Neutron Refraction in Ferromagnets

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THE disagreement between the results of Halpern[1] and of Ekstein[2] concerning the index of refraction of neutrons in ferromagnets can be summarized as follows. (1) Halpern claims that the change in neutron index of refraction is determined by the scattering amplitude in the forward direction. (2) Ekstein points out that Halpern's calculation of the forward scattering amplitude is incorrect and that the magnetic scattering of a neutron by an ion vanishes in the forward direction. (3) Ekstein shows that the change in index of refraction does not vanish but is determined by the magnetic field in the magnet.

I shall show here that *all* of these statements are correct. The procedure is based on a general theory of the multiple scattering of waves to be published soon. The chief result of this theory is that the coherent wave $\langle\psi(\mathbf{r})\rangle$ obeys an equation

$$(E-H-c\bar{T})\langle\psi(\mathbf{r})\rangle=0, \tag{1}$$

$$\bar{T}=\int_\tau T(\mathbf{r}_j)n(\mathbf{r}_j)d\mathbf{r}_j, \tag{2}$$

where $n(\mathbf{r}_j)$ is the density of scatterers. Here c is a correction factor for the relation between the "effective" and total neutron wave and for present purposes may be set equal to unity. $T(\mathbf{r}_j)$ is the transition operator[3] whose matrix elements $T_{ba}(\mathbf{r}_j)$ are proportional to the scattering amplitude from direction a to direction b for a scatterer at position $\mathbf{r}_j$. These differ by a phase factor from the case in which the scatterer is located at the origin:

$$T_{ba}(\mathbf{r}_j)=\exp[i(\mathbf{k}_a-\mathbf{k}_b)\cdot\mathbf{r}_j]T_{ba}(0). \tag{3}$$

$\bar{T}$ represents the sum of the scattering amplitudes of the scatterers in a volume τ equal to the volume of quantization. It follows from (2) and (3) that $\bar{T}$ is already diagonal and independent of τ if plane waves quantized in τ are used as the eigenfunctions. Since $E=\hbar^2k^2/2m$ and $H=-\hbar^2\nabla^2/2m=\hbar^2k'^2/2m$, the propagation constant k' in the magnet is related to the vacuum case by:

$$k'^2=k^2-c(2m/\hbar^2)(\bar{T})_{aa}. \tag{4}$$

This establishes the first point that the index of refraction k'/k is determined by the forward scattered amplitude.

Since both Halpern and Ekstein treat the scattering in Born approximation, we can replace T by the corresponding interaction potential:

$$T(\mathbf{r}_j)=V(\mathbf{r}-\mathbf{r}_j)-\mu\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}-\mathbf{r}_j), \tag{5}$$

where $V(\mathbf{r}-\mathbf{r}_j)$ is the pseudopotential of the jth nucleus and $\mathbf{B}(\mathbf{r}-\mathbf{r}_j)$ is the magnetic field associated with the corresponding ion. The forward scattered amplitude of a single scatterer is then proportional to:

$$\lim_{\tau\to\infty}\int_\tau[V(\mathbf{r}-\mathbf{r}_j)-\mu\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}-\mathbf{r}_j)]d\mathbf{r}. \tag{6}$$

The second term vanishes in the limit, since the complete space integral of a solenoid vector $\mathbf{B}$ vanishes,[4] providing $\lim r\mathbf{B}\to 0$ as $r\to\infty$. This establishes the second point that the Born approximation to the magnetic forward scattered amplitude vanishes.

