ON THE DEMAGNETIZATION OF IRON AND STEEL RODS BY STRAIN AND IMPACT.

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THE following is an investigation concerning the redistribution of magnetic flux through the various cross-sections of magnetized rods, after they have been subjected to torsion, flexure, or to the impact of a hammer.

The rods used had a diameter of about 1.25 cm., and were 112 cm. in length. They were first completely demagnetized by means of a gradually diminishing alternating current passing through the turns of a helix of about the same length as the rod. Then by means of a direct current through the same helix, the rods were magnetized to saturation, or nearly so.

In order to determine the number of lines of magnetic force passing through any cross-section of the magnet, a narrow spool of small diameter, wound with fine copper wire, was employed, the terminals being connected to a sensitive ballistic galvanometer. The spool, being placed on the rod at the section desired, was slid off from that point entirely over the end of the rod, thus cutting all the lines of force which escaped from the magnet in the length passed over. If the spool is moved with sufficient rapidity, the throw produced in the ballistic galvanometer is proportional to the total magnetic flux at the cross-section specified.

The dimensions of the spool were as follows: diameter 4 cm., width of windings 1 cm., and the wire was chosen of such a diameter that its total resistance was equal to that of the galvanometer, the condition for a maximum induced current.

In brief, then, the method was to determine the induction flux at various sections along the magnetized rod, after which the specimen was subjected to some form of mechanical strain and then again explored by the sliding coil; the remaining induction flux being measured at the same points on the rod as before. The decrease in this magnetic flux at any cross-section, occasioned by the mechanical strain to which the magnet was subjected, is here defined as the *demagnetization* at that section.

If we represent by x the linear coördinate specifying the particular section of the rod investigated, by ϕ_x the flux at that section before strain, and by ϕ_x' the flux there after strain, we have

$$D_x = \phi_x - \phi_x', \tag{1}$$

where D_x is the demagnetization over the section x.

Torsion.

Perhaps the simplest case to investigate in this connection is that of uniform torsion of the magnet.

The specimen was held securely at one end in a horizontal position by means of a heavy brass clamp which gripped about one centimeter of the rod. Emery dust sprinkled over the rounded jaws of the brass vise was sufficient to prevent all slipping.

Near the free end of the magnet another heavy brass clamp was placed to prevent flexure in any direction, the jaws being left loose enough to allow the rod to rotate easily during torsion. At the extreme free end was a strong brass lever, fastened at right angles to the specimen by means of a brass key-pin passing through a small hole in the iron. During torsion, especially for large strains, it was found necessary to use a loose-fitting clamp at the middle to do away with buckling of the specimen which had caused large errors in the results. The angle of torsion was measured with sufficient accuracy by using a light metal pointer of length 30 cm., fastened at right angles to the rod, and moving over a scale.

There was no difficulty found in demagnetizing a specimen, and remagnetizing it to a standard distribution, but it did require practice to be able to subject the specimen to a given torsion and leave it in the same state of demagnetization time after time. Especially was this true of Norway iron, which may be appreciably demagnetized merely by stroking. This is due perhaps to vibrations set up within the metal, and not merely to surface agitation.

The rods were investigated when in the direction of magnetic east and west, but this was found to be a hardly necessary precaution.

In Tables I. and II. are shown the results of uniform torsion on a magnet of Norway iron. The clamp was applied from $x = 0$, to $x = 1$ cm., and the torsion at $x = 112$ cm.

The angle of torsion here merely represents the angular displacement of the free end of the specimen with respect to the fixed one. Results are given for six different angles, for the highest of which the rod showed an appreciable set after being released from torsion.

It should be stated at this point that the rods were studied not while under strain, but after release.

The first column in the tables specifies the section of the magnet, the second gives the original magnetic flux at that section. In the first

TABLE I.

NORWAY IRON.

Demagnetization by Uniform Torsion.

x Cm.	ϕ_x.	Torsion 1.5°. Factor 0.216. Observed D_x.	Torsion 1.5°. Factor 0.216. Computed D_x.	Torsion 2.4°. Factor 0.320. Obs. D_x.	Torsion 2.4°. Factor 0.320. Comp. D_x.	Torsion 3.6°. Factor 0.436. Obs. D_x.	Torsion 3.6°. Factor 0.436. Comp. D_x.
2	145	28	31	37	46	63	63
5	315	68	68	97	101	139	137
8	464	102	100	150	149	206	202
11	599	133	129	198	192	265	261
14	723	160	156	241	231	319	315
17	837	187	181	281	268	368	365
23	1,037	226	224	351	332	453	452
29	1,200	261	259	397	384	524	523
35	1,331	288	288	451	426	580	581
44	1,465	317	316	494	469	634	639
56	1,532	331	331	513	490	661	668
68	1,474	318	318	492	472	635	643
77	1,345	290	290	448	430	580	586
83	1,215	263	262	405	389	525	530
89	1,043	228	225	349	334	455	455
95	850	186	184	281	272	369	371
98	739	162	160	241	237	320	322
101	615	136	133	199	197	265	268
104	490	106	106	150	157	206	214
107	329	73	71	98	105	140	143
110	161	36	35	37	52	67	70

of each of the double columns following is the corresponding demagnetization as measured for the torsional angle specified.

It will be found that in each case the demagnetization, D_x, divided by the original flux, ϕ_x, gives nearly a constant ratio, R, for a fixed angle of torsion.

The mean of these ratios was taken over the entire rod for each torsion angle, θ, and the values of R and θ were found to be quite accurately related by the equation

$$aR = \theta(b - R^2) \quad \text{(see Fig. 1)}, \tag{2}$$

where a and b are constants depending upon the material. For the specimen of Norway iron used, the values of these constants were found to be $a = 7.585$; $b = 1.115$.

In Tables I. and II., following, the column showing the values for the

TABLE II.

NORWAY IRON.

