tion of each electron line is presented in column 3 and the resultant gamma-energies in column 4. Arbitrary superscripts are used to denote the gamma-rays increasing in order with the energy. Seven of the low energy electron lines are of Auger origin, as noted in column 3. A summary of the energies of the gamma-rays associated with each process is offered in Table II.

It is now possible to arrange a scheme of 6 levels for the excited platinum 192 nucleus as shown in Fig. 1, which is satisfied remarkably well by 10 of the observed gamma-rays. A proposed level scheme for the osmium 192 nucleus is portrayed in Fig. 2. The K-capture process is also substantiated by the presence of Auger lines characteristic of osmium. Six of the weaker electron lines do not enter in K–L combinations and are assumed to be K lines for either osmium or platinum. A choice is made for the two gamma-rays designated as 6 and 14, since their energies fit satisfactorily in Fig. 2.

The early exposures of the freshly irradiated specimen showed a few electron lines which did not appear on later photographic plates, indicating that they were associated with the 19-hour radioactivity in Ir^{194}. From the electron energies, a single gamma-ray of energy 327.5 kev is indicated for the Pt^{194} nucleus. A similar energy has been known to exist in the K-capture decay of Au^{194}.

* This project was made possible by the joint support of the ONR and AEC.

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[2] M. Deutsch, Phys. Rev. **64**, 265 (1943); J. Cork, Phys. Rev. **72**, 581 (1947); M. Levy, Phys. Rev. **72**, 352 (1947); R. Hill and W. Meyerhof, Phys. Rev. **73**, 812 (1948).

On Some Recent Calculations on Cascade Shower Theory

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THERE have recently appeared a number of works[1–3] on the electron-photon cascade theory. In each case the authors were concerned with the solution of the diffusion equations of the cascade theory of electron showers when the ionization loss term was included. Bhabha and Chakrabarty[4] developed an elegant and simple series solution of the diffusion equations and showed that the first two terms of the series gave results which were quite satisfactory for all practical purposes. It was also pointed out by Jánossy and Messel[5] that the solutions given by B.C. could be quickly and easily evaluated. Their solutions have also the added advantages:

(a) the first-order correction for the effects of ionization loss is expressed as a shift of the energy spectrum by an amount of the order of the critical energy,

(b) the series may be used either to evaluate the number of electrons $[N(E, t)]$ or photons $[\gamma(E, t)]$ above a specified energy E (and this is the physically important quantity), or to evaluate the total number of electrons $[N(0, t)]$ or photons $[\gamma(0, t)]$ at various depths t.

In the meantime Snyder[2] developed solutions of the cascade equations using a different approach. He showed that the solution of the cascade equations could be reduced to the solution of certain difference equations. The method is fairly long and is far from yielding solutions which lend themselves to easy computation. In fact, the solutions given by Snyder are of practical value only for computing the total number of electrons or photons for $E=0$, at various depths. In order to obtain results for any other value of E, the evaluation of a triple complex integral is required. This in itself is a serious drawback. Snyder also pointed out that his solutions yielded values for $N(0, t)$ at the cascade maximum which were about 35 percent higher than those obtained by B.C., who used the first two terms of their series. Hence, it was concluded that the results of B.C. were inaccurate. At the same time Snyder pointed out that for a shower initiated by an electron of energy E_0