We shall now show that (4) yields a non-vanishing magnetic contribution to the index of refraction providing we calculate $(\bar{T})_{aa}$ rather than $(T_{aa})_{\text{Av}}$. In other words, we must add the contributions of the individual scatterers before calculating the forward amplitude. Usually the two procedures would be in agreement, but here they differ because of the long-range nature of the magnetic forces:

$$(\bar{T})_{aa}=\frac{1}{\tau}\int_\tau d\mathbf{r}\int_\tau d\mathbf{r}_j[V(\mathbf{r}-\mathbf{r}_j)-\mu\boldsymbol{\sigma}\cdot\mathbf{B}(\mathbf{r}-\mathbf{r}_j)]n(\mathbf{r}_j) \tag{7}$$

$$=\lim_{\tau\to\infty}\frac{1}{\tau}\int_\tau d\mathbf{r}[V_t(\mathbf{r})-\mu\boldsymbol{\sigma}\cdot\mathbf{B}_t(\mathbf{r})] \tag{8}$$

$$=V_{\text{Av}}-\mu\boldsymbol{\sigma}\cdot\mathbf{B}_{\text{Av}}, \tag{9}$$

where $V(\mathbf{r})$ and $\mathbf{B}_t(\mathbf{r})$ represent the total potential and total magnetic field at $\mathbf{r}$ and V_{Av}, $\mathbf{B}_{\text{Av}}$ are the corresponding macroscopic average potential and field. Since the eigenvalues of $\boldsymbol{\sigma}\cdot\mathbf{B}_{\text{Av}}$ are $\pm|\mathbf{B}_{\text{Av}}|$ for neutrons polarized parallel or antiparallel to the magnetic field, we obtain Ekstein's result:

$$k'^2=k^2-(2m/\hbar^2)[V_{\text{Av}}\pm\mu|\mathbf{B}_{\text{Av}}|] \tag{10}$$

and establish the third point.

Equation (8) embodies carefully the principle that an index of refraction should be computed for a medium of infinite volume. The space integral of $\mathbf{B}_t(\mathbf{r})$ does not vanish for an infinite magnet because $r\mathbf{B}_t(\mathbf{r})$ does not vanish as $r\to\infty$.

[1] O. Halpern, Phys. Rev. **76**, 1130 (1949). Halpern, Hamermesh, and Johnson, Phys. Rev. **59**, 981 (1941).
[2] H. Ekstein, Phys. Rev. **78**, 731 (1950); **76**, 1328 (1949).
[3] M. Lax, Phys. Rev. **78**, 306 (1950).
[4] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill Book Company, Inc., New York, 1942), p. 111.

Cation Distribution in Copper Zinc Ferrite

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A NOTE in this Journal by Brockman[1] on the effect of low temperature heat treatment on the Curie temperature of copper zinc ferrite calls for comment.

The experimental results are not disputed; in fact, they agree with some qualitative observations made at an earlier date by the author of the present note.

The interpretation seems to me to be open to question, however. In view of the strong preference of zinc for the tetrahedral position, as shown by the fact that a quenching from 1200°C brings about only a weak ferromagnetism in pure zinc ferrite, and given the great sensitivity of the saturation magnetization of pure copper ferrite to variations in the heat treatment,[2] it seems to be much more likely that changes in the distribution of the copper atoms over the available positions in the lattice are responsible for the observed variations. Obviously, accurate measurements of the saturation magnetization would give the answer to this problem at once.

[1] F. G. Brockman, Phys. Rev. **77**, 841 (1950).
[2] L. Néel, Comptes Rendus **230**, 190 (1950).

Crystals and Geiger Counters for Scintillation Counting*

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IN three previous communications,[1-3] the writers have discussed the detection of alpha-, beta-, and gamma-rays by recording scintillations from crystals in photo-sensitive Geiger counters. A survey has been conducted wherein a large number of inorganic crystals have been produced with emphasis upon the silver-activated alkali halides. Some organic materials have also been considered. Using a scintillation Geiger counter as a detector, effects have been observed as shown in Table I.

TABLE I.

Scintillator	Particles detected in the photon Geiger counter
NaCl-Ag	alpha, beta, gamma
NaBr-Ag	alpha, beta
NaBr-Tl	alpha
Powdered durene	alpha
KCl-Ag	alpha
LiCl-Ag	alpha
LiBr-Ag	alpha