Demagnetization by Uniform Torsion.

x Cm.	ϕ_x.	Torsion 5.2°. Factor 0.556. Obs. D_x	Torsion 5.2°. Factor 0.556. Comp. D_x.	Torsion 8.1°. Factor 0.687. Obs. D_x.	Torsion 8.1°. Factor 0.687. Comp. D_x.	Torsion 11.3°. Factor 0.773. Obs. D_x.	Torsion 11.3°. Factor 0.773. Comp. D_x.
2	145	85	81	86	100	101	112
5	315	184	175	204	216	235	243
8	464	251	258	307	319	352	359
11	599	329	333	402	412	458	463
14	723	400	402	486	497	554	559
17	837	464	465	564	575	642	647
23	1,037	574	577	700	713	797	802
29	1,200	666	667	813	825	927	928
35	1,331	740	740	904	914	1,029	1,029
44	1,465	816	814	999	1,006	1,136	1,132
56	1,532	854	852	1,045	1,052	1,190	1,184
68	1,474	820	819	1,006	1,013	1,142	1,139
77	1,345	747	748	922	924	1,041	1,039
83	1,215	676	676	838	835	943	939
89	1,043	587	580	730	717	809	806
95	850	478	473	598	584	668	657
98	739	415	411	522	508	581	571
101	615	346	342	438	423	485	475
104	490	269	272	344	337	381	378
107	329	183	183	240	226	266	255
110	161	85	90	120	111	135	124

demagnetization as measured for a particular torsion angle, is a column in which the computed value $D = R\varphi$ of the demagnetization is shown. The factor R, specified at the top of the column, is computed from equation (2) for each torsion angle.

It will be seen that the agreement between the observed and calculated values of D is good in general, the variation being not often greater than 1 or 2 per cent. of the maximum demagnetization measured. Of course the greatest deviations are at the ends where the quantity measured is least. Also the magnetic distribution at the ends is somewhat changed by the clamped portion and by the torsion device.

From this it may be seen that if we have given the original flux curve and the two constants a and b, the demagnetization curve for any angle θ (within the elastic limit) is determined by the two equations

$$aR = \theta(b - R^2) \quad (3)$$
$$D_x = R\phi_x,$$

or by

$$D_x = \frac{\phi_x}{2\theta}\left[\sqrt{4b\theta^2 + a^2} - a\right].$$

A similar set of experiments was performed also with a specimen of Viking steel, and the results are here tabulated (Table III.).

Table III.

Viking Steel.

Demagnetization by Uniform Torsion.

x Cm.	ϕ_x.	Torsion 4.1°. Factor 0.128. Obs. D_x.	Comp. D_x.	Torsion 8.1°. Factor 0.234. Obs. D_x.	Comp. D_x.	Torsion 12.6°. Factor 0.328. Obs. D_x.	Comp. D_x.	Torsion 16.5°. Factor 0.390. Obs. D_x.	Comp. D_x.	Torsion 20.6°. Factor 0.440. Obs. D_x.	Comp. D_x.
2	628	73	80	95	147	135	206	262	245	313	276
5	1,319	175	169	283	309	394	433	555	514	641	580
8	1,912	259	245	440	448	612	627	800	746	919	841
11	2,437	330	312	579	570	803	799	1,012	950	1,161	1,072
14	2,903	390	372	700	679	968	952	1,196	1,132	1,371	1,277
17	3,321	442	423	804	777	1,113	1,089	1,358	1,295	1,555	1,461
23	4,032	526	515	975	943	1,349	1,319	1,620	1,568	1,844	1,774
29	4,571	589	585	1,100	1,070	1,524	1,499	1,814	1,783	2,049	2,011
35	4,991	636	639	1,186	1,168	1,649	1,637	1,951	1,947	2,189	2,196
44	5,403	679	692	1,261	1,264	1,757	1,772	2,070	2,107	2,316	2,377
56	5,605	698	717	1,295	1,312	1,802	1,838	2,122	2,186	2,374	2,466
68	5,400	675	691	1,263	1,264	1,761	1,771	2,071	2,106	2,318	2,376
77	4,994	626	639	1,190	1,169	1,663	1,638	1,954	1,948	2,192	2,197
83	4,585	575	587	1,108	1,073	1,547	1,504	1,819	1,788	2,052	2,017
89	4,044	510	518	992	946	1,376	1,326	1,626	1,577	1,848	1,779
95	3,352	424	429	835	784	1,147	1,100	1,365	1,307	1,567	1,475
98	2,938	374	376	739	688	1,011	964	1,205	1,146	1,394	1,293
101	2,471	318	316	629	578	852	811	1,023	964	1,193	1,087
104	1,947	254	249	502	456	670	639	811	769	967	857
107	1,354	183	173	353	317	461	444	568	528	706	596
110	671	99	86	177	157	215	220	281	262	395	295

The relation between R and θ was found to be of the same form as that for Norway iron (equation 2), the values of the constants a and b being respectively 18.241 and 0.583.

The agreement between the observed and the calculated values of the demagnetization is not so good as with Norway iron, the discrepancies in a few instances being as great as 4 or 5 per cent. of the maximum demagnetization measured. However, the angles of torsion ran higher, and on account of the necessarily greater torsional moment required the specimen showed the effect of buckling between the clamp-guides to a greater extent than did the first specimen.

The curves representing R as a function of θ for the two materials are shown in Fig. 1.

It is evident at once that the soft iron is far more subject to torsional demagnetization than the steel. For an angle of 1.75° the Norway iron had lost at each cross-section, one fourth of its original flux at that section; for a torsion of 4.4°, one half; and for 10.3°, three fourths. While the Viking steel had lost only one fourth at 8.8°, and as far as 20° had not lost half the original flux.

In Fig. 2 are shown the demagnetization curves for Viking steel; *G*, *F*, *E*, *D* and *C* corresponding to angles of torsion of 4.1°, 8.1°, 12.6°,

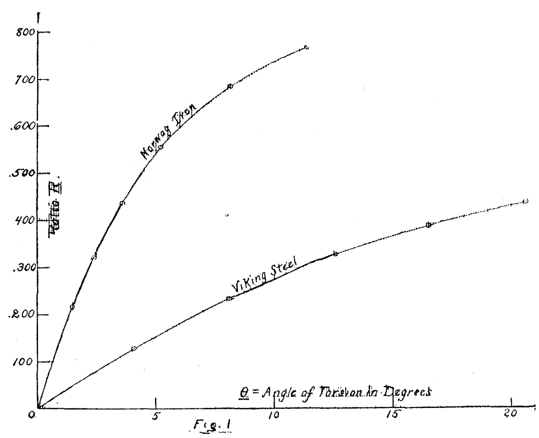

Fig. 1.

16.5° and 20.6° respectively. Since these angles vary by increments of about 4°, it may be seen, since the curves become nearer together for the greater angles, that the demagnetization is relatively greater for the small angles of torsion.

The curve A represents the flux before torsion as a function of x. B is the flux-curve after torsion, for an angle of 16.5°. Curve D, then, is the difference of the curves A and B.

A is approximately a parabola, and is represented by the equation

$$Ax^2 + Bx + C = \phi_x, \tag{4}$$

where A, B and C are constants. Since it was shown that $D_x = R\phi_x$, where R is a constant for a fixed angle of torsion, it follows that

$$D_x = A'x^2 + B'x + C', \tag{5}$$

where $A' = AR$; $B' = BR$; $C' = CR$, and so the demagnetization curves are also approximate parabolas.

