ cm, we obtain for the total cross section

$$\sigma=0.044(Z/A^{\frac{1}{3}})^2\int_0^{x_1}Pdx, \quad (10)$$

where P is the penetration probability, x is the energy of the outgoing protons measured in units of the barrier height, and x_1 is the maximum energy of the outgoing protons.

In Table I we compare the experimental cross sections found by Hirzel and Wäffler to cross sections computed from (10), cross sections from the alpha-particle model (see below), and cross sections from the statistical theory assuming neutron binding energies of 7.5 Mev, as computed by Hirzel and Wäffler. The "experimental" cross sections were obtained from Hirzel and Wäffler's work, together with the absolute value of 100 millibarns for the $Cu^{63}(\gamma,n)$ reaction at 17.6 Mev recently obtained by Diven and Almy.[8]

The experimental cross sections vary considerably from nucleus to nucleus. This may be ascribed partly to the fluctuations in the factor (7), to variations in the proton binding energy, and to variations in the proton energy distribution near the top of the distribution.

In addition, the theoretical cross sections are consistently smaller than the experimental ones, although the agreement is much better than for the statistical theory.

This disagreement with experiment is not surprising, in view of the crudity of the model used. In the first place, the individual particle model is only an approximation, especially for the final state. The actual wave functions will therefore be considerably more com-

[13] V. F. Weisskopf, MDDC-1175, *Lecture Series in Nuclear Physics* (U. S. Government Printing Office, 1947), p. 105.

plicated than those taken here and may well lead to different overlap integrals. Furthermore, even if the single particle model is accepted, the best well may be different from the one taken here, and may in fact be different for the initial and final states. Finally, the method used for estimating the effect of the coulomb barrier—which is the critical factor in reducing proton emission below neutron emission—may lead to errors; it would be better to take exact coulomb wave functions.

The dependence of the cross section on the well depth for a square well can be inferred from (9). The cross section for high energy photons increases with well depth, not only because of the factor TW but also because a deeper well requires a smaller radius in order that the top of the Fermi distribution remain at the binding energy. On the other hand, as the well depth increases, so does the barrier height (owing to the decrease in radius), leading to decreased penetrability; the net result is that for the square well the calculated (γ,p) cross section is roughly independent of well depth. However, if the shape of the well is changed, the cross section will change. Specifically, if the assumed potential inside the nucleus is less smooth than the square well, the wave functions will be less similar to those of free particles and the matrix elements may be expected to be larger. In addition, the energy level distribution will favor large internal kinetic energies more strongly than the $T^{\frac{1}{2}}$ distribution. Thus, if the square well is replaced by the "wine bottle" potential suggested by Feenberg and Hammack,[14] the theoretical cross sections are increased. A detailed calculation (see Appendix) shows that the cross section per proton may be as much as 4 times greater when an infinitely high central elevation is assumed for $0<r<R/4$, which leads to cross sections in better agreement with experiment.

Additional support for the central elevation model comes from the angular distribution of the photoprotons from Rh and Ag.[7,8] As we shall see shortly, the observed angular distributions in these elements indicates that s-protons contribute considerably to the total (γ,p) cross sections, which means that an s shell, presumably the $2s$ shell, lies near the top of the distribution for Z just below 50. This agrees with the predictions of the central elevation model but not with the simple square well which is the basis of Mayer's[15] nuclear shell model.

The agreement between the calculated and observed total cross sections is also improved if an alpha-particle model is assumed for the nucleus. In the alpha-particle model the radius R in the expression for the cross section per proton should be taken to be the radius of the alpha-particle. If we make this assumption and assume at the same time that the energy distribution in the nucleus is as in the Fermi gas model we obtain the cross sections in the fourth column of Table I.

[14] E. Feenberg and K. C. Hammack, Phys. Rev. **75**, 1877 (1949).
[15] M. G. Mayer, Phys. Rev. **78**, 16 (1950).