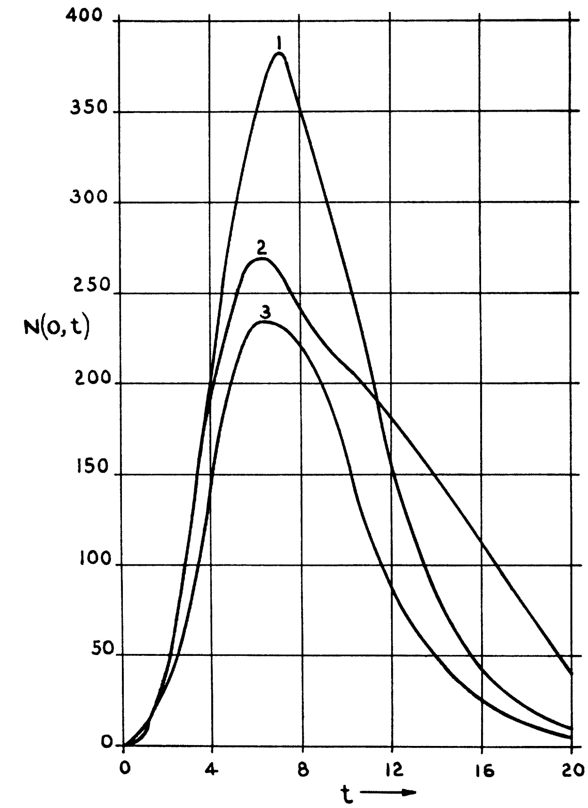

FIG. 1. $N(0, t)$, the average number of electrons induced by an incident electron of energy E_0, plotted against the depth t in cascade units. We have marked the curve given by Snyder's solution 1, that given by Bernstein for lead 2, and that by B.C. 3. In each case $\ln E_0/\beta = 8$.

$$\int_0^t N(0, t)dt = E_0/\beta, \tag{1}$$

where β is the ionization loss. He then mentioned that the first two terms of the B.C. solution contribute only 70 to 85 percent of E_0/β and hence these authors did not use a sufficient number of terms of their series. B.C. had given in their paper a very satisfactory explanation for the above, which appears to have been overlooked by both Snyder and Bernstein.[3] We quote from reference 1.

"From the physical point of view, however, Eq. (1) must be taken with caution, especially in substances of high atomic number where the critical energy is low. It is not true that all the energy of a cascade is dissipated by the collision loss of cascade electrons alone. A good deal of energy is lost in the form of quanta of energy less than 2 mc^2 which are incapable of further pair creation. Thus the complete series for $N(0, t)$ must give too many cascade electrons of low energy at large thicknesses, and the first two or three terms of the series may well give a truer picture of the physical process in substances of high atomic number."

Bernstein lately has taken Snyder's solution and applied to it a correction by means of a perturbation method. He used a more refined approximation to the Bethe-Heitler cross sections than had hitherto been used. The method employed by Bernstein is straightforward but naturally even more tedious than that of Snyder—and again of the same limited applicability. From Bernstein's results there evolves an interesting feature. He finds that Snyder's results give a value for $N(0, t)$ which is much too high at the cascade maximum. We have plotted in Figs. 1 and 2 the results

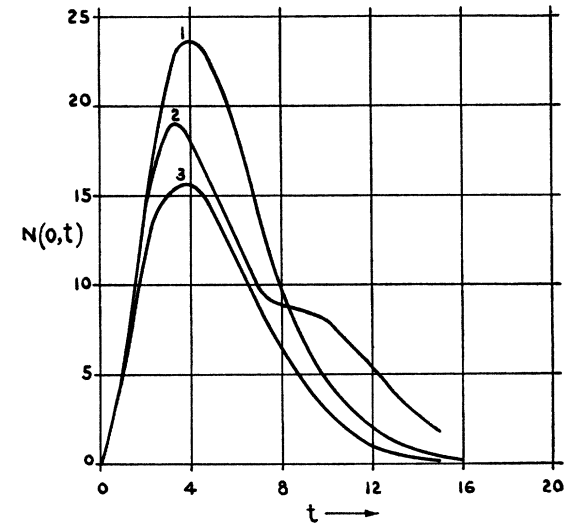

FIG. 2. $N(0, t)$, the average number of electrons induced by an incident electron of energy E_0, plotted against the depth t in cascade units. We have marked the curve given by Snyder's solution 1, that given by Bernstein for lead 2, and that by B.C. 3. In each case $\ln E_0/\beta=5$.

found by Bernstein, Snyder, and B.C. for $N(0, t)$ for $\ln E_0/\beta=8$ and 5. The curves taken from Bernstein are those for lead.

It becomes immediately evident that the B.C. solutions (which can be used with ease) are not as inaccurate as hitherto made out. We also wish to point out that Bernstein, like Snyder, has failed to take into account the facts given in the quotation above. When this is taken into consideration, it appears likely that the pronounced "tail" of the curves given by Bernstein, beyond the cascade maximum, will be decreased to a value close to that given by B.C. It thus now appears that the series solution of B.C. can be used with confidence.

I am indebted to Professor H. J. Bhabha for valuable discussion of the above.

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[3] I. B. Bernstein, Phys. Rev. **80**, 995 (1950).
[4] Reference 1, henceforth shortened to B.C.
[5] L. Jánossy and H. Messel, Proc. Phys. Soc. (London) **A64**, 1 (1951).