The following table (IV) shows the agreement between the observed data and the parabolic equation for the original flux curve with Norway iron.

By the method of least-squares the constants best fitting the observations were found to be $A = -0.4737$; $B = +53.18$; $C = +54.65$.

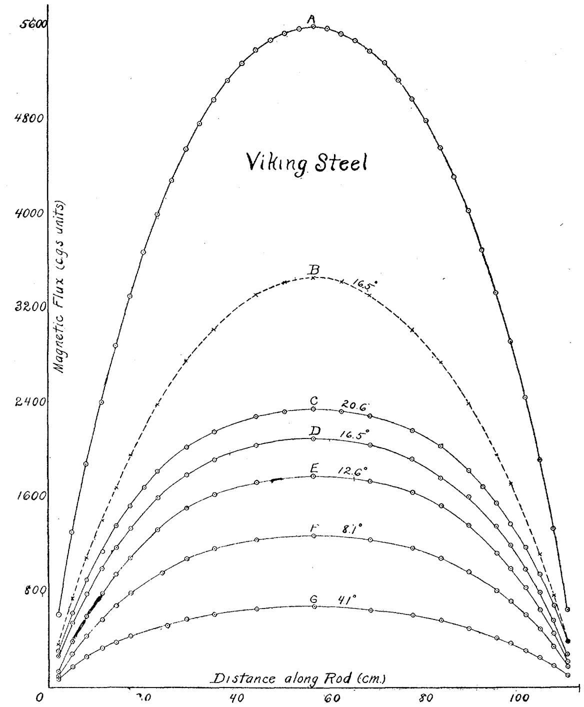

Fig. 2.

In Table IV. are also shown three sets of data for the demagnetization curves, the constants of the parabolic equation being found in each case by multiplying the values of A, B and C above by the factor R corresponding to the particular torsion angle. In other words the columns of computed values of demagnetization are derived for the torsion angles 1.5°, 3.6° and 5.2°, by multiplying the corresponding values of ϕ by the factors 0.216, 0.436, 0.556 respectively, as derived from Equation 2.

Thus the entire family of demagnetization curves corresponding to values of θ within the elastic limit may, by knowing the values of five constants, A, B, C, a, b, be constructed from the two equations

$$aR = \theta(b - R^2), \tag{6}$$

$$D = ARx^2 + BRx + CR.$$

In Table V. is shown the behavior of the specimen of Viking steel when subjected to a fixed torsion, 8.1°, but starting with different intensities of magnetization. In the first case the magnetizing current was 0.2

Table IV.

Norway Iron.

x Cm.	ϕ_x Obs.	ϕ_x Comp.	1.5°.		3.6°.		5.2°.	
			D_x Obs.	D_x Comp.	D_x Obs.	D_x Comp.	D_x Obs.	D_x Comp.
2	145	159	28	34	63	69	85	89
5	315	307	68	66	139	134	184	171
11	599	582	133	126	265	254	329	324
14	723	706	160	153	319	308	400	393
23	1,037	1,027	226	222	453	448	574	571
29	1,200	1,199	261	259	524	523	666	666
35	1,331	1,336	288	289	580	582	740	743
44	1,465	1,478	317	319	634	644	816	822
56	1,532	1,547	331	334	661	675	854	871
68	1,474	1,481	318	320	635	646	820	823
77	1,345	1,341	290	290	580	585	747	746
83	1,215	1,205	263	260	525	526	676	670
89	1,043	1,036	228	224	455	452	587	576
98	739	717	162	155	320	313	415	399
101	615	594	136	128	265	259	346	330
107	329	320	73	69	140	140	183	178
110	161	171	36	37	67	75	85	95

Table V.

Viking Steel.

Torsion Angle = 8.1°.

x Cm.	ϕ.	$R=0.272$.	ϕ.	$R=0.245$.	ϕ.	$R=0.234$.
		D.		D.		D.
2.	266	50	579	92	628	95
5	487	115	1,141	259	1,319	283
8	694	179	1,642	410	1,912	440
11	890	241	2,100	545	2,437	579
14	1,075	298	2,520	664	2,903	700
17	1,248	351	2,903	769	3,321	804
23	1,560	440	3,721	941	4,032	975
29	1,821	511	4,121	1,066	4,571	1,100
35	2,031	564	4,542	1,152	4,991	1,186
44	2,255	615	4,957	1,223	5,403	1,261
56	2,366	637	5,155	1,247	5,605	1,295
68	2,270	613	4,957	1,197	5,400	1,263
77	2,065	571	4,542	1,085	4,994	1,190
83	1,863	525	4,124	1,035	4,585	1,108
89	1,607	460	3,583	913	4,044	992
95	1,303	377	2,920	750	3,352	835
98	1,131	328	2,540	653	2.938	739
101	946	275	2,125	544	2,471	629
104	747	216	1,670	422	1,947	502
107	533	154	1,163	285	1,354	353
110	304	90	608	130	671	177

ampère; in the second 0.5 ampère, and in the third, 4.0 ampères, sufficient to produce approximate saturation.

The flux curves as well as the demagnetization curves are all approximate parabolas as before, and so there is nearly a constant ratio between the demagnetization at any point and the original flux corresponding. The mean ratios were found to be respectively 0.272, 0.245, 0.234, showing that the lower the original intensity of magnetization, the greater the *relative* demagnetization. However, the actual demagnetization at every section of the magnet is seen to be greater for higher intensities.

If these values of R are considered as functions of the original flux at the middle section, the relation is found to be linear, but the data at hand are not sufficient to consider this as accurately true over a wide range of values of initial flux.

A much more complicated problem arises when the magnetized rod is clamped at some point other than the end, and torsion applied at one of the free ends. In this case the portion behind the clamp is not subjected to strain at all, while that in front of the clamp undergoes uniform torsion. The demagnetization curve, of course, is continuous as before, but it is no longer symmetrical.

In Table VI. are shown three sets of data representing the demagnetization corresponding to each of three different torsion angles, when the specimen is clamped at the middle point ($x = 56$ cm.) and twisted at the free end, $x = 112$ cm.

The second column of the table gives the values of the initial flux for the various values of x. There does not seem to be any simple relation between D_x and ϕ_x in this case, although it was found that the values of D at $x = 56$ are accurately proportional to the ratio of the corresponding angles of torsion. This ratio is nearly the same for all values of x, and the lack of uniformity is perhaps due to slight flexure during torsion.