The alpha-particle model is also supported by the calculations of Levinger and Bethe,[16] who find that something like an alpha-particle model can account for the behavior of the total photodisintegration cross section as a function of energy much better than can the Fermi gas model. Furthermore, Cheston and Goldfarb and Tamor[17] find that the energy spectrum of protons produced by nuclear capture of π-mesons is most satisfactorily explained by means of a mechanism based on the alpha-particle model.

We have also neglected the existence of exchange forces, which tend to increase the cross sections by factors of the order of 1.5 to 2.[16]

The remaining discrepancy, if any, may be removed by assuming a mechanism intermediate between that of direct one-step interaction as assumed here and evaporation from the nucleus as a whole, namely, "local heating" of a portion of the nucleus and evaporation from the heated region before the excitation spreads over the whole nucleus.[18]

III. ANGULAR DISTRIBUTION

Irrespective of the detailed shape of the nuclear potential the angular distribution of the protons will depend only on the angular momentum of the proton before and after the absorption. For a transition $l\rightarrow l+1$ it will be proportional to

$$\sum_{m=-l-1}^{l+1}[l(l+1)+m^2]|\Theta_{l+1}{}^m(\theta,\phi)|^2 = l(l+1)+\tfrac{1}{2}(l+1)(l+2)\sin^2\theta;$$

for $l\rightarrow l-1$, to

$$\sum_{m=-l+1}^{l-1}[l(l+1)+m^2]|\Theta_{l-1}{}^m(\theta,\phi)|^2 = l(l+1)+\tfrac{1}{2}l(l-1)\sin^2\theta.$$

In general, the angular distribution should, therefore, be of the form $A+B\sin^2\theta$. The ratio B/A depends on the particular angular momentum of the protons involved. In particular, if $B/A>1.5$, i.e., if the intensity at 90° is more than 2.5 times that at 0°, the reaction must necessarily involve at least some protons making $l=0\rightarrow l=1$ transitions, since $B/A\leqslant 1.5$ for $l\geqslant 1$. If $B/A<1.5$, it is more difficult to draw any conclusions regarding the angular momenta, since for $l\geqslant 1$ the values of B/A may range anywhere between $\frac{1}{2}(l-1)/(l+1)$ and $\frac{1}{2}(l+2)/l$, which are rather wide limits for all angular momenta ($l\leqslant 5$) likely to exist inside a nucleus.

Both Curtis *et al.*[7] and Diven and Almy[8] find $B/A>1.5$, for the highest energy protons ejected from Rh and Ag, respectively. It follows, as remarked before, that an s-shell is at or near the top in the region $Z\approx 45$.

An angular distribution of the form $A+B\sin^2\theta$ is also

[16] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[17] S. Tamor, Phys. Rev. **77**, 412 (1950); W. B. Cheston and L. J. B. Goldfarb, Phys. Rev. **78**, 682 (1950).
[18] L. Rosenfeld, *Nuclear Forces* (Interscience Publishers, New York, 1948), p. 198.

consistent with the alpha-particle model. For an alpha-particle at rest a pure $\sin^2\theta$ distribution is to be expected; this will be modified by the internal motion of the alpha-particle inside the nucleus.

We have assumed here that we are dealing with pure dipole transitions. Quadrupole transitions would modify the angular distribution. However, Levinger and Bethe[16] have shown that in the energy range considered dipole absorption must be predominant; quadrupole effects can only account for a small fraction of the total cross section.

IV. DISCUSSION

The values obtained here for $(\gamma-p)$ cross sections agree fairly well with those obtained experimentally by Hirzel and Wäffler[2] at a γ-ray energy of 17.6 Mev. However, the experiments on the energy spectrum of the protons, performed with betatron x-rays, show more low energy protons than expected from our direct interaction mechanism. In particular, Diven and Almy[8] found with 21-Mev x-rays on Ag that the energy distribution and absolute yield (or rather yield relative to the neutron yield) agreed quite well with the statistical theory except near the upper energy limit, where more protons were found than expected statistically. The Rh energy spectrum found by the Brookhaven group[7] is similar, so that the same conclusion applies.

