for body- and face-centered cubic lattices. In the present note a general method of treatment is outlined which introduces the short range order parameters in a particularly straightforward way. For this purpose we introduce a parameter $\mu_k{}^L$ which has the value $+1$ if an A atom is at the kth site in the Lth cell, and the value -1 if a B atom is at this site. This is a device similar to that used in the statistics of the Ising lattice.[2] The atomic structure factor at the k, L site can then be represented as

$$f_k{}^L=\tfrac{1}{2}(f_A+f_B)+\tfrac{1}{2}(f_A-f_B)\mu_k{}^L, \tag{1}$$

where f_A and f_B are the structure factors for the two types of atoms. The mean value of $\mu_k{}^L$ is

$$\bar{\mu}_k=(1/N)\sum_L \mu_k{}^L=x_A{}^k-x_B{}^k, \tag{2}$$

where $x_A{}^k$ and $x_B{}^k$ are the fractions of the N sites of type k which are occupied by A and B atoms, respectively. $\bar{\mu}_k$ is obviously a measure of the long range order in the kth sublattice. The mean structure factor at the kth type of site is

$$\bar{f}_k=(1/N)\sum_L f_k{}^L=f_Ax_A{}^k+f_Bx_B{}^k, \tag{3}$$

while the *fluctuation* at the k, L site may be written in the form,

$$\phi_k{}^L=(f_k{}^L-\bar{f}_k)=\tfrac{1}{2}(f_A-f_B)(\mu_k{}^L-\bar{\mu}_k). \tag{4}$$

We now follow closely the method of Zachariasen[3] to obtain the following formula for the intensity of diffuse scattering,

$$I_D=\tfrac{1}{4}NI_e(f_A-f_B)^2\sum_{k,k'}\exp[i\mathbf{s}\cdot(\mathbf{r}_k-\mathbf{r}_{k'})]\times\sum_M \alpha_{kk'}{}^M\exp(-i\mathbf{s}\cdot\mathbf{A}_M). \tag{5}$$

(I_e=scattering by a single electron; $\mathbf{A}_M=M_1\mathbf{a}_1+M_2\mathbf{a}_2+M_3\mathbf{a}_3$=direct lattice vector; $\mathbf{r}_k$=position vector of kth site; $\mathbf{s}=2\pi(\mathbf{k}-\mathbf{k}_0)$ =Bragg vector).[3] In the present case $\mathbf{s}$ is to be regarded as a continuously variable vector in reciprocal space. The intensity I_D is periodic in reciprocal space provided the sites, k, k', $\cdots$, occur at rational positions in the unit cell (e.g., body or face centering). The fourier coefficients of the diffuse scattering, $\alpha_{kk'}{}^M$, are related to the $\mu_k{}^L$'s as follows,

$$\alpha_{kk'}{}^M=\mu_{kk'}{}^M-\bar{\mu}_k\bar{\mu}_{k'} \tag{6}$$

$$\mu_{kk'}{}^M=(1/N)\sum_L \mu_k{}^L\mu_{k'}{}^{L-M}. \tag{7}$$

These parameters will fall into equivalent sets if the crystal has nontrivial symmetry. $\mu_{kk'}{}^M$ has the properties of a short range order parameter. In fact, if the fraction of the N pairs of sites, k, L and k', $L-M$ (all L), which are occupied by *like* atoms (AA or BB, as opposed to AB or BA) is denoted by $z_{kk'}{}^M$, it is obvious that

$$\mu_{kk'}{}^M=z_{kk'}{}^M-(1-z_{kk'}{}^M)=2z_{kk'}{}^M-1. \tag{8}$$

For $M=0$, $k=k'$, $\mu_{kk}{}^0=1$, and $z_{kk}{}^0=1$. For *perfect* long range order, $\alpha_{kk'}{}^M=0$, $\mu_{kk'}{}^M=\pm1$, and $z_{kk'}{}^M=0$ or 1, depending on the pair of sites in question. The diffuse scattering, I_D, vanishes identically for a perfectly ordered lattice.[4] The limiting values of these parameters as $|M|\to\infty$ are: $\alpha_{kk'}{}^\infty=0$; $\mu_{kk'}{}^\infty=\bar{\mu}_k\bar{\mu}_{k'}$; and $z_{kk'}{}^\infty=x_A{}^kx_A{}^{k'}+x_B{}^kx_B{}^{k'}$. These are also their *average* values (over M).