One point of interest may be noticed, that the maximum demagnetization for all angles of torsion occurs at $x = 77$, which is three eighths of the distance from the clamp to the twisted end of the magnet. It will be seen later that in several other instances under torsion and fluxure the maxima occur at multiples of 7. (7 is one sixteenth the length of the rod.)

A similar set of data is shown in Table VII. The specimen in this case was clamped at the quarter-length ($x = 28$ cm.), and torsion applied at the free end, $x = 112$ cm. As before the values of the demagnetization at the mid-section, $x = 56$, are accurately proportional to the angles of torsion; and, in fact, the proportionality is fairly good throughout the length of the rod.

TABLE VI.

Torsion (*Viking Steel*).

Rod clamped at middle.

x Cm.	C.G.S.	1.58°.	3.16°.	4.74°.
	ϕ.	*D.*	*D.*	*D.*
2	720	81	82	167
8	2,036	112	150	282
14	3,036	136	208	381
20	3,838	158	262	467
26	4,452	178	315	547
32	4,936	198	370	628
38	5,291	238	427	693
44	5,541	249	491	795
50	5,688	287	567	896
56	5,731	337	666	1,019
62	5,679	370	746	1,129
68	5,533	385	791	1,196
74	5,279	391	815	1,230
77	5,118	392	817	1,236
80	4,928	389	812	1,227
86	4,447	370	775	1,172
92	3,828	338	703	1,069
98	3,041	292	595	907
104	2,059	232	446	688
110	778	147	245	386

TABLE VII.

Torsion (*Viking Steel*).

Rod clamped at quarter-length. (x = 28 cm.)

x Cm.	ϕ.	4.33°.	6.08°.	7.70°.
		D.	*D.*	*D.*
2	752	113	120	185
8	2,048	219	279	382
14	3,030	320	426	562
20	3,813	423	572	740
26	4,432	535	725	927
32	4,913	641	876	1,112
38	5,265	705	976	1,244
44	5,509	748	1,045	1,334
50	5,659	779	1,095	1,397
56	5,714	798	1,125	1,439
63	5,645	811	1,138	1,458
68	5,521	802	1,130	1,450
74	5,276	789	1,112	1,419
80	4,922	760	1,068	1,363
86	4,444	709	981	1,266
92	3,825	632	893	1,124
98	3,047	530	722	936
104	2,062	423	606	684
110	769	230	247	335

The maximum demagnetization for each of the angles occurs at $x = 63$, which is five twelfths of the distance from the clamp to the twisted end of the rod. Or, in other terms, the maxima in the two cases respectively occur at eleven and nine sixteenths the length of the rod.

In Fig. 3 are shown the initial flux curve, the flux curve after torsion, and their difference, the demagnetization curve, for the case of the magnet clamped at the middle and twisted at the end, $x = 112$.

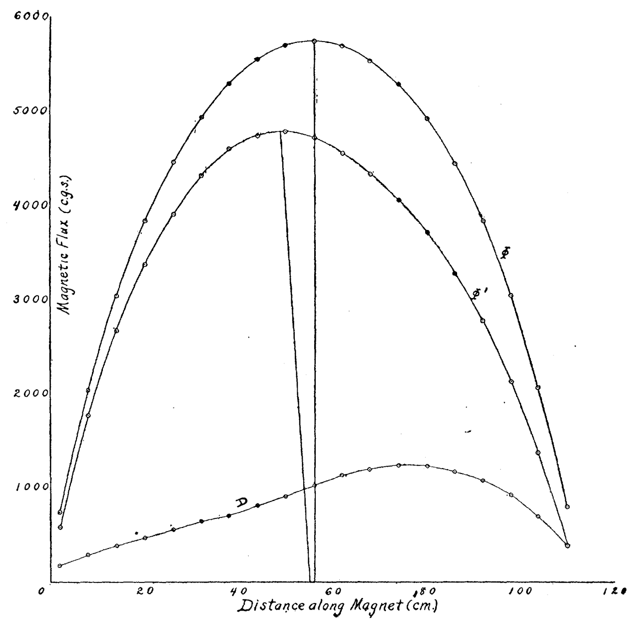

Fig. 3.

The first of these curves was found, as before, to be an approximate parabola of the form $\phi_x = Ax^2 + Bx + C$. The constants were found by the method of least squares to be: $A = -1.669$; $B = 187.8$; $C = 566.0$.

The second flux curve also resembles a parabola, but one which has had its axes rotated with respect to the first. Upon trial it is seen that this second curve is at least as nearly parabolic as the first, if not more accurately so. The equation of the parabola in this case is

$$(x + D\phi')^2 + Ex + F\phi' + G = 0,$$

where D, the tangent of the angle of rotation of the axes has a value of .00147 when ϕ' and x are plotted to the same scale. The other values of the constants found by least-squares are:

$$E = -113.66;\ F = 0.51431;\ G = -66.2.$$

In Table VIII. the observed and the computed values are given for the ordinates of these three curves. The demagnetization values agree

Table VIII.

Torsion (Viking Steel).

Rod clamped middle, torsion at end. Torsion angle 4.73°.

x	ϕ.		ϕ'.		$D\ \phi - \phi' = D$.	
Cm.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.
2	720	935	553	555	167	380
8	2,036	1,961	1,754	1,681	282	280
14	3,036	2,858	2,655	2,605	381	253
20	3,828	3,654	3,361	3,342	467	312
26	4,452	4,321	3,905	3,915	547	406
32	4,936	4,867	4,308	4,340	628	527
38	5,291	5,292	4,599	4,624	693	668
44	5,541	5,598	4,746	4,787	795	811
50	5,688	5,783	4,792	4,836	896	947
56	5,731	5,849	4,712	4,780	1,019	1,069
62	5,679	5,794	4,550	4,627	1,129	1,167
68	5,533	5,619	4,337	4,385	1,196	1,234
74	5,279	5,327	4,049	4,051	1,230	1,276
77	5,118	5,131	3,882	3,858	1,236	1,273
80	4,928	4,908	3,701	3,642	1,227	1,266
86	4,447	4,373	3,275	3,135	1,172	1,238
92	3,828	3,718	2,759	2,698	1,069	1,020
98	3,041	2,941	2,134	1,984	907	957
104	2,059	2,045	1,371	1,304	688	741
110	778	970	392	558	386	412

fairly well over a large range of values of x, but there are a few discrepancies for the smaller values of x. This need not be considered as fatal to the proposition that the demagnetization curve is the difference of two parabolas, one of which has been rotated with respect to the other. For in the first place, the original flux curve was not an exact parabola, and so the difference of two approximately fitted curves might show considerable discrepancies with respect to the observed data. It might be possible to produce an accurately parabolic distribution of flux by choosing properly the ratio of the diameter to the length of the magnet, or by varying the length of the magnetizing helix. In such a case the second flux curve might be, also, an accurate parabola.