It should be noted, however, that Rh and Ag are odd Z-even N nuclei, leading to stable even-even end products; the proton binding energy is therefore relatively low (about 6 Mev). On the other hand, Hirzel and Wäffler were necessarily restricted by their activation method to nuclei which would lead to radioactive products; the proton binding energies in those cases are considerably higher (8–9 Mev). Therefore, it is not surprising that in the cases of Ag and Rh the statistical cross section is comparable to or even larger than the direct cross section, while in the cases studied by Hirzel and Wäffler the statistical cross section is negligible.

The betatron results are thus explained by postulating that the low energy protons are produced primarily by nuclear evaporation, while the high energy tail is caused by the direct photoelectric effect discussed in this paper. This view is supported by the angular distribution, which is nearly (but not quite, in the case of Rh) isotropic for low energy protons and markedly anisotropic for the high energy protons. It would be desirable to repeat the angular and energy distribution experiments with some of the elements bombarded by Hirzel and Wäffler; if our view is correct, the energy distribution should be markedly shifted towards the high energy end and the angular distribution should be markedly anisotropic at all energies.

It would also be of interest to perform the energy and angular distribution experiments with monochromatic gamma-rays such as those obtained from the $Li^7(p,\gamma)$ or $H^3(p,\gamma)$ reactions. We would expect discrete lines each with its own angular distribution characteristic of the angular momentum of the corresponding nuclear shell.

Our theory applies to neutron emission as well as to proton emission, with the neutron having an effective charge $-eZ/A$. It is, therefore, to be expected that the photoneutron spectrum will contain, in addition to the relatively low energy neutrons from evaporation, a high energy "tail" containing a few percent of all the neutrons emitted, with an anisotropic distribution of the high energy neutrons. This is confirmed by an experiment by Poss,[19] who irradiated Bi, Pb, and W with 20-Mev x-rays and found high energy neutrons (>4 Mev) with an intensity ratio $I(90°)/I(0°)\approx 2$. Similar results have also been obtained at the University of Illinois.[20] On the other hand, Price and Kerst[9] found that if all photoneutrons are detected, the angular distribution is isotropic, which means that the anisotropic high energy tail can only represent a small fraction of all the photoneutrons produced.

Our conclusions may be compared with Schiff's theory.[3] Both theories lead to γ,p cross sections and (γ,p) to (γ,n) ratios of the right order of magnitude. But Schiff predicts a rather different energy distribution; his energy distribution, both for protons and for neutrons, should be much like the statistical one but shifted towards higher energies. Diven and Almy[8] compared their energy distribution with Schiff's theory and found that the low and intermediate energy yields were lower than predicted by Schiff's theory, agreeing much better with Weisskopf's statistical theory, while the high energy tail appeared to be near that predicted by Schiff. The anisotropy of the high energy protons is also most easily explained by the direct interaction theory, although, as Schiff has pointed out,[21] his theory does not necessarily require isotropy.

Another test of these competing theories is afforded by the study of other reactions involving proton emission. Schiff's theory is based on the supposition that photon excitation can lead only to a restricted subset of all the states of the compound nucleus. This feature is supposed to be peculiar to photon excitation and not to apply to particle bombardment. Accordingly, the competition between proton and neutron emission following particle bombardment should be governed by Weisskopf's statistical theory. On the other hand, the direct interaction mechanism should work with particle bombardment as well as with photon bombardment.[22] Thus, an incident neutron or proton could knock a proton out of the nucleus without greatly perturbing the residual nucleus (as is known to happen in the 100-Mev range). There is evidence that this is the case. Wäffler[23] finds that $n-p$ cross sections in elements between Fe and La, with neutrons having maximum energies of about 14

[19] H. L. Poss, Phys. Rev. **79**, 539 (1950).
[20] Private communication from M. Goldhaber.
[21] L. I. Schiff, private communication.
[22] This possibility has also been suggested by Weisskopf (reference 10).
[23] H. Wäffler, Helv. Phys. Acta **23**, 238 (1950).