Photonuclear Stars in Emulsions*

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ILFORD C2 emulsions, 200 microns thick, were exposed in the collimated x-ray beam from the Berkeley 322-Mev synchrotron. Exposures were made at synchrotron energies of 322, 242, and 161 Mev, and the monitor ionization chamber was calibrated relative to pair production cross sections at each of the three energies by the method described by Blocker, Kenney, and Panofsky.[1]

The complete area covered on each plate by the beam ($\frac{3}{8}$-in. circle) has been scanned for stars with two and more prongs. Because of the impossibility of distinguishing between a two-prong star and a scatter in the track of a single particle in a large number of cases, two-prong stars were classified as either possible or probable. Because of the arbitrariness and the difficulty of setting sufficiently precise criteria for this distinction, the two-prong data are regarded as essentially qualitative. All events within 5.5 microns of either surface of the emulsion after processing were discarded, and the minimum prong range counted was 3 microns.

TABLE I. Relative yields of stars.

No. prongs \ X-ray energy	322 Mev	242 Mev	161 Mev
possible 2	73±7	81±8	34±5
probable 2	91±8	70±7	36±5
3	137±6	109±9	39±5
4	88±5	57±6	29±4
5	37±3	14±3	8±2
6	13±2	3.6±1.6	0.6±0.6
7	3±1		
8	0.9±0.5		
≥3	280±9	184±12	77±9
≥5	54±4	18±4	9±2

The yields of stars with various numbers of prongs are shown in Table I. The 322-Mev yields are normalized to an emulsion thickness before processing of 200 microns, and to an exposure of 10^8 "equivalent quanta," or 322×10^8-Mev total energy in the complete bremsstrahlung spectrum. The yields at the other energies are normalized to the same thickness and to exposures that contain the same number of quanta per Mev interval at 32 Mev. The uncertainties indicated in the table are the standard deviations due to counting statistics only. The prong spectrum of the 322-Mev yield is similar to that reported by Kikuchi;[2] the difference in the minimum prong length counted probably accounts for the discrepancies.

Using the nominal partial densities of emulsion elements given by the manufacturer, and assuming that star production cross sections are proportional to mass number, the 322-Mev yield of 3 and more prong stars gives a cross section for silver, integrated over the bremsstrahlung spectrum, of 6.5×10^{-27} cm^2 per "equivalent quantum." This figure is reduced to 5.6×10^{-27} if the cross section is assumed proportional to $A^{2/3}$.

The energy distribution of the quanta responsible for the differences in yields from the different energy exposures is shown in Fig. 1. Also shown, at one-tenth the scale for the difference

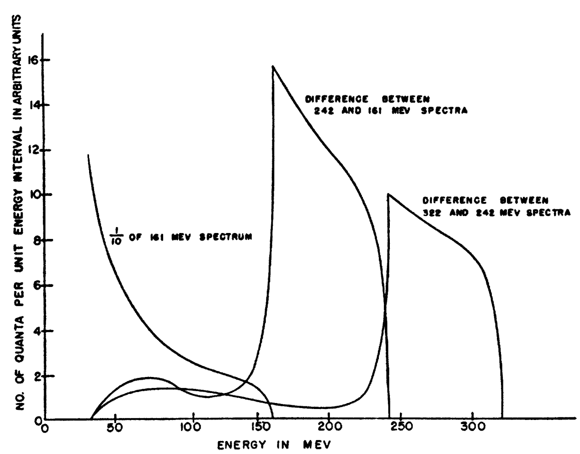

FIG. 1. Differences between spectral distributions of bremsstrahlung from electrons of 322, 242, and 161 Mev, when the spectra have been normalized to contain the same number of quanta per unit energy interval at 32 Mev. The 161-Mev bremsstrahlung spectrum is shown at one-tenth the ordinate scale for the difference spectra.

spectra, is the spectrum of 161-Mev bremsstrahlung. Because most of the difference quanta are in the energy range between the two upper limits of the bremsstrahlung spectra, it is possible to calculate a cross section averaged over this energy interval. Estimating that that part of the yield due to the low energy tail in the 161- to 242-Mev difference spectrum is 10 percent of the 161-Mev yield, and making similar corrections for the yield from the tail in the 242- to 322-Mev difference spectrum, the average cross