It may be worth while to mention that the greater the angle of torsion the greater the inclination of the axis of the second flux curve, but this inclination is relatively greater for small angles of torsion than for large values. Also, the inclination of the axis of the parabola is greater for the first method of clamping than for the second, when the strains are equal in the two cases; but the two methods of applying torsion produce curves having the same general characteristics.

Flexure.

The first mode of flexure studied was with the magnet clamped at one end, the other end being slightly displaced. The curvature is then a linear function of the coördinate x, being proportional to the distance of the point from the free end of the rod.

In Table IX. the demagnetization is shown for displacements of the end of the rod of 2.3 cm., 3.6 cm., and 4.7 cm. In every instance there is a maximum in the value of D at about $x = 28$, or one fourth of the

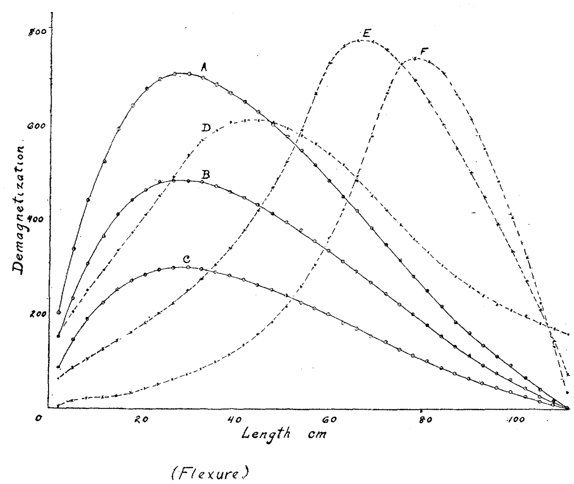

Fig. 4.

length of the rod. That is, the point of maximum demagnetization is midway between the point of maximum curvature and the point of maximum original flux. (See Fig. 4, A, B, and C.)

From this consideration it would seem reasonable that the demagnetization at any point is proportional both to the curvature and the original flux at the point. Or that the relative demagnetization, $R = D/\varphi$, is proportional to the curvature.

In column 5 of the table the values of R are shown for one of the experiments, and the variation is nearly linear toward zero at the free end.

Thus, we may write

$$D = k\phi(l - x), \tag{7}$$

where k is a constant, and l is the length of the rod under strain. However, if l is assumed to be a constant whose value, with that of k is to be

TABLE IX.

Tool Steel.

Rod clamped at end, $x = 0$; displaced at end, $x = 112$ cm.

x Cm.	ϕ.	Displacement at End, 2.3 Cm.			3.6 Cm.		4.7 Cm.	
		D Observed.	D Computed.	$R = D/\phi$.	D Observed.	D Computed.	D Observed.	D Computed.
2	1,420	88	80	.0620	140	141	203	190
5	2,425	149	133	.0612	235	220	339	316
8	3,278	192	174	.0587	308	289	445	415
11	4,021	227	208	.0564	367	344	529	495
14	4,680	253	234	.0540	412	389	599	559
17	5,259	273	255	.0519	445	423	648	609
20	5,766	287	271	.0498	468	450	682	646
23	6,212	296	282	.0477	483	468	702	673
26	6,601	300	290	.0455	489	483	711	691
29	6,938	300	294	.0433	489	488	711	701
32	7,229	297	295	.0411	484	490	704	704
35	7,477	292	294	.0377	475	487	692	700
38	7,684	285	290	.0371	464	481	676	690
41	7,851	276	284	.0351	450	471	656	677
44	7,894	266	277	.0333	436	459	633	660
47	8,081	254	268	.0315	419	444	607	638
50	8,148	242	257	.0297	399	427	580	613
53	8,182	230	246	.0281	380	408	551	586
56	8,191	216	233	.0264	360	387	520	556
59	8,168	202	220	.0247	338	365	488	525
62	8,116	187	206	.0231	316	342	454	492
65	8,030	173	192	.0215	293	318	421	457
68	7,912	158	177	.0200	270	293	385	421
71	7,762	144	161	.0185	248	268	350	385
74	7,575	129	146	.0171	225	242	317	347
77	7,353	115	130	.0157	202	216	284	310
80	7,091	102	115	.0143	180	190	251	273
83	6,788	89	99	.0131	159	164	220	236
86	6,446	77	84	.0120	136	140	189	201
89	6,054	67	70	.0110	111	116	163	166
92	5,608	57	56	.0101	95	93	138	133
95	5,101	48	43	.0093	79	71	114	103
98	4,530	39	31	.0085	62	52	91	74
101	3,880	29	21	.0076	47	34	68	50
104	3,142	19	12	.0061	29	20	41	29
107	2,290	10	5	.0043	14	9	20	12
110	1,276	1	1	.0010	2	2	3	2

determined, we get by least squares, $l = 111.4$. Now the portion of the specimen under the clamp is 1 cm. long, and a certain amount of flexure undoubtedly takes place for some distance under the clamp; therefore we might expect the effective bending length to be between 111 and 112 cm.

Taking then the value of l as 111.4, the values of k for the three cases are respectively .000514, .000853, and .001226. The computed values of D are arranged in Table IX. for comparison with the observed values.

It will be seen that the agreement is good at every point.

Besides the study of demagnetization, or decrease in the number of lines of force passing through the iron, it is also of considerable interest to investigate the lines of force which escape into the air from the magnetic material.

If we define the *Leakage* as the flux escaping from the metal per centimeter of its length, then it is found that this quantity may decrease at some points of the magnet and increase at others. Of course the leakage as defined may have both positive and negative values. It is measured by sliding the coil over a small constant distance between guides at various points of the magnet, both before and after the flexure takes place. The difference between these two sets of readings gives the *decrease in leakage* occasioned by flexure.

Corresponding to the mode of clamping just discussed the leakage decrease has its greatest value at the position of the clamp ($x = 0$). Its value is zero at $x = 28$, the point of maximum demagnetization; while there is a maximum at $x = 70$ ($\frac{5}{8}l$); and a minimum at $x = 98$ ($\frac{7}{8}l$).