Mev, are also higher than predicted by the statistical theory, though not by factors as large as for γ–p reactions. On the other hand, he finds that n–$2n$ cross sections agree fairly well with statistical theory; thus, a modification of the level density in the statistical theory, such as proposed by Schiff, could lead to agreement with n–p cross sections only at the expense of the agreement with n–$2n$ cross section. The most likely explanation would seem to be a direct interaction between the incident neutron and a proton of the nucleus. It might be desirable to check this hypothesis by investigating the energy distribution of the protons. Gugelot[24] has measured the neutron spectra from reactions induced by 16-Mev protons and obtains good agreement with a statistical theory except at the high energy end of the neutron spectrum, where he finds somewhat more neutrons than expected. This may again be accounted for by the direct interaction hypothesis.

A modified direct interaction mechanism is also called for by the experiments on meson absorption by nuclei as interpreted by Tamor.[17]

In the case of alpha-particle bombardment, on the other hand, direct energy transfer to one nucleon would appear difficult. Therefore, (α,p) to (α,n) ratios should be as predicted by the statistical theory. It would be interesting to check this experimentally.

It would thus seem that, for nuclear excitation energies of the order of 20 Mev or more, reactions can proceed by direct interaction as well as by way of an intermediate compound nucleus. This does not affect the total cross section or the cross sections for the most prevalent modes of disintegration to any large degree, but the probabilities of rarer processes such as proton emission may be greatly enhanced by the direct mechanism.

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APPENDIX

(a) Square Well

The normalization constant A_l is determined from the condition

$$A_l^2\left[\int_0^R r^2 j_l^2(ar)dr+\{j_l^2(aR)/k_l^2(bR)\}\int_R^\infty r^2k_l^2(br)dr\right]=1. \quad \text{(A1)}$$

Carrying out the integrations and making use of the continuity conditions at $r=R$, we obtain

$$A_l^2=\frac{2b^2}{(a^2+b^2)R^3}\frac{1}{[-j_{l-1}(aR)j_{l+1}(aR)]}. \quad \text{(A2)}$$

The constant C_l is determined from the conditions

$$\begin{aligned}C_lj_l(cR)&=d[\alpha j_l(dR)+\beta n_l(dR)],\\ cC_lj_{l-1}(cR)&=d^2[\alpha j_{l-1}(dR)+\beta n_{l-1}(dR)]\end{aligned} \quad \text{(A3)}$$

[24] P. C. Gugelot, Phys. Rev. **81**, 51 (1951).

with $\alpha^2+\beta^2=1$. Solving for α, β, and C_l we obtain

$$C_l=R^{-2}\{[dj_l(cR)j_{l-1}(dR)-cj_{l-1}(cR)j_l(dR)]^2 +[dj_l(cR)n_{l-1}(dR)-cj_{l-1}(cR)n_l(dR)]^2\}^{-\frac{1}{2}}. \quad \text{(A4)}$$

The matrix element

$$(0|r|1)=\int_0^\infty r^3\phi_0\phi_1 dr$$

is most easily evaluated by noting that the equation of motion

$$M\ddot{\mathbf{r}}=-\nabla V \quad \text{(A5)}$$

holds as a matrix equation. Thus, we have

$$(0|\mathbf{r}|1)=-(1/\omega^2)(0|\ddot{\mathbf{r}}|1)=(1/M\omega^2)(0|\nabla V|1). \quad \text{(A6)}$$

In our case $\nabla V=(\mathbf{r}/r)W\delta(r-R)$, so that we have, for the radial part of the matrix element,

$$\begin{aligned}(0|r|1)&=(WR^2/M\omega^2)\phi_0(R)\phi_1(R)\\ &=(WR^2/M\omega^2)A_lC_{l'}j_l(aR)j_{l'}(cR),\end{aligned} \quad \text{(A7)}$$

which leads to the expression (6) for the cross section.