In the absence of long range order $x_A{}^k$, $x_A{}^{k'}$, and $x_B{}^k$, $x_B{}^{k'}$, may be replaced by x_A and x_B, the over-all fractions of A and B atoms, in all the preceding formulas. In addition, all sites are then usually equivalent, so that a primitive cell containing a single site may be selected. Setting $k=k'=1$, Eq. (5) then reduces to

$$I_D=\tfrac{1}{4}NI_e(f_A-f_B)^2\sum_M \alpha_{11}{}^M\exp(-i\mathbf{s}\cdot\mathbf{A}_M). \tag{9}$$

Inversion of Eq. (9) yields

$$\alpha_{11}{}^M=(4V/N)\int(I_D/I_e)(f_A-f_B)^{-2}\exp(i\mathbf{s}\cdot\mathbf{A}_M)d\mathbf{s}, \tag{10}$$

where the integration is over a single "phase cell" surrounding any reciprocal lattice point[3] (V=volume of primitive cell in direct lattice).

A detailed investigation of diffuse x-ray scattering by the disordered alloy Cu_3Au (face-centered cubic lattice) has been carried out by Cowley.[5] Cowley's interpretation of the fourier coefficients of the diffuse scattering is not entirely in accord with that arrived at here. A consideration of his method of normalization indicates that his α_{lmn} (as tabulated in Table I of reference 5) are equal to $(\alpha_{11}{}^M/\alpha_{11}{}^0)$ in the present notation. From the preceding equations, $\alpha_{11}{}^0=\frac{3}{4}$, and $\alpha_{11}{}^M=2(z_{11}{}^M-z_{11}{}^\infty)$. Therefore, the quantities tabulated in the last three columns of Cowley's Table I need only be multiplied by $\frac{3}{8}$ to obtain $z_{11}{}^M-z_{11}{}^\infty$, the fluctuation of $z_{11}{}^M$ about its limiting (or average) value. The latter is $z_{11}{}^\infty=\frac{5}{8}$, in this case (in the absence of long range order). As an example, at 405°C $z_{11}{}^M$ has the values 0.568 for nearest neighbors, and 0.695 for next-nearest neighbors, as compared with the limiting value 0.625, and the values $\frac{1}{2}$ and 1, respectively, for perfect order.[6]

[1] C. H. MacGillavry and B. Strijk, Physica **11**, 369 (1946).
[2] G. H. Wannier, Revs. Modern Phys. **17**, 50 (1945).
[3] W. H. Zachariasen, *Theory of X-Ray Diffraction in Crystals* (John Wiley and Sons, Inc., New York, 1945), pp. 213–216.
[4] Other forms of disorder scattering may occur, of course (e.g., that due to thermal vibrations). The intensity I_D of Eq. (5) does not include the superlattice reflections which may appear if long range order is present, and which are essentially as sharp as ordinary Laue-Bragg reflections (in the absence of complicating factors, such as *anti-phase* domains). The intensities of these reflections may be calculated from the *mean structure factors* of Eq. (3).
[5] J. M. Cowley, J. Appl. Phys. **21**, 24 (1950).
[6] The fractional value $\frac{1}{2}$ is not inconsistent with the previous statement that $z=0$ or 1 for perfect order. Inspection of the ordered Cu_3Au structure will show that in averaging over all pairs of nearest neighbors, like pairs ($z=1$) and unlike pairs ($z=0$) occur with equal frequency, leading to the average value $\frac{1}{2}$.