When the clamp is placed at the quarter-length of the magnet, that is at $x = 28$, there is a maximum decrease in leakage beneath the clamp; another at 70, as in the preceding case; a minimum at $x = 14$ ($\frac{1}{8}l$); and another at or near 105 ($\frac{15}{16}l$). The corresponding demagnetization curve is shown in Fig. 4(D). It has a maximum at 42, half-way between the clamp and the point of greatest original flux. At the same point the leakage decrease is, of course, zero. In fact we have in general

$$L = \frac{d\phi}{dx}; \quad L' = \frac{d\phi'}{dx},$$

where L and L' represent the leakage before and after flexure. Also

$$\Delta L = L - L' = \frac{d\phi}{dx} - \frac{d\phi'}{dx} = \frac{dD}{dx},$$

since $\phi - \phi' = D$. Therefore ΔL is zero whenever D is a maximum.

A summary is given for comparison (in Table X.) of the points of maxima and minima in the values of D, R, and ΔL, for four different positions of the clamp. The displacement of the end of the magnet ($x = 112$) was 2.3 cm. in each case, making the curvatures at the clamp proportional to 3, 4, 6, and 8 respectively.

It will be noticed that when the clamp is placed at an aliquot part

TABLE X.

Position of Clamp.	Curvature at Clamp.	D	R		ΔL	
		Max.	Max.	Min.	Max.	Min.
0	3	$\frac{l}{4}$	none	none	$\frac{5l}{8}$	$\frac{7l}{8}$
$\frac{l}{4}$	4	$\frac{3l}{8}$	$\frac{5l}{16}$	$\frac{5l}{32}$; $\frac{13l}{16}$	$\frac{l}{4}$; $\frac{5l}{8}$	$\frac{l}{8}$; $\frac{15l}{16}$
$\frac{l}{2}$	6	$\frac{19l}{32}$	$\frac{5l}{8}$	$\frac{l}{8}$	$\frac{l}{2}$	$\frac{l}{8}$
$\frac{5l}{8}$	8	$\frac{11l}{16}$	$\frac{l}{16}$; $\frac{3l}{4}$	$\frac{3l}{32}$	$\frac{5l}{8}$	$\frac{3l}{32}$

of the length of the rod, the maxima and minima are also at simple fractional parts of the length.

Table XI. shows the actual values of R in the four cases just mentioned.

TABLE XI.

Tool Steel.

Displacement of 2.3 cm. at end, $x = 112$.

x.	Clamp at 0.	$\frac{l}{4}$.	$\frac{l}{2}$.	$\frac{5l}{8}$.	x.	Clamp at 0.	$\frac{l}{4}$.	$\frac{l}{2}$.	$\frac{5l}{8}$.
	R.	R.	R.	R.		R.	R.	R.	R.
2	.06204	.11232	.04423	.00423	59	.02471	.06674	.09034	.04447
5	.06124	.08586	.03612	.00643	62	.02309	.06414	.09532	.05251
8	.05867	.07712	.03313	.00632	65	.02154	.06107	.09802	.06197
11	.05635	.07326	.03180	.00602	68	.02001	.05791	.09890	.07386
14	.05402	.07143	.03132	.00622	71	.01851	.05454	.09858	.08735
17	.05191	.07097	.03143	.00673	74	.01707	.05170	.09726	.09673
20	.04979	.07158	.03202	.00759	77	.01565	.04911	.09521	.10180
23	.04770	.07302	.03302	.00848	80	.01433	.04686	.09264	.10445
26	.04549	.07526	.03442	.00942	83	.01314	.04514	.08978	.10526
29	.04328	.07790	.03628	.01055	86	.01201	.04392	.08672	.10438
32	.04114	.07935	.03857	.01187	89	.01098	.04328	.08358	.10248
35	.03772	.07977	.04129	.01344	92	.01011	.04339	.08042	.09963
38	.03707	.07949	.04452	.01529	95	.00931	.04454	.07724	.90586
41	.03513	.07862	.04835	.01754	98	.00850	.04698	.07400	.09022
44	.03325	.07730	.05288	.02013	101	.00755	.05158	.07088	.08260
47	.03145	.07570	.05831	.02338	104	.00605	.05983	.06782	.07288
50	.02975	.07359	.06479	.02700	107	.00428	.07647	.06489	.05913
53	.02809	.07169	.07268	.03203	110	.00102	.12594	.06050	.02821
56	.02636	.06930	.08227	.03765					

Since the strained portion of the rod is shorter in each succeeding case, for equal displacements of the free end the curvature is, therefore, becoming each time greater at the position of the clamp, and also at all points in the part under flexure. At the displaced end there is no curvature in any case. (See Col. 2, Table X.)

The effect of this increasing stiffness is shown at $x = 56$, the point of maximum initial flux. However, in the last case the portion under strain is so small as to more than compensate for the increased curvature, and R does not exceed in value that of the other case until the point 74 is reached. This is near the point of maximum D. (F, Fig. 4.)

It may be noted also that the value of R at the position of the clamp is greater in each succeeding case.

An interesting relation is found to hold between the position of the clamp and that of the maximum D, but it fails when the clamp is at the middle (E, Fig. 4). It may be stated thus: The position of the maximum demagnetization is half as far ahead of the clamp as the clamp is distant from the middle of the specimen.

From this it would follow that the maximum D would fall at the same

TABLE XII.

Tool Steel.

Rod supported at ends; loaded at middle.

x.	Displacement of Middle = 0.50 Cm.		Displacement = 0.95 Cm.		Displacement = 1.60 Cm.	
	D.	R.	D.	R.	D.	R.
2	81	.0568	169	.1192	287	.2018
5	106	.0438	235	.0968	374	.1542
8	128	.0390	290	.0885	465	.1418
11	150	.0374	344	.0855	533	.1326
14	174	.0372	397	.0848	610	.1303
17	199	.0378	450	.0855	686	.1304
23	251	.0403	554	.0892	836	.1345
29	305	.0440	658	.0948	982	.1415
35	361	.0483	757	.1012	1,120	.1498
41	413	.0526	847	.1079	1,245	.1586
47	454	.0561	912	.1128	1,344	.1663
53	476	.0582	947	.1157	1,402	.1713
59	476	.0583	946	.1158	1,401	.1715
65	452	.0562	909	.1132	1,338	.1667
71	408	.0526	841	.1083	1,233	.1589
77	355	.0482	747	.1016	1,102	.1499
83	297	.0437	645	.0950	955	.1407
89	242	.0399	541	.0894	799	.1320
95	192	.0376	437	.0856	640	.1254
98	169	.0373	384	.0848	557	.1229
101	147	.0379	332	.0856	478	.1233
104	126	.0402	279	.0886	396	.1259
107	106	.0464	222	.0969	309	.1347
110	82	.0643	152	.1194	212	.1660

distance from the clamp no matter which end of the magnet was displaced.