(b) Central Elevation

In the case of an infinite central elevation extending to $r=R_0$ the wave functions ϕ_0 and ϕ_1 inside the nucleus take the form

$$\begin{aligned}\phi_0&=A_l[\alpha_0j_l(ar)+\beta_0n_l(ar)] \quad R_0<r<R,\\ \phi_1&=C_{l'}[\alpha_1j_{l'}(cr)+\beta_1n_{l'}(cr)] \quad R_0<r<R,\end{aligned}$$

where α_0, α_1, β_0, and β_1 must be chosen so as to make $\phi_0=\phi_1=0$ for $r=R_0$. For convenience, let us write $\alpha_0j_l(ar)+\beta_0n_l(ar)=y_l(ar)$; $\alpha_1j_{l'}(cr)+\beta_1n_{l'}(cr)=y_{l'}(cr)$. Then the normalization condition gives

$$\tfrac{1}{2}A_l^2R^3[-(a^2+b^2)b^{-2}y_{l-1}(ar)y_{l+1}(aR) +(R_0/R)^3y_{l-1}(aR_0)y_{l+1}(aR_0)]=1. \quad \text{(A8)}$$

If the asymptotic expressions for the bessel functions are valid, i.e., if $aR_0>l$, we have

$$\begin{aligned}y_{l-1}(aR)y_{l+1}(aR)&=-b^2/[(a^2+b^2)(aR)^2],\\ y_{l-1}(aR_0)y_{l+1}(aR_0)&=-1/(aR_0)^2\end{aligned}$$

so that

$$A_l^2=2a^2/(R-R_0). \quad \text{(A9)}$$

The expressions for C_l are the same as in the case of the square well, except that $j_l(cR)$, $j_{l-1}(cR)$ must be replaced by $y_l(cR)$, $y_{l-1}(cR)$.

In computing the matrix element the formula (A7) must be modified by the addition of a term arising from the infinite potential jump at $r=R_0$. This is most easily done by first considering a finite jump W_0 and then letting W_0 tend to infinity. We have, then,

$$(0|r|1)=(1/M\omega^2)[WR^2\phi_0(R)\phi_1(R)-W_0R_0^2\phi_0(R_0)\phi_1(R_0)]. \quad \text{(A10)}$$

As W_0 tends to infinity, $\phi_0(R_0)$ and $\phi_1(R_0)$ tend to zero, and we must evaluate the limit of the product. From the continuity conditions at $r=R_0$ we have, for large W_0,

$$\phi_0'(R_0)/\phi_0(R_0)\approx\phi_1'(R_0)/\phi_1(R_0)\approx(2MW_0/\hbar^2)^{\frac{1}{2}}, \quad \text{(A11)}$$

so that

$$\lim_{W_0\to\infty} W_0R_0^2\phi_0(R_0)\phi_1(R_0)=(\hbar^2/2M)R_0^2\phi_0'(R_0)\phi_1'(R_0) \quad \text{(A12)}$$

and

$$(0|r|1)=(WR^2A_lC_{l'}/M\omega^2)[y_l(aR)y_{l'}(cR) -(\hbar^2/2MR^2W)R^2acy_l'(aR_0)y_{l'}'(cR_0)]. \quad \text{(A13)}$$

If asymptotic formulas hold throughout, $y_l(aR)=(a^2+b^2)^{-\frac{1}{2}}R^{-1}$, $y_{l'}(cR)=[\sin(cR+\alpha)]/cR$, $y_l'(aR_0)=1/aR$, $y_{l'}'(cR_0)=1/cR_0$, and

$$(0|r|1)=\frac{WA_lC_{l'}}{M\omega^2}\left[\frac{\sin(cR+\alpha)}{c(a^2+b^2)^{\frac{1}{2}}}-\frac{\hbar^2}{2MW}\right] \quad \text{(A14)}$$

with

$$C_{l'}=cd[c^2-(c^2-d^2)\sin^2(cR+\alpha)]^{-\frac{1}{2}}, \quad \text{(A15)}$$

where α is a phase angle. Squaring and averaging over the phase angle as before we find

$$\langle |(0|r|1)|^2\rangle_{Av}=\frac{W^2A^2}{M^2\omega^4}\frac{c^2d^2R^2}{c^2(a^2+b^2)}\left[\frac{1}{d(c+d)}-\frac{\gamma^2}{cd}\right], \qquad \text{(A16)}$$

where $\gamma^2=c^2/(a^2+b^2)$.