Emulsion Studies of Cosmic-Ray Stars Produced in Metal Foils*

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WE have described, and given preliminary results[1] from a method in which cosmic-ray induced disintegrations of nuclei of several elements are studied with metal foils "sandwiched" between pairs of G5 emulsions placed face-to-face. After high altitude exposure, removal of foils, and development, the original alignment of the plates is reconstructed so that they can be scanned, and groups of tracks originating in disintegrations of foil nuclei can be projected back to common centers between the emulsions. Hodgson has recently reported[2] analysis of stars from disintegration of nuclei of a lead foil in contact with a single photographic plate. Advantages of the "sandwich" method, besides the doubled probability for a particle to enter the emulsion, are (a) elimination of frequent spurious coincidences of unrelated cross tracks; (b) lower probability of missing a star, since the scanner not only looks for converging tracks, but also, in the case of every track seen, looks for its continuation in the opposite emulsion which would eliminate it from consideration as a possible member of a foil disintegration; (c) the probability that a particle from an observed foil star not be detected (because it stops in the foil or strikes the emulsion too far from the field-of-view) is almost independent of the point within the foil at which the disintegration occurs, since the increased detection probability for tracks in the plate nearer the disintegrating nucleus is compensated for by the decreased detection probability in the further plate.

Figure 1 summarizes the uncorrected cross-section data to date for six elements ranging in atomic weight from 27 to 197. Typical statistical probable errors are indicated at intervals for each element. The differential distributions (given by the *slopes* of the curves in Fig. 1) are of the form e^{-kn} in the region of larger stars ($n>Z/4$), where for $Z>40$, $k\sim0.2$, increasing for low Z to perhaps twice that value for Al; for smaller n the differential distributions tend to level off to about the same value for all the elements, though correction for stars completely missed would somewhat increase the values for small stars, especially $m=3$ and 4. In Fig. 2

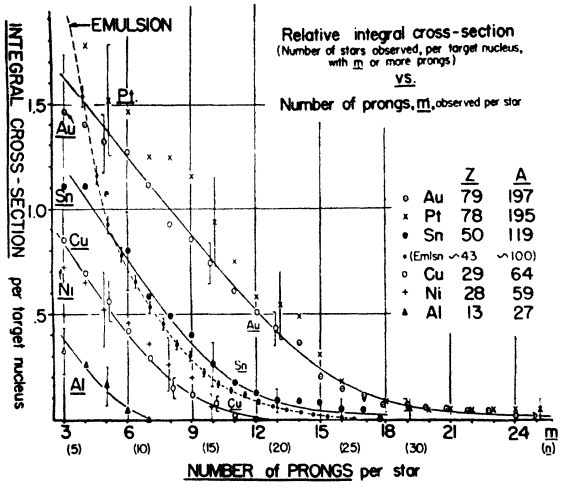

FIG. 1. Relative integral cross sections (number of stars observed, per target nucleus, with m or more prongs) are plotted for several elements against m, the number of prongs observed. Data for emulsion stars are plotted on a scale of n, the number of prongs actually leaving the disintegrating nucleus (see text).

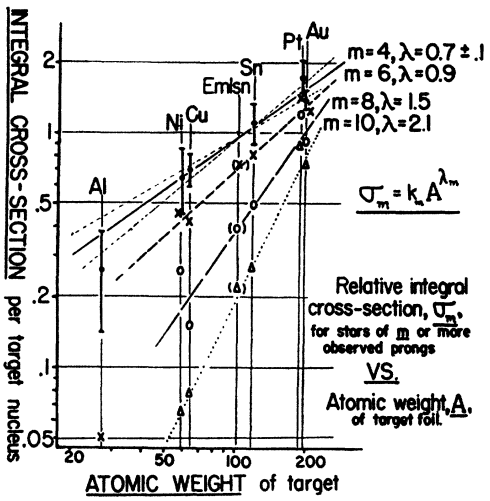

FIG. 2. Integral cross sections *vs* atomic weight of foil for stars of m or more prongs are shown on a double-log plot for four values of m.

the data have been replotted against atomic weight for stars of different sizes. The slopes of the lines are indicated, decreasing as smaller stars are included, and probable errors are shown for stars of 4 or more observed prongs. Estimated extrapolation to include stars of *all* sizes indicates a total cross section close to $A^{2/3}$.