Two cases of symmetrical curvature were tried. In the first the rod was supported on knife-edges at the ends and bent by displacing the middle; in the second the ends were similarly clamped and the middle displaced.

Tables XII. and XIII. show the values of D and R. The demagnetization curves for the first case resemble hyperbolas, having only one maximum and no points of inflection; those for the second case have a maximum at the same point as the others, and also points of inflection at $3l/16$ and $13l/16$. Both sets of curves are, of course, symmetrical about a vertical axis.

Table XIII.

Tool Steel.

Rod clamped at ends; loaded at middle.

x.	Displacement of Middle = 0.42 Cm.		Displacement = 0.65 Cm.		Displacement = 1.60 Cm.	
	D.	R.	D.	R.	D.	R.
2	260	.1828	375	.2639	562	.3959
5	320	.1321	466	.1920	739	.3048
8	361	.1103	526	.1604	858	.2617
11	391	.0971	574	.1428	946	.2352
14	412	.0881	614	.1311	1,012	.2162
17	429	.0815	647	.1230	1,069	.2032
23	457	.0736	707	.1138	1,136	.1828
29	489	.0705	773	.1114	1,260	.1816
35	535	.0716	850	.1137	1,401	.1874
41	586	.0746	929	.1183	1,553	.1978
47	632	.0782	1,001	.1238	1,698	.2108
53	660	.0807	1,048	.1281	1,802	.2203
59	658	.0805	1,043	.1277	1,802	.2206
65	622	.0775	980	.1220	1,701	.2118
71	568	.0732	893	.1150	1,554	.2002
77	511	.0695	804	.1093	1,396	.1898
83	465	.0684	729	.1074	1,253	.1845
89	440	.0726	676	.1116	1,144	.1889
95	423	.0829	630	.1234	1,060	.2077
98	411	.0907	603	.1330	992	.2191
101	393	.1012	569	.1466	923	.2379
104	366	.1163	524	.1667	830	.2642
107	324	.1415	463	.2020	715	.3124
110	262	.2051	372	.2916	531	.4161

In the second case the curvature at the middle point is twice as great as in the first case, for the same displacement. However, the value of D at the mid-point is not nearly twice as great. This is because there are two points of zero curvature, $l/4$ and $3l/4$, to offset the higher curvatures of the middle and ends.

It is evident that the relative demagnetization will be a maximum also at the middle point; but there are minima in both sets of R curves while there were none for the D curves.

When the ends of the magnet were clamped and the middle displaced, the curvature fell linearly toward the quarter points where it became zero, then rose linearly again to the middle. In the corresponding curves for R the minima fall at these quarter points as might be expected, and the high points are the middle and ends. Nevertheless, the values of R cannot be proportional to the curvature as in the first mode of flexure since R does not become zero, but only a minimum where the curvature is zero. This shows conclusively that original flux at the point and curvature are not the only factors to be considered. It seems clear from the investigation up to this point that the demagnetization

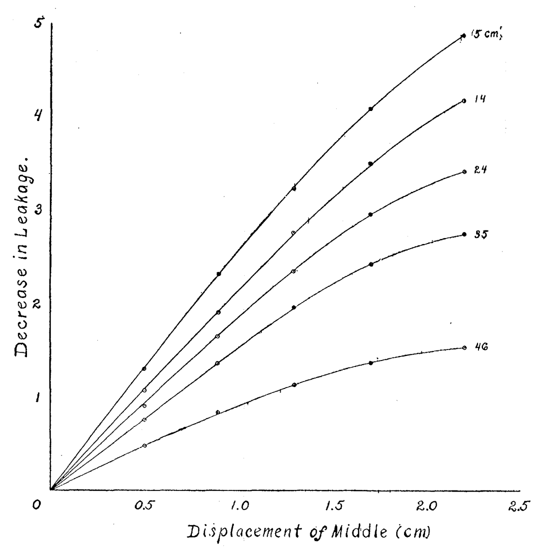

Fig. 5.

taking place at any point is influenced largely also by that at the surrounding points.

When the specimen merely rests at the ends without clamping the minimum values of R occur at $l/8$ and $7l/8$. The reason for these minima is not so clear as before, for the curvature increases linearly from end to middle. The curves for decrease in leakage (ΔL) when the ends of the magnet are supported, show very much the same form whatever the strain; but unlike any of the other modes of flexure it is found that distinct maxima and minima appear only under small strains. This change of characteristic form is, of course, a gradual one.

When the ends are clamped ΔL becomes a maximum at $11l/32$ and $21l/32$; and a minimum at $3l/16$ and $13l/16$. Such a curve is shown in Fig. 6 (middle curve).

The variation of ΔL with the curvature is represented in Fig. 5 for the case where there is no clamping at the ends. This variation is not the same for all points on the rod, as was the similar variation of R in the torsional case (see Fig. 1), but is greater as the end is approached. Five points are studied in the figure, $x = 5, 14, 24, 35, 46$ cm. respectively, and for each one the curve is found to become more nearly horizontal for large strains.

This is also true for the demagnetization in all cases.

In this instance, however, the relation between leakage decrease and curvature is more nearly a linear one near the ends of the specimen.

Impact.

The effect of impact upon a magnetized rod may be observed and measured during the instant it is taking place by means of a coil of wire slipped over the rod, the terminals being connected to a ballistic galvanometer. The coil remains stationary at any chosen position while the blow is struck at any other point desired. The rapid decrease in the flux through the coil produces an induced current whose integral over the interval of the blow is sufficiently large to enable it to be measured with accuracy.

Such a device may also be used to show demagnetization due to the rapid heating of a magnet by a flame, but the effect is immensely smaller.

The hammer used in this series of experiments consisted of a brass cylinder which could be fastened by means of a screw at any point along a brass rod, which in turn swung upon a horizontal pivot.

The energy of the blow could thus be varied in several ways and easily computed.

It was found by the method described above that whenever a fixed blow was delivered repeatedly at the same point on the magnet the demagnetization produced at any particular section grew less and less at each blow. In fact the metal rapidly approaches a state in which it is no longer demagnetizable by the blow which had been operating upon it. However, it is by no means completely demagnetized, and a heavier blow will at once show its effect. It may be stated here that it is not possible, seemingly, to render a magnet neutral by means of torsion, flexure, or impact.

In order to study the effect of longitudinal vibration in the magnetized rod, free from lateral vibration, the magnet was grasped tightly in the hands and struck a solid blow by the hammer at one end and lengthwise of the rod. The original flux and the resulting demagnetization for such an impact are shown in Table XIV. It may be seen at once that the demagnetization is considerably greater at the end struck than at the other end. This would seem to show that the compression wave travelling along the rod is considerably damped in traversing the length of the specimen.