Thus, the cross section is increased over that for the square well by the factor

$$[R/(R-R_0)][1+c(c+d)/(a^2+b^2)]. \qquad \text{(A17)}$$

To compare the modified cross sections with experiment, we must choose a value for R_0. We choose $R_0=R/4$, since then the 2s level is pushed up near the top of the distribution for Z near 45, as required by the angular distributions of Rh and Ag photoprotons. For this value of R_0 the depth of the well need not be increased substantially over that of the square well. We then find, for the parameters chosen in the text, an increase in the cross section by a factor of 3.7 as compared with (9). This asymptotic estimate is, however, good only for $l=0$ or 1, since it is predicated on the assumption $aR_0>l$. For higher values of l the change in the cross section will be less pronounced, since the wave functions do not differ as much from those for the square well.

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Photoprotons from Magnesium

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The distributions in energy and in angle of the protons ejected from magnesium by the bremsstrahlung x-rays of a 24-Mev betatron have been observed in nuclear emulsions. Both normal magnesium and enriched Mg^{25} show a uniform angular distribution for all proton energies. The energy distribution of the protons from enriched Mg^{25} is compared to the distributions calculated from an evaporation process using various energy level densities. Good agreement is obtained using the known Na^{24} level densities. The integrated cross section of the $Mg^{25}(\gamma,p)$ reaction is determined as 0.056 ± 0.03 Mev-barn.

I. INTRODUCTION

Early measurements[1] of the ratio of (γ,p) to (γ,n) cross sections have indicated larger values than expected from the statistical theory of nuclei.[2] The observation of a broad resonance in the photoexcitation curves was also surprising.[3–5] The explanations[6,7] which were advanced for these phenomena seemed to differ in the angular and energy distributions to be expected of the ejected protons. Consequently, this investigation was initiated to examine such characteristics of the photoprotons in the hope that they would help determine the predominant mechanisms. Magnesium was chosen as the first element to be investigated since its photoproton yield was known to be appreciable.[1,5] Initial results on the angular distribution of photoprotons from normal magnesium indicated no appreciable direct photoelectric effects.[8] Enriched magnesium was secured from the AEC, and further work was undertaken to refine the observations. Meanwhile other experimental data[9,10] and theoretical calculations[11–15] have been published.

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II. APPARATUS

The x-ray beam from an Allis-Chalmers betatron is defined, as shown in Fig. 1, by a collimator which has about 16 in. of lead plus 16 in. of steel and lead with a magnetic field to remove the electrons. The x-ray beam which emerges from the collimator is $\frac{1}{4}$ in. in diameter and is diverging at an angle of only eleven minutes. The shielding afforded by the collimator is at least a factor of 10^4 between the beam intensity and the stray radiation. A concrete wall 16 to 24 in. thick surrounds the collimator to reduce the neutron background. Under normal running conditions, the beam intensity at the camera is about 25 roentgens/min as measured by a Victoreen and has passed through 0.7 cm of

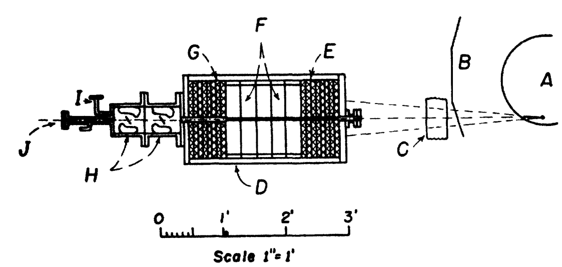

Fig. 1. Experimental arrangement showing A—betatron doughnut, B—coil box, C—monitor, D—collimator, E and G—lead collimating disks, F—steel and lead disks holding Alnico magnets, H—double camera, I—to vacuum pump, and J—window.

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