Emulsion stars are shown in Fig. 1 for comparison; while their statistical errors are small, for 1630 stars were analyzed, uncertainties exist in (a) the ordinate, which depends on the assumed thickness of the emulsion, and its effective number of atoms per cm². For the latter we have used the *equivalent* number of atoms of atomic weight 100 (intermediate between Ag and Br) that would give the same cross section as the emulsion constituents, assuming the manufacturer's analysis and a dependence on atomic weight as discussed above. (This equivalent number is not very sensitive to the exponent.) (b) For comparison with the foil curves, the emulsion stars should be plotted on an abscissa of n, the actual number of prongs in the disintegration. The probability, m/n, of detecting a particle from the observed disintegration of a foil nucleus depends on the stopping power and thickness of the foil and the diameter of the field of view, and varies slightly with the position of the disintegrating nucleus in the foil and with the size of the star; but for these foils calculations of this ratio did not vary greatly from an average value of about 2/3.

The resulting emulsion curve approximates, for $n>6$, that which one might expect for $A=100$. The greater number of smaller stars can readily be accounted for by the known percentages of lighter elements in the emulsion. The average slope of the integral distribution of emulsion stars of 4, 5, and 6 prongs is about 3 times that of the line obtained by projecting to small n the emulsion curve for larger stars (that is, the curve which one would anticipate for $A=100$ by comparison with the foil curves). This would indicate that of the small stars found in emulsion, approximately twice as many can be attributed to the lighter elements present as to Ag and Br. There is, however, considerable uncertainty in this value, because the data for small stars in the foils are less reliable; a differential prong distribution will be given upon completion of scanning and rechecking and more careful analysis of corrections. There will also appear an analysis of measurements made on the energy of each particle in the foil stars and the angular distribution to determine their dependence upon atomic weight as compared with theoretical calculations. Only a few "showers" of minimum-ionization particles, usually interpreted as mesons, have been observed, but these yield some information concerning the variation with atomic weight of meson-production cross sections at these energies. Several students have assisted in this work: James Bailey, Milton Meux, Mel Reed, and Dick Wilson.

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Natural Spread of the Conic Distribution of the Čerenkov Radiation

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BECAUSE of the existence of a high degree of partial coherence of the wavelets from succeeding free paths, the natural spread of the Čerenkov radiation was very much overestimated in a previous letter.[1] The coherent path length of an electron is much closer to the actual length of travel in the material than it is to the mean free path between emissions of quanta. R. L. Mather of Berkeley has observed with proton beams that the half-breadth of the ray directions is less than the minimum predicted in the letter, when the effects of dispersion are removed. He has also observed a rainbow spectrum.

My sincere thanks are due Professor L. I. Schiff and Mr. R. L. Mather for their kind communications.

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Theoretical Low Temperature Spectra of the Thallium Activated Potassium Chloride Phosphor

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A FUNDAMENTAL calculation of the absorption and emission spectra of KCl:Tl has been recently reported.[1] From the properties of the constituent ions the potential energy was computed for $^1S_0$ and $^3P_1{}^0Tl^+$ substituted for K^+ in KCl as a function of a precise configuration coordinate, namely, the Tl^+ nearest Cl^- neighbor distance with the condition that the remainder of the lattice rearranges to minimize the total energy. A classical distribution of atomic configurations characteristic of