In order to find an equation for this curve it was assumed that D decreases linearly with x; that is that D is proportional to $(k - x)$, where k is a constant greater than the length of the magnet. We may then write $D = c\phi(k - x)$, where ϕ is the initial flux, and c is a constant.

TABLE XIV.

Tool Steel.

Impact (blow at end $x = 0$).

x.	ϕ.	D Observed.	D Computed.	x.	ϕ.	D Observed.	D Computed.
2	1,296	143	175	59	8,533	980	990
5	2,572	332	323	62	8,476	971	975
8	3,349	476	445	65	8,387	957	956
11	4,159	588	548	68	8,260	935	934
14	4,864	675	636	71	8,101	907	908
17	5,484	744	712	74	7,906	875	874
20	6,025	799	776	77	7,675	840	845
23	6,497	842	830	80	7,399	802	807
26	6,912	876	877	83	7,082	761	766
29	7,266	903	915	86	6,719	717	720
32	7,572	924	946	89	6,301	669	669
35	7,831	941	970	92	5,826	616	613
38	8,047	954	989	95	5,282	556	550
41	8,220	964	1,002	98	4,666	489	482
44	8,355	970	1,010	101	3,960	411	405
47	8,456	976	1,014	104	3,122	318	316
50	8,522	980	1,014	107	2,189	211	219
53	8,554	983	1,009	110	1,089	89	108
56	8,559	984	1,001				

When determined by least squares the values of these constants are $k = 412$; $c = .0003286$. From these constants the fourth column of Table XIV. was computed, and it is seen to compare very well with the column of observed values of D. The maximum variation is not over 4 per cent. of the maximum value of D, and over a large portion of the curves it is not over one half per cent.

The form of the leakage decrease curve when a blow is struck at the middle of a magnet which is not allowed to vibrate transversely is shown in Fig. 6 (A). The value of ΔL is nearly a linear function of x, except near the ends.

When a similar blow is struck with the rod clamped only at the ends the resulting curve for ΔL has maxima and minima, and resembles one of the forms discussed under flexure. Nevertheless the two are not alike. This suggested the possibility of analyzing the general impact effect into components, each of which should represent a distinct demagnetizing effect.

In the first place the magnet was clamped throughout its length, except for a few centimeters near the middle, to a heavy, smooth stone slab. A blow of known strength was then delivered at the middle of the specimen. This method of clamping allows no appreciable vibration

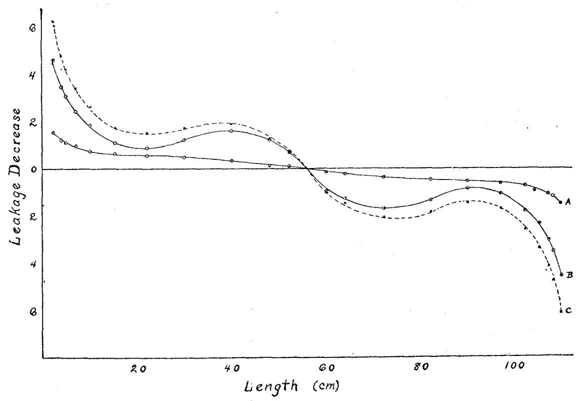

Fig 6.

of the rod, and the level, smooth surface enables the magnet to be securely fastened without any demagnetization due to flexure. In fact the demagnetization produced in this case can be traced solely to the compression wave which emanates from the point of impact of the hammer and spreads out over the entire length of the rod.

The curve for ΔL in this case is shown in Fig. 6 (A).

Next the specimen was completely demagnetized by an alternating current and remagnetized with the same initial distribution as in the first case. The ends of the rod only were clamped this time, and the same blow struck at the middle; the rod being allowed afterward to vibrate as a whole. The magnitude of the depression of the middle of the specimen was determined by taking the impression of the middle of the rod, when the blow was delivered, in a block of wax. This deflection amounted to .68 cm. (See Fig. 6, points marked x.)

Again the magnet was completely demagnetized and then brought to the same initial state as in the former cases. It was clamped as before at the ends; the mid-point was depressed by .68 cm., but without any blow; then released, and the rod allowed to vibrate as a whole. The leakage decrease curve thus produced is shown in Fig. 6 (B).

Now the sum of curves A and B is given by the dotted curve C, and it is seen that it passes very accurately through the set of points marked by the x's.

This seems to prove rather conclusively that, in general, demagnetization by impact is due to two distinct effects. The predominant one is caused by vibration of the magnet either transversely or longitudinally, particularly the former. The second effect in importance is that of the wave of compression set up by the hammer at the point of impact.

The same proof in terms of demagnetization is exhibited in Table XV.

Table XV.

Tool Steel.

Impact (blow at middle).

x.	Blow at Middle.	No Blow; Vibration Only.	Sum of Columns 2 and 3.	Blow at Middle.
	Rod Clamped to Stone Slab. (No Flexure.)	Clamped Ends; Mid-point Deflected, and Rod Vibrates.		Clamped Ends.
2	253	625	878	887
5	291	732	1,023	1,031
8	320	806	1,126	1,135
11	343	864	1,207	1,215
14	363	901	1,264	1,275
17	383	936	1,319	1,328
20	400	965	1,365	1,374
23	418	988	1,406	1,417
26	432	1,017	1,449	1,460
29	446	1,045	1,491	1,506
32	461	1,083	1,544	1,558
35	472	1,123	1,595	1,613
38	484	1,169	1,653	1,671
41	492	1,215	1,707	1,728
44	501	1,259	1,760	1,765
47	507	1,299	1,806	1,823
50	513	1,331	1,844	1,860
53	516	1,354	1,870	1,886
56	516	1,362	1,878	1,895
59	516	1,351	1,867	1,881
62	510	1,325	1,835	1,849
65	504	1,287	1,791	1,809
68	498	1,244	1,742	1,757
71	490	1,195	1,685	1,702

It would be difficult to say what would be the form of the analytic expression which would describe accurately and in general the final distribution of magnetic flux in a magnet under strain. Nevertheless, there seems to be sufficient evidence brought out in this investigation to lead us to suppose that such an expression would depend fundamentally upon the initial flux and the curvature at each point.

The experiments, particularly those involving flexure where only a part of the specimen was under strain, show conclusively that curvature does not in general enter the expression as a direct factor; otherwise there would be no demagnetization at points of zero curvature.

No doubt it will be necessary before arriving at a complete solution of the problem to know the general conditions for the equilibrium of lines of